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Anisotropic diffusion filter for 3D full waveform inversion: application to a North Sea dataset

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Main objectives

To present a regularization strategy for seismic imaging using the full waveform which naturally accounts for the structure information contained in the gradient. A comparison with a conventional Gaussian smoothing strategy on a 3D field data from the North sea emphasizes the more accurate reconstruction of the subsurface parameters.

New aspects covered

The regularization strategy is based on an anisotropic diffusion filter presented in 2D at last EAGE (Madrid 2022). Here the filter is extended to 3D, a specific discretization scheme is used to ensure numerical stability, and the method is applied for the first time to a 3D field data to illustrate its interest for industry scale applications.

Summary (200 words)

Regularization is central in full waveform inversion, specifically to remove spurious oscillations due to noisy data and to compensate the uneven illumination of the subsurface. Conventional strategies directly apply smoothing operators at each full waveform inversion iteration. Numerous of operators have been proposed in the literature, from Gaussian operators to more evolved structure preserving Laplacian operators. The former are easy to implement but might destroy the structural information embedded in the gradient. The latter require an external source of information related to the structure to be preserved. We propose to use an anisotropic diffusion filter which can directly enhance the structure of the gradient without using external prior information. We extend the 2D method presented at 2022 EAGE meeting in Madrid to 3D, use an adapted discretization scheme which guarantees numerical stability, and apply it to a 3D field data from the North Sea. We compare the results of inversion up to 10 Hz between a Gaussian smoothing and our anisotropic diffusion smoothing. The enhancement of the resolution and the structural coherence is striking, making full wavefield images much easier to interpret geologically. We believe this technique could be applied to a wide variety of field data in the future.

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Introduction

Regularization is a central topic in Full Waveform Inversion (FWI), a high resolution seismic imaging method formulated as a data-fitting problem, resulting in an ill-posed inverse problem. For this category of problems, regularization strategies are needed to restrict the size of the subspace of acceptable solutions by introducing a prior information on the solution.

Regularization strategies in FWI are implemented following two main approaches. In the first approach, penalization terms are added to the misfit function to prevent the solution to depart from a prior model and/or to enforce the smoothness of the solution by constraining the amplitude of its spatial derivatives. This is also known as Tikhonov regularization strategy. The second consists in smoothing/filtering the gradient of the misfit function which is iteratively minimized, so as to remove unwanted features, for instance fast spatial oscillations, or promote specific ones, for instance sharp edges, or piece-wise constant structures, directly in the model updates.

The second approach has the specific advantage over the first that the prior information is imposed at each iteration, therefore generating a sequence of models which satisfies the prior information. In the first approach, this is ensured only at convergence, when the weight given to the prior information starts to be of the same order of magnitude than the weight given to the data misfit term. This requires a careful tuning of the hyper-parameters controlling the relative amplitude of the data misfit and regularization terms. In the second approach, hyper-parameters defining the smoothing/filtering operator which acts on the gradient are easier to control as they are directly related to the expected resolution of the final model. The latter can be estimated from the frequency band of the data which is inverted. For instance, (Operto et al., 2006) apply a non-stationary Gaussian convolution filter to the gradient of the misfit function at each iteration. The coherence lengths of the filter are computed from an estimation of the local wavelength using the current velocity model and a reference frequency.

Such non-stationary Gaussian filter however ignores the structural information present in the gradient and might result in oversmoothing specific features one would like to keep or even enhance in the model update. Structure preserving filters have thus been proposed to overcome this issue, using for instance directional Laplacian or Bessel functions (see Guitton et al., 2012, for instance). More recently, we have proposed to use a filter based on an anisotropic diffusion process (Métivier and Brossier, 2022). The interest for such a strategy is to extract directly the structural information through a local Singular Values Decomposition (SVD) of the structure tensor associated with the gradient, and to enhance all the coherent features which can be detected from this SVD decomposition. Previously proposed structure preserving smoothing techniques requires a prior information on the structure coming from an external source (geological interpretation, well logs, etc...) which might not always be available.

In this study, we extend our structure enhancing filter to 3D and apply it to the multi-parameter reconstruction of P-wave velocity and density models from a 3D field dataset from the North Sea. By comparison with a conventional Gaussian filter, we show how using our structure enhancing filter results in an increased resolution for both P-wave velocity and density models, and hence the impedance model. This is especially the case at depth where the wavelength increases with higher velocities, resulting in an increased smoothing length when using the non-stationary Gaussian filter.

FWI with a 3D anisotropic diffusion filter

We cast the FWI problem as

$$\min_m f(m) = \frac{1}{2} \sum_{s=1}^S \|d_{cal,s}[m] - d_{obs,s}\|^2, \quad d_{cal,s}[m] = Ru_s[m], \quad A(m)u_s = b_s, \quad (1)$$

where m is the subsurface model, $f(m)$ is the misfit function between observed $d_{obs,s}$ and calculated $d_{cal,s}[m]$ data, for $s = 1, \dots, S$ shots, and R is a restriction operator mapping the wavefield $u_s[m]$ solution of the wave equation $A(m)u_s = b_s$ to the data space by restraining the wavefield values at the receivers

location. We solve the FWI problem through local optimization: from m_0 we build the sequence $m_{k+1} = m_k + \alpha_k \Delta m_k$, where $\Delta m_k = -Q_k \nabla f(m_k)$, with Q_k an approximation of the inverse Hessian $\nabla^2 f(m_k)^{-1}$. Regularization through smoothing/filtering consists in composing Q_k with a smoothing/filtering operator \mathcal{F} such that

$$\Delta m_k = -Q_k \mathcal{F}(\nabla f(m_k)). \quad (2)$$

In this work, the filter \mathcal{F} we consider is given by the following anisotropic diffusion process

$$\begin{cases} \partial_\tau v - \text{div}(D(v)\nabla v) = 0 \\ v(x, 0) = \nabla f(m), \end{cases} \quad (3)$$

with Neumann boundary conditions. The filter \mathcal{F} is defined such that $\mathcal{F}(\nabla f(m)) = v(x, T)$ for a given diffusion time T . Here $D(v)$ is a 3x3 diffusion matrix which we build as follows, extending the 2D approach introduced in [Weickert \(1998\)](#). Given two reference scales ℓ and σ , we consider the matrix

$$S_{\ell, \sigma} = K_\ell .* ((K_\sigma * \nabla f(m)) (K_\sigma * \nabla f(m))^T), \quad (4)$$

where K_l denotes a Gaussian convolution kernel with coherence length l , $*$ is the convolution operation, and $.*$ denotes a term-by-term convolution. Let $P(v)^T \Lambda(v) P(v)$ be the SVD decomposition of $S_{\ell, \sigma}$. Λ is a diagonal matrix

$$\Lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3), \quad (5)$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3$ are the eigenvalues of $S_{\ell, \sigma}$. The columns $P_i, i = 1, \dots, 3$ of the matrix P are the associated eigenvectors. We build the 3D diffusion matrix as $D(v) = P(v)^T \Sigma(v) P(v)$ where

$$\Sigma(v) = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & g((\lambda_1(v) - \lambda_2(v))^2) & 0 \\ 0 & 0 & g((\lambda_1(v) - \lambda_3(v))^2) \end{pmatrix}, \quad (6)$$

with g a threshold function taking values from α to 1

$$g(x) = \begin{cases} \alpha & \text{if } x = 0 \\ \alpha + (1 - \alpha) \exp\left(\frac{-x}{C}\right) & \text{else.} \end{cases} \quad (7)$$

In (7), α and C are user-defined scalar values, typically $\alpha = 10^{-2}$ and $C = 10^{-8}$. The direction $P_1(x)$ is the direction of main variations in the gradient $\nabla f(m)$. Defined as such, the diffusion matrix $D(v)$ implies a weak diffusion in the direction P_1 while the diffusion increases as soon as a coherent feature is detected in the direction P_2 (resp. P_3) detected by an increase of the quantity $(\lambda_1(v) - \lambda_2(v))^2$ (resp. $(\lambda_1(v) - \lambda_3(v))^2$). We implement this anisotropic diffusion filter using the dedicated numerical scheme proposed by [Fehrenbach and Mirebeau \(2014\)](#) which ensures stability properties even for strongly 3D anisotropic diffusion.

Field data application

We consider an Ocean Bottom Cable (OBC) dataset from the North sea with 2048 receivers deployed on the sea floor and approximately 50,000 shots positions. The water column depth is constant, equal to 70 m only (shallow water environment). The main challenge for this field data is to image a 3D large scale low velocity anomaly in the center of the model as well as the structure below this low velocity anomaly.

We use a minimal data preprocessing. After despiking, the near offset data is muted to remove what we identify as Schölte waves propagating at approximately 300 m.s^{-1} along the sea bottom interface. We make use of the reciprocity to decrease the computational cost of FWI which scales linearly with the number of shots. We filter the data in three frequency bands, 2.5 - 5 Hz, 2.5 - 7 Hz, 2.5 - 10 Hz and invert them in a multiscale manner. We use a 3D visco-acoustic VTI modeling engine to invert this data. We have been provided a reflection tomography initial vertical velocity model as well as simple blocky anisotropy models for ε and δ Thomsen's parameters. The quality factor we consider is constant and equal to 200 everywhere except in the water column where it is set to 1000.

We use a random subsampling strategy to decrease the computational cost, considering 16 random batches of 128 shots (after reciprocity) such that one shot belongs to a single batch. For each batch we perform 3 *l*-BFGS iterations, leading to 48 FWI iterations in total per frequency band. From the estimated velocity v_P and density ρ we compute the impedance $I = \rho v_P$ and we display the reflectivity images derived from the impedance volume I as

$$\frac{\partial I}{\partial n} = \frac{\partial I}{\partial x} \sin \theta \cos \phi + \frac{\partial I}{\partial y} \sin \theta \sin \phi + \frac{\partial I}{\partial z} \cos \theta, \quad (8)$$

where θ and ϕ are dip and azimuth angles normal to the subsurface reflectors.

We provide the comparison between the reflectivity images obtained with the Gaussian filter and the anisotropic diffusion filter in Figures 1 and 2. One can appreciate the finer level of details brought to the edges and the inner layering structure of the central low velocity anomaly. More strikingly, a strong reflector appearing below the low velocity anomaly, at approximately 3.5 km depth, is made much more clear and continuous using the anisotropic diffusion filter. This represents a significant improvement in the imaging of this challenging zone. This improvement is entirely due to the use of the anisotropic diffusion filter as this is the only part of the workflow which changes between the two images.

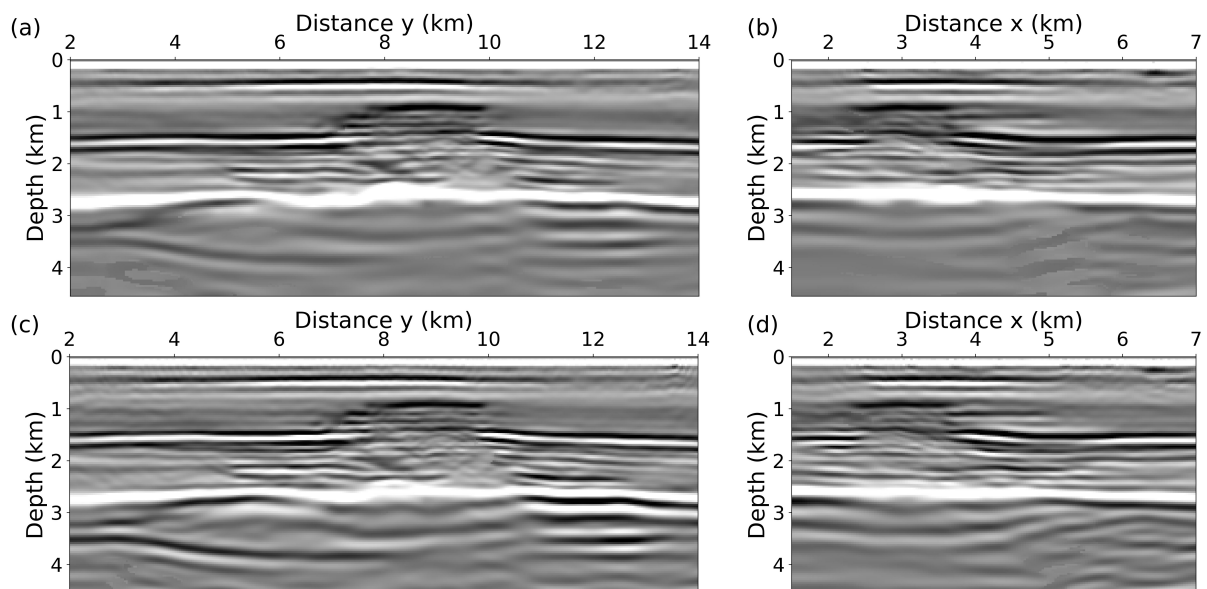


Figure 1 Comparison between the results obtained with the non-stationary Gaussian filter (a-b) and the anisotropic diffusion filter (c-d). Constant x slices at $x = 2.95$ km (a,c), constant y slices at $y = 9$ km (b,d).

This improvement in resolution can be linked to the better interpretation of short-offset reflections, as can be observed in Figure 3. The non-stationary Gaussian filter applied to the velocity and density reconstruction has a tendency to oversmooth the models, which is detrimental to the reconstruction of these reflections and hence the reflectivity model. On the contrary the diffusion filter enhances continuous features in the velocity and density models which result in a better interpretation of the reflection and a higher resolution reflectivity model.

Conclusion

We have successfully extended to 3D the 2D anisotropic diffusion filter presented by Weickert (1998) for image processing and we have applied it to the inversion of a field data set from the North sea. We observe that the anisotropic diffusion filter makes it possible to enhance coherent structures, which results in an increased resolution of the final reflectivity images, especially at depth, where a deep reflector (3.5 km) is made apparent and continuous. Compared with a conventional non-stationary Gaussian smoothing strategy the improvement is significant. We believe that this strategy could offer similar advantages on a variety of field data applications especially when inverting data in high frequency bands.

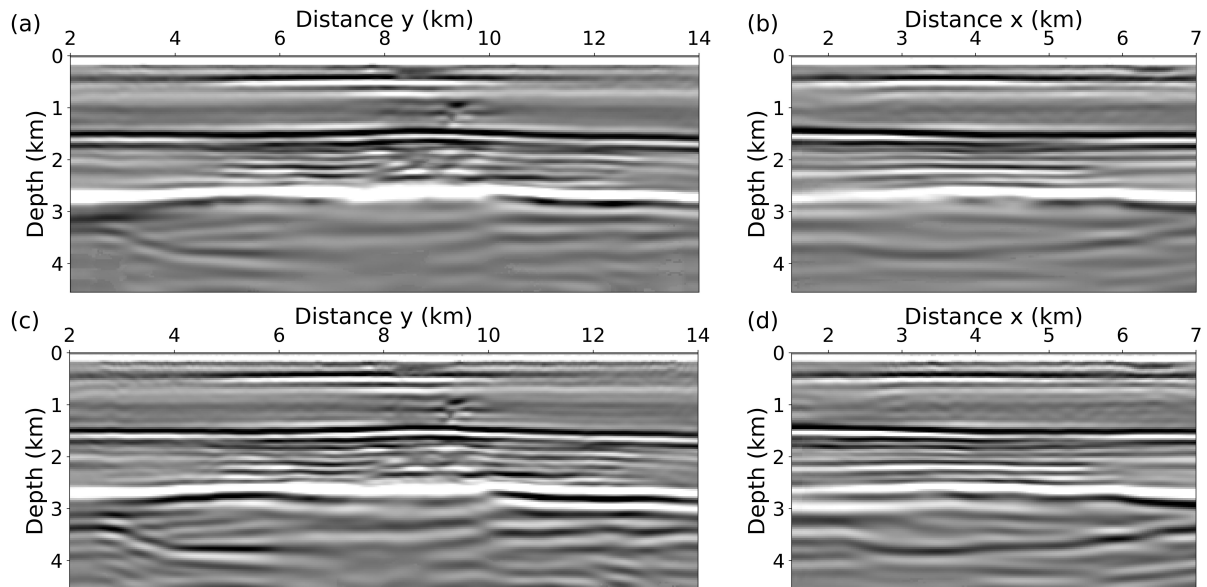


Figure 2 Same as Figure 1 with constant x slices at $x = 3.95$ km (a,c), and constant y slices at $y = 6$ km (b,d).

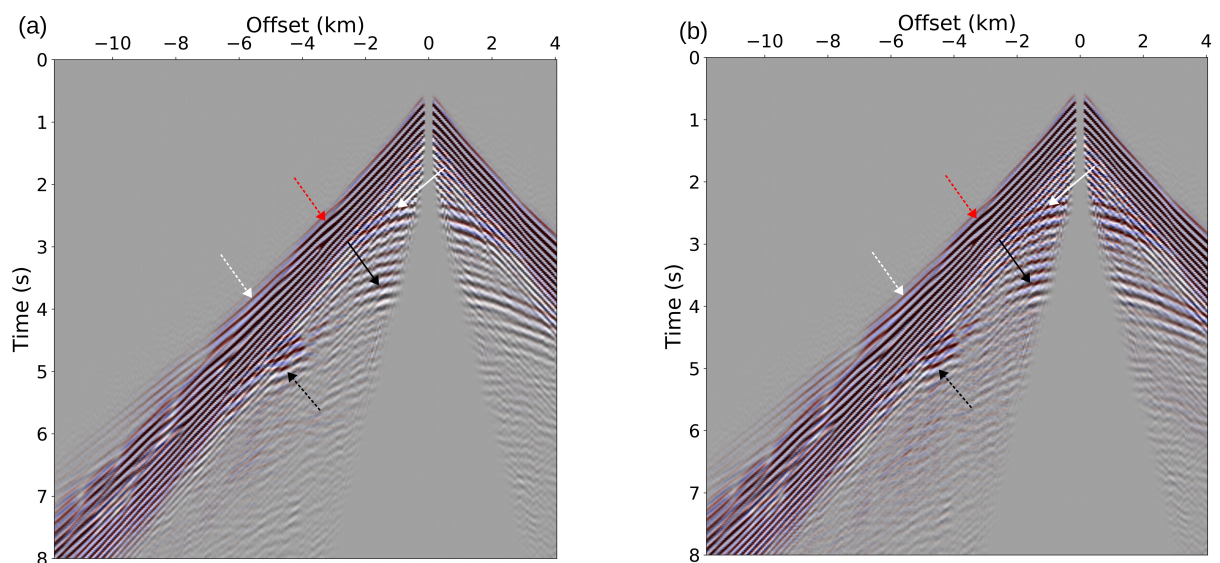


Figure 3 Superposition of field data (black/white colorscale) and predicted data (red/blue colorscale) obtained with (a) the Gaussian filter (b) the anisotropic diffusion filter. A good match means absence of white and red. The red, white, black arrows point on the reflection from a shallow reflector, the top of the low velocity anomaly and the bottom of the low velocity anomaly, respectively. The solid arrows point on the pre-critical reflections, while the dashed ones point on the post-critical reflections.

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