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On the Optimal Power Allocation at the MAC layer in the Asymptotic and in the Finite Blocklength

Mouktar Bello*, Arsenia Chorti* and Inbar Fijalkow*

ETIS UMR8051, CY Cergy Paris Université, ENSEA, CNRS, F-95000, Cergy, France

Abstract—In this paper, we investigate the optimal power allocation at the MAC layer in both the asymptotic and the finite blocklength regimes in a two-user NOMA uplink network, under statistical delay constraints captured through the link-layer rate. Using the link-layer effective rate analytic expressions, we provide closed-form expressions of the optimal power coefficients in low and high signal-to-ratio (SNR) regimes. These are validated by an extensive set of simulations, showing that the proposed power allocation policy optimizes the performance of NOMA compared to OMA at Layer 2, in a two-user network.

Index Terms—Effective capacity, NOMA, power allocation, finite blocklength.

I. INTRODUCTION

The exponential growth of Internet of Things (IoT) devices is categorically changing the wireless communications landscape. Serving this growing number of network devices and meeting their requirements are major challenges in the design of future networks. The beyond 5G (B5G) is purposely seen as a paradigm of extreme requirements, in the sense that the transmissions are made with extremely low latency ($< 1\text{ms}$), with extremely high reliability, with extreme massive connectivity [1], [2].

The non-orthogonal multiple access (NOMA) [3], [4] is one of the new technologies that have emerged and is gaining a lot of attention, to satisfy these various constraints. Power-domain NOMA has positioned itself as a serious candidate to be the multiple access technique for future generations of wireless networks. It relies typically on superposition coding (SC) and successive interference cancellation (SIC), whose implementation is made possible today by the enormous progress made in computing power [5]. Its importance is even more pronounced in uplink networks, where the need is to handle large number of users.

Moreover, the use of short packets is one of the enablers of low latency communications. This is made possible through the recent progress in the information theory field, precisely in the finite blocklength regime [6], [7].

With respect to the latency, several parameters come into play in the calculation of the E2E latency, including the delay at the layer 2 (L2) scheduler. The theory of the effective capacity (EC) becomes very relevant to capture this statistical delay. The EC is defined as the maximum constant arrival rate which can be served by a given service process, while guaranteeing the required statistical delay quality of service (QoS) provisioning [8], [9]. The interest of this metric lies in the fact that maximizing it lowers the queuing delay of

the scheduler, which can contribute to the reduction of the communication overall latency.

In this paper we investigate the optimal power allocation at the medium access control (MAC) layer for communications under statistical delay constraints, subject to the requirement that the individual users have at least the performance they would achieve using OMA.

The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, we formalize the optimization problem whose solutions are presented in the form of propositions. Section IV presents numerical results. We conclude the paper in Section V.

II. SYSTEM MODEL

We assume a two-user NOMA uplink network with users U_1 and U_2 in a Rayleigh fading propagation channel, with respective channel gains during a transmission block denoted by $|h_1|^2 < |h_2|^2$. The users transmit corresponding symbols s_1, s_2 respectively, with $\mathbb{E}[|s_i|^2] = 1$.

The base station (BS) observes the following superimposed signal,

$$z = \sum_{i=1}^2 \sqrt{\alpha_i P_b} h_i s_i + w, \quad i = 1, 2, \quad (1)$$

where w denotes a zero mean circularly symmetric complex Gaussian random variable with variance σ^2 , i.e., $w \sim \mathcal{CN}(0, \sigma^2)$. P_b is the total available power, while α_i is the NOMA power coefficient for $U_i, i = 1, 2$, with $\alpha_1 + \alpha_2 = 1$.

In the asymptotic regime, the NOMA achievable rates for U_1 and U_2 are respectively given by

$$R_1 = \log_2 (1 + \rho \alpha_1 |h_1|^2), \quad (2)$$

$$R_2 = \log_2 \left(1 + \frac{\rho \alpha_2 |h_2|^2}{1 + \rho \alpha_1 |h_1|^2} \right), \quad (3)$$

assuming perfect SIC. Furthermore, $\rho = \frac{P_b}{\sigma^2}$ denotes the transmit SNR. The OMA achievable rates are given as

$$\tilde{R}_i = \frac{1}{2} \log_2 (1 + \rho |h_i|^2), \quad i = 1, 2. \quad (4)$$

assuming equal resource sharing between the users.

The EC is defined as follows for a user U_i in NOMA [8]

$$E_c^i = -\frac{1}{\theta_i T_f B} \ln (\mathbb{E} [e^{-\theta_i T_i B R_i}]), \quad (\text{in b/s/Hz}), \quad (5)$$

where T_f is the duration of each fading-block, B is the bandwidth, $\mathbb{E}[\cdot]$ denotes expectation over the channel gains

and θ_i the statistical delay QoS exponent (For $\theta_i \rightarrow 0$ the system is delay-tolerant, however as θ_i increases the delay becomes more stringent). The EC of a user U_i in OMA is defined accordingly by

$$\tilde{E}_c^i = -\frac{1}{\theta_i T_f B} \ln \left(\mathbb{E} \left[e^{-\theta_i T_f B \tilde{R}_i} \right] \right), \quad i = 1, 2. \quad (6)$$

In the finite blocklength regime, the user U_i Layer 1 rates for NOMA and OMA are respectively given by [6], [10], [11]

$$r_i \approx \log_2(1 + \gamma_i) - \sqrt{\frac{V(\gamma_i)}{N}} Q^{-1}(\epsilon_i), \quad (7)$$

$$\tilde{r}_i \approx \frac{1}{2} \left(\log_2(1 + \tilde{\gamma}_i) - \sqrt{\frac{V(\tilde{\gamma}_i)}{N}} Q^{-1}(\epsilon_i) \right), \quad (8)$$

where the $\frac{1}{2}$ indicates the OMA resource sharing; $\gamma_1 = \rho \alpha_1 |h_1|^2$ and $\gamma_2 = \frac{\rho \alpha_2 |h_2|^2}{1 + \rho \alpha_1 |h_1|^2}$, $\tilde{\gamma}_i = \rho |h_i|^2$ $i = 1, 2$. N denotes the blocklength, ϵ_i the packet error probability of U_i , Q^{-1} the inverse of the Q -Gaussian function defined as $Q(x) = \frac{1}{2\pi} \int_x^\infty e^{-\frac{u^2}{2}} du$ and V_i the channel dispersion given by $V(\gamma_i) = (1 - (1 + \gamma_i)^{-2}) (\log_2 e)^2$, which is approximated in the following by

$$\sqrt{V_i} \approx \left(1 - \frac{1}{2} (1 + \gamma_i)^{-2} \right) \log_2 e. \quad (9)$$

The ER for NOMA/OMA U_i is defined in [10] as follows

$$E_R^i = -\frac{1}{\theta_i N T_f B} \ln \left(\mathbb{E} \left[\epsilon + (1 - \epsilon) e^{-\theta_i N T_f B r_i} \right] \right). \quad (10)$$

Furthermore, it is worth noting that in (7) not all combinations of (ϵ, γ_i, N) are feasible. For some combinations, we have a negative rate values in (7), and therefore a negative L2 rate. Specifically,

$$\forall \epsilon < Q \left(\frac{\sqrt{N} \log_2(1 + \gamma_i)}{\sqrt{(1 - (1 + \gamma_i)^{-2}) (\log_2 e)^2}} \right) \implies r_i < 0. \quad (11)$$

III. OPTIMAL POWER ALLOCATION AT THE MAC LAYER

In this Section we investigate the optimal power allocation problem at the MAC layer.

A. Asymptotic regime

We formalize the L2 sum rate maximization optimization problem as follows

$$[\mathbf{P1}] \max_{\alpha_2} E_c^1 + E_c^2 \quad (12)$$

$$\text{s.t. } E_c^1 \geq \tilde{E}_c^1 \quad (13)$$

$$E_c^2 \geq \tilde{E}_c^2 \quad (14)$$

$$0 < \alpha_2 < 1,$$

where the aim is to maximize the L2 sum rate, subject to the constraints that U_1 and U_2 to achieve at least the performance they would have using OMA as indicated by (13) and (14).

The objective function (12) is an increasing function for α_2 as we have $\frac{\partial(E_c^1 + E_c^2)}{\partial \alpha_2} > 0$, thus the solution of $[\mathbf{P1}]$ can be directly obtained from the constraints. However, due to the

complexity of the closed-form expressions of E_c^1 and E_c^2 [12], it is almost impossible to find the optimal power coefficients for the whole range of SNRs. We propose here to solve this problem in the extreme SNRs, i.e. in low and high SNRs, specifically using (13) as it gives the lower bound of α_1 , thus the upper bound on α_2 .

1) *At low SNRs:* Using Maclaurin's series of E_c^1 , we get the low-SNR approximation of the E_c^1

$$E_c^1 \approx \bar{C}_w + \rho \dot{C}_w + \frac{\rho^2}{2} \ddot{C}_w = \alpha_1 \frac{2\rho}{\ln 2} \Gamma(2) \mathbb{U}(2, 3, 2) + \alpha_1^2 \frac{\rho^2}{\ln 2} \left(\frac{\beta_1}{2} - 1 \right) \Gamma(3) \mathbb{U}(3, 4, 2). \quad (15)$$

where $\beta_i = -\frac{\theta T_f B}{\ln 2}$, $\bar{C}_w = E_c^1|_{\rho=0} = 0$; $\dot{C}_w = \frac{\partial E_c^1}{\partial \rho}|_{\rho=0}$; and $\ddot{C}_w = \frac{\partial^2 E_c^1}{\partial \rho^2}|_{\rho=0}$.

Using (15), solving $E_c^1 \geq \tilde{E}_c^1$ gives the optimal power allocation in low SNRs as stated in the following Proposition.

Proposition 1: In a two-user NOMA uplink network, the optimal power allocation that maximizes the sum EC while ensuring each user to have at least the OMA performance is given as follows in low SNRs:

$$\alpha_1^{opt} \approx -2 \sqrt{\frac{1 + (2\beta_1 - 4) \ln 2 \tilde{E}_c^1}{\rho^2 (\beta_1 - 2)^2}} - \frac{2}{\rho (\beta_1 - 2)}, \quad (16)$$

$$\alpha_2^{opt} \approx 1 - \alpha_1^{opt}. \quad (17)$$

Proof: The proof is provided in Appendix I. ■

2) *At high SNRs:* On the other hand, at high SNRs, E_c^1 is approximated as follows

$$E_c^1 \approx \frac{1}{\beta_1} \times \log_2 \left((\rho \alpha_1)^{\beta_1} 2 \Gamma(1 + \beta_1) U(1 + \beta_1, 2 + \beta_2, 2) \right). \quad (18)$$

(18) is used to solve $E_c^1 \geq \tilde{E}_c^1$ and get the optimal power allocation in high SNRs as stated in the following proposition.

Proposition 2: In a two-user NOMA uplink network, the optimal power allocation that maximizes the sum EC while ensuring each user to have at least the OMA performance is given as follows in high SNRs:

$$\alpha_1^{opt} \approx \beta_1 \sqrt{\frac{2^{\beta_1} \tilde{E}_c^1}{2 \rho^{\beta_1} \Gamma(1 + \beta_1) U(1 + \beta_1, 2 + \beta_2, 2)}}, \quad (19)$$

$$\alpha_2^{opt} \approx 1 - \alpha_1^{opt}. \quad (20)$$

Proof: The proof is provided in Appendix II. ■

B. Finite blocklength regime

Similarly, in the finite blocklength regime the optimization problem is formalized as follows

$$[\mathbf{P2}] \max_{\alpha_2} E_R^1 + E_R^2 \quad (21)$$

$$\text{s.t. } E_R^1 \geq \tilde{E}_R^1 \quad (22)$$

$$E_R^2 \geq \tilde{E}_R^2 \quad (23)$$

$$0 < \alpha_2 < 1,$$

Here again we suppose a constrained sum power, i.e., $\alpha_1 + \alpha_2 = 1$ for the sake of simplicity. Likewise, solving [P2] comes down to solving (22) as the objective function (21) is also an increasing function of α_2 .

1) *Low SNRs*: the low-SNR approximation of E_R^1 is given by (24) using Maclaurin's series, where $\bar{C}_f = E_R^1|_{\rho=0}$; $\dot{C}_f = \frac{\partial E_R^1}{\partial \rho}|_{\rho=0}$; $\ddot{C}_f = \frac{\partial^2 E_R^1}{\partial \rho^2}|_{\rho=0}$ and $a = \sqrt{N}\theta_1 Q^{-1}(\epsilon) \log_2 e$.

Proposition 3: In a two-user NOMA uplink network, considering the finite blocklength regime, the weak user alone requires (25), which is more than the total available power, for NOMA to achieve at least the performance of OMA in low SNRs, thus [P2] does not have a solution.

Proof: The proof is provided in Appendix III. ■

2) *High SNRs*: the analytic expression of the high-SNR approximation of E_R^1 is given as follows

$$E_R^1 \approx \frac{1}{\beta_1^F} \log_2 \left(\epsilon + 2(1 - \epsilon)(\rho\alpha_1)^{\beta_1^F} e^{\sqrt{N}\theta_1 Q^{-1}(\epsilon) \log_2 e} \right) \times \Gamma(1 + \beta_1^F) \mathcal{U}(1 + \beta_1^F, 2 + \beta_1^F, 2), \quad (26)$$

and is used to get the optimal power allocation as stated in the following.

Proposition 4: In a two-user NOMA uplink network, considering the finite blocklength regime, the optimal power that maximizes the L2 rate (ER) while ensuring each user to have at least the performance of OMA is given as follows in high SNRs

$$\alpha_1^{opt} \approx \frac{1}{\rho^{\beta_1^F} \sqrt{2(1 - \epsilon_1)}} \times \beta_1^F \sqrt{\frac{2\beta_1^F \widehat{E}_R^1 - \epsilon_1}{e^{\sqrt{N}\theta_2 Q^{-1}(\epsilon) \log_2 e} \Gamma(1 + \beta_1^F) \mathcal{U}(1 + \beta_1^F, 2 + \beta_1^F, 2)}}, \quad (27)$$

$$\alpha_2^{opt} \approx 1 - \alpha_1^{opt}. \quad (28)$$

Proof: The proof is provided in the Appendix IV. ■

IV. NUMERICAL RESULTS

In Fig.1 we provide the numerical validation of the proposed analytic expressions in (15) and (18) through Monte Carlo simulations.

Fig.2 depicts the comparison of NOMA and OMA in terms of the EC, using the proposed power allocation policy. In fact, we showed in [12] that with fixed power coefficients, $\alpha_1 = 0.2$ and $\alpha_2 = 0.8$, OMA is more advantageous than NOMA for low-medium transmit SNRs, while NOMA outperforms OMA at high transmit SNRs for the weak user U_1 . Reverse conclusions is drawn for the strong user U_2 . We noticed also that the EC of the strong user converges at high SNRs due to inference U_2 suffers from U_1 . These conclusions highlighted the importance of the optimal power allocation in optimizing the performance of NOMA at L2. As can be seen, using the derived optimal power coefficients, (16) and (17) in low SNRs, (19) and (20) in high SNRs, NOMA has at least a similar performance to OMA. This validates propositions 1 and 2.

Moreover, at high SNRs, we find that the performance gain of NOMA compared to OMA is at the expense of the delay.

In fact, in (19) the term inside $\Gamma(\cdot)$ should be positive, i.e., $\theta < \ln 2$; This indicates that for more stringent delays beyond that threshold, not all the constraints in [P1] are met in high SNRs. In other words, reducing the delay beyond a certain threshold is at the expense of the individual performance gain of NOMA compared to OMA.

Fig.3 and Fig.4 represent numerical results in the finite blocklength regime. Fig.3 validates the novel analytic expressions of E_R^1 and E_R^2 at the extreme SNRs in the finite blocklength regime, i.e., (24) and (26), through Monte-Carlo simulations (10^5 iterations).

Fig.4 displays the comparison of NOMA versus OMA in the finite blocklength regime validating Propositions 3 and 4. On the one hand, the Fig.4-(a) shows that [P2] does not have a solution in low SNRs because both constraints are not met using (25). In fact, at low SNRs (25) might be greater than one, meaning that the weak user may need more than the total available power for NOMA to be able to achieve the performance of OMA. This can be seen in Fig.5 where the amount of power required for NOMA to have similar performance as OMA, for U_1 , decreases with the transmit SNRs increasing. What emerges is that it is more challenging for NOMA to outperform OMA in low SNRs for U_1 , considering a constrained sum power scheme. It would be interesting to know if this is also the case where there is no constraint on the sum power, i.e., when users are only limited by their own power budget. We will deal with this case in future works.

On the other hand, Fig.4-(b) shows the comparison of NOMA versus OMA in high SNRs using the proposed optimal power coefficients in the finite blocklength regime, (27) and (28). As can be seen, with the optimal power allocation, for U_1 NOMA performs as just as OMA; while for U_2 there is a clear performance gain of NOMA compared to OMA. Similarly as in the asymptotic regime, at high SNRs, reducing the delay beyond the threshold, $N\theta < \ln 2$, is at the expense of the individual performance gain of NOMA compared to OMA, for the same reason explained above. This means that seeking to meet more stringent delays will result in an outperformance of OMA over NOMA.

V. CONCLUSION

We investigate the optimal power allocation at the MAC layer in a two-user NOMA uplink network. We derived closed-form expressions of the optimal power coefficients in low and high SNRs. We show that with a constrained sum power setting in the finite blocklength regime, at low SNRs, NOMA cannot outperform OMA for both users because the weak user alone might require more than the available total power. Furthermore, our analysis show that at high SNRs, reducing the delay beyond a certain threshold is at the expense of the individual performance gain of NOMA compared to OMA. This is true for both the asymptotic and the finite blocklength regimes.

$$\begin{aligned}
E_R^1 &\approx \bar{C}_f + \rho \dot{C}_f + \frac{\rho^2}{2} \ddot{C}_f, \\
&\approx \frac{1}{\beta_1} \log_2 \left(\epsilon + (1-\epsilon) e^{\frac{\alpha}{2}} \right) + \frac{\rho}{\ln 2 \beta_1} \left(\frac{(1-\epsilon) e^{\frac{\alpha}{2}} (\beta_1 + a) \alpha_1 Y}{\epsilon + (1-\epsilon) e^{\frac{\alpha}{2}}} \right) + \frac{\rho^2}{2 \ln 2 \beta_1} \times \\
&\left(\frac{\left((1-\epsilon) e^{\frac{\alpha}{2}} (\beta_1 (\beta_1 - 1) + a(2\beta_1 - 3) + a^2) \alpha_1^2 W \right) (\epsilon + (1-\epsilon) e^{\frac{\alpha}{2}}) - \left((1-\epsilon) e^{\frac{\alpha}{2}} (\beta_1 + a) \alpha_1 Y \right)^2}{(\epsilon + (1-\epsilon) e^{\frac{\alpha}{2}})^2} \right). \tag{24}
\end{aligned}$$

$$\alpha_1^{opt} \approx \frac{e^{-\frac{\alpha}{2}} \beta_1 \ln 2 (\epsilon - e^{\frac{\alpha}{2}} (\epsilon - 1))^2 \left(\sqrt{t_0^2 - t_1 \frac{2e^{\frac{\alpha}{2}} \rho^2 (1-\epsilon) \left(\frac{\ln(\epsilon - e^{\frac{\alpha}{2}} (\epsilon - 1))}{\beta_1 \ln 2} - \bar{E}_R^1 \right)}{\beta_1 \ln 2 (\epsilon - e^{\frac{\alpha}{2}} (\epsilon - 1))^2}} - t_0 \right)}{\rho^2 (1-\epsilon) t_1}, \tag{25}$$

where $Y = 2\Gamma(2)U(2, 3, 2)$; $W = 2\Gamma(3)U(3, 4, 2)$; $t_0 = \frac{e^{\frac{\alpha}{2}} Y (\epsilon - 1) (a + \beta_1) \rho}{\beta_1 \ln 2 (\epsilon - e^{\frac{\alpha}{2}} (\epsilon - 1) - \epsilon)}$
 $t_1 = W (a^2 + a(2\beta_1 - 3) + (\beta_1 - 1)\beta_1) (\epsilon - e^{\frac{\alpha}{2}} (\epsilon - 1)) + e^{\frac{\alpha}{2}} Y^2 (\epsilon - 1) (a + \beta_1)^2$.

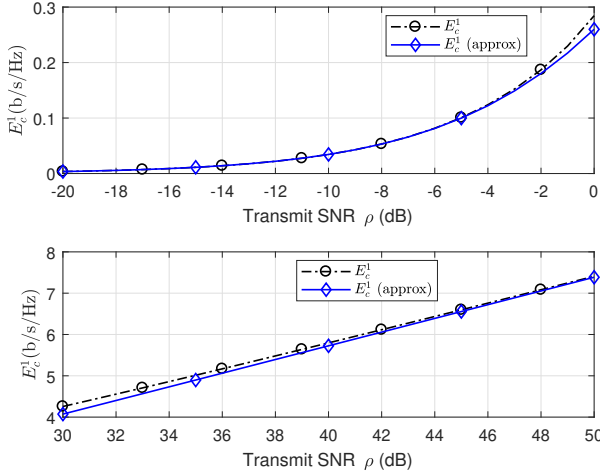


Fig. 1. Numerical validation of the E_c^1 approximation (asymptotic block-length) in low and high SNRs given respectively in (15) and (18) through Monte-Carlo simulations (10^5 iterations).

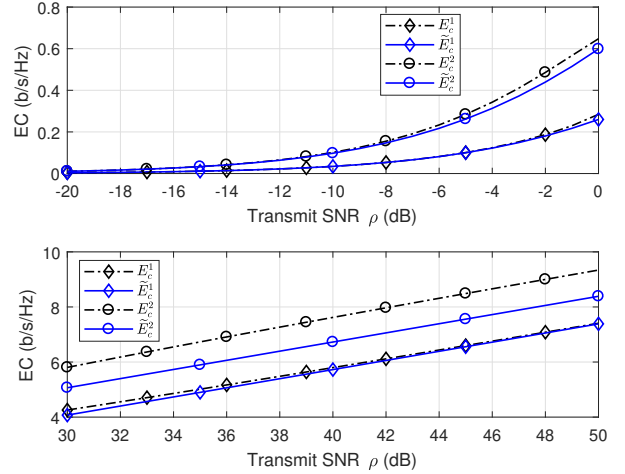


Fig. 2. Comparison of NOMA versus OMA, considering the derived optimal power allocation, (a)- given in (16) and (17); (b)- given in (19) and (20).

APPENDIX I: PROOF OF PROPOSITION 1

From (5) we have that for U_1

$$\bar{C}_w = E_c^1|_{\rho=0} = 0. \tag{29}$$

The first derivative with respect to ρ is given as follows

$$\begin{aligned}
\frac{\partial E_c^1}{\partial \rho} &= \frac{1}{\beta_1 \ln 2} \frac{\left(\mathbb{E} \left[\frac{(1 + \rho \alpha_1 |h_1|^2)^{\beta_1}}{\mathbb{E} \left[(1 + \rho \alpha_1 |h_1|^2)^{\beta_1} \right]} \right] \right)'}{\mathbb{E} \left[(1 + \rho \alpha_1 |h_1|^2)^{\beta_1} \right]} \\
&= \frac{\alpha_1 \mathbb{E} \left[|h_1|^2 (1 + \rho \alpha_1 |h_1|^2)^{\beta_1 - 1} \right]}{\ln 2 \mathbb{E} \left[(1 + \rho \alpha_1 |h_1|^2)^{\beta_1} \right]}, \tag{30}
\end{aligned}$$

which gives the following

$$\dot{C}_w = \frac{\partial E_c^1}{\partial \rho} |_{\rho=0} = \frac{\alpha_1}{\ln 2} \mathbb{E} \left[|h_1|^2 \right]. \tag{31}$$

Similarly we have that

$$\begin{aligned}
\ddot{C}_w &= \frac{\partial^2 E_c^1}{\partial \rho^2} |_{\rho=0} \\
&= \frac{\alpha_1}{\ln 2} \left(\alpha_1 (\beta_1 - 1) \mathbb{E} \left[|h_1|^4 \right] - \beta_1 \alpha_1 \left(\mathbb{E} \left[|h_1|^2 \right] \right)^2 \right). \tag{32}
\end{aligned}$$

$\mathbb{E} \left[|h_1|^4 \right]$ is compute using the following result,

$$\begin{aligned}
\mathbb{E} \left[(x_1)^n \right] &= \int_0^\infty x_1^n f_{x(1)}(x_1) dx_1 = \int_0^\infty x_1^n 2e^{-2x_1} dx_1 \\
&= 2\Gamma(1+n)U(1+n, 2+n, 2), \tag{34}
\end{aligned}$$

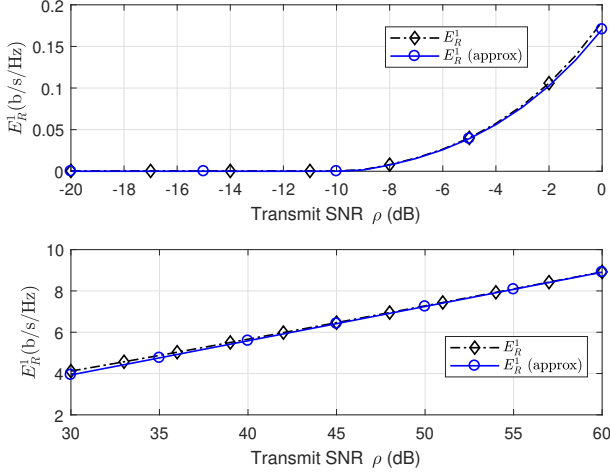


Fig. 3. Numerical validation of the E_R^1 approximation (finite blocklength regime) in low and high SNRs given respectively in (24) and (26), through Monte-Carlo simulations (10^5 iterations).

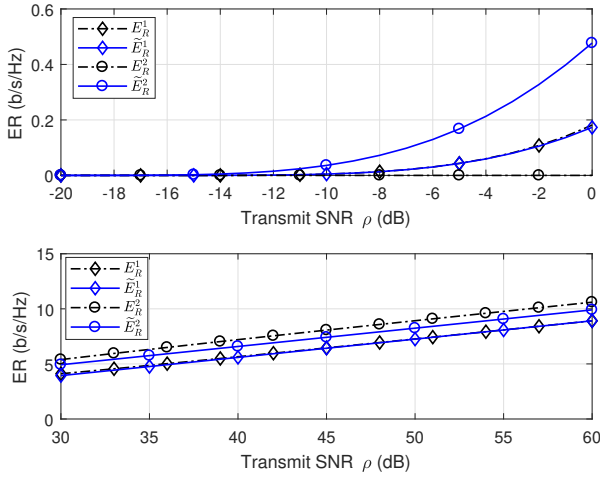


Fig. 4. Comparison of NOMA versus OMA, considering the power coefficients, (a)- in low SNRs given by (25) ; (b)- in high SNRs given by (27) and (28). $\theta = 0.0001$, $N = 600$, $\epsilon = 10^{-6}$.

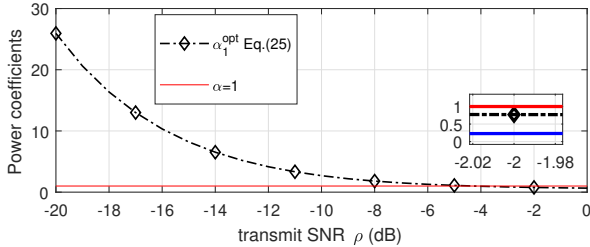


Fig. 5. Power coefficient for U_1 given in (25) versus ρ .

where $f_{x(1)}(x_1) = 2e^{-2x_1}$ is the probability density function (PDF) of the 1-th ordered random variable in a population of $M = 2$, with $x_1 = |h_1|^2$ [13]. We notice that $\mathbb{E}[|h_1|^4] = 2(\mathbb{E}[|h_1|^2])^2$, then

$$\ddot{C}_w = \frac{\partial^2 E_c^1}{\partial \rho^2} \Big|_{\rho=0} = \frac{\alpha_1^2}{\ln 2} \left(\frac{\beta_1}{2} - 1 \right) \mathbb{E}[|h_1|^4] \quad (35)$$

$$= 2 \frac{\alpha_1^2}{\ln 2} \left(\frac{\beta_1}{2} - 1 \right) \Gamma(3) \mathbb{U}(3, 4, 2). \quad (36)$$

From the above, the low-SNR approximation of the E_c^1 is given as follows

$$E_c^1 \approx \bar{C}_w + \rho \dot{C}_w + \frac{\rho^2}{2} \ddot{C}_w, \quad (37)$$

$$\approx \alpha_1 \frac{2\rho}{\ln 2} \Gamma(2) \mathbb{U}(2, 3, 2) + \alpha_1^2 \frac{\rho^2}{\ln 2} \left(\frac{\beta_1}{2} - 1 \right) \Gamma(3) \mathbb{U}(3, 4, 2). \quad (38)$$

As the objective function is increasing with α_2 (decreasing with α_1), the solution of the problem is the upper bound of α_2 (lower bound of α_1) given by the first constraint

$$E_c^1 \geq \tilde{E}_c^1 \iff \alpha_1 \frac{2\rho}{\ln 2} \Gamma(2) \mathbb{U}(2, 3, 2) + \alpha_1^2 \frac{\rho^2}{\ln 2} \left(\frac{\beta_1}{2} - 1 \right) \Gamma(3) \mathbb{U}(3, 4, 2) - \tilde{E}_c^1 \geq 0. \quad (39)$$

The solution of the previous inequality is as follows, if $0 < \tilde{E}_c^1 < \frac{1}{4 \ln 2 - 2 \ln 2 \beta_1} \implies$ the solution is

$$-2 \sqrt{\frac{1 + (2\beta_1 - 4) \ln 2 \tilde{E}_c^1}{\rho^2 (\beta_1 - 2)^2}} - \frac{2}{\rho (\beta_1 - 2)} \leq \alpha_1 \leq 2 \sqrt{\frac{1 + (2\beta_1 - 4) \ln 2 \tilde{E}_c^1}{\rho^2 (\beta_1 - 2)^2}} - \frac{2}{\rho (\beta_1 - 2)}. \quad (40)$$

The optimal solution is the lower bound on α_1 ,

$$\alpha_1 \geq -2 \sqrt{\frac{1 + (2\beta_1 - 4) \ln 2 \tilde{E}_c^1}{\rho^2 (\beta_1 - 2)^2}} - \frac{2}{\rho (\beta_1 - 2)}. \quad (41)$$

APPENDIX II: PROOF OF PROPOSITION 2

Using (2) and (5), the EC of U_1 is given by

$$E_c^1 = \frac{1}{\beta_1} \log_2 \left(\mathbb{E} \left[(1 + \rho |h_1|^2 \alpha_1)^{\beta_1} \right] \right). \quad (42)$$

For $\rho \gg 1$, we have that

$$E_c^1 \approx \frac{1}{\beta_1} \log_2 \left((\rho \alpha_1)^{\beta_1} \mathbb{E} \left[(|h_1|^2)^{\beta_1} \right] \right), \quad (43)$$

$$\approx \frac{1}{\beta_1} \log_2 \left((\rho \alpha_1)^{\beta_1} 2\Gamma(1 + \beta_1) U(1 + \beta_1, 2 + \beta_2, 2) \right). \quad (44)$$

The constraint $E_c^1 \geq \tilde{E}_c^1$ gives the following inequality

$$E_c^1 \geq \tilde{E}_c^1 \iff \frac{1}{\beta_1} \times \quad (45)$$

$$\log_2 \left((\rho \alpha_1)^{\beta_1} 2\Gamma(1 + \beta_1) U(1 + \beta_1, 2 + \beta_2, 2) \right) \geq \tilde{E}_c^1 \quad (46)$$

From there we only have to express α_1 as a function of the other parameters and take the lower bound to obtain (19) and then (20).

APPENDIX III: PROOF OF PROPOSITION 3

The low-SNR approximation of E_R^1 is obtained using the Maclaurin series expansion.

First, by inserting (9) in (10) we have that for U_1

$$E_R^1 \approx \frac{1}{\beta_1} \log_2 \left(\mathbb{E} \left[\epsilon + (1 - \epsilon) (1 + \rho \alpha_1 |h_1|^2)^{\beta_1} e^{a - \frac{a}{2}(1 + \rho \alpha_1 |h_1|^2)^{-2}} \right] \right), \quad (47)$$

where $a = \sqrt{N} \theta_1 Q^{-1}(\epsilon) \log_2 e$. So, we have that $\bar{C}_f = E_R^1|_{\rho=0} = \frac{1}{\beta_1} \log_2 \left(\epsilon + (1 - \epsilon) e^{\frac{a}{2}} \right)$.

The first derivative with respect to ρ is given by

$$\frac{\partial E_R^1}{\partial \rho} = \frac{1}{\beta_1 \ln 2} \left(\frac{Num}{Deno} \right), \quad (48)$$

with

$$Num = \mathbb{E} \left[(1 - \epsilon) \left(|h_1|^2 \alpha_1 \beta_1 (1 + \rho \alpha_1 |h_1|^2)^{\beta_1 - 1} + \alpha_1 |h_1|^2 a (1 + \rho \alpha_1 |h_1|^2)^{\beta_1 - 3} \right) e^{a - \frac{a}{2}(1 + \rho \alpha_1 |h_1|^2)^{-2}} \right], \quad (49)$$

$$Deno = \mathbb{E} \left[\epsilon + (1 - \epsilon) (1 + \rho \alpha_1 |h_1|^2)^{\beta_1} e^{a - \frac{a}{2}(1 + \rho \alpha_1 |h_1|^2)^{-2}} \right]. \quad (50)$$

Thus we have that

$$\begin{aligned} \dot{C}_f &= \frac{\partial E_R^1}{\partial \rho} \Big|_{\rho=0} \\ &= \frac{1}{\beta_1 \ln 2} \left(\frac{(1 - \epsilon) e^{\frac{a}{2}} (\beta_1 + a) \alpha_1 \mathbb{E} [|h_1|^2]}{\epsilon + (1 - \epsilon) e^{\frac{a}{2}}} \right). \end{aligned} \quad (52)$$

where $Y = \mathbb{E} [|h_1|^2]$ which can be calculated using (34).

From (48) we get the second derivative with respect to ρ

$$\frac{\partial^2 E_R^1}{\partial \rho^2} = \frac{1}{\beta_1 \ln 2} \left(\frac{\frac{\partial Num}{\partial \rho} Deno - \frac{\partial Deno}{\partial \rho} Num}{Deno^2} \right). \quad (53)$$

Knowing that $\frac{\partial Deno}{\partial \rho} = Num$, we have

$$\begin{aligned} \ddot{C}_f &= \frac{\partial^2 E_R^1}{\partial \rho^2} \Big|_{\rho=0} \\ &= \frac{1}{\beta_1 \ln 2} \left(\frac{\frac{\partial Num}{\partial \rho} \Big|_{\rho=0} Deno|_{\rho=0} - (Num|_{\rho=0})^2}{(Deno|_{\rho=0})^2} \right), \end{aligned} \quad (54)$$

where

$$\begin{aligned} \frac{\partial Num}{\partial \rho} \Big|_{\rho=0} &= (1 - \epsilon) e^{\frac{a}{2}} \times \\ & \left(\beta_1 (\beta_1 - 1) + a (\beta_1 - 3) + a \beta_1 + a^2 \right) \alpha_1^2 \mathbb{E} [|h_1|^4]. \end{aligned} \quad (55)$$

By using \bar{C}_f , (52) and (54) we obtain the Maclaurin series of E_R^1 given in (24). Then the lower bound of α_1 obtained from the inequality $E_R^1 \geq \bar{E}_R^1$ corresponds to (25).

APPENDIX IV: PROOF OF PROPOSITION 4

For $\rho \gg 1 \implies \sqrt{V_1} \rightarrow \log_2 e$, so that (10) can be approximated as follows for U_1 ,

$$E_R^{1'} \approx \frac{1}{\beta_1^F} \log_2 \left(\epsilon + (1 - \epsilon) (\rho \alpha_1)^{\beta_1^F} e^{\sqrt{N} \theta_1 Q^{-1}(\epsilon) \log_2 e} \mathbb{E} \left[(|h_1|^2)^{\beta_1^F} \right] \right). \quad (56)$$

$\mathbb{E} \left[(x_1)^{\beta_1^F} \right]$ is computed using (34) and inserted in (56) to get (26), which is used to solve the inequality given by the first constraint

$$E_R^1 \geq \bar{E}_R^1 \iff \quad (57)$$

$$\begin{aligned} & \frac{1}{\beta_1^F} \log_2 \left(\epsilon + 2(1 - \epsilon) (\rho \alpha_1)^{\beta_1^F} e^{\sqrt{N} \theta_1 Q^{-1}(\epsilon) \log_2 e} \right. \\ & \left. \times \Gamma(1 + \beta_1^F) \mathbb{U}(1 + \beta_1^F, 2 + \beta_1^F, 2) \right) \geq \bar{E}_R^1. \end{aligned} \quad (58)$$

From there we get the lower bound of α_1 , thus the upper bound of α_2 , that correspond to the optimal power for U_1 and U_2 given respectively in (27) and (28).

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