

Remote Colloquium on Vortex Dominated Flows

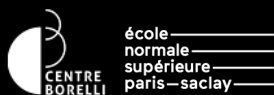
Long-wave instability of three interlaced helical vortices

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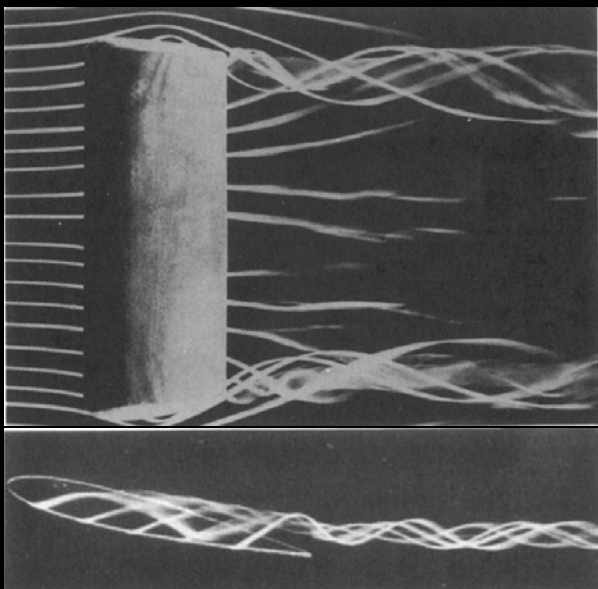
Context
●●○○○

Water channel experiments
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Wake Models
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Simpler Wake Models
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Examples of elongated vortices



Rolled-up tip vortices behind a rectangular wing, taken from Rom, J. (2012).



Propeller vortex on a F4U Corsair

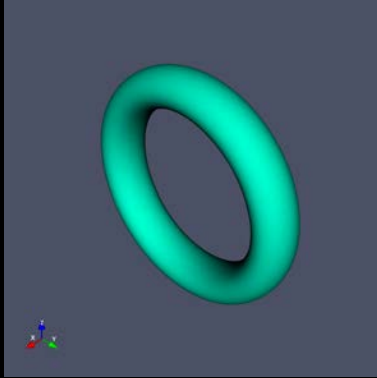


Smoke ring produced by Mount Etna



Examples of **steadily translating and rotating elongated vortices**

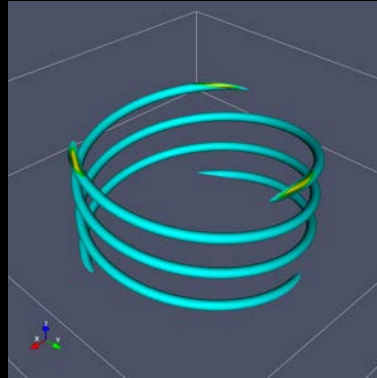
Vortex rings



Radius R , core size a

Lord Kelvin (1883), Lamb (1932), Widnall & Bliss (1971) Moore & Saffman (1972), ...

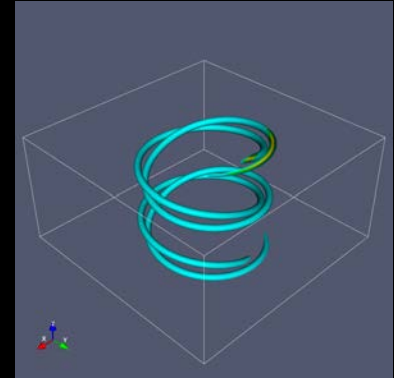
Uniform helical vortices



+ pitch H , number of helices N

Kawada (1936), Hardin (1949), Boersma & Wood (1999), Selçuk, Delbende & Rossi (2017), Okulov & Sørensen (2020)

Helical vortex pairs



+ separation distance d , twist parameter β

Durán Venegas & Le Dizès (2019), Schröder et al (2021), Castillo-Castellanos & Le Dizès (2022)

Complexity →

— Advective regime if $a \ll$ all other scales

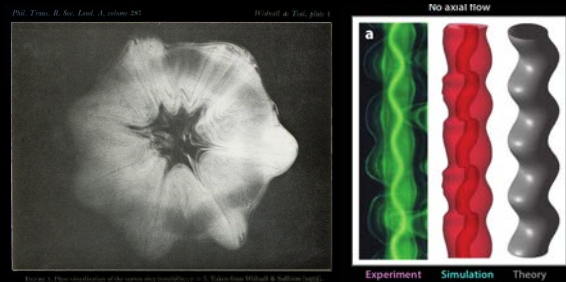
— Global translation and rotation → Structure is "steady" in a certain reference frame.

Vortex structures might be **steady** but not **stable** w/ respect to external perturbations

Short-wave instabilities

- Wavelength comparable to core size
- Internal deformation of the vortex core (e.g. elliptical, curvature instabilities)
- Interaction of vortex Kelvin modes

Kerswell (2002), Hattori & Fukumoto (2003), Fukumoto & Hattori (2005) Leweke et al. (2016), Blanco-Rodríguez & S Le Dizès (2016,2017), ...

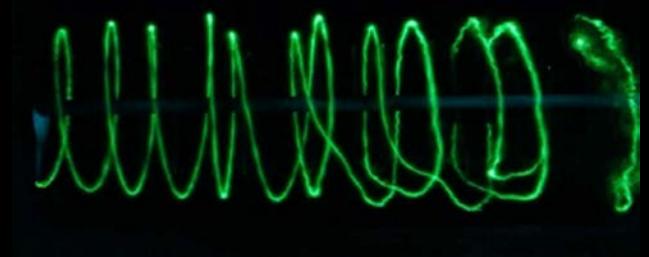


(Left) Vortex ring instability Widnall & Tsai (1977), (Right) Elliptical instability in a vortex Leweke et al. (2016)

Long-wave instabilities

- Wavelength is large relative to core size
- Local and global pairing instabilities
- Description by a filament approach

Widnall (1972), Gupta & Loewy (1974), Quaranta et al. (2015,2019), Durán Venegas et al. (2021), Castillo-Castellanos & Le Dizès (2022)



Helical pairing instability Quaranta et al. (2015)

Hydrodynamic instabilities are **relevant** for rotor systems like wind turbines

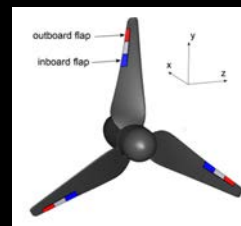
- Unperturbed wakes extend for long stretches
- Need for speed-up wake recovery
- Mitigate noise, mechanical wear, fatigue



Wake interactions in Horns Rev Offshore Wind Farm

Active control

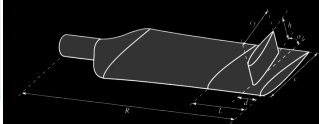
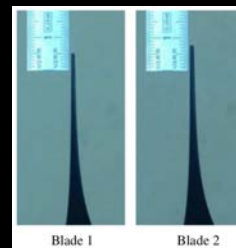
(Rotation rate, Flaps, Yaw, Angle-of-attack, ...)



(Left) Flap vanes from Huang et al. (2019). (Right) Optimal yaw control from B&R Automation.

Passive control

(Blade Geometry)



(Left) Radial asymmetry from Quaranta et al. (2019). (Right) Vane tip design from Schröder et al. (2021).

Water channel experiments at IRPHÉ

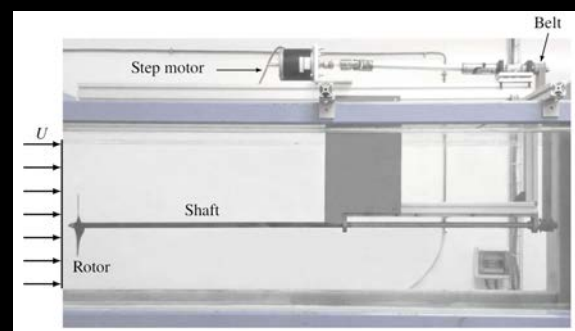
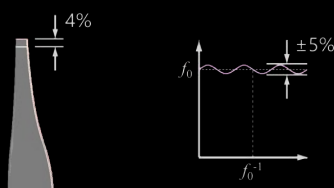
Experimental set-up

- recirculating, free surface
- 150 cm long, 38 cm wide and 50 cm high
- free stream velocity $U_z^\infty = 56$ cm/s
- $f = 3$ Hz, or tip-speed ratio $\lambda = 2\pi f R_{tip} / U_z^\infty = 3$

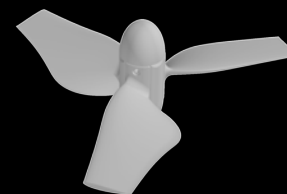
3-Bladed rotor, 3-D printed

- NACA2414, $Re_{chord} \sim 40000$
- $N = 3$ blades, $R_{tip} = 9$ cm, $c_{tip} = 2.3$ cm

2 ways to excite the unstable modes

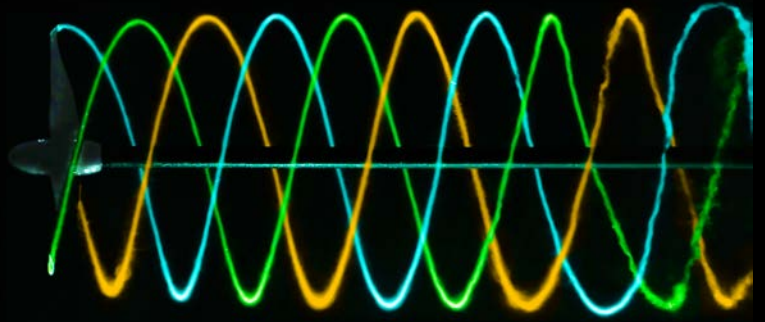


Side view of the water channel test section.



NACA2414 airfoil

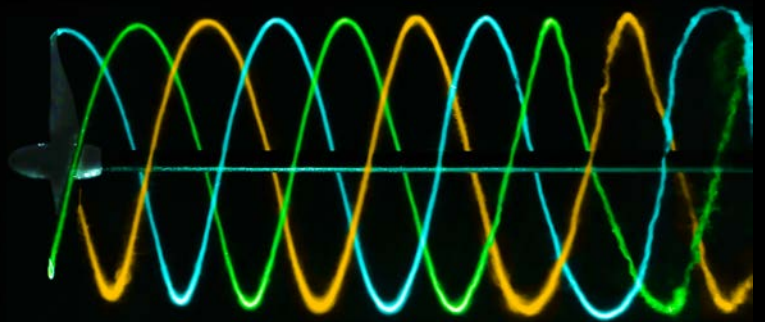
Baseline wake: symmetric rotor



* fluorescent dye painted on the tips, T. Leweke added colors by hand for clarity

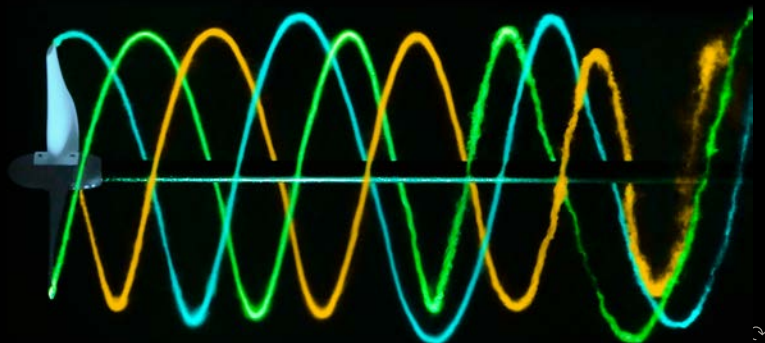


Baseline wake: symmetric rotor



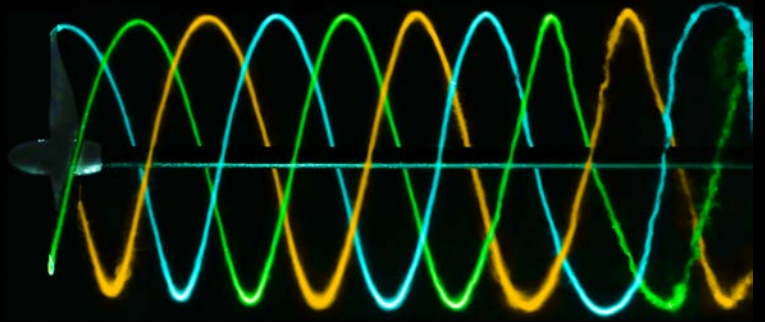
Perturbed wake: radial asymmetry

- Uniform expansion
- Leapfrogging wake



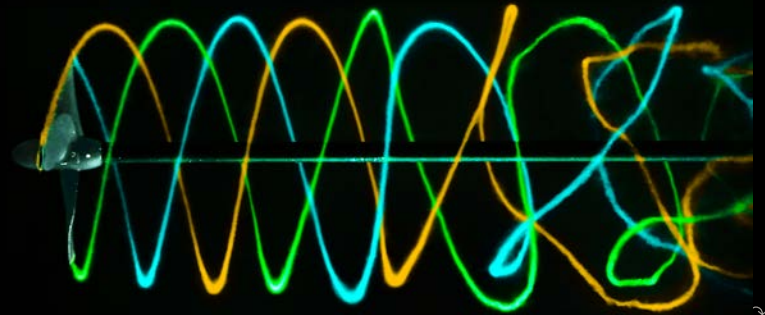
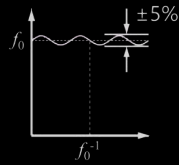
* fluorescent dye painted on the tips, T. Leweke added colors by hand for clarity

Baseline wake: symmetric rotor



Perturbed wake: variable rotation rate

- Complex distortions
- Local pairing and leapfrogging

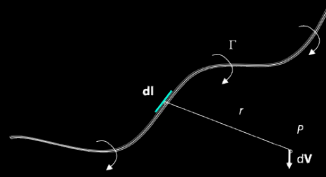


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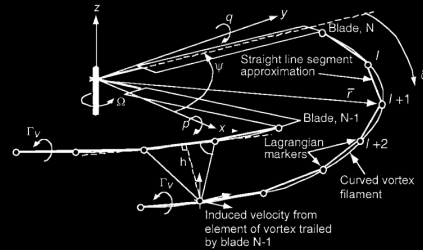
Can we propose a wake model?

Vortex filament approach

- Vortices represented as 1-D space-curves + vortex core model



Representation of a curved vortex filament, from Pinzón (2015)



from Leishman et al (2002)

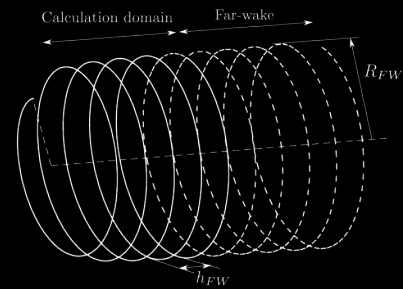
- Parametrization in terms of Lagrangian coordinates

Rotor angle: $\psi = \Omega_R t$ (proxy of time)

Wake age: $\zeta = \Omega_R (t - t_0)$ (proxy of space)

$$\frac{\partial \mathbf{x}_j}{\partial \psi} + \frac{\partial \mathbf{x}_j}{\partial \zeta} = \frac{1}{\Omega^R} [\mathbf{u}_j^{BS}(\mathbf{x}_j) + \mathbf{u}^{ext}]$$

- Boundary conditions:
 - Fixed position at $\zeta = 0$ (rotor plane)
 - Match helices at $\zeta \rightarrow \infty$ (semi-infinite helices)



Calculation domain from Durán Venegas & Le Dizès (2020)

This is old school. Why should we use these models?

$$\frac{\partial \mathbf{x}_j}{\partial \psi} + \frac{\partial \mathbf{x}_j}{\partial \zeta} = \frac{1}{\Omega^R} [\mathbf{u}_j^{BS}(\mathbf{x}_j) + \mathbf{u}^{ext}]$$

1. Search for stationary solutions (in the rotor frame)

$$\frac{\partial \mathbf{x}_j^B}{\partial \zeta} = \frac{1}{\Omega^R} [\mathbf{u}_j^{BS}(\mathbf{x}_j) + \mathbf{u}^{ext}]$$

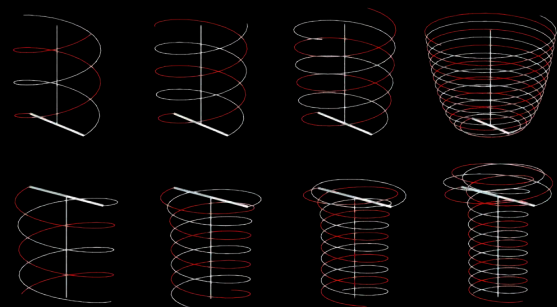
2. Linear stability analysis

$$\mathbf{x}_j(\psi, \zeta) = \mathbf{x}_j^B(\zeta) + \mathbf{x}'_j(\psi, \zeta)$$

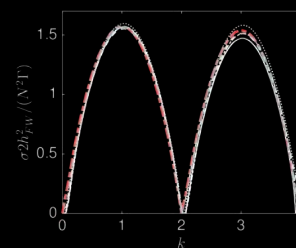
$$\frac{\partial \mathbf{x}'_j}{\partial \psi} = \left[-\frac{\partial}{\partial \zeta} + \frac{1}{\Omega^R} \mathbf{J}(\mathbf{u}_j, \mathbf{x}_j^B) \right] \mathbf{x}'_j = \mathbf{L} \mathbf{x}'_j$$

3. Non-linear dynamics

$$\mathbf{x}_j(\psi = 0, \zeta) = \mathbf{x}_j^B(\zeta) + \delta$$



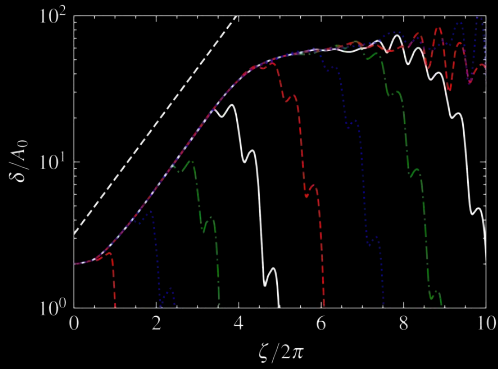
Wind turbine and helicopter regimes for 2-bladed-rotors Durán Venegas & Le Dizès (2021)



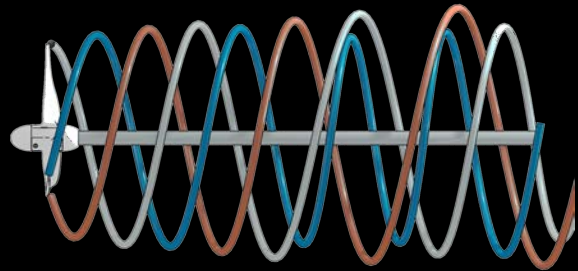
Stability diagram from linear-impulse analysis Durán Venegas & Le Dizès (2021)

Introduce a small perturbation at the rotor plane and follow its evolution

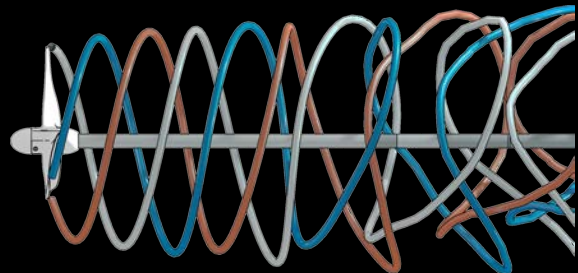
Exponential growth, then non-linear dynamics



Perturbed wake: radial asymmetry



Perturbed wake: variable rotation rate



Can we do something simpler?

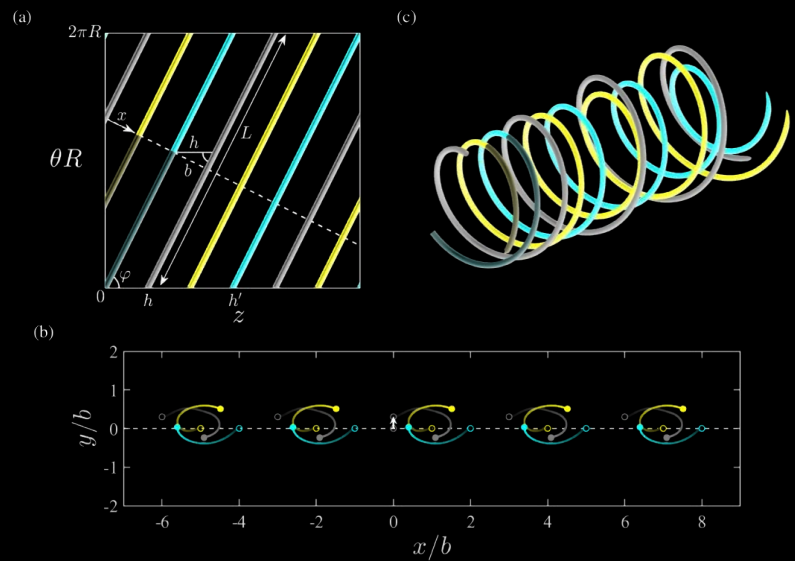
Time evolution of uniform vortices → Spatial evolution of stationary vortices

Helices as parallel lines in a developed plan

Use a point vortex model to approximate dynamics

- Periodic array of $N = 3$ point vortex
- Add a small perturbation
- (Non-linear) temporal evolution

Then, reconstruct the helical system



Taken from Abraham, Castillo-Castellanos & Lewke (2023)



Time evolution of spatially periodic vortices → Spatial evolution of time periodic vortices

Use a filament model to approximate dynamics

- Periodic array of 3 interlaced helical vortices
- Add a small spatially periodic perturbation
- (Non-linear) temporal evolution

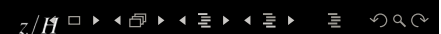
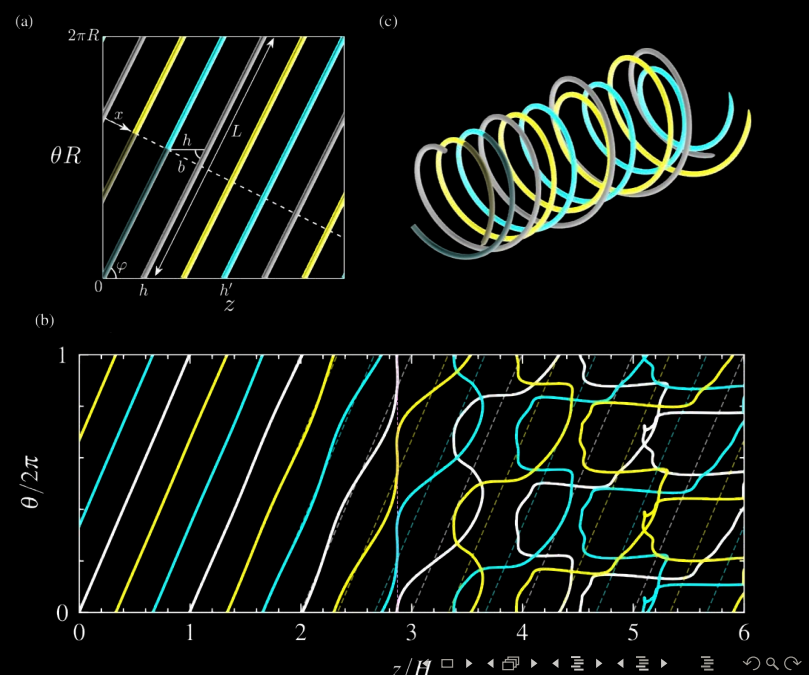
$$\mathbf{x}'_j(\psi = 0, \zeta) = \delta \cos(2\pi k \zeta + \varphi)$$

$$\mathbf{x}'_j(\psi, \zeta)$$

Then, reconstruct the helical system

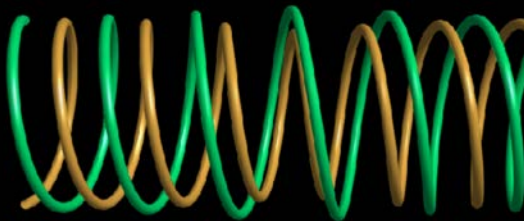
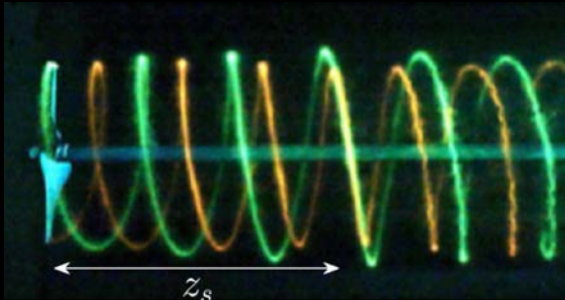
$$\mathbf{x}_j^*(\zeta, \psi) = \mathbf{x}'_j(\psi, \zeta)$$

$$\tilde{\mathbf{x}}_j(\zeta, \psi) = \mathbf{x}_j^B(\zeta) + \mathbf{x}_j^*(\zeta, \psi)$$

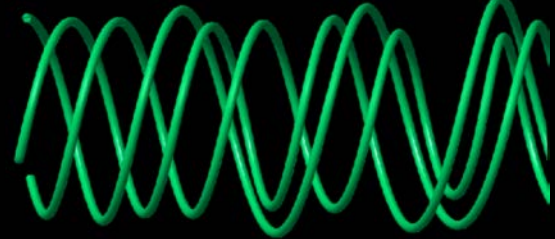
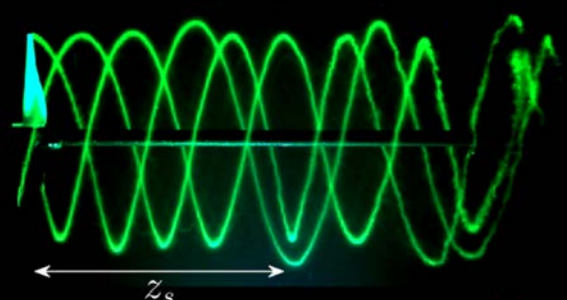


Comparison between experiments and point vortex model

2-bladed rotor



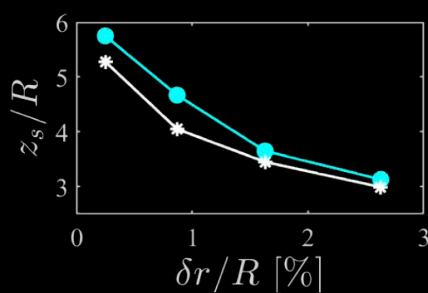
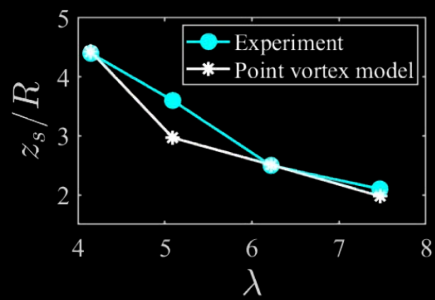
3-bladed rotor



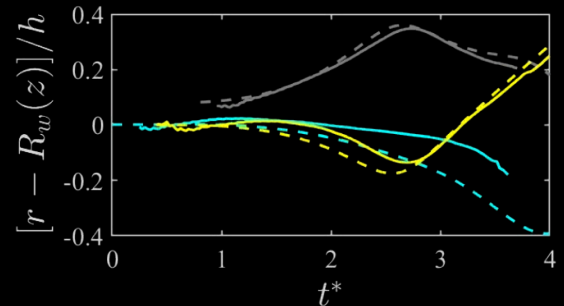
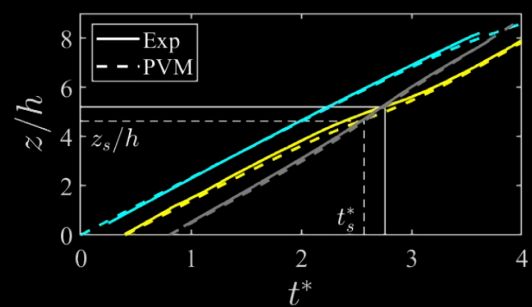
Taken from Abraham, Castillo-Castellanos & Leweke (2023)



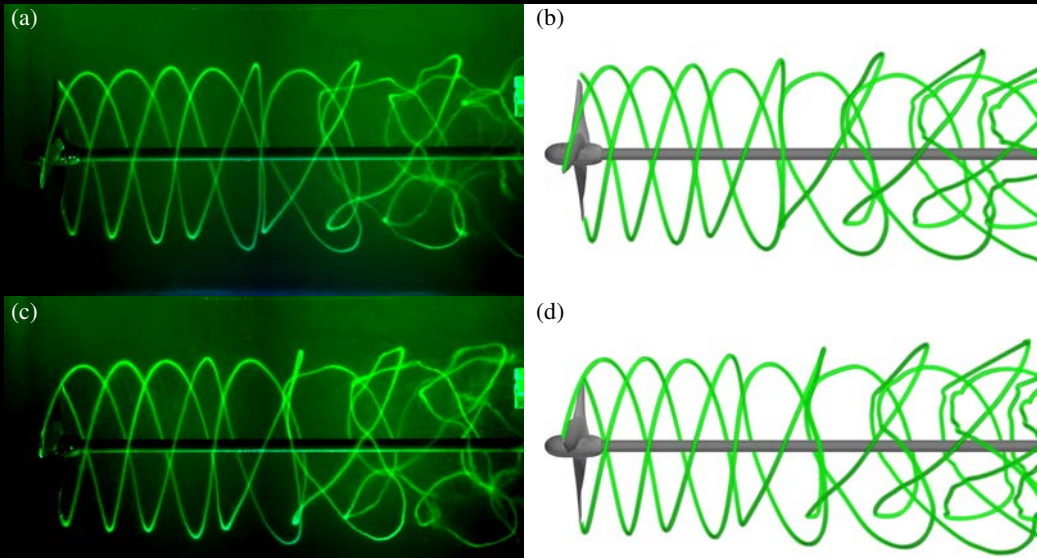
Comparison between experiments and point vortex model

Leapfrogging distance z_s 

Vortex trajectories



Comparison between experiments and simplified filament model

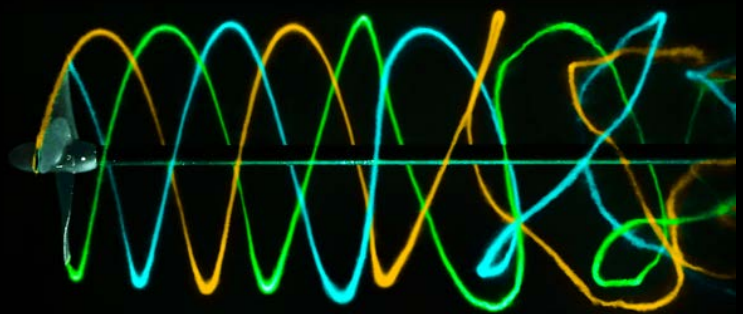
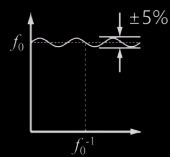


Comparison between experiments with $k = 3/2$ and simplified model at two different instants.



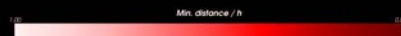
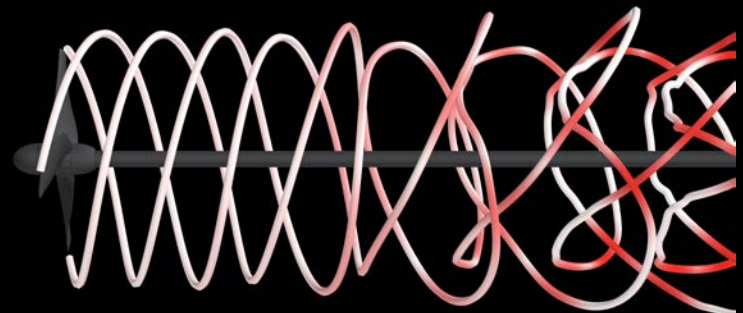
Experimental results $k = 3/2$

- Complex distortions
- Local pairing and leapfrogging



Vortex filament model $k = 3/2$

- Good agreement with experiments
- Low computational cost
- Model provides additional information E.g. minimum distance w/respect to neighbours (in colour)



Conclusions

A general approach for wake models

- Modelling approach based on filament vortices
- Boundary problem approach for semi-infinite helices
- Easily extended to other systems

Interest in studying non-linear regime

- A simplified model takes into account the non-linear regime
- Good quantitative agreement with experiments
- We may infer some quantities that are not easily measured from the model