



HAL
open science

Thermomagnetic convective instabilities in the Taylor-Couette ferrofluid flow.

Anupam Mahantayya Hiremath, Antoine Meyer, Harunori Yoshikawa,
Innocent Mutabazi

► **To cite this version:**

Anupam Mahantayya Hiremath, Antoine Meyer, Harunori Yoshikawa, Innocent Mutabazi. Thermomagnetic convective instabilities in the Taylor-Couette ferrofluid flow.. 25e Congrès Français de Mécanique,, Aug 2022, Nantes, France. hal-04280168

HAL Id: hal-04280168

<https://hal.science/hal-04280168>

Submitted on 10 Nov 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Thermomagnetic convective instabilities in the Taylor-Couette ferrofluid flow

A M. HIREMATH^a, A. MEYER^a, H N. YOSHIKAWA^b and I. MUTABAZI^a

a. Normandie Université, UNIHAVRE, LOMC UMR CNRS 6294,
anupam-mahantayya.hiremath@etu.univ-lehavre.fr

b. Université de Nice Côte d'Azur, Institut de Physique de Nice, UMR 7010,
harunori.yoshikawa@univ-cotedazur.fr

Résumé : L'analyse de stabilité linéaire (LSA) des ferrofluides dans le système Taylor-Couette a été effectuée. Le cylindre intérieur à température fixe T_1 a un rayon R_1 et tourne avec une vitesse angulaire Ω_1 et le cylindre extérieur de rayon R_2 est maintenu à température fixe T_2 . Le champ magnétique est fourni par l'empilement d'aimants placés à l'intérieur du cylindre intérieur [1].

Le ferrofluide dans l'entrefer des cylindres concentriques est soumis à la pousée magnétique centripète grâce au champ magnétique appliqué et, à la pousée centrifuge grâce à la rotation des cylindres.

Les paramètres de contrôle sont le rapport de rayon $\eta = R_1/R_2$, le nombre de Taylor Ta qui tient compte de la rotation du cylindre intérieur et le nombre magnétique de Rayleigh Ra_m qui tient compte du champ magnétique. La configuration d'écoulement est supposée de longueur axiale infinie. De petites perturbations sont superposées à l'écoulement de base, les équations résultantes sont linéarisées et les perturbations sont développées sous forme de modes normaux $\exp[st + i(kz + n\varphi)]$, où $s = \sigma + i\omega$ est le taux de croissance des perturbations, k est le nombre d'onde axial et n est le nombre d'onde azimutal. Les états marginaux de stabilité sont déterminés avec la condition ($\sigma = 0$).

À partir des courbes de stabilité marginale, les états critiques, c'est-à-dire les états avec les valeurs les plus basses de Ra_m et Ra_{mc} augmente avec η avec $Ra_{mc} = 1708$ pour $\eta \rightarrow 1$. En négligeant l'effet de la gravité terrestre, les modes critiques sont hélicoïdaux ($k_c \neq 0, n_c \neq 0$) et stationnaires ($\omega = 0$) pour $Ta = 0$ et ils sont oscillatoires ($\omega \neq 0$) pour $Ta \neq 0$ [2].

Abstract : A linear stability analysis (LSA) of ferrofluid in the Taylor-Couette system has been conducted to understand the critical instability modes in the system. The inner cylinder has a radius R_1 at a fixed temperature T_1 and rotates with an angular velocity Ω_1 and the outer cylinder of radius R_2 is maintained at a fixed temperature T_2 . The magnetic field is furnished by the stack of magnets placed inside of the inner cylinder [1].

The ferrofluid in the gap between the cylinders is subject to the centripetal magnetic buoyancy towards the inner hot cylinder due to the applied magnetic field and the centrifugal buoyancy due to the rotation of the cylinders.

The flow control parameters are the radius ratio $\eta = R_1/R_2$, the Taylor number Ta which accounts for the rotation of the inner cylinder and the Rayleigh magnetic number Ra_m which accounts for the magnetic buoyancy. The flow configuration is assumed of infinite axial length. Small perturbations are

superimposed to the base flow, the resulting equations are linearized and the perturbations are expanded in form of normal modes $exp[st+i(kz+n\varphi)]$, where $s = \sigma + i\omega$ is the growth rate of the perturbations, k is the axial wavenumber and n is the azimuthal wavenumber. The marginal stability states are determined with the condition ($\sigma = 0$).

From marginal stability curves, the critical states i.e. those states with the lowest values of Ra_m is defined as Ra_{mc} and found that Ra_{mc} increases with η with $Ra_{mc} = 1708$ for $\eta \rightarrow 1$. Neglecting the effect of terrestrial gravity, the critical modes are helical ($k_c \neq 0, n_c \neq 0$) and stationary ($\omega = 0$) for $Ta = 0$ and they are oscillatory ($\omega \neq 0$) for $Ta \neq 0$ [2].

Mots clés : Thermomagnetic convection, ferrofluids, hydrodynamic instability, Taylor-Couette flow.

1 Introduction

The ferrofluids are a colloidal solutions consisting of the ferric particles of nearly $10nm$ coated with surfactants of $2nm$. These particles are either fabricated using a water based ($Pr = 15$), if not oil based solutions ($Pr = 350$). The stability is governed by the Brownian motion.

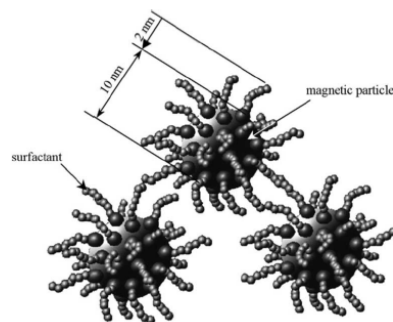


FIGURE 1 – Description of a ferromagnetic particle.[3]

In the presence of an external magnetic field they act like a paramagnetic material and tend to align in the direction of the applied field. The magnetization of these fluids is a function of applied field and the temperature. The Kelvin force acting on these particles arises due to the magnetization and the external field.

2 Flow modeling

In this section the flow configuration and the governing equations of our system will be described.

2.1 Flow configuration and body force

The fluid is between the gap of the cylinders. The inner cylinder of radius R_1 maintained at temperature T_1 and the outer cylinder of radius R_2 and temperature T_2 . This is a configuration of outwards heating i.e. $T_1 > T_2$, and for inwards heating $T_1 < T_2$.

Due to the rotation of the inner cylinder at a rate of Ω_1 there arises a centrifugal buoyancy $\rho\alpha\theta g_c$ which is associated with the centrifugal force \vec{g}_c . The centripetal magnetic gravity \vec{g}_m is the consequence of

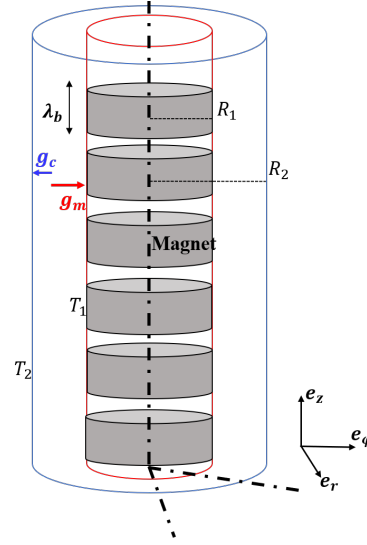


FIGURE 2 – Configuration of the ferrofluid flow in the considered system

magnetic buoyancy $\rho\alpha\theta g_m$ due the interaction between the magnetization of the ferrofluid and the applied magnetic field. This field arises from the magnets which are stacked at an axial spacing of λ_b in the inside of inner cylinder.

The magnetic gravity \vec{g}_m arises from the Kelvin force given by

$$\vec{F}_B = M_0 \vec{\nabla} |\vec{B}| + |\vec{B}| \vec{\nabla} M_0 \quad (1)$$

In the Boussinesq approximation the second term in RHS of Eqn. 1 is neglected. The magnetization is a function of temperature $M_0 = M_{Ref} [1 - \alpha_m \theta]$, where $\alpha_m = K/M_{Ref}$ and $K = -\partial M/\partial T$ is the pyromagnetic coefficient. After simplification the Eqn.1 develops as,

$$\vec{F}_B = -\alpha_m M_{Ref} B_0 \kappa K_1(\kappa r) \theta \vec{e}_r + \vec{\nabla} (M_{Ref} B_0 K_0(\kappa r)) \quad (2)$$

The gradient term is integrated into the pressure term and the first term on the RHS of Eqn.2 is defined as the magnetic buoyancy $\rho\alpha\theta g_m$ where the magnetic gravity \vec{g}_m is given by,

$$\vec{g}_m = \frac{\alpha_m M_{Ref} B_0 \kappa_b K_1(\kappa_b r)}{\alpha \rho_{Ref}} \vec{e}_r \quad (3)$$

2.2 Governing equations

The governing equations for the configuration under study are defined as

$$\vec{\nabla} \cdot \vec{v} = 0 \quad (4)$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\vec{\nabla} \pi + \Delta \vec{v} - \gamma_a \frac{v^2}{r} \theta \vec{e}_r + \frac{Ra_m}{Pr} \frac{K_1(\kappa_b r)}{C_{Mag}} \theta \vec{e}_r \quad (5)$$

$$\frac{\partial \theta}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \theta = \frac{1}{Pr} \Delta \theta \quad (6)$$

The constant in the Eqn.5 $C_{Mag} = \int_{R_1}^{R_2} K_1(\kappa_b r)$. The non-slip boundary conditions at the surfaces of the cylinders are,

$$\vec{v} = Re\vec{e}_\varphi; \quad \theta = 1; \quad \text{at } r = \eta / (1 - \eta), \quad (7)$$

$$\vec{v} = 0; \quad \theta = 0; \quad \text{at } r = 1 / (1 - \eta), \quad (8)$$

The control parameters arising in the system are defined as, Prandtl number $Pr = \nu / \kappa$, magnetic Rayleigh number $Ra_m = \alpha \Delta T g_m d^3 / \nu \kappa$ and the dimensionless thermal expansion parameter $\gamma_a = \alpha \Delta T$.

The base state is steady, axisymmetric and axially invariant and depends on the radial position. The base state for temperature Θ and the azimuthal velocity V are defined as,

$$\Theta = \frac{\ln[(1 - \eta)r]}{\ln \eta} \quad (9)$$

$$V = r\Omega = \frac{Ta \eta^{3/2}}{(1 - \eta)^{5/2} (1 + \eta)} \left[\frac{1}{r} - (1 - \eta)^2 r \right] \quad (10)$$

where Ω is the angular velocity and Ta is the Taylor number.

3 Results

The magnitude of the magnetic gravity is calculated in the flow system using the Eqn.(3). The value $\lambda_b = 3.54d$ for the axial spacing of the magnets is considered as suggested by Tagg and Weidman [1], where the dimensional value of the gap between the cylinders is assumed to be $d = 7mm$. Data of the water-based ferrofluid Fe_3O_4 from [4, 5] are used to calculate the values of the magnetic averaged gravity $\langle g_m \rangle = \frac{1}{R_2 - R_1} \int_{R_1}^{R_2} g_m dr$. The ratio of the average $\langle g_m \rangle$ with respect to the terrestrial gravity g is given in table.1.

η	$\langle g_m \rangle / g$
0.65	15.5371
0.7	15.4513
0.95	15.1262

TABLE 1 – Ratio of the average magnetic gravity $\langle g_m \rangle$ to the terrestrial gravity g , for each value of η and an applied field of $B = 0.005T$.

For linear stability analysis small perturbations are superimposed to the base state flow and resulting equations are linearized, the perturbations are expanded into normal modes $exp[st + i(kz + n\varphi)]$, where $s = \sigma + i\omega$ is the growth rate of perturbations, k as the axial wave number and n the number of modes in the azimuth.

The marginal states are investigated for the case $\sigma = 0$. For a given set of control parameters the critical magnetic Rayleigh number Ra_{mc} is considered for the lowest value on the stability curve. For the case of inner rotating cylinder figure.3 shows the stability curves for different n . the critical parameters are $Ra_{mc} = 1595.6$, $k_c = 1.149$, $n_c = 13$ and $\omega_c = 0$.

The variation of the critical parameters are investigated depending on the rotation rates and the situation of outwards heating ($\gamma_a > 0$) and inwards heating ($\gamma_a < 0$), the figure.4 by Meyer et al.[2] explains the two cases for two different ferrofluids of $Pr = 15$ and $Pr = 350$. The discontinuity in the curves is due to the abrupt change in the value of γ_a [2]. In the situation of outwards heating there is a decrease in the

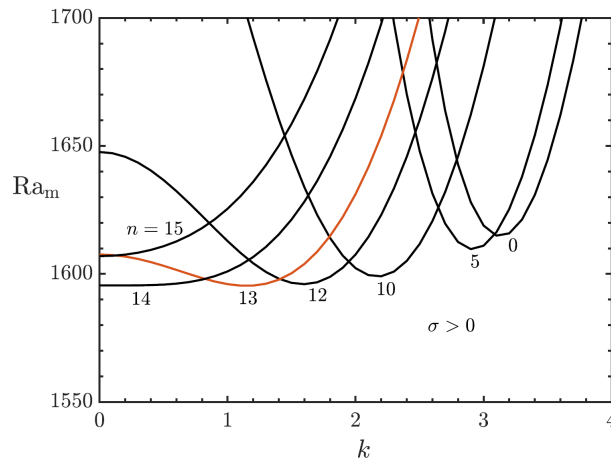


FIGURE 3 – The marginal stability curves for $\eta = 0.8$, $Pr = 15$, $\gamma_a = 0.01$ and $Ta = 0$ [2]

value of Ra_{mc} as the value of Ta is increasing indicating the stabilizing effect of the rotation of cylinder. The inwards heating has the inverse dynamics where the rotation of cylinders causes the destabilization in the system.

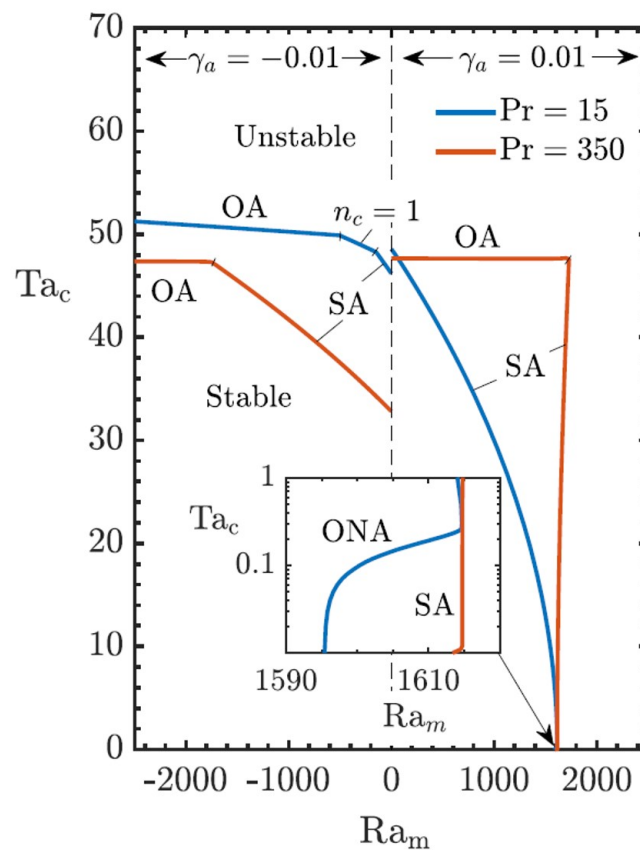


FIGURE 4 – The variation of the critical Ra_{mc} according to the rotation rates and the system of heating[2]

In the microgravity situation a similar investigation is made for solid body rotation, where both the cylinders rotate with the same angular velocity $\Omega_1 = \Omega_2$. The variation of the threshold in case of $\gamma_a = 0.001$ is evolving with respect to the radius ratio η and the rotation of the cylinders Ta .

The eigenfunctions are used to understand the types of modes in the regime of solid body rotation, figure.5 shows the modes for $\eta = 0.8$ and $Ta = 0$. The modes in this condition of zero rotation are stationary figure.5c ($\omega = 0$) and are helical figure.5b ($k_c \neq 0, n_c \neq 0$). Figure. 6 shows the modes when the rotation is applied. The modes are changed to columnar structures ($k_c = 0$) and are oscillatory i.e. ($\omega \neq 0$).

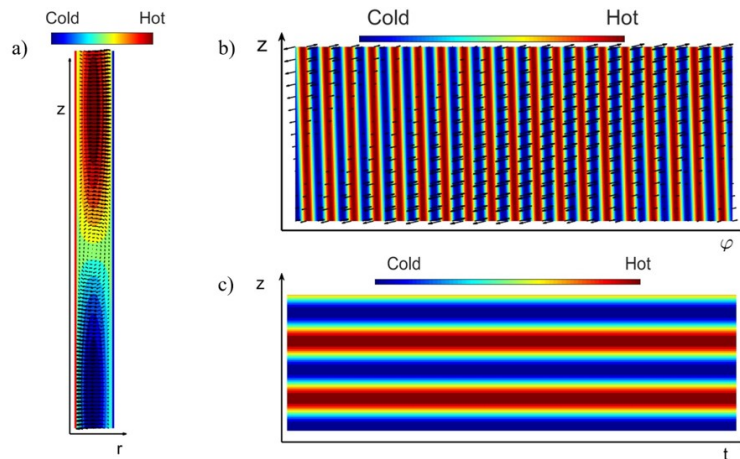


FIGURE 5 – Eigenfunctions for $\eta = 0.8$ and $Ta = 0$.

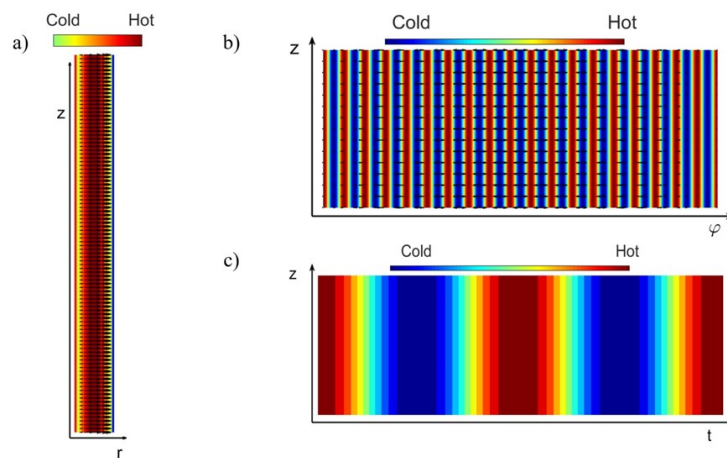


FIGURE 6 – Eigenfunctions for $\eta = 0.8$ and $Ta = 40$.

4 Discussion and conclusion

In the current system for a small value of applied magnetic field there is generation of \vec{g}_m which is nearly 15 times larger than \vec{g} in the fluid where the magnetization is considered to be close to the saturation value.

In microgravity conditions the ferrofluid in the gap is subject to two radial forces. The centrifugal gravity g_c and the centripetal force the magnetic gravity g_m . For the situation of outwards heating $\gamma_a > 0$ the instability modes are created due to the magnetic gravity g_m . figure.4 prove that the rotation of cylinder for a fixed η and $\gamma_a > 0$ increases the threshold, hence indicating that the centrifugal gravity is the

stabilizing force in the system. The case of inner rotating cylinder and $\gamma_a < 0$ shows that the centrifugal gravity is the destabilizing force.

Earlier studies of dielectrophoretic fluids under similar configuration have shown that for higher value of electric field the modes are electric and have similar dynamics.[6]

In the current configuration the modes are magnetic. In case of the inner rotating cylinder the modes are either stationary axisymmetric or oscillatory axisymmetric depending on the rotation rate and the applied field. The eigenfunctions as shown in figure.5 and figure.6 for the solid body rotation manifest the nature of the instability modes. For $Ta = 0$ stationary cylinders they are stationary helical and for applied rotation $Ta \neq 0$ they are oscillatory columnar.

Acknowledgments

The present work has benefited from the financial support from the French Space Agency (CNES) and the French National Research Agency (ANR) through the program “Investissements d’Avenir (No. ANR-10 LABX-09-01),” LABEX EMC3 INFEMA.

Références

- [1] R. Tagg and P.D. Weidman, Linear stability of radially-heated circular Couette flow with simulated radial gravity, *Z. angew. Math. Phys.*, **58**, 431-456 (2007).
- [2] A. Meyer, A. Hiremath and I. Mutabazi, Thermomagnetic instability of a ferrofluid in a differentially heated Taylor-Couette system, *Phy. Rev. Fluids*, **7**, 023901 (2022).
- [3] S. Odenbach, Ferrofluids—Magnetically controlled suspensions, *Colloids and Surfaces A : Physicochemical. Eng. Asp.*, **217(1-3)**, 171–178 (2003).
- [4] H. Rahman and S. A. Suslov, Thermomagnetic convection in a layer of ferrofluid placed in a uniform oblique external magnetic field, *J. Flu. Mech.*, **764**, (2015) 316–348.
- [5] G. K. Auernhammer and H. R. Brand, Thermal convection in a rotating layer of a magnetic fluid, *Eur. Phys. J. B-Cond. Mat. and Comp. Sys.*, **16(1)**, (2000) 157–168.
- [6] C. Kang, A. Meyer, H. N. Yoshikawa, and I. Mutabazi, Numerical study of thermal convection induced by centrifugal buoyancy in a rotating cylindrical annulus, *Phys. Rev. Fluids*, **4**, 043501 (2019).