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Sandro Trindade Mordente Gonçalves

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# Thin-Wire EFIE-MoM Formulation for Fast Computational Analysis of Microstrip Metasurfaces

Ícaro V. Soares  
IETR - UMR 6164  
Université de Rennes 1  
Rennes, France  
icaro.soares@ieee.org

Felipe M. Freitas  
Dept. of Electrical Engineering  
CEFET-MG  
Belo Horizonte, Brazil  
felipe.machado@ieee.org

Úrsula C. Resende  
Dept. of Electrical Engineering  
CEFET-MG  
Belo Horizonte, Brazil  
resendeursula@cefetmg.br

Sandro T. M. Gonçalves  
Dept. of Electrical Engineering  
CEFET-MG  
Belo Horizonte, Brazil  
sandro@cefetmg.br

**Abstract**—This work adapts the thin-wire approximation to the integro-differential formulation that models electromagnetic wave propagation through thin microstrip metasurfaces. A computationally efficient approach is achieved by reducing a surface current distribution to a filamentary electric current immersed in an effective dielectric medium. Besides, this is a general formulation that can be applied to wire and microstrip structures, even with lumped elements. Finally, this formulation is used to calculate the magnetic field through a magnetic metasurface lens excited by a wire loop. The results obtained agree on the overall behavior with full-wave simulation in commercial software, with almost 33 times faster computational time.

**Index Terms**—EFIE, lens, Method of Moments, metasurfaces.

## I. INTRODUCTION

Resonant metasurfaces (MTSs) are artificial materials composed of a periodic arrangement of subwavelength unit cells on a thin layer that presents an electromagnetic behavior that cannot be found in nature [1]. Nowadays, these devices have been used in frequency selective surfaces [2], electromagnetic band-gap structures [3], and superlenses [4], among other applications [5].

Once most MTSs are composed of a substantial number of small unit cells, the computational simulation and optimization of these devices is still a time-consuming task, even with a computer with high computational capability. Although the most common option is the full-wave simulation in commercial software, simulating these structures requires a highly dense mesh to obtain accurate results. Another option is to apply stratified media theory and asymptotic expansions to get a semi-analytical solution [6]. However, this second approach is mathematically complex and less general. In this scope, the present work proposes an alternative approach to the analysis and design of resonant MTSs by proposing approximations to reduce the computational complexity of solving a tridimensional integro-differential equation, thus reducing the processing time. Then, this formulation is applied to a magnetic MTS lens with lumped capacitors excited by a wire loop, and the results and computational cost are compared with full-wave analysis from commercial software.

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## II. NUMERICAL FORMULATION

MTSs are composed of sub-wavelength unit cells leading to a full structure's physical size  $L$ , much smaller than the wavelength ( $L \ll \lambda$ ). Besides, these devices are fabricated with microstrip technology so the thickness  $t_c$  of the microstrip line is much smaller than its length  $a$ . Based on these characteristics, it is reasonable to analyze this problem through a Thin-Wire (TW) formulation in which the surface current  $\mathbf{J}_s(\mathbf{r})$  is approximated by an electric current  $\mathbf{I}(t') = I(t')\hat{t}$  that flows in the direction  $\hat{t}$  tangential to the microstrip wire with an equivalent diameter  $d = \sqrt{(4w_f t_c)/\pi}$ , where  $w_f$  is the microstrip line width as indicated in Fig. 1(a).

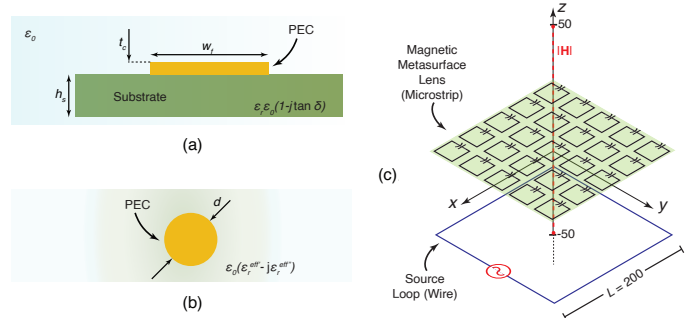


Fig. 1: TW approximation and problem geometry (units in mm).

However, the standard TW approach considers that the radiator is in free space. So, in order to apply this approximation to microstrip structures, the dielectric characteristics of the substrate, namely its relative electric permittivity  $\epsilon_r$  and its loss tangent  $\tan \delta$ , must be taken into account. It is accomplished by analyzing the equivalent problem represented in Fig.1(b), in which the radiation is immersed in an effective homogeneous medium in which the radiator presents the same properties, particularly the propagation constant  $k$ , of the corresponding microstrip line [7]. The effective electric permittivity  $\epsilon_r^{eff}$  of this medium can be calculated through the procedure presented in [7], and its dispersive nature is taken into account through the multipole Debye model as in [8].

Based on these approximations and by imposing the tangential boundary conditions, the current that flows through the MTS unit cells can be calculated by solving the following

Electric Field Integro-differential Equation (EFIE):

$$\hat{\mathbf{n}} \times \mathbf{E}_{\text{inc}}(\mathbf{r}) = \hat{\mathbf{n}} \times \left[ \frac{j}{4\pi\omega\epsilon} \int_{t'} k^2 \mathbf{I}(t') \Psi(\mathbf{r}, \mathbf{r}') - [\nabla \cdot \mathbf{I}(t')] \nabla \Psi(\mathbf{r}, \mathbf{r}') \right] dt' \quad (1)$$

where  $\hat{\mathbf{n}}$  is the normal direction,  $\mathbf{E}_{\text{inc}}$  is the incident electric fields that excites the structure,  $\omega$  is the angular frequency, and  $\Psi(\mathbf{r}, \mathbf{r}')$  is the Green's function given the position vector for the observer  $\mathbf{r}$  and source  $\mathbf{r}'$ .

The EFIE is numerically solved in this work by applying the Method of Moments (MoM). In this case, the unknown current  $\mathbf{I}(t')$  is expanded in a finite sum of  $N$  triangular basis functions  $\mathbf{u}_j(t') = u_j(t') \hat{\mathbf{t}}'$ :  $\mathbf{I}(t') = \sum_{j=1}^N I_j \mathbf{u}_j(t')$ , and by employing the Galerkin method, the weighting functions  $\mathbf{w}_i(t) = w_i(t) \hat{\mathbf{t}}$  are also triangular. Therefore, the EFIE becomes a linear system equation  $\mathbb{V}_{(N \times 1)} = \mathbb{Z}_{(N \times N)} \cdot \mathbb{I}_{(N \times 1)}$ , where the  $i$ -element of the excitation matrix  $\mathbb{V}$  is given by:

$$\mathbb{V}_i = \int_t \mathbf{w}_i(t) \cdot [\hat{\mathbf{n}} \times \mathbf{E}_{\text{inc}}(\mathbf{r})] dt. \quad (2)$$

The  $i$ -element of impedance matrix  $\mathbb{Z}$  can be calculated as:

$$\mathbb{Z}_{i,j} = \frac{j}{4\pi\omega\epsilon} \int_t \int_{t'} k^2 w_i(t) u_j(t') (\hat{\mathbf{t}} \cdot \hat{\mathbf{t}}') \Psi(\mathbf{r}, \mathbf{r}') - \frac{dw_i(t)}{dt} \frac{du_j(t')}{dt'} \Psi(\mathbf{r}, \mathbf{r}') dt' dt. \quad (3)$$

Moreover, lumped impedances can be included in this formulation as proposed in [8]. Finally, the unknown elements of the current matrix  $\mathbb{I}$  are found by solving this linear system.

### III. ANALYSIS OF MAGNETIC METASURFACE LENS

A magnetic MTS lens is used as a case study for validating the proposed numerical formulation. The detailed design procedure, dimensions, and performance of the analyzed uniform lens are described in [9]. As shown in the problem geometry in Fig. 1(c), a square wire loop 50 mm apart from the lens generates the magnetic field that excites the MTS. For comparison, the original problem with microstrip structure was simulated using the full-wave solver in CST<sup>®</sup> Studio Suite.

Fig. 2 presents the magnetic field amplitude  $|\mathbf{H}|$  evaluated over the dashed red line in Fig. 1(c), from the center of the source loop ( $z = -50$  mm), passing through the center of the MTS lens ( $z = 0$  mm), until the position  $z = 50$  mm, antipodal to the source. As it can be seen, the smooth results obtained with the proposed TW-EFIE formulation are due to the fact that it considers an effective homogeneous medium, and the reflections at the interface air-dielectric, which are noticed in the full-wave results, are disregarded. However, the proposed approach leads to a reasonable numerical approximation, and it is able to describe the lens behavior accurately, amplifying the evanescent  $\mathbf{H}$  and reconstructing the  $\mathbf{H}$  produced by the source at the opposite side. Regarding the computational time, the full-wave simulation took about 248s, whereas the TW-EFIE solved in 8.63s, i.e., almost 33 times faster.

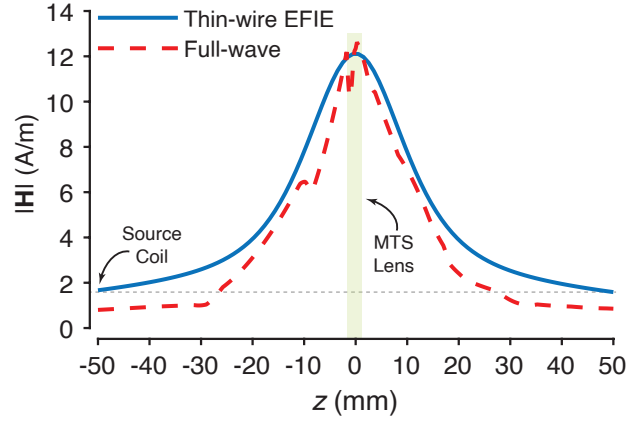


Fig. 2:  $|\mathbf{H}|$  results for the TW-EFIE and full-wave simulations.

### IV. CONCLUSION

This work proposes a TW approximation combined with effective medium treatment of dielectric substrates for reducing the computational cost of solving the EFIE that models the electromagnetic propagation through resonant MTS composed of subwavelength unit cells, also taking into account filamentary structures and lumped elements. This formulation was then applied to analyze a magnetic MTS lens illuminated by a wire loop antenna. The results have shown that the proposed approach can reproduce the lens's behavior, amplifying the evanescent near-field and reconstructing the field generated by the source at the opposite side. Besides that, the numerical values obtained with the TW EFIE agree with full-wave simulations in commercial software, with a computational time speedup of almost 97%. Finally, this formulation can be further extended to synthesize MTS-based devices.

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