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► **To cite this version:**

Mohamed Amine Abdellaziz, Romain Brossier, Ludovic Métivier, Edouard Oudet. Optimal Experimental Design in Full Waveform Inversion. 84th EAGE Annual Conference & Exhibition, Jun 2023, Vienne, Austria. pp.1-5, 10.3997/2214-4609.202310605 . hal-04278865

**HAL Id: hal-04278865**

**<https://hal.science/hal-04278865v1>**

Submitted on 10 Nov 2023

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# Optimal Experimental Design in Full Waveform Inversion

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January 22, 2023

## Main objectives

To propose a new criterion for the optimal positioning of sources and receivers in the context of Full Waveform Inversion.

## New aspects covered

Our new approach is based on the theoretical wavenumber spectrum of the FWI gradient and uses concepts and results borrowed from numerical geometry to model and solve the problem.

## **Summary (197 words)**

In this work we propose a new optimality criterion for the design of acquisition geometries in the context of Full Waveform Inversion, which is based on the wavenumber spectrum of the gradient. We suppose an acquisition is composed of  $N_s$  sources and  $N_r$  receivers and where each source is fired with all the receivers, generating  $N_s \times N_r$  wavenumbers. We are looking for the positions of sources and receivers that would optimize the distribution of wavenumbers. We are able to outline the envelope of the wavenumber cloud so this allows us to define the underlying problem as an instance of Centroidal Voronoi Tessellation and to propose a new energy function to minimize. To illustrate our method we show the optimized geometry for surface, circular and in transmission types of acquisitions. We compare them and the wavenumber spectrum generated with the evenly-spaced equivalents. We also show the impact of the optimized acquisition on a gradient for the reconstruction of a scattering point in a homogeneous medium. The result of our optimization fulfills the criterion of optimal distribution of the wavenumbers and this has the consequence to produce gradients that are more focused at the position of the anomaly.

## Optimal Experimental Design in Full Waveform Inversion

### Introduction

Optimal Experimental Design (OED) is a field related to finding the best experimental design for a specific purpose. In the context of geophysics it has been used to design seismic stations for the localization of earthquakes (Rabinowitz and Steinberg, 1990) and to design acquisition geometries for seismic imaging (Curtis, 2004), notably Full Waveform Inversion (FWI) (Krampe et al., 2021).

Most often in OED techniques used for FWI, the optimality criterion taken into account for the choice of a good acquisition design is based on the eigenvalue spectrum of the approximate Hessian (Maurer et al., 2009). In this work we propose to use an alternative optimality criterion based on the wavenumber content of the gradient.

The FWI gradient can be computed as the zero-lag crosscorrelation between an adjoint wavefield and an incident wavefield. Under the planewave approximation we can estimate the wavenumber spectrum of the gradient. For an evenly spaced acquisition geometry we remark that those wavenumber points are not optimally distributed, as illustrated by the example in Figure 2. We are interested in finding positions of sources and receivers for which these points are regularly and densely distributed in the wavenumber space.

### Method

Let us consider a 2D seismic acquisition. We suppose that each source is fired with all the receivers so an acquisition of  $N_s$  sources and  $N_r$  receivers provides  $N = N_s \times N_r$  wavenumbers. For a couple source/receiver ( $\mathbf{s}, \mathbf{r}$ ), the corresponding wavenumber of the FWI gradient at spatial position  $\mathbf{x}$  is a planewave approximate on (Wu and Toksöz, 1987)

$$k(\mathbf{s}, \mathbf{r}) = k_0 (p_s + p_r) \quad (1)$$

where  $k_0 = \frac{\omega}{c_0}$  is the wavenumber of the incident and scattered waves in the homogeneous medium, with  $\omega$  the angular frequency of the planewaves and  $c_0$  the velocity of the medium supposed constant,  $p_s$  (resp.  $p_r$ ) are the unit-length directions of the rays connecting the sources (resp. receivers) to  $\mathbf{x}$ . Here, under the assumption of homogeneity of the medium, the rays are straight lines. In the nonhomogeneous case we would need to solve an eikonal equation to recover the rays and the corresponding directional vectors.

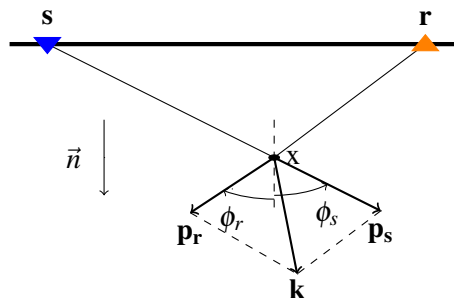
We can also express the wavenumber as a function of the angles  $\phi_s$  and  $\phi_r$  that the directional vectors make with the vertical axis  $\vec{n}$  (Figure 1) as follows

$$k_\phi(\phi_s, \phi_r) = 2k_0 \cos\left(\frac{\phi_s - \phi_r}{2}\right) \begin{bmatrix} \sin\left(\frac{\phi_s + \phi_r}{2}\right) \\ -\cos\left(\frac{\phi_s + \phi_r}{2}\right) \end{bmatrix}. \quad (2)$$

For an acquisition with a single source the wavenumbers associated to it form a circular arc. Indeed, if for example we fix a single source  $\mathbf{s}$  then we would fix the value of  $\phi_s$  and let the  $\phi_r$ 's vary. We find that we can rewrite formula (2) as follows

$$k_\phi(\phi_s, \phi) = k_0 \begin{bmatrix} \sin \phi_s \\ -\cos \phi_s \end{bmatrix} + k_0 \begin{bmatrix} \cos(\phi - \frac{\pi}{2}) \\ \sin(\phi - \frac{\pi}{2}) \end{bmatrix}. \quad (3)$$

The set of points  $Arc_{\phi_s} = \{k_\phi(\phi_s, \phi) \mid \phi \in [-\phi_{\max}, \phi_{\max}]\}$  obtained from the derived formula (3) when fixing the value of  $\phi_s$  defines a circular arc of center  $k_0(\sin \phi_s, -\cos \phi_s)$ , radius  $k_0$  and length  $2\phi_{\max}k_0$ .



**Figure 1** A source  $\mathbf{s}$  and a receiver  $\mathbf{r}$  are positioned at the surface of a homogeneous medium. We denote by  $p_s$  and  $p_r$  the unit-length directions of the rays connecting respectively  $\mathbf{s}$  and  $\mathbf{r}$  to the diffraction point  $\mathbf{x}$  along with the angles  $\phi_s$  and  $\phi_r$  that the rays make with the vertical axis.

Now suppose we know the type of geometry that the acquisition will take (on the surface, circular, in transmission etc.) and the allowed range on which we can position our devices. Then using the arc circles we would be able to draw the envelope that delimits the domain  $\mathcal{W}$  inside which the wavenumber points are generated. If we further fix the number of sources and receivers and the temporal frequency in advance then the choice of their positions is the only parameter available to affect the distribution of the points in the wavenumber cloud. Our aim is thus to find the optimal positions of sources and receivers to get the most regular distribution of wavenumbers possible. A geometrical tool that would help us achieve this inside the envelope is the Centroidal Voronoi Tessellation (CVT).

A Voronoi Tessellation of a bounded domain  $\Omega \subset \mathbb{R}^2$  generated by points  $X = \{x_i\}_{i=1,\dots,n} \subset \Omega$ , which we will suppose distinct and we will call seeds, is a partition  $\{\Omega_i, i = 1, \dots, n\}$  of  $\Omega$  where each subset is associated to a seed  $x_i$ . A subset  $\Omega_i$  is the set of points that are closer to  $x_i$  than any other seed. Those subsets are called Voronoi cells. A CVT is a particular instance of Voronoi Tessellation where the seeds coincide with the centroids of their respective Voronoi cells. Examples of Voronoi Tessellations can be seen in Figure 2, plots (b)-(d), with (c) illustrating a CVT of a disk.

Given a domain  $\Omega$  and an integer  $N$  a natural question is to find the seeds  $X$  which give us a CVT of  $\Omega$ . In addition to its geometric definition, this CVT problem can be characterized analytically by the energy function (Liu et al., 2009)

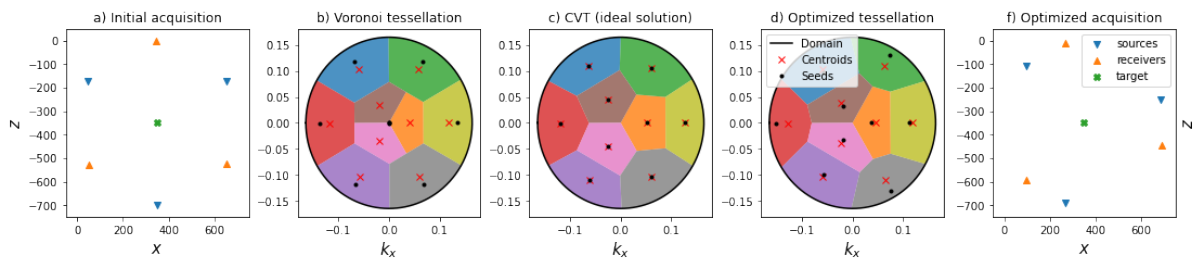
$$F(X) = \sum_{i=1}^N \int_{y \in \Omega_i} \|y - x_i\|^2 dy. \quad (4)$$

Thus the search for a CVT can be expressed as a minimization problem of the energy function (4).

Our approach takes advantage of this analytical characterization. We define a function  $G : (S, R) \mapsto \{k(s, r) \mid s \in S, r \in R\}$  which for sets  $S$  of sources and  $R$  of receivers computes the wavenumbers through formula (1). To assess the quality of an acquisition we apply the energy function  $F$  in (4) to  $G(S, R)$ . This will tell us how close to a CVT the distribution of the corresponding wavenumbers is inside the envelope  $\mathcal{W}$ . With this we have got a new energy function  $H : (S, R) \mapsto F \circ G(S, R)$  to express the quality of an acquisition with regards to our criterion of regularity of the wavenumber sampling and our problem is expressed as

$$\min_{(S,R)} H(S, R). \quad (5)$$

Knowing the analytical expression of the gradient of  $F$  (Liu et al., 2009) and deriving the one for  $G$  we can use Newton-based methods to solve the newly defined minimization problem.



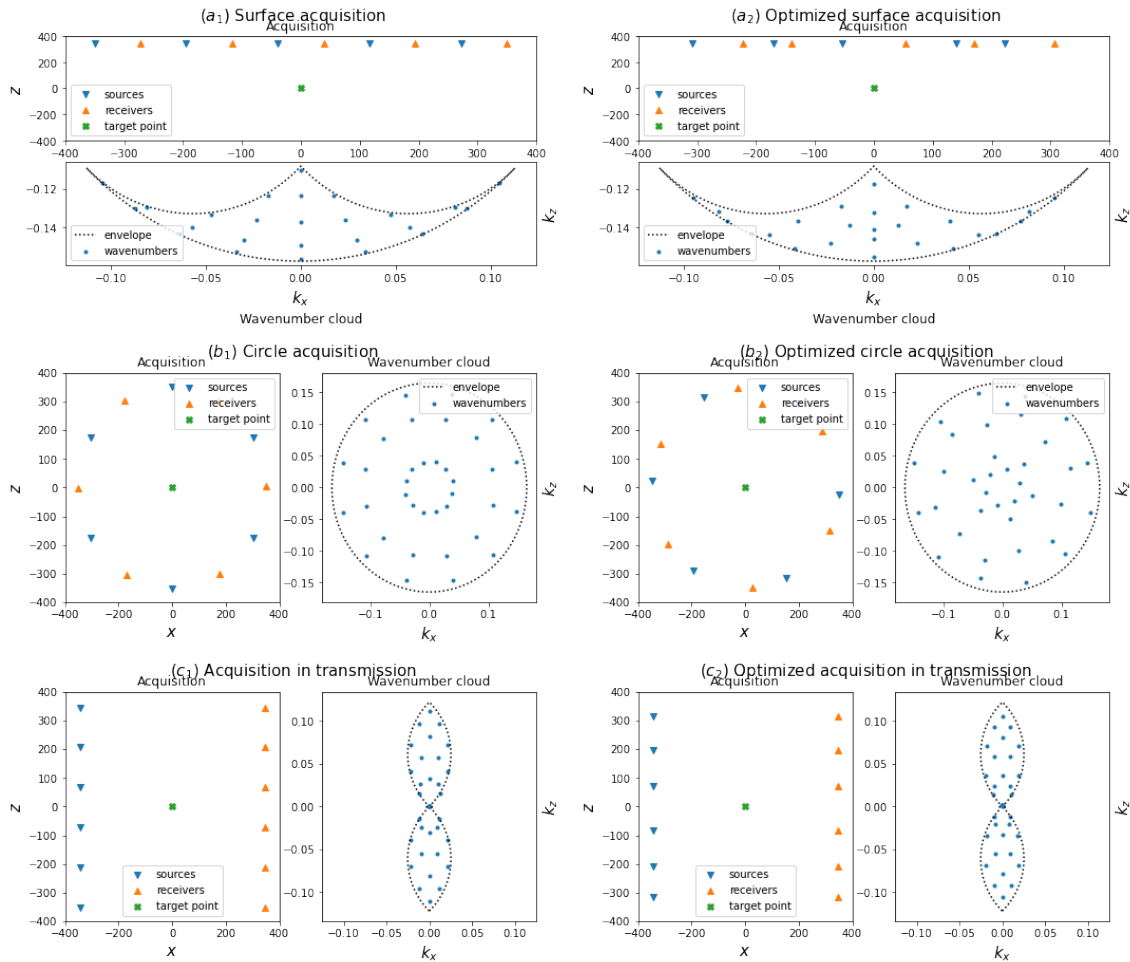
**Figure 2** An example for our proposed approach. Here we consider a simple acquisition in circle as shown in (a). We can compute the wavenumbers using formula (1) and generate their Voronoi tessellation inside their envelope  $\mathcal{W}$ , as in (b). Compare this tessellation with the one in (c) which corresponds to a CVT. Notice that here the wavenumbers are regularly distributed inside the envelope. However this represents an ideal solution because it is unachievable due to the interdependence of the wavenumbers through their source/receiver formulation. Yet, solving (5) still yields an acquisition (f) that provides a wavenumber cloud (d) the Voronoi Tessellation of which is significantly closer to a CVT.

### Numerical examples

Let us consider a 2D medium of size  $700\text{m} \times 700\text{m}$  and a diffraction point  $\mathbf{x}$  in the center. We want to optimally position  $N_s$  sources and  $N_r$  receivers with regards to our proposed criterion. We will test our optimization procedure on three types of acquisition : a) on the surface above the anomaly b) in circle around the anomaly and c) in transmission (cross-hole acquisition).

Figure 3 shows the initial evenly spaced acquisitions and the optimized acquisitions resulting from our method. Each acquisition is plotted alongside its corresponding wavenumber cloud. For sources and receivers on the surface the difference between the geometry of the initial acquisition ( $a_1$ ) and the

optimized one ( $a_2$ ) results in the improvement of the regularity of the wavenumber sampling, with points filling up more space in the center of the envelope in ( $a_2$ ) compared to the initial cloud. The improvement is more noticeable for an acquisition in circle with the wavenumbers forming a hexagon in the center. For acquisitions in transmission the result of the optimization is less perceptible and is mainly the spacing between between the devices rather than a drastic change in geometry.



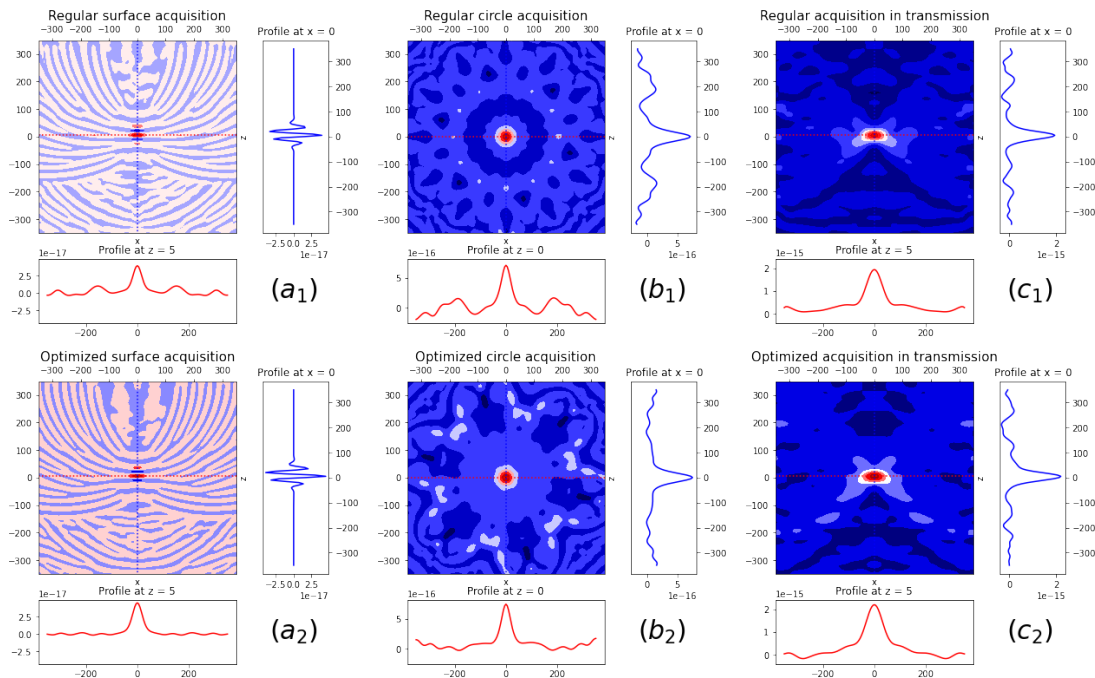
**Figure 3** Acquisitions of different types alongside their corresponding wavenumber clouds. In the left column are initial acquisitions where the sources and receivers are evenly spaced, whereas in the right column are the acquisitions optimized by our method.

To complement these results, we study the effect of the acquisition geometry on the shape of the FWI gradient in the context of the reconstruction of a scattering point in a homogeneous medium. We consider an initial medium which is homogeneous of constant velocity 2000m/s and a true model which is the same except for a Gaussian anomaly of peak velocity 3000m/s in the center and which we would like to recover. Using TOYxDAC\_TIME, a finite-difference based full waveform modeling and inversion code developed in the SEISCOPE project (Yang et al., 2018) we can construct the first FWI gradient associated with each acquisition. The gradients are shown in Figure 4 with profiles on the  $x$  and  $z$ -axes. The first thing that we notice is the difference in smoothness of the curves between initial and optimized acquisitions. For the acquisition in circle the area around the center is flat in the optimized case ( $b_2$ ) compared with the initial one ( $b_1$ ) where there exists a valley right next to it. The most striking result might be the surface acquisition where we can see a flatter curve on the  $z$ -profile in ( $a_2$ ) compared to ( $a_1$ ), with the gradient of the initial acquisition having several oscillations along the curve. All things considered, the optimized acquisition geometries appear to provide gradients that are more focused on the anomaly compared to non-optimized acquisitions.

### Conclusion and perspective

In our work we have proposed a new approach to design better acquisition geometries in the context of FWI. It consists in selecting optimal locations for sources and receivers that would improve the wavenumber content of the FWI gradient. This strategy takes advantage of new results in the characteri-

zation of the CVT and apply them to get a formulation of the regularity of the wavenumber cloud which can be used to solve our optimization problem using classical descent methods. It provides optimized acquisition design that fulfill the requirement of a more even distribution of the wavenumbers and the preliminary results in the construction of more focused gradients are promising. A next step of this work is to extend and apply it to realistic models to explore its potential. Future improvements of the method include taking into account features of the terrain, costs associated with the choice of some locations and target-oriented FWI.



**Figure 4** The gradients for each acquisition in Figure 3 with profiles on the  $x$  and  $z$ -axis. The first row are gradients associated to initial acquisitions and on the second are gradients associated to optimized acquisitions.

### Acknowledgements

This study was funded by the SEISCOPE consortium (<http://seiscope2.osug.fr>), sponsored by AKERBP, CGG, EXXON-MOBIL, GEOLINKS, JGI, PETROBRAS, SHELL, SINOPEC and TOTAL ENERGIES. This study was granted access to the HPC resources provided by the GRICAD infrastructure (<https://gricad.univ-grenoble-alpes.fr>).

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