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# Electromagnetic Analysis of Microstrip Hybrid Circuits Using Thin-Wire EFIE Formulation

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**Abstract**—This work presents a computationally efficient approach for the numerical analysis of the integro-differential formulation that models electromagnetic radiation by thin microstrip structures, including the contribution of lumped elements. This proposal is based on the thin-wire approximation to reduce a surface current distribution to a filamentary electric current thus, reducing a tridimensional problem to a unidimensional formulation. In addition, the effects of the dielectric are taken into account by considering the effective electric permittivity and the multipole Debye model that does this correction over the analyzed frequency band. Finally, the results obtained with this proposed technique are compared with measurements and simulation in commercial software, and an error lower than 1.5% was verified for all the analyzed cases with a processing time approximately eight times lower when compared with a full-wave simulation.

**Index Terms**—EFIE, lumped elements, Method of Moments, microwave circuits, thin-wire approximation.

## I. INTRODUCTION

The analysis and design of electronic circuits for microwave applications present an increased complexity when compared with those that operate in lower frequencies once the size of the circuit is in the order of magnitude of the wavelength ( $\lambda$ ) or even smaller. In this case, the electromagnetic behaviour of these circuits are not determined only by the lumped elements, but the distributed impedance in its physical structure [1]. Therefore, the wave equation must be solved to model these circuits taking into account the boundary conditions.

There are different numerical formulations for analyzing hybrid circuits composed by lumped and distributed impedances, each one being more or less suitable depending on the nature of the device and its application. For instance, the transmission line modelling allows a more straightforward solution, and at the same time, provides a clear insight of the physical behaviour of these circuits [1]. However, this approach does not take into account the radiation and scattering in these devices, which is accomplished by full-wave analysis in frequency or time domain. Frequency domain techniques are indicated to time-harmonic and resonant structures [2], whereas time domain enables the modelling of wideband and non-linear circuits, as well as to describe transient behaviour [3], [4]. Both techniques provide a complete description of the

electromagnetic dynamic of these circuits with the drawback of being numerically complex and computationally expensive.

In many applications, microstrip structures operate at a frequency range in which the thin-wire approximation conditions are satisfied; that is, the radius  $a$  of microstrip line cross-section is such that  $a \ll \lambda$ , and the structure size  $L \ll \lambda$ . Therefore, the thin-wire approximation is applied in this work to simplify the integro-differential formulation that models the wave radiation by microstrip devices. In addition, the model is extended to take into account the dielectric properties of the substrate and the contribution of lumped impedances that allows the analysis of complete microwave circuits. In the next sections this approach is detailed and validated through measurements and simulations for microstrip structures with and without lumped elements.

## II. NUMERICAL FORMULATION

In the electromagnetic modelling of filamentary structures, if the radius  $a$  of the wire cross-section is much less than the wavelength ( $a \ll \lambda$ ) and the wire length ( $a \ll L$ ), the surface current density  $\mathbf{J}_s(\mathbf{r})$  can be approximated by an electric current that flows on the direction  $\hat{t}$  tangential to the wire, according to  $\mathbf{J}_s(\mathbf{r}') = I(t')/(2\pi a)\hat{t}$ . In this case, it is assumed that the current does not vary in the  $\hat{\phi}$  direction and that it goes to zero at the wire extremities [5]. Therefore, by applying this approximation and imposing the tangential boundary conditions, it is possible to define the thin-wire approximation for the Electric Field Integro-differential Equation (EFIE) as:

$$\hat{\mathbf{n}} \times \mathbf{E}_{\text{inc}}(\mathbf{r}) = \hat{\mathbf{n}} \times \left[ \frac{j}{4\pi\omega\epsilon} \int_{t'} k^2 \mathbf{I}(t') \Psi(\mathbf{r}, \mathbf{r}') - [\nabla \cdot \mathbf{I}(t')] \nabla \Psi(\mathbf{r}, \mathbf{r}') dt' \right], \quad (1)$$

where  $\hat{\mathbf{n}}$  is the normal direction,  $\mathbf{E}_{\text{inc}}$  is the incident electric fields that excites the structure,  $\omega$  is the angular frequency,  $\epsilon$  is the electric permittivity,  $k$  is the propagation constant,  $\mathbf{I}(t') = I(t')\hat{t}$  and  $\Psi(\mathbf{r}, \mathbf{r}')$  is the Green's function for the free space given the position vector for the observer  $\mathbf{r}$  and source  $\mathbf{r}'$ . For the sake of notation simplicity, the use of apostrophe indicates quantities and operations related to the sources, while the others are related to the observer.

In order to apply the Method of Moments (MoM) to the EFIE in (1), the current  $\mathbf{I}(t')$ , which is the unknown function, is expanded in a finite sum of  $N$  triangular basis functions  $\mathbf{u}_j(t') = u_j(t')\hat{t}'$ :  $\mathbf{I}(t') = \sum_{j=1}^N I_j \mathbf{u}_j(t')$ , where  $I_j$  are the unknown that can be determined by taking the inner product with the triangular weighting function  $\mathbf{w}_i(t) = w_i(t)\hat{t}$  on both sides of the EFIE and by integrating it along the observer. The EFIE (1) reformulated by the application of MoM can be written in matrix form:  $\mathbb{V}_{(N \times 1)} = \mathbb{Z}_{(N \times N)} \cdot \mathbb{I}_{(N \times 1)}$ , where the elements of the excitation matrix  $\mathbb{V}$  are given by:

$$\mathbb{V}_i = \int_t \mathbf{w}_i(t) \cdot [\hat{\mathbf{n}} \times \mathbf{E}_{\text{inc}}(\mathbf{r})] dt, \quad (2)$$

and the equation for the elements of the impedance matrix  $\mathbb{Z}$  can be written as:

$$\mathbb{Z}_{i,j} = \frac{j}{4\pi\omega\epsilon} \int_t \int_{t'} k^2 w_i(t) u_j(t') (\hat{t} \cdot \hat{t}') \Psi(\mathbf{r}, \mathbf{r}') - \frac{dw_i(t)}{dt} \frac{du_j(t')}{dt'} \Psi(\mathbf{r}, \mathbf{r}') dt' dt. \quad (3)$$

Finally, the elements of the current matrix  $\mathbb{I}$ , that is, the unknown coefficients, can be found by solving the linear system.

#### A. Thin-wire approximation for microstrip structures

Consider a microstrip structure surrounded by free-space and constituted by a conductive line with width  $w_f$ , conductive layer thickness  $t_c$  and arbitrary geometry printed over a dielectric substrate with thickness  $h_s$ , relative electric permittivity  $\epsilon_r$  and loss tangent  $\tan \delta$ , as represented in Fig. 1(a). Since in most printed circuit boards, copper is used in the conductive layer, and it has one of the highest conductivities among the metals, it is reasonable to assume the conductive layer as PEC. However, the microstrip line cross-section is rectangular, whereas the cross-section considered in the thin-wire modelling is circular. As it can be seen in Fig. 1(b), a way to get around this is by calculating the equivalent diameter  $d = \sqrt{(4w_f t_c)/\pi}$  that leads to a circular cross-section with the same area as the microstrip line cross-section [6].

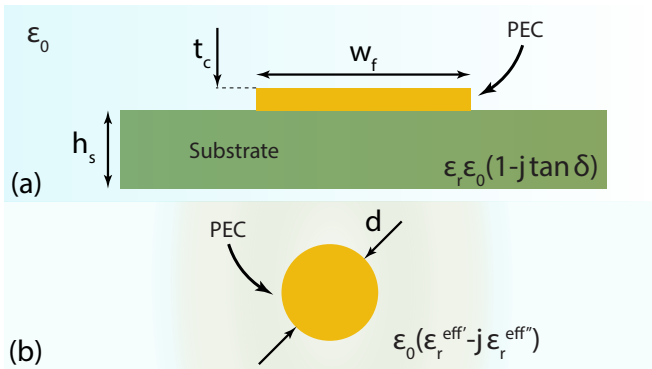


Fig. 1: Electromagnetic modelling of thin microstrip structures: (a) cross-section of a real microstrip line, and (b) thin-wire approximation.

Two different media surround the microstrip line, the dielectric substrate with relative electric permittivity  $\epsilon_r$  and

the free space. An effective medium can be defined as a homogeneous dielectric in which a transmission line presents the same properties, particularly the propagation constant, of the corresponding microstrip line [7]. This effective medium is characterized by its effective electric permittivity  $\epsilon_r^{\text{eff}}$  given in [7], [8].

However, the effective electric permittivity depends on the frequency, and the dispersive effects represented by the loss tangent must be taken into account. It can be accomplished by applying the multipole Debye model for the complex electric permittivity in the frequency domain as [9]:

$$\epsilon = \epsilon_\infty + \sum_{n=0}^{M-1} \frac{\epsilon_\infty (k-1) k^n}{1 + \frac{j\omega k^{nm}}{\omega_0}}, \quad (4)$$

where  $m = \pi/2\delta$ ,  $k = 10^{1/m\varsigma}$  is the spacing factor related to the number of substrate characterization samples per decade  $\varsigma$ ,  $\tan \delta$  is the substrate loss tangent, and  $M$  that is the Debye model order, that must be chosen in order to increase the model accuracy [9]. The parameter  $\epsilon_\infty$  is the asymptotic permittivity, that is, the electric permittivity at very high frequencies given by:

$$\epsilon_\infty = \frac{2\epsilon_r^{\text{eff}} \epsilon_0}{k^{\frac{M-1}{2}} (k+1)} \quad (5)$$

The upper frequency bound  $\omega_0$  is related to the bandwidth centre frequency as follows  $\omega_0 = \omega_c 10^{\frac{M-1}{2\varsigma}}$ , where  $\omega_c$  is the angular frequency in which the substrate is characterized.

#### B. Lumped element modelling

A lumped element with impedance  $Z_{\text{load}}$  leads to a reduction of  $\mathbf{E}_{\text{load}} = Z_{\text{load}} \mathbf{J}_s$  in the total electric field. Consequently, this lumped impedance contribution must be considered in the thin-wire EFIE formulation [5] and the impedance matrix becomes a sum of the distributed impedance matrix which elements can be found through (3), with the lumped impedance matrix  $\mathbb{Z}^{\text{load}}$ . The elements of the lumped impedance matrix, in turn, are given by:

$$\mathbb{Z}_{i,j}^{\text{load}} = \int_t Z_{\text{load}} w_i(t) u_j(t') (\hat{t} \cdot \hat{t}') dt. \quad (6)$$

Notice that the single integral in the equation for the lumped impedance matrix is carried out only in the observer segment which contains the lumped element. Thus, this is a sparse matrix with non-zero elements only in the rows related to that segment.

### III. RESULTS

In order to validate the thin-wire approximation for thin microstrip structures proposed in this work, two prototypes based on the double Split Ring Loop (SRL) topology were built: one open and the other one with a lumped inductor ( $7.5 \text{ nH}$ ,  $53 \text{ m}\Omega$ ) in the slit ring gap. The geometry and dimensions of these Devices Under Test (DUTs) are presented in Fig. 2. The DUTs were printed over a single-sided copper fibreglass substrate (FR-4) with the dielectric properties shown in Table I [10]. These properties were used as sample measurements for the multipole Debye model implementation.

These DUTs were experimentally characterized by measuring their reflection coefficient with the Vector Network Analyser (VNA) Agilent® E5071C in the range of 1 – 500 MHz. In addition, the open SRL case was simulated by using the full-wave frequency-domain solver of the simulation suite Computer Simulation Technology (CST®), which is based on the finite element method with adaptive tetrahedral mesh refinement.

TABLE I: DIELECTRIC PROPERTIES OF FR-4

Frequency	100 MHz	500 MHz	1 GHz
$\epsilon_r$	4.60	4.27	4.25
$\tan \delta$	0.016	0.015	0.015

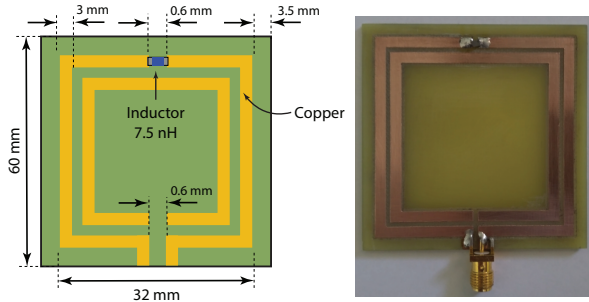


Fig. 2: Geometry of the printed split ring loop DUT.

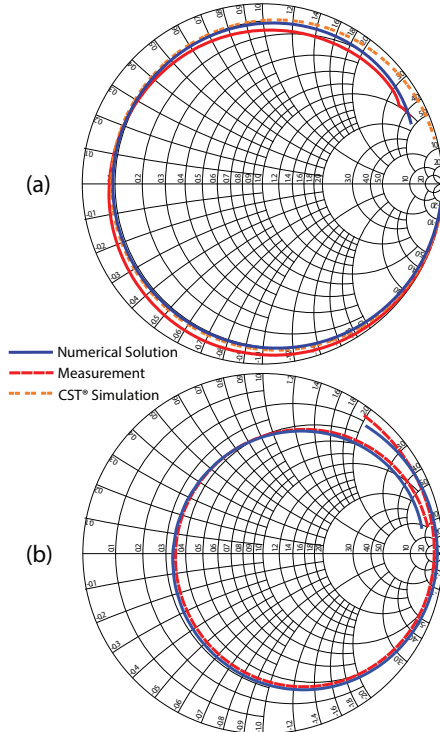


Fig. 3: Results for the reflection coefficient for the: (a) open SRL, and (b) SRL with lumped inductor.

The results for the reflection coefficient obtained with the proposed implementation are presented in Fig. 3 alongside the measured and simulated results. As seen in both cases, the numerical results are in close agreement with measurements and simulations, and the differences are mainly due to constructive uncertainties and the presence of the SMA connector that is

not considered in the numerical model and the simulation. The error, defined as the normalized difference between the Euclidean norms of the measured and numerical results, is 0.9706% for the open SRL case and 1.4344% for the SRL with the lumped inductor. It is also important to notice that the proposed implementation leads to a significantly reduction in the computational cost with a total processing time of 84 s, whereas the simulation in CST® takes about 665 s, both in the same computer with processor Intel® Core™ i7-8750H, 32 GB RAM, and 2.21 GHz clock.

#### IV. CONCLUSION

In this work, the thin-wire approximation is applied to the EFIE that models the electromagnetic radiation by hybrid microstrip circuit in which the microstrip line width and the structure size are much smaller than the wavelength at its operating frequency. This consideration holds for many applications in the microwave band, from radio-frequency circuits, wireless power transfer devices, metamaterials, among others.

From the comparison between the numerical results for the reflection coefficient obtained with the proposed implementation, measurements and simulations, it was verified that they are in close agreement in the entire analyzed band, with a maximum error equal to 1.4344%. Besides, a significant reduction in the processing time was verified, being around eight times lower than the simulation time in CST®.

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