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Abstract

We study an interconnection between a switched affine system and a fast LTI dynamics. Often, the classical stabilization method based on the existence of a stable convex combination is directly used for systems on two time scales while using only the slow dynamics. Here, we provide conditions for checking the Uniform Ultimate Boundedness when the fast dynamics is taken into account. In addition, criteria for verifying asymptotical stability are provided for a special case. Numerical examples illustrate the main results.

Key words: switched affine system, multi time-scale, Lyapunov method

1 Introduction

Switched affine systems, consisting of a family of continuous-time dynamics and a switching law managing the switch among them, are a class of hybrid systems [1]. This class of systems is interesting since it has applications in various domains of physics and engineering. For example, in DC-DC power converters, which are widely used in electronic devices, the switched circuit indicates different modes of operation [2, 3, 4]. However, the stabilization of such switched affine systems is challenging since one has to handle some complex phenomena such as non standard equilibrium points, fast switching, zeno behaviour, etc.. Different approaches have been considered to deal with such problems. Control design methods have been proposed based on the existence of Hurwitz convex combinations [5, 2], generalized relays [6, 7], adaptive control methods [8] and hybrid approaches [9]- see also the pioneering work in [10, 11].

In large number of switching electronic devices [12], the analysis and design of the system is complex since the system evolve on multiple time scales. We may encounter simultaneously fast dynamics (for example current in inductances) and slow system (for example voltage in capacitors), interactions with mechanical parts. Taking into account switching controllers in such problem settings is an open challenging problem. Singular perturbation methods [13, 14] which are widely used for two time-scale continuous-time systems are not applicable to the case under study. In this work we focus on switched affine systems with *state-dependent switching* laws, which result in complex phenomena such as sliding modes and non unique sliding solutions. To the best of our knowledge, the case (multiple time scales dynamics with *state-dependent switching* laws) considered in this work has not been studied in the literature. However, some results are available for a related problem of two time-scale switched systems with *time-dependent switching* laws [15, 16, 17, 18, 19].

The main contribution of this work is to provide a new method for the stability analysis of an interconnection between switched affine system and fast LTI dynamics. Often the classical stabilization method based on the existence of a stable convex combination [5] is used while ignoring the fast dynamics. Here, we propose simple conditions for checking the stability (in the sense of Uniform Ultimate Boundedness) when the fast dynamics is taken into account. A preliminary version of this work has been presented in [20]. Compared to the preliminary version, in the present work we provide an estimation of the upper bound of the perturbation parameter ε^* such that the stability property holds for all $\varepsilon \in (0, \varepsilon^*)$. In addition, we prove the asymptotical stability for a special case. From a theoretical point of view, the results in this work can also be related with the studies concerning the Uniform Ultimate Boundedness of switched affine systems with dwell-time switching [9, 21, 22].

The paper is organized as follows. Section 2 presents

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motivations and the system under consideration. Section 3 states the stability properties of such system under a state-dependent switching law. In Section 4, the main results are illustrated by numerical examples. Finally, concluding remarks end the paper.

Notation. Given a matrix K, K^{-1} and $K^\top$ represent the inverse and the transpose matrix of K respectively. For a symmetric matrix, $\star$ denotes the symmetric part. $K \succeq 0$ ($K \preceq 0$) means that the matrix K is positive (negative) semi-definite. $K \succ 0$ ($K \prec 0$) means that the matrix K is positive (negative) definite. $\|\cdot\|$ denotes the usual Euclidean norm in $\mathbb{R}^n$. For a positive integer N, $\mathcal{I}_N$ denotes the set $\{1,\dots,N\}$. The convex closure of the set S is denoted by $\text{Conv}\{S\}$. The simplex is presented by

$$\Delta_N=\left\{v=\begin{pmatrix} v_1 & \cdots & v_N \end{pmatrix}^\top \in \mathbb{R}^N : \sum_{i=1}^{N} v_i = 1, v_i \geqslant 0, \forall i \in \mathcal{I}_N\right\}.$$

For a set of scalars γ_i with $i \in \mathcal{I}_N$, we denote

$$\arg\min_{i\in\mathcal{I}_N} \gamma_i = \{i \in \mathcal{I}_N : \gamma_i \leq \gamma_j, \forall j \in \mathcal{I}_N\}.$$

2 Problem statement

2.1 Motivation

Consider the following system

$$\begin{cases} \dot{x}_s = A^{11}_{\sigma(x_s)} x_s + A^{12}_{\sigma(x_s)} x_f + b_{\sigma(x_s)}, & \text{(1a)} \\ \varepsilon \dot{x}_f = A^{21} x_s + A^{22} x_f, & \text{(1b)} \end{cases}$$

where $x_s \in \mathbb{R}^n$, $x_f \in \mathbb{R}^m$ are the state variables and the switching law is $\sigma : \mathbb{R}^n \to \mathcal{I}_N$, with $\mathcal{I}_N = \{1,\dots,N\}$. ε is a fixed system's parameter which is positive and sufficiently small, i.e., $0 < \varepsilon \ll 1$. The affine term is $b_i \in \mathbb{R}^n$, and the matrices $A^{11}_i \in \mathbb{R}^{n\times n}$, $A^{12}_i \in \mathbb{R}^{n\times m}$ for $i \in \{1,\dots,N\}$, $A^{21} \in \mathbb{R}^{m\times n}$, $A^{22} \in \mathbb{R}^{m\times m}$.

Since $0 < \varepsilon \ll 1$, the dynamics of system (1) evolve according to two time scales. We adopt the standard singular perturbation approach [13] to decompose (1) into the slow and fast subsystems. Assuming A^{22} invertible and setting $\varepsilon = 0$ in (1b), we have

$$x_f = -{A^{22}}^{-1} A^{21} x_s. \tag{2}$$

Replacing x_f in (1a) by the right-hand side of (2), we get the slow switched affine subsystem

$$\dot{x}_s = A^s_{\sigma(x_s)} x_s + b_{\sigma(x_s)}, \tag{3}$$

where

$$A^s_{\sigma(x_s)} = A^{11}_{\sigma(x_s)} - A^{12}_{\sigma(x_s)} {A^{22}}^{-1} A^{21}. \tag{4}$$

In the following, we compute the fast subsystem. Performing the following change of variable

$$y = x_f + {A^{22}}^{-1} A^{21} x_s,$$

we can write

$$\varepsilon \dot{y} = A^{22} y + \varepsilon(G^1_{\sigma(x_s)} x_s + G^2_{\sigma(x_s)} y + {A^{22}}^{-1} A^{21} b_{\sigma(x_s)}),$$

where $G^1_{\sigma(x_s)} = {A^{22}}^{-1} A^{21} A^s_{\sigma(x_s)}$, $G^2_{\sigma(x_s)} = {A^{22}}^{-1} A^{21} A^{12}_{\sigma(x_s)}$. Using a new time scale $\tau = \frac{t}{\varepsilon}$ and $\varepsilon \to 0$, we get the fast LTI subsystem

$$\frac{dy}{d\tau} = A^{22} y. \tag{5}$$

We first consider the following assumption.

Assumption 1 *Assume that there exists* $\alpha = \begin{pmatrix} \alpha_1 \cdots \alpha_N \end{pmatrix}^\top \in \Delta_N$ *such that the convex combinations* $b(\alpha) = \sum_{i=1}^{N} \alpha_i b_i = 0$, *and* $A^s(\alpha) = \sum_{i=1}^{N} \alpha_i A^s_i$ *is Hurwitz, with* A^s_i *defined in (4) for* $i \in \{1,\dots,N\}$, *that is*

$$A^s(\alpha)^\top P + P A^s(\alpha) \preceq -c_s P, \tag{6}$$

with a matrix $P = P^\top \succ 0$, *and a scalar* $c_s > 0$. □

Remark 1 Assumption 1 is widely used in the literature [9]. Let us remark that condition $b(\alpha) = \sum_{i=1}^{N} \alpha_i b_i = 0$ is a necessary condition [6] for the existence of a switching law $\sigma(x_s)$ such that 0 is an equilibrium of the differential inclusion associated with system (3). Note that α satisfying condition (6) should be the same as the one which satisfies $b(\alpha) = 0$. Moreover, it is worth to emphasize that condition (6) does not require the stability of each mode of the slow subsystem (3). ∘

Based on Assumption 1, we consider the following state-dependent switching law of the following form

$$\sigma(x_s) \in \arg\min_{i\in\mathcal{I}_N} x_s^\top P(A^s_i x_s + b_i), \tag{7}$$

with P defined in (6). Note that it is always possible to transform inclusion (7) into an identity by choosing either the smallest or the largest index.

Remark 2 The state-dependent switching law (7) is obtained by using the classical method based on the existence of a stable convex combination [5] when ignoring the fast dynamics. This switching law is widely used in the literature and we believe that it is important to study it when slow and fast dynamics occur simultaneously. ∘

Remark 3 In the literature, for switched affine systems (with only one time scale) there exist other types of switching laws, and other, less restrictive, design conditions. For instance, the method in [23] is able to handle systems where the convex combination is rank-deficient. When the study is restricted to local stability, condition (6) can be replaced by a less restrictive one such that the Hurwitz convex combination is not necessary [6]. However, for the sake of simplicity in the case of systems with two time scales, we focus on this classical assumption (Assumption 1) and on the switching law (7). ∘

Remark 4 It is well known that the switching law (7) may lead to chattering and sliding dynamics. In some practical cases, a sampled-data implementation of switching laws is preferable. This guarantees a minimum dwell time between two switching instants. From a theoretical point of view, it was shown that the sampled-data implementation of the switching law results in practical stability. Moreover, it has been shown that the domain of chattering is of the order of the sampling interval [22, 9]. One could imagine an extension of the problem setting we propose here, where a sampled-data implementation of the switching law (7) is considered. However, in this work we prefer to focus on a basic problem setting which highlights the effects of interconnected switched systems on two time scales. ∘

We next state a second assumption.

Assumption 2 *A^{22} is Hurwitz, that is*

$$A^{22\top}Q + QA^{22} \preceq -c_f Q, \tag{8}$$

with a matrix $Q = Q^\top \succ 0$, and a scalar $c_f > 0$. □

Note that condition (8) is a standard stability condition for the fast subsystem.

Considering a sufficiently small positive scalar ε^*, for two time-scale linear continuous-time systems, the stability of slow and fast subsystems implies the stability of the whole system for any $\varepsilon \in (0, \varepsilon^*]$ [13]. This property does not hold for two time-scale switched systems with time-dependent switching laws. Additional constraints are needed to ensure the stability [24, 17, 18].

For switched systems with state-dependent switching laws, the analysis is more complex. We need to handle phenomena such as non unique equilibria and sliding modes. Under the state-dependent switching law (7), the interconnection of switched affine system (1a) and fast LTI dynamics (1b) can be unstable for small value of ε, even if both slow and fast subsystems are stable. We use the following numerical example to illustrate this fact.

Example 2.1. Let us consider system (1) with $N = 2$ modes, and matrices given as

$$A_1^{11} = \begin{pmatrix} 2 & 2 \\ -3 & 1 \end{pmatrix}, \; A_2^{11} = \begin{pmatrix} -3 & -3 \\ 2 & -1 \end{pmatrix},$$

$$A_1^{12} = \begin{pmatrix} 2 & -3 \\ -1 & -2 \end{pmatrix}, \; A_2^{12} = \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, b_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$A^{21} = \begin{pmatrix} -0.5 & 0.3 \\ -0.5 & 0.4 \end{pmatrix}, \; A^{22} = \begin{pmatrix} -0.03 & 0.02 \\ -0.02 & 0.01 \end{pmatrix},$$

and $\varepsilon = 1 \times 10^{-3}$. With $\alpha = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}^\top$, we get $A^s(\alpha) = \sum_{i=1}^{N} \alpha_i A_i^s = \begin{pmatrix} -0.5 & -25.5 \\ 74.5 & -75 \end{pmatrix}$ is Hurwitz, such that Assumption 1 is satisfied with $P = \begin{pmatrix} 0.86 & -0.07 \\ \star & 0.16 \end{pmatrix}$. Thus, the slow subsystem (3) under the switching law (7) is stable. Moreover, A^{22} is also Hurwitz, that is Assumption 2 is verified with $Q = \begin{pmatrix} 17.11 & -12.84 \\ \star & 15.20 \end{pmatrix}$. Therefore, the fast subsystem (5) is also stable. However, the interconnected system (1), (7) diverges. Figure. 1 illustrates the time evolution of the system with the initial condition $x_s(0) = (0\ 1)^\top$, $x_f(0) = (0\ 1)^\top$.

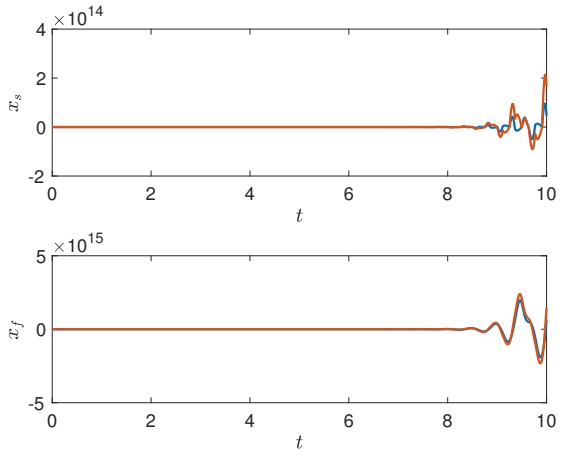

Fig. 1. Time evolution of system (1) with (7) (Example 2.1)

In this work, we aim to provide conditions for checking the stability of switched affine system (1a) interconnected with fast LTI system (1b) under the state-dependent switching law (7). Under Assumptions 1-2 (ensuring the stability of both fast and slow subsystems), an additional constraint is proposed to guarantee the stability of the closed-loop system (1) and (7).

2.2 Main problem

In the following, we rewrite the interconnected system (1) in the coordinate (x_s, y) as

$$\begin{pmatrix}\dot{x}_s\\ \dot{y}\end{pmatrix} = H(x_s,y) = \begin{cases}h_1(x_s,y,\sigma(x_s))\\ h_{2\varepsilon}(x_s,y,\sigma(x_s))\end{cases}$$
$$:= \begin{cases}A^s_{\sigma(x_s)}x_s + A^{12}_{\sigma(x_s)}y + b_{\sigma(x_s)},\\ \frac{1}{\varepsilon}A^{22}y + \Big(G^1_{\sigma(x_s)}x_s + G^2_{\sigma(x_s)}y + {A^{22}}^{-1}A^{21}b_{\sigma(x_s)}\Big).\end{cases} \tag{9}$$

The objective of this work is to study the stability properties of the closed-loop system (9) and (7). The stability notion adopted in this work is given as follows.

Definition 1 *[UUB] Let $\mathcal{S}$ be a neighborhood of the origin. The closed-loop system (9) and (7) is said to be Uniformly Ultimately Bounded in $\mathcal{S}$ if for all $a > 0$ there exists $T(a) > 0$ such that, for any $\left\| \begin{matrix} x_s(0) \\ y(0) \end{matrix} \right\| \leq a$, the solutions $\begin{pmatrix} x_s(t) & y(t) \end{pmatrix}^\top \in \mathcal{S}$, for all $t \geq T(a)$. The set $\mathcal{S}$ is called Ultimately Bounded set.*

Definition 2 *The closed-loop system (9) and (7) is said to be asymptotically stable at the origin if for each $\mu > 0$ there exists $\delta > 0$ such that for each $\left\| \begin{matrix} x_s(0) \\ y(0) \end{matrix} \right\| < \delta$, the solutions $\left\| \begin{matrix} x_s(t) \\ y(t) \end{matrix} \right\| < \mu, \forall t \geq 0$, and $\lim_{t\to+\infty} \left\| \begin{matrix} x_s(t) \\ y(t) \end{matrix} \right\| = 0$.*

Note that in the above definitions, solutions can be considered in the sense of Krasovskii. As any Filippov solution is also a Krasovskii solution, the result could also be applied when the Filippov regularization procedure is used [25, 26, 27].

3 Stability analysis

In this section, we provide conditions for checking the stability properties of system (9) with the switching law (7). The main results are stated as follows.

Theorem 1 *Consider symmetric positive definite matrices P, Q satisfying Assumptions 1-2 for some positive scalars c_s, c_f. For given $c > 0$ and $0 < \varepsilon \ll 1$, if there exists $\theta > 0$, such that*

$$\begin{pmatrix} -c_sP + \theta P & 0 & \sqrt{\varepsilon}(PA_i^{12} + {G_i^1}^\top Q) \\ \star & -\theta cI & \sqrt{\varepsilon}(QA^{22^{-1}}A^{21}b_i)^\top \\ \star & \star & -c_fQ + \varepsilon({G_i^2}^\top Q + QG_i^2) + \theta Q \end{pmatrix} \prec 0, \tag{10}$$

for all $i \in \mathcal{I}_N$, then, system (9) with the state-dependent switching law (7) is Uniformly Ultimately Bounded in

$$\mathcal{D}(c) = \left\{ \begin{pmatrix} x_s \\ y \end{pmatrix} \in \mathbb{R}^{n+m} \ : \begin{pmatrix} x_s \\ y \end{pmatrix}^\top \begin{pmatrix} P & 0 \\ 0 & Q \end{pmatrix} \begin{pmatrix} x_s \\ y \end{pmatrix} \leq c \right\}. \tag{11}$$

Proof. The proof is based on the Lyapunov analysis.
Step 1 First, for each $x_s \in \mathbb{R}^n$, we define the subset of index $\mathcal{I}^*(x_s) \subseteq \mathcal{I}_N$ that minimize the term $x_s^\top P(A_i^s x_s + b_i)$,

$$\mathcal{I}^*(x_s) = \arg\min_{i\in\mathcal{I}_N} x_s^\top P(A_i^s x_s + b_i). \tag{12}$$

Note that (12) corresponds to the set in which the switching law (7) takes values. Since the closed-loop system (9) and (7) is a discontinuous ODE system, we study the following enlarged differential inclusion

$$\begin{pmatrix}\dot{x}_s\\ \dot{y}\end{pmatrix} \in \mathcal{F}^*(x_s,y) := \mathrm{Conv}\Big\{ f_i(x_s,y), i \in \mathcal{I}^*(x_s)\Big\}, \tag{13}$$

with

$$f_i(x_s,y) = \begin{pmatrix} A_i^s x_s + A_i^{12}y + b_i \\ \frac{A^{22}}{\varepsilon}y + (G_i^1 x_s + G_i^2 y + {A^{22}}^{-1}A^{21}b_i) \end{pmatrix}. \tag{14}$$

This allows to show UUB when the Krasovskii or Filippov regularization procedure is used [25, 26, 27].

Step 2 The objective is to show that the following candidate Lyapunov function (15) decreases along the solutions of (13) whenever it is greater than c.

$$V(x_s,y) = x_s^\top P x_s + y^\top Q y, \tag{15}$$

with P and Q satisfying (6) and (8) respectively.

Let us consider vectors $(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathbb{R}^n \times \mathbb{R}^m$. We need to prove that

$$\sup_{(\bar{h}_1,\bar{h}_{2\varepsilon})\in\mathcal{F}^*(x_s,y)} \left\{ \frac{\partial V}{\partial x_s}\bar{h}_1 + \frac{\partial V}{\partial y}\bar{h}_{2\varepsilon} \right\} < 0,$$

whenever $V(x_s, y) \geqslant c$.

We first define the set $\Delta^*_{N(x_s)}$ such that

$$\Delta^*_{N(x_s)} = \left\{ \beta \in \Delta_N : \beta_i = 0, i \notin \mathcal{I}^*(x_s) \right\}. \quad (16)$$

Then $\mathcal{F}^*(x_s, y)$ in (13) becomes

$$\mathcal{F}^*(x_s, y) = \left\{ \sum_{i=1}^{N} \beta_i f_i(x_s, y), i \in \mathcal{I}^*(x_s) : \beta_i \in \Delta^*_{N(x_s)} \right\}. \quad (17)$$

In view of (14), with using

$$\begin{pmatrix} \bar{h}_1 \\ \bar{h}_{2\varepsilon} \end{pmatrix} \in \begin{pmatrix} A^s_i x_s + A^{12}_i y + b_i \\ \frac{A^{22}}{\varepsilon} y + (G^1_i x_s + G^2_i y + A^{22^{-1}} A^{21} b_i) \end{pmatrix}, \quad (18)$$

it holds that

$$\begin{aligned} &\sup_{(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathcal{F}^*(x_s, y)} \left\{ \frac{\partial V}{\partial x_s} \bar{h}_1 + \frac{\partial V}{\partial y} \bar{h}_{2\varepsilon} \right\} \\ &= \sup \left\{ \text{Conv} \left\{ \Omega^1_i + \Omega^2_i, i \in \mathcal{I}^*(x_s) \right\} \right\}, \end{aligned} \quad (19)$$

where $\Omega^1_i = \frac{\partial V}{\partial x_s} \times (A^s_i x_s + A^{12}_i y + b_i)$ and $\Omega^2_i = \frac{\partial V}{\partial y} \times \left(\frac{A^{22}}{\varepsilon} y + (G^1_i x_s + G^2_i y + A^{22^{-1}} A^{21} b_i) \right)$.

Due to the compactness of the set $\Delta^*_{N(x_s)}$ defined by (16), the following holds for $i \in \mathcal{I}^*(x_s)$

$$\begin{aligned} &\sup \left\{ \text{Conv} \left\{ \Omega^1_i + \Omega^2_i, i \in \mathcal{I}^*(x_s) \right\} \right\} \\ &= \sup_{\beta \in \Delta^*_{N(x_s)}} \left\{ \sum_{i=1}^{N} \beta_i (\Omega^1_i + \Omega^2_i) \right\} \\ &\leq \max_{\beta \in \Delta^*_{N(x_s)}} \left\{ \sum_{i=1}^{N} \beta_i (\Omega^1_i + \Omega^2_i) \right\}. \end{aligned} \quad (20)$$

In the following, let us overbound Ω^1_i, Ω^2_i. For all $i \in \mathcal{I}^*(x_s)$,

$$\begin{aligned} \Omega^1_i &= \frac{\partial V}{\partial x_s} \times (A^s_i x_s + A^{12}_i y + b_i) \\ &= x_s^\top (A^{s\top}_i P + P A^s_i) x_s + 2 x_s^\top P A^{12}_i y + 2 x_s^\top P b_i. \end{aligned} \quad (21)$$

From (12), since $i \in \mathcal{I}^*(x_s)$, we have

$$\begin{aligned} &x_s^\top (A^{s\top}_i P + P A^s_i) x_s + 2 x_s^\top P b_i \\ &\leq x_s^\top (A^{s\top}_j P + P A^s_j) x_s + 2 x_s^\top P b_j, \forall j \in \mathcal{I}_N. \end{aligned} \quad (22)$$

Let $\alpha \in \Delta_N$ such that Assumption 1 is verified. We deduce from equation (22)

$$\begin{aligned} &\sum_{j=1}^{N} \alpha_j (x_s^\top (A^{s\top}_i P + P A^s_i) x_s + 2 x_s^\top P b_i) \\ &\leq \sum_{j=1}^{N} \alpha_j (x_s^\top (A^{s\top}_j P + P A^s_j) x_s + 2 x_s^\top P b_j). \end{aligned}$$

Then using $\sum_{j=1}^{N} \alpha_j = 1$, we have

$$\begin{aligned} &x_s^\top (A^{s\top}_i P + P A^s_i) x_s + 2 x_s^\top P b_i \\ &\leq x_s^\top (A^s(\alpha)^\top P + P A(\alpha)) x_s + 2 x_s^\top P b(\alpha). \end{aligned}$$

Using Assumption 1, we get, for $i \in \mathcal{I}^*(x_s)$

$$\begin{aligned} \Omega^1_i &= \frac{\partial V}{\partial x_s} \times (A^s_i x_s + A^{12}_i y + b_i) \\ &= x_s^\top (A^{s\top}_i P + P A^s_i) x_s + 2 x_s^\top P A^{12}_i y + 2 x_s^\top P b_i \\ &\leq x_s^\top (-c_s P) x_s + 2 x_s^\top P A^{12}_i y. \end{aligned} \quad (23)$$

Similarly, we overbound Ω^2_i as follows, for $i \in \mathcal{I}^*(x_s)$

$$\begin{aligned} \Omega^2_i &= \frac{\partial V}{\partial y} \times \left(\frac{A^{22}}{\varepsilon} y + (G^1_i x_s + G^2_i y + A^{22^{-1}} A^{21} b_i) \right) \\ &= \frac{1}{\varepsilon} y^\top \left(A^{22\top} Q + Q A^{22} + \varepsilon (G^{2\top}_i Q + Q G^2_i) \right) y \\ &\quad + 2 x_s^\top G^{1\top}_i Q y + 2 y^\top Q A^{22^{-1}} A^{21} b_i. \end{aligned} \quad (24)$$

Under Assumption 2, we deduce from inequality (24)

$$\begin{aligned} \Omega^2_i &= \frac{\partial V}{\partial y} \times \left(\frac{A^{22}}{\varepsilon} y + (G^1_i x_s + G^2_i y + A^{22^{-1}} A^{21} b_i) \right) \\ &\leq \frac{1}{\varepsilon} y^\top \left(-c_f Q + \varepsilon (G^{2\top}_i Q + Q G^2_i) \right) y \\ &\quad + 2 x_s^\top G^{1\top}_i Q y + 2 y^\top Q A^{22^{-1}} A^{21} b_i. \end{aligned} \quad (25)$$

In view of (23) and (25), we have, for $i \in \mathcal{I}^*(x_s)$

$$\begin{aligned}
&\Omega_i^1 + \Omega_i^2 \\
&\leq x_s^\top(-c_sP)x_s + \frac{1}{\varepsilon}y^\top\left(-c_fQ + \varepsilon(G_i^{2\top}Q + QG_i^2)\right)y \\
&+2x_s^\top\left(PA_i^{12} + G_i^{1\top}Q\right)y + 2y^\top QA^{22^{-1}}A^{21}b_i \\
&= \begin{pmatrix} x_s \\ 1 \\ y \end{pmatrix}^\top \begin{pmatrix} -c_sP & 0 & PA_i^{12} + G_i^{1\top}Q \\ \star & 0 & (QA^{22^{-1}}A^{21}b_i)^\top \\ \star & \star & \frac{-c_fQ}{\varepsilon} + G_i^{2\top}Q + QG_i^2 \end{pmatrix} \begin{pmatrix} x_s \\ 1 \\ y \end{pmatrix} \\
&=: \phi_i(x_s, y). \qquad (26)
\end{aligned}$$

Recall (19)-(20), therefore

$$\begin{aligned}
&\sup_{(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathcal{F}^*(x_s, y)} \left\{ \frac{\partial V}{\partial x_s}\bar{h}_1 + \frac{\partial V}{\partial y}\bar{h}_{2\varepsilon} \right\} \\
&\leq \max_{\beta \in \Delta^*_{N(x_s)}} \left\{ \sum_{i=1}^{N} \beta_i \phi_i(x_s, y) \right\}, \qquad (27)
\end{aligned}$$

with $\phi_i(x_s, y)$ defined in (26).

In the following, we aim at proving that under condition (10), $\sup_{(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathcal{F}^*(x_s, y)} \left\{ \frac{\partial V}{\partial x_s}\bar{h}_1 + \frac{\partial V}{\partial y}\bar{h}_{2\varepsilon} \right\} < 0$, whenever $V(x_s, y) \geqslant c$. Using convexity arguments it is sufficient to show that $\phi_i(x_s, y) < 0$, for all $i \in \mathcal{I}^*(x_s)$ whenever $V(x_s, y) \geqslant c$. By using the Schur complement, condition (10) is equivalent to

$$\begin{pmatrix} -c_sP + \theta P & 0 \\ \star & -\theta cI \end{pmatrix} \prec 0, \qquad (28)$$

and

$$\begin{aligned}
&-c_fQ + \varepsilon(G_i^{2\top}Q + QG_i^2) + \theta Q \\
&-\varepsilon \begin{pmatrix} PA_i^{12} + G_i^{1\top}Q \\ (QA^{22^{-1}}A^{21}b_i)^\top \end{pmatrix}^\top \\
&\begin{pmatrix} -c_sP + \theta P & 0 \\ \star & -\theta cI \end{pmatrix}^{-1} \begin{pmatrix} PA_i^{12} + G_i^{1\top}Q \\ (QA^{22^{-1}}A^{21}b_i)^\top \end{pmatrix} < 0. \qquad (29)
\end{aligned}$$

The above inequality (29) can be rewritten as follows

$$\begin{aligned}
&\varepsilon \left\{ \left(-\frac{c_fQ}{\varepsilon} + G_i^{2\top}Q + QG_i^2 \right) \right. \\
&- \begin{pmatrix} PA_i^{12} + G_i^{1\top}Q \\ (QA^{22^{-1}}A^{21}b_i)^\top \end{pmatrix}^\top \\
&\left. \begin{pmatrix} -c_sP + \theta P & 0 \\ \star & -\theta cI \end{pmatrix}^{-1} \begin{pmatrix} PA_i^{12} + G_i^{1\top}Q \\ (QA^{22^{-1}}A^{21}b_i)^\top \end{pmatrix} \right\} + \theta Q < 0. \qquad (30)
\end{aligned}$$

Since $0 < \varepsilon \ll 1$, $\theta > 0$, and $Q \succ 0$, inequality (30) implies

$$\begin{aligned}
&\left(-\frac{c_fQ}{\varepsilon} + G_i^{2\top}Q + QG_i^2 \right) - \begin{pmatrix} PA_i^{12} + G_i^{1\top}Q \\ (QA^{22^{-1}}A^{21}b_i)^\top \end{pmatrix}^\top \times \\
&\begin{pmatrix} -c_sP + \theta P & 0 \\ \star & -\theta cI \end{pmatrix}^{-1} \begin{pmatrix} PA_i^{12} + G_i^{1\top}Q \\ (QA^{22^{-1}}A^{21}b_i)^\top \end{pmatrix} + \theta Q < 0. \qquad (31)
\end{aligned}$$

In view of (28) and (31), using again the Schur complement, we have

$$\begin{pmatrix} -c_sP + \theta P & 0 & PA_i^{12} + G_i^{1\top}Q \\ \star & -\theta cI & (QA^{22^{-1}}A^{21}b_i)^\top \\ \star & \star & \frac{-c_fQ}{\varepsilon} + G_i^{2\top}Q + QG_i^2 + \theta Q \end{pmatrix} \prec 0. \qquad (32)$$

Multiplying both sides of (32) by the vector $\begin{pmatrix} x_s & 1 & y \end{pmatrix}$, we get

$$\phi_i(x_s, y) + \begin{pmatrix} x_s \\ 1 \\ y \end{pmatrix}^\top \begin{pmatrix} \theta P & 0 & 0 \\ 0 & -\theta cI & 0 \\ 0 & 0 & \theta Q \end{pmatrix} \begin{pmatrix} x_s \\ 1 \\ y \end{pmatrix} < 0, \qquad (33)$$

with $\phi_i(x_s, y)$ defined in (26).

In addition, let us remark that if $V(x_s, y) \geqslant c$, it holds

$$\begin{pmatrix} x_s \\ 1 \\ y \end{pmatrix}^\top \begin{pmatrix} P & 0 & 0 \\ 0 & -cI & 0 \\ 0 & 0 & Q \end{pmatrix} \begin{pmatrix} x_s \\ 1 \\ y \end{pmatrix} \geq 0. \qquad (34)$$

Since $\theta > 0$ and in view of (33) and (34), it holds $\phi_i(x_s, y) < 0$ (given in (26)), whenever $V(x_s, y) \geqslant c$, for $i \in \mathcal{I}^*(x_s)$. Thus, from (19) and (27), we have

$$\sup_{(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathcal{F}^*(x_s, y)} \left\{ \frac{\partial V}{\partial x_s}\bar{h}_1 + \frac{\partial V}{\partial y}\bar{h}_{2\varepsilon} \right\} < 0, \qquad (35)$$

whenever $V(x_s, y) \geqslant c$. Then, system (9) with the state-dependent switching law (7) is uniformly ultimately bounded. □

Remark 5 Theorem 1 provides a simple condition for checking the UUB property of the closed-loop system

(9), (7). In the following, let us summarize the tuning parameters. The matrices P and Q are the corresponding Lyapunov matrices, which are given such that Assumptions 1-2 are satisfied. For a given ε, θ is solved as an LMI parameter and a line search can be performed to minimize the ultimate bound c.
Note that in the present work, we are mainly interested in analysing the UUB for a given switching law, that is, when the matrices P, Q are given such that Assumptions 1-2 are satisfied. We emphasize that condition (10) is sufficient only for the UUB, not necessary, that is, the existence of a parameter θ satisfying condition (10) is only a sufficient condition for the UUB of the closed-loop system. ∘

Remark 6 The result of Theorem 1 can also be turned into a control design condition. In order to find a switching law (7) which guarantees that the ultimate bound set (11) is contained in a ball of desired radius ρ, we need to solve simultaneously conditions (6), (8), (10) and $\begin{pmatrix} P & 0 \\ 0 & Q \end{pmatrix} \geq \frac{c}{\rho^2} I$. In this case, without loss of generality, c can be fixed as 1, (P, Q) are considered as LMI parameters and a line search can be performed for θ. ∘

Remark 7 In the singular perturbation theory [13], it is usually interesting to study the influence of variations of the perturbation parameter ε on the stability of the system. It is a useful information when choosing system parameters (for instance, inductances, resistances in electrical systems). Let us remark that if the conditions in Theorem 1 are satisfied for systems (9) and (7) with $\varepsilon = \varepsilon^*$, then the UUB is also guaranteed for systems with the same matrices A_i^{11}, A_i^{12}, A^{21}, A^{22}, b_i and for all $\varepsilon \in (0, \varepsilon^*]$. This can be shown using simple arguments. Note that if condition (10) holds for $\varepsilon = \varepsilon^*$, then (31) also holds for the same ε^*. In addition, (31) holds for all $\varepsilon \in (0, \varepsilon^*]$, when it holds for $\varepsilon = \varepsilon^*$. Therefore, using similar arguments as the ones around equations (32)-(35) in the proof of Theorem 1, the UUB can be guaranteed for all $\varepsilon \in (0, \varepsilon^*]$. ∘

Remark 8 Let us remark that if we set $\varepsilon = 0$, condition (10) becomes

$$\begin{pmatrix} -c_s P + \theta P & 0 & 0 \\ \star & -\theta c I & 0 \\ \star & \star & -c_f Q + \theta Q \end{pmatrix} \prec 0. \qquad (36)$$

Note that for any given ultimate bound c, there always exists a positive scalar $\theta < \min\{c_s, c_f\}$ such that (36) holds. Then, by continuity, we can also find a sufficiently small ε^* such that (10) is satisfied with the same c and θ. Therefore, if Assumptions 1-2 are satisfied, then we can always find a system in the form of (9), (7) which guarantees a desired Ultimate Bound. In particular, in Example 2.1, stabilization is guaranteed if one takes ε sufficiently small. ∘

In the following, we provide explicitly an upper bound of the perturbation parameter ε^* such that, for a desired ultimate bound c, the UUB is guaranteed or all $\varepsilon \in (0, \varepsilon^*)$.

Corollary 1 *Consider symmetric positive definite matrices P, Q verifying Assumptions 1-2 for some positive scalars c_s, c_f. For any given positive constant c, there exists*

$$\varepsilon^* = \sup_{\theta \in (0, c_s)} \min_{i \in \mathcal{I}_N} \frac{c_f \lambda_{\min}(Q)}{\|M_i^3 - {M_i^2}^\top {M^1}^{-1} M_i^2 + \theta Q\|}, \qquad (37)$$

where

$$M^1 = \begin{pmatrix} -c_s P + \theta P & 0 \\ \star & -\theta c I \end{pmatrix}, \qquad (38)$$

$$M_i^2 = \begin{pmatrix} P A_i^{12} + {G_i^1}^\top Q \\ (Q {A^{22}}^{-1} A^{21} b_i)^\top \end{pmatrix}, \qquad (39)$$

$$M_i^3 = \left({G_i^2}^\top Q + Q G_i^2 \right), \qquad (40)$$

such that for any $\varepsilon \in (0, \varepsilon^*)$, the closed-loop system (9) and (7) is Uniformly Ultimately Bounded in the set $\mathcal{D}(c)$ as defined in (11).

Proof. The proof is based on the same steps as the proof of Theorem 1. Since symmetric positive definite matrices P, Q satisfy Assumptions 1-2, inequality (27) holds with $\phi_i(x_s, y)$ given in (26). In the following, the objective is to show that $\phi_i(x_s, y) < 0$ whenever $V(x_s, y) \geqslant c$. This implies $\sup_{(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathcal{F}^*(x_s, y)} \left\{ \frac{\partial V}{\partial x_s} \bar{h}_1 + \frac{\partial V}{\partial y} \bar{h}_{2\varepsilon} \right\} < 0$, whenever $V(x_s, y) \geqslant c$.

Consider ε^* defined in (37) and $\bar{\varepsilon} \in (0, \varepsilon^*)$. From the definition of ε^* and from the fact that the function $\min_{i \in \mathcal{I}_N} \frac{c_f \lambda_{\min}(Q)}{\|M_i^3 - {M_i^2}^\top {M^1}^{-1} M_i^2 + \theta Q\|}$ is continuous in θ on the interval $(0, c_s)$, there exists $\theta \in (0, c_s)$ such that $\bar{\varepsilon} = \min_{i \in \mathcal{I}_N} \frac{c_f \lambda_{\min}(Q)}{\|M_i^3 - {M_i^2}^\top {M^1}^{-1} M_i^2 + \theta Q\|}$. Consider such a $\theta \in (0, c_s)$ and $c > 0$, we have that condition (28) holds. Moreover, for any $\varepsilon \in (0, \bar{\varepsilon}]$, the following holds, for $i \in \mathcal{I}^*(x_s)$

$$\begin{aligned} &\varepsilon (M_i^3 - {M_i^2}^\top {M^1}^{-1} M_i^2 + \theta Q) \\ &\leq \varepsilon \|M_i^3 - {M_i^2}^\top {M^1}^{-1} M_i^2 + \theta Q\| I \\ &\leq \bar{\varepsilon} \|M_i^3 - {M_i^2}^\top {M^1}^{-1} M_i^2 + \theta Q\| I \\ &\leq c_f \lambda_{\min}(Q) I \\ &\leq c_f Q, \end{aligned} \qquad (41)$$

where M^1, M_i^2, M_i^3 are defined in (38)-(40).

Condition (41) implies that (31) holds. Then, following similar arguments as in the proof of Theorem 1, we get that for any $0 < \varepsilon \leq \bar{\varepsilon} < \varepsilon^*$, condition (35) holds whenever $V(x_s, y) \geqslant c$. Therefore, system (9) with the state-dependent switching law (7) is uniformly ultimately bounded. □

In the following, we show that asymptotical stability can be guaranteed for a special case in which the effect of the affine term b_i in the slow dynamics can be rejected by the fast dynamics.

Corollary 2 *Consider symmetric positive definite matrices P, Q satisfying Assumptions 1-2 for some positive constants c_s, c_f. Assume that there exist positive constants c, θ, $\varepsilon^* \ll 1$ such that condition (10) holds for all $i \in \mathcal{I}_N$. If*

$$\underset{i\in\mathcal{I}_N}{Span}(b_i) \subset Ker(QA^{22^{-1}}A^{21}), \tag{42}$$

then, for all $\varepsilon \in (0, \varepsilon^*)$, the closed-loop system (9) and (7) is asymptotically stable.

Proof. Following *Step 2* in the proof of Theorem 1, under Assumption 2 and using (42), inequality (25) becomes

$$\begin{aligned}\Omega_i^2 &= \frac{\partial V}{\partial y} \times \left(\frac{A^{22}}{\varepsilon} y + (G_i^1 x_s + G_i^2 y + A^{22^{-1}} A^{21} b_i) \right) \\ &\leq \frac{1}{\varepsilon} y^\top \left(-c_f Q + \varepsilon ({G_i^2}^\top Q + Q G_i^2) \right) y + 2 x_s^\top {G_i^1}^\top Q y,\end{aligned} \tag{43}$$

Under Assumption 1, in view of (23) and (43), we have, for $i \in \mathcal{I}^*(x_s)$

$$\begin{aligned}&\Omega_i^1 + \Omega_i^2 \\ &\leq x_s^\top (-c_s P) x_s + \frac{1}{\varepsilon} y^\top \left(-c_f Q + \varepsilon ({G_i^2}^\top Q + Q G_i^2) \right) y \\ &+ 2 x_s^\top \left(P A_i^{12} + {G_i^1}^\top Q \right) y \\ &= \begin{pmatrix} x_s \\ y \end{pmatrix}^\top \begin{pmatrix} -c_s P & P A_i^{12} + {G_i^1}^\top Q \\ \star & -\frac{c_f Q}{\varepsilon} + {G_i^2}^\top Q + Q G_i^2 \end{pmatrix} \begin{pmatrix} x_s \\ y \end{pmatrix} \\ &=: g_i(x_s, y).\end{aligned} \tag{44}$$

Then, the inequality (27) becomes

$$\begin{aligned}&\sup_{(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathcal{F}^*(x_s, y)} \left\{ \frac{\partial V}{\partial x_s} \bar{h}_1 + \frac{\partial V}{\partial y} \bar{h}_{2\varepsilon} \right\} \\ &\leq \max_{\beta \in \Delta^*_{N(x_s)}} \left\{ \sum_{i=1}^N \beta_i g_i(x_s, y) \right\},\end{aligned} \tag{45}$$

where $g_i(x_s, y)$ defines in (44).

In the following, we prove that under condition (10) $g_i(x_s, y) < 0$, for $i \in \mathcal{I}^*(x_s)$. Using (42), we deduce from inequality (31)

$$\begin{aligned}&-\frac{c_f Q}{\varepsilon} + ({G_i^2}^\top + Q G_i^2) \\ &+ (P A_i^{12} + {G_i^1}^\top Q)^\top ((c_s - \theta) P)^{-1} (P A_i^{12} + {G_i^1}^\top Q) < 0.\end{aligned} \tag{46}$$

Since $c_s > (c_s - \theta) > 0$, inequality (46) implies

$$\begin{aligned}&-\frac{c_f Q}{\varepsilon} + ({G_i^2}^\top + Q G_i^2) \\ &+ (P A_i^{12} + {G_i^1}^\top Q)^\top (c_s P)^{-1} (P A_i^{12} + {G_i^1}^\top Q) < 0.\end{aligned} \tag{47}$$

Using the Schur complement, we have

$$\begin{pmatrix} -c_s P & P A_i^{12} + {G_i^1}^\top Q \\ \star & -\frac{c_f Q}{\varepsilon} + {G_i^2}^\top Q + Q G_i^2 \end{pmatrix} \prec 0. \tag{48}$$

Multiplying both sides of (48) by the vector (x_s, y), we get $g_i(x_s, y) < 0$, $i \in \mathcal{I}^*(x_s)$. Thus

$$\sup_{(\bar{h}_1, \bar{h}_{2\varepsilon}) \in \mathcal{F}^*(x_s, y)} \left\{ \frac{\partial V}{\partial x_s} \bar{h}_1 + \frac{\partial V}{\partial y} \bar{h}_{2\varepsilon} \right\} < 0,$$

Therefore, system (9) with the state-dependent switching law (7) is asymptotically stable. □

Remark 9 Corollary 2 provides a simple assumption to verify asymptotical stability for a switched affine system interconnected with fast LTI dynamics. ∘

Numerical examples will be shown in the following section to illustrate the main results.

4 Numerical example

In this section we use academic numerical examples to illustrate the main results.

Example 4.1. We consider system (1) with $N = 2$ modes, and matrices defined as

$$A_1^{11} = \begin{pmatrix} 2 & 2 \\ -3 & 1 \end{pmatrix}, \; A_2^{11} = \begin{pmatrix} -3 & -3 \\ 2 & -1 \end{pmatrix},$$

$$A_1^{12} = \begin{pmatrix} 2 & -3 \\ -1 & -2 \end{pmatrix}, \; A_2^{12} = \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \; b_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$A^{21} = \begin{pmatrix} -5 & 3 \\ -5 & 4 \end{pmatrix}, \; A^{22} = \begin{pmatrix} -3 & 2 \\ -2 & 1 \end{pmatrix}, \text{ and } \varepsilon = 1 \times 10^{-2}.$$

Considering $\alpha = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}^\top$, we have $b(\alpha) = \sum_{i=1}^{N} \alpha_i b_i = 0$. From the definition of the matrices A_i^s, $i \in \mathcal{I}_N$ in (4), we get that $A^s(\alpha) = \sum_{i=1}^{N} \alpha_i A_i^s = \begin{pmatrix} -0.5 & -3 \\ 7 & -7.5 \end{pmatrix}$ is Hurwitz, which means that Assumption 1 is satisfied. Furthermore, we can see that A^{22} is also Hurwitz, which means that Assumption 2 is also satisfied.

It is important to note that in this example, none of the A_i^s matrices for $i \in \mathcal{I}_N$ is Hurwitz, which means that even if the fast subsystem (with state y) converges, the slow subsystem (with state x_s) will converge only by using of a switching law (*i.e.* staying only in one mode will lead to the unstability of the slow subsystem). Consider a state-dependent switching law proposed in (7), with $P = \begin{pmatrix} 0.5513 & -0.0697 \\ \star & 0.1627 \end{pmatrix}$, $c_s = 1$, and $Q = \begin{pmatrix} 0.7571 & -0.7462 \\ \star & 0.8842 \end{pmatrix}$, $c_f = 1$ satisfying Assumptions 1-2. We are able to show that the closed-loop system is UUB, and the best estimate of the ultimate bound found by using a line search as $c = 0.8$ with $\theta = 10^{-2}$. Figures 2-4 illustrate the evolution of the closed-loop system (1) and (7) with the initial condition $x_s(0) = (2 \; 2)^\top$, $x_f(0) = (5 \; 5)^\top$. The time evolution of the Lyapunov function is shown in Figure 5.

Example 4.2. In order to show the asymptotical stability, we consider the following example that verifies condition (42) with

$$A_1^{11} = \begin{pmatrix} 3 & 0 \\ -1 & -17 \end{pmatrix}, \; A_2^{11} = \begin{pmatrix} -9 & 0 \\ 1 & 3 \end{pmatrix},$$

$$A_1^{12} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; A_2^{12} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; b_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix},$$

$$A^{21} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \; A^{22} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ and } \varepsilon = 1 \times 10^{-3},$$

and $\alpha = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}^\top$. The time evolution of the closed-loop system (1) and (7) with the initial condition $x_s(0) = (15 \; 15)^\top$, $x_f(0) = (5 \; 10)^\top$ is given in Figure 6.

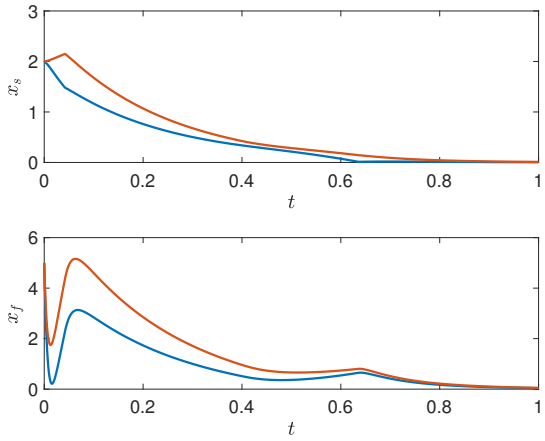

Fig. 2. System's evolution (Example 4.1)

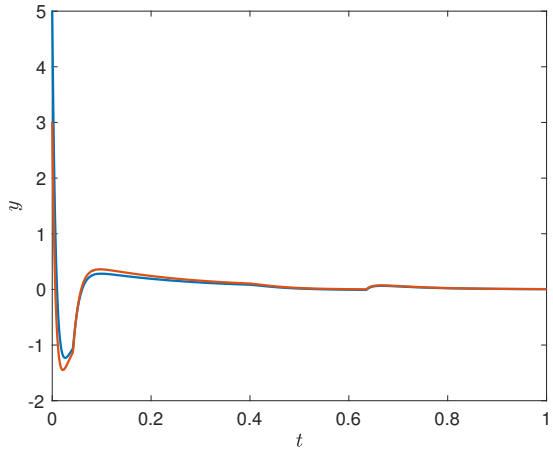

Fig. 3. Evolution of the fast dynamics y (Example 4.1)

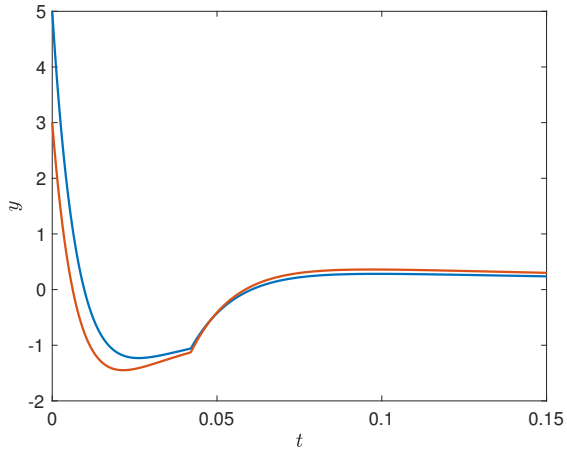

Fig. 4. Zoom of the fast dynamics y (Example 4.1)

5 Conclusion

In this work, we studied a switched affine system interconnected with a fast LTI dynamics. We considered a

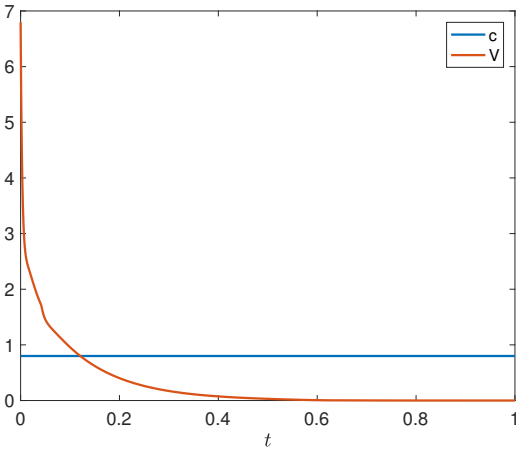

Fig. 5. Time evolution of the Lyapunov function V (Example 4.1)

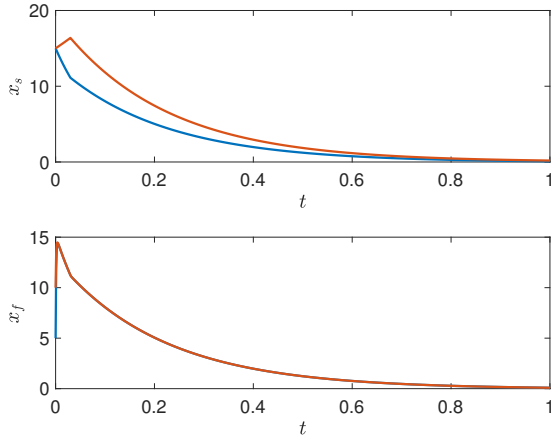

Fig. 6. System's evolution (Example 4.2)

state-dependent switching law (depending only on the slow dynamics) based on the classical convex combination method and we provided simple conditions for checking stability when the fast dynamics has been taken into account. Moreover, we proposed an additional condition for ensuring the asymptotical stability for a special case.

Perspectives for future works are numerous. Further extensions will include the study of sampled-data implementations, the generalization to the case of systems with switching in both fast and slow dynamics, as well as the applications of the results to DC-DC power converters.

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