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Modal analysis of waveguide for the study of frequency bandgaps of a bounded periodic medium

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Abstract

A novel method based on waveguide modal analysis is presented to evaluate the effectiveness in terms of frequency bandgaps of bounded periodic metamaterials. The effect of the size of the bounded medium, i.e., the number of periodic unit cells, on the frequency bandgaps predicted by Floquet-Bloch theory in the infinite case is investigated and quantified. The waveguides considered in this work have a bounded cross-section and an infinite longitudinal axis simulated by Perfectly Matched Layers. Its useful area is made of a bounded 1D or 2D-periodic matrix-inclusion metamaterial. In the 2D-periodic case, an approach combining the waveguide modal analysis with the Floquet-Bloch transform is further proposed. Applying the proposed method, the SH-waves propagating in a waveguide are studied by using the finite element method. The discrete spectrum of the waveguide and the associated attenuated or trapped wave modes are calculated and analyzed, which provides, within a semi-analytic framework, an exact characterization regarding the attenuation coefficient and the frequency bandgaps of the studied bounded periodic medium. More particularly, effectiveness indicators are defined to compare the width and the position of frequency bandgaps between the bounded and infinite cases as a function of the cell number. Last but not least, the proposed method is also successfully applied to bounded periodic media with local resonators to characterize their filtering and attenuation effectiveness, which shows its interest in such a case of great practical interest.

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1. Introduction

In order to reduce the induced damage from waves generated principally by ambient vibrations or earthquakes in structures (e.g., the ones designed before the implementation of seismic standards), a promising strategy is to reduce the amplitude of seismic waves by locally treating the subsoil to modify its mechanical characteristics [1–13, among others]. Avilés and Sánchez-Sesma [14] show that it is possible to modify and reduce the seismic energy distribution for appropriately selected frequency ranges to limit the damage to the structure. In practice, the principal solution is the installation of multiple rows of barriers between a vibration source and the area of protection. Among the various types of barriers, trenches, solid or hollow piles are often used. These kind of solutions is commonly called seismic metamaterials.

Previous numerical and experimental works have been conducted in order to estimate the amplitude reduction or the shielding effectiveness as a function of both the mechanical and geometric characteristics of the inclusion (e.g., diameter, shape, solid or hollow, material properties' contrast between inclusion and soil matrix), the number of rows, the distance between two rows compared to the wavelength of the incoming waves [1, 6, 15–20, among others]. However, to measure the effectiveness of such a protection system, it is necessary to study the wave diffraction phenomena within a bounded domain made of periodic material [21].

In practice, the dispersive characteristics of a periodic medium with respect to the propagation of elastic waves are usually studied within the framework of an infinite domain using the Floquet-Bloch (F-B) theory [17, 18, 22–26, among others]. The F-B transform makes it possible to reduce the analysis of the behavior of an infinite periodic media to the modal analysis of a single elementary unit cell. In the published studies, the key point concerns the assessment of both the geometrical and mechanical parameters of the periodic rows of pile/trench barriers on the apparition of filtering/attenuation frequency ranges (also named bandgaps). Therefore, due to the fact that actually, the number of periodic matrix-inclusion cells is limited to a finite number, the identified beneficial properties of the assumption of an infinite domain, such as the existence of frequency bandgaps and their characteris-

34 tics (e.g., width and center frequency), must be double-checked against the
35 dimension of the bounded periodic domain [26–28].

36 Several numerical parametric studies have been conducted in order to as-
37 sess the differences between these two hypotheses [20, 29–31]. Previous works
38 on soil media subjected to surface waves principally evaluate the effective-
39 ness of a periodic medium as a function of the number of elementary cells or
40 rows that compose the barriers by comparing the values of the attenuation
41 in the frequency bandgaps [18, 25, 32, 33, among others]. They have been
42 mostly focused on the direct use of the finite element method (FEM) or the
43 boundary element method (BEM) to understand the isolation mechanism of
44 wave barriers. Most of them can be qualified as measurement-based, i.e.,
45 with a chosen recording zone separated by wave barriers from the sources,
46 they define an amplitude reduction factor in terms of the ratio between sur-
47 face displacement or acceleration amplitude with and without wave barriers.
48 However, an issue with this factor is that it does not provide an exact char-
49 acterization of wave barriers; for instance, it depends on the chosen studied
50 zone and is generally not uniform, so an averaged amplitude reduction factor
51 must be defined. Another issue concerns the meshing complexity associated
52 with elastic properties discontinuities due to the barrier inclusions and the
53 large size of the numerical model imposed by the number of propagated wave-
54 lengths, which drastically increases the computation time. As an example,
55 Jenson studied in [34] the vibrational response of finite 1-D and 2-D mass-
56 spring periodic lattice structures subjected to periodic loadings. One of his
57 results is to show the convergence of the response in the bandgap frequency
58 range as a function of the unit cell number by considering the frequency
59 response functions (FRF) of the structure recorded at the end of the finite
60 periodic barrier. He studied, in the 1D case, a periodic structure composed
61 of up to 10 unit cells and, in the 2D case, up to 7x7 unit cells. In the 2D
62 case, only the case of loadings concentrated at a point is considered and the
63 finite periodic structure is characterized by the FRF response at only one
64 point located on the other side. Furthermore, the mass-spring framework of
65 Jenson’s study cannot be easily extended to periodic structures exhibiting
66 complex microstructural geometry.

67 Thus, in order to assess for a bounded periodic metamaterial the ef-
68 fectiveness of the analytically predicted frequency bandgaps using the F-B
69 theory, a novel method based on the waveguide modal analysis is proposed
70 in this work. The waveguides considered in this work have a bounded cross-
71 section and an infinite longitudinal axis which is simulated with Perfectly

72 Matched Layers (PMLs) [35, 36, among others]. The waveguide modal anal-
73 ysis method was initially developed and used to highlight modes trapped by
74 irregularities [37–40] or to find defects in the propagation media [41, 42]. In
75 this paper, this approach is extended and adapted to characterize a heteroge-
76 neous bounded periodic medium, e.g., its discrete spectrum, and attenuated
77 or trapped modes. Hence, in contrast to the measurement-based direct nu-
78 merical modeling, the proposed approach provides exact characterization,
79 within a semi-analytic framework, in terms of attenuation coefficient and
80 bandgaps for the studied problem.

81 Applying the proposed method, parametric studies can be carried out so
82 as to quantitatively assess both the filtering and attenuation effectiveness of
83 bounded periodic systems, which should converge to the modal solution of
84 the F-B analysis when the system becomes infinite. Indeed, wave modes in
85 a bounded periodic medium are calculated and characterized, and the effect
86 of the latter’s size (i.e., the number of cells) on their attenuation or filtering
87 is quantified regarding both the position and width of bandgaps compared
88 with those of the corresponding infinite domain predicted by the F-B theory.

89 Otherwise, in many practical applications, it is also important to limit the
90 size of periodic cells (especially in the case of urban installations). However,
91 Bragg scattering bandgaps have their frequency ranges directly linked to the
92 size of periodic cells. Thus, to obtain bandgaps at frequencies much lower
93 than those expected by the Bragg diffraction phenomenon, local resonance
94 bandgaps were introduced [43], which have since aroused great interest, for
95 instance, in the field of civil engineering for the design of metabarriers. The
96 proposed waveguide-base modal analysis approach is also applied to bounded
97 periodic media with local resonators to characterize their filtering and atten-
98 uation effectiveness.

99 The outline of the paper is as follows. In Section 2, after the definition of
100 the model problem considered in this study and a brief introduction of the
101 Floquet-Bloch theory, the modal analysis of waveguides is presented, and its
102 extension to a bounded periodic domain is proposed. In Section 3, using the
103 proposed waveguide-based approach, the numerical investigation of bounded
104 1D-periodic media, *i.e.*, periodic media with a periodicity in one direction,
105 is conducted and compared to the infinite case. In Section 4, to study a 2D-
106 periodic medium, *i.e.*, a periodic medium with a periodicity in two directions,
107 a new strategy to combine the F-B theory and the waveguide method is
108 presented. Then again, parametric studies are carried out to characterize
109 the studied bounded 2D-periodic medium. Finally, Section 5 applies the

110 proposed method to a bounded 1D-periodic matrix-inclusion medium with
 111 local resonators and shows its interest in this case of great practical utility.

112 2. Model wave propagation problem and modal analysis of waveg- 113 uides applied to bounded periodic media

114 The model problem considered herein is the propagation of SH waves in
 115 an elastic medium Ω , which is governed by the following equation in the
 116 frequency domain:

$$\nabla_{\mathbf{x}} \cdot (\mu(\mathbf{x}) \nabla_{\mathbf{x}} u(\mathbf{x})) = -\rho \omega^2 u(\mathbf{x}). \quad (1)$$

117 where $\mathbf{x} = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2$, $u(\mathbf{x}) \equiv u_z(\mathbf{x})$ is the out-of-plane displacement re-
 118 duced to a scalar component, and $\mu(\mathbf{x})$ and $\rho(\mathbf{x})$ are respectively the shear
 119 modulus and the density.

120 In this study, it is assumed that the model problem Eq.(1) is defined in a
 121 2D periodic medium, infinite or bounded. More particularly, a medium with
 122 periodically located circular inclusions is considered without loss of generality
 (Figure 1).

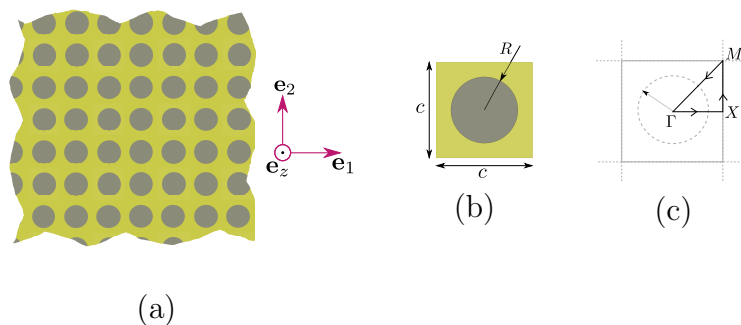


Figure 1: (a) A 2D-periodic medium with unit cells consisting of a matrix and a circular inclusion; (b) Unit cell Q_0 in physical space; (c) Corresponding reciprocal cell, the first Brillouin zone Q^0 in reciprocal space, on which the contour of the Brillouin irreducible zone, $\Gamma \rightarrow X \rightarrow M \rightarrow \Gamma$, is represented.

123 In the case of an infinite periodic medium, the analysis of its dispersion
 124 properties is usually performed using the F-B transform, which makes it
 125 possible to reduce the analysis of the behavior of an infinite periodic media to
 126 the modal analysis of a single unit cell, denoted Q_0 hereafter (Figure 1(b) for
 127

128 an example of Q_0 composed of a matrix and an inclusion for the 2D studied
 129 case in the present work). Indeed, the F-B transform of a non periodic
 130 function $u(\mathbf{x})$ defined on Ω reads as:

$$U^B(\mathbf{x}, \mathbf{k}) = \sum_{\{m_j\} \in Z^d} u(\mathbf{x} + \sum_{j=1,d} m_j \mathbf{g}_j) e^{i\mathbf{k} \cdot (\mathbf{x} + \sum_{j=1,d} m_j \mathbf{g}_j)} \quad (2)$$

131 with $\{\mathbf{g}_j\}_{j=1,d}$ the basis of periodicity vectors of Ω , in the present work $d = 1$
 132 or 2. In Eq.(2), $\mathbf{k} \in Q^0$, is the Bloch wave vector that evolves in the reciprocal
 133 space limited to Q^0 the Brillouin irreducible zone (Figure 1(c) for an example
 134 of Q^0 for the 2D studied case in the present work).

135 Applying the Floquet-Bloch transform Eq.(2) to Eq.(1) gives rise to the
 136 following eigenvalue problem:

$$(\nabla_{\mathbf{x}} + i\mathbf{k}) \cdot (\mu(\mathbf{x})(\nabla_{\mathbf{x}} + i\mathbf{k})U^B(\mathbf{x}, \mathbf{k})) = -\rho(\mathbf{x})\omega^2 U^B(\mathbf{x}, \mathbf{k}) \quad (3)$$

137 then, dispersion surfaces $\omega(\mathbf{k})$ can be obtained by solving this eigenvalue
 138 problem for the wave vector $\mathbf{k} \in Q^0$. Furthermore, if Eq.(3) is solved only
 139 for the wave vector \mathbf{k} belonging to the contour of the irreducible Brillouin
 140 zone, dispersion curves are obtained instead [44].

141 For the eigenvalue problem Eq.(3) defined on the unit cell Q_0 , it is gener-
 142 ally necessary to use numerical methods to calculate its solutions. Then,
 143 the dispersion curves of the medium under consideration can be plotted. In
 144 the present work, for a given value of \mathbf{k} , the computed eigenvalue is $\omega^2(\mathbf{k})$,
 145 solved using the commercial finite element code, Comsol-Multiphysics. The
 146 dispersion curves obtained from infinite periodic media will serve as reference
 147 results, useful for comparison studies with bounded periodic media.

148 A real protection system exploiting a periodic medium is generally not
 149 infinite, and the modal characterization of such a bounded system cannot
 150 be done with the F-B transform approach. To characterize such a bounded
 151 system, surrounded by an infinite domain like the soil, a temporal analysis
 152 could be done. However, depending on the size of the bounded periodic
 153 medium to be considered and the number of cells, the numerical simulation
 154 becomes very expensive due to the fact that the element size must be refined
 155 with respect to the characteristic length of heterogeneities.

156 Now, a method based on the waveguide modal analysis [37, 39, 40] is
 157 proposed. It provides a semi-analytic solution for the bounded system sur-
 158 rounded by an infinite domain. It allows a quantitative assessment of the
 159 filtering and attenuation effectiveness of bounded periodic systems, which

160 should converge to the modal solution of the F-B analysis when the system
 161 becomes infinite.

162 The waveguides considered in this work are a system that allows waves to
 163 be guided and has a bounded cross-section and an infinite longitudinal axis.
 164 One of the methods to study this family of waveguides is the simulation of
 165 infinite media using Perfectly Matched Layers (PMLs) [35, 36, among others].
 166 The main idea is based on the fact that the modes obtained by modal analysis
 167 of a waveguide could describe both the behavior of the infinite medium and
 168 the defects included in a localized zone. Thanks to the use of the PMLs, it is
 169 possible to separate the modes corresponding to the behavior of the medium
 170 at infinity from those localized because trapped and/or attenuated due to
 171 the presence of local defects.

172 Therefore, the first goal of this section is to distinguish the modes corre-
 173 sponding to the behavior of the infinite medium and those trapped and/or
 174 attenuated by a bounded periodic medium containing a finite number of cells.

175 *2.1. Elements of the spectral theory for waveguides including a specific zone*
 176 *of interest*

177 In this part, some basic elements of the spectral theory for partial differ-
 178 ential operators are provided. Let us consider the eigenvalue problem of a
 179 waveguide with a specific zone of interest governed by an equation of type:

$$\mathbb{A}(\mathbf{u}) = \lambda \mathbf{u} \tag{4}$$

180 where \mathbb{A} is a self-adjoint linear operator. The specific zone is the one that
 181 contains irregularities and breaks the homogeneity of the waveguide. For the
 182 studied case defined by Eq.(1), we have $\lambda = \omega^2$ and $\mathbb{A}(\mathbf{u}) = -\frac{1}{\rho} \nabla_{\mathbf{x}} \cdot (\mu(\mathbf{x}) \nabla_{\mathbf{x}} u(\mathbf{x}))$.

183 The modal analysis of a waveguide allows the calculation of the spectrum
 184 of the operator \mathbb{A} , denoted $\sigma(\mathbb{A})$, which is the complement of the set of
 185 complexes $\rho(\mathbb{A})$, the resolvent set of \mathbb{A} , defined as:

$$\rho(\mathbb{A}) = \{\Lambda \in \mathbb{C}, \mathbb{A} - \Lambda \mathbb{I} \text{ is invertible}\}. \tag{5}$$

186 For a bounded system, $\sigma(\mathbb{A})$ is in fact the set of eigenvalues defined as:

$$\sigma(\mathbb{A}) = \{\lambda \in \mathbb{C}, \mathbb{A} \mathbf{u} = \lambda \mathbf{u} \text{ et } \mathbf{u} \neq \mathbf{0}\} \tag{6}$$

187 However, for an open system like the waveguide, it is considered that \mathbb{A} is
 188 an unbounded operator. Therefore, it is not always possible to have equality

189 for the eigenvalue system, but only a limit. This is the case of the essential
 190 spectrum of \mathbb{A} , denoted $\sigma_{ess}(\mathbb{A})$ and defined below.

191 Indeed, the spectrum $\sigma(\mathbb{A})$ can be decomposed into two sets: $\sigma_{disc}(\mathbb{A})$ the
 192 discrete spectrum (i.e., the set consisting of all the isolated eigenvalues of the
 193 spectrum) and $\sigma_{ess}(\mathbb{A})$ the essential spectrum (i.e., the complement of σ_{disc}
 194 in the spectrum):

$$\sigma_{ess}(\mathbb{A}) = \{\lambda \in \mathbb{C}, \mathbb{A} - \lambda\mathbb{I} \text{ is not a Fredholm operator of index } 0\} \quad (7)$$

195 Then, the behavior of the waveguide is studied by separating $\sigma_{disc}(\mathbb{A})$
 196 from $\sigma_{ess}(\mathbb{A})$. To accomplish this, the idea is to use the PMLs to represent
 197 the infinite waveguide. It consists of performing a change of geometric vari-
 198 ables to complex that directs the essential spectrum in the complex plane
 199 and reveals the discrete eigenvalues distributed on or around the real axis.
 200 Hence, the modal analysis of waveguides allows the differentiation between
 201 the discrete spectrum of the modes in the bounded domain, which is inter-
 202 esting for the study, and the essential spectrum of the modes in the PMLs,
 203 which represent the infinite medium (i.e., the geometrical configuration of
 204 the waveguide).

205 The modes describing the solution of the wave propagation inside the
 206 PMLs are not considered in the following. The domain of interest being in
 207 the bounded domain, only the discrete spectrum modes are considered in the
 208 sequel.

209 In this work, so as to perform the modal analysis of the studied waveg-
 210 uide, contrary to the usual definition of PMLs [35, 45], the properties in the
 211 subdomains represented by the PMLs are independent of the wave frequency
 212 [46]. This modification eliminates the frequency dependence of the mass and
 213 stiffness matrices, making modal analysis possible.

214 *2.2. Configuration of the bounded periodic matrix/inclusion medium with a* 215 *waveguide approach*

216 As recalled before, in this work the approach is applied to quantitatively
 217 characterize a bounded periodic matrix/inclusion medium. The aim is to
 218 quantify the effect of the finiteness of the periodic medium on its filtering
 219 properties and to overcome the limitation of the F-B theory that is only
 220 capable of dealing with an infinite number of cells.

221 For this purpose, a 1D waveguide model of a total length L_{tot} and with a
 222 finite number n_{cells} of matrix/inclusion unit cells is defined (Figure 2). The

223 central part of the waveguide is the domain of interest of length L_u , also
 224 called the useful area, which is delimited on both sides by two PMLs of
 225 length L_{PML} . The useful area is made up of n_{cells} cells in its center and a
 226 marginal zone of length L_m on each side:

$$L_{tot} = L_u + 2L_{PML}, \quad L_u = n_{cells}c + 2L_m \quad (8)$$

227 The role of the marginal zones between the edge of the bounded periodic
 228 medium and the PMLs is to limit the effects of the periodic medium on
 229 the PMLs. The different parameters, L_{tot} , L_u , L_m , and L_{PML} , need to be
 230 adjusted and are presented below.

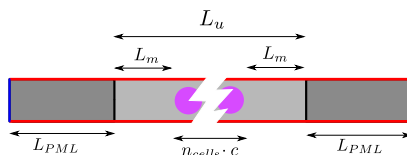


Figure 2: 1D waveguide model composed of: 1) a useful area of length L_u , which contains, at the center, n_{cells} matrix-inclusion unit cells of size c consisting of a matrix (light grey) and a circular inclusion (purple), and at each end, a marginal zone of length L_m ; 2) two PMLs (dark grey) of length L_{PML} , with free boundary conditions (on red edges) and clamped boundary conditions (on blue edges).

231 As stated above, the PMLs are used, and they are defined through the
 232 following change of variables to complex [39]:

$$\chi_\theta(x_1) = \begin{cases} -\frac{L_u}{2} + (x_1 + \frac{L_u}{2})e^{\eta^{left}} & \text{if } x_1 \leq -\frac{L_u}{2} \\ x_1 & \text{si } |x_1| < \frac{L_u}{2} \\ +\frac{L_u}{2} + (x_1 - \frac{L_u}{2})e^{\eta^{right}} & \text{si } x_1 \geq \frac{L_u}{2} \end{cases} \quad (9)$$

233 where $\eta^{left/right} = \pm i\theta$ allows to choose the rotation of the essential spectrum
 234 in the complex plane. In the following, the two PMLs are said to be con-
 235 jugated if they are defined such that $\eta^{left} = \overline{\eta^{right}}$, while in the case where
 236 $\eta^{left} = \eta^{right}$, they are said to be classical. In an arbitrary way having no
 237 impact on the results, we choose $\eta^{left} = -i\theta$. In what follows, the results
 238 are mainly obtained with conjugated PMLs, unless it is explicitly stated that
 239 classical PMLs are used.

240 As previously mentioned, contrary to the usual properties of PMLs, the
 241 damping of the absorbing layer does not depend on the frequency (i.e., θ is

242 independent of the frequency). The change of variables in Eq.(9) modifies the
 243 differential operator of the problem Eq.(1) only in the PMLs. The resolution
 244 of the eigenvalue problem associated with the modified operator then consists
 245 of two parts, one giving the modal solutions of the finite problem, (i.e., in
 246 the area where the bounded periodic medium is located), and the other
 247 concerning the modal response of the PMLs.

248 2.3. Spectrum and characteristic modes in a homogeneous waveguide

249 In the following, for the numerical simulations, the geometry parameters
 250 and the material properties of the studied periodic media are given in Ta-
 251 ble I. They are chosen to be more in agreement with the Civil Engineering
 252 applications. For instance, inclusions are more rigid than the matrix, and c
 253 the unit cell’s size is equal to 1 m, which corresponds to what is feasible in
 practice.

c (m)	R (m)	μ_m (GPa)	μ_i (GPa)	ρ_m (kg/m ³)	ρ_i (kg/m ³)
1	0.346	0.233	30	1750	2400

Table I: Summary of the data used for the simulation of the 2D-periodic medium, where
 subscripts “ m ” and “ i ” represent the soil matrix and the inclusions, respectively.

254
 255 The Figure 3 represents the case of a uniform waveguide (i.e., a waveguide
 256 without any matrix-inclusion cells). The homogeneous material behavior is
 257 defined by the shear modulus μ_m and mass density ρ_m while the elastic
 258 inclusion by corresponding quantities μ_i and ρ_i (Table I). The dimensions
 259 chosen for this homogeneous waveguide are $L_{tot} = 40$ m, $L_u = 20$ m, and
 260 $L_{PML} = L_u/2 = 10$ m. In the following, we choose to always have $L_{PML} =$
 261 $L_u/2$. The solution of this homogeneous case serves as a reference to see the
 262 evolution of the behavior of the waveguide when a bounded periodic medium
 263 is added and to account for new phenomena.

264 In summary, two types of modes are observed in the homogeneous wave-
 265 guide. The first type groups the modes associated with complex eigenvalues
 266 belonging to $\sigma_{ess}(\mathbb{A})$ the essential spectrum of the operator \mathbb{A} (displayed in
 267 black in the Figure 3), which is directed in the complex plane because of
 268 the definition of the PMLs. These modes evolve in the PMLs, and an exam-
 269 ple of them is given in the Figure 4(a). The change of variable to complex
 270 performed in PMLs modifies the geometry to model an infinite domain by a
 271 finite model. The parameter θ in Eq.(9) is, in fact, the angle made in the

272 complex plane by the first branch of the essential spectrum with the real axis
 273 (which corresponds to the spectrum in an infinite homogeneous domain).

274 The second type groups are the modes associated with real eigenvalues
 275 (displayed in cyan in the Figure 3), and an example of them is shown in the
 276 Figure 4(b). These modes are propagative within the useful area without
 277 attenuation or reflection. They are evanescent in the PMLs because of their
 278 complex part. Indeed, without heterogeneity, waves propagate in the wave-
 279 guide without attenuation or reflection, which results in real eigenvalues. For
 280 numerical simulation, an eigenvalue ω is considered a real number when its
 imaginary part is between -0.01 and 0.01 .

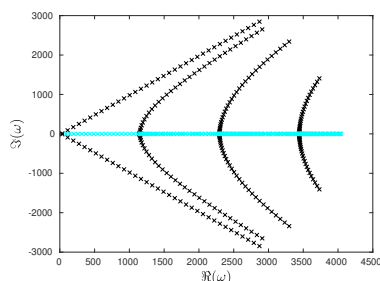


Figure 3: Spectrum of the homogeneous waveguide in the complex plane: σ_{disc} with real eigenvalues (cyan) and σ_{ess} (black).

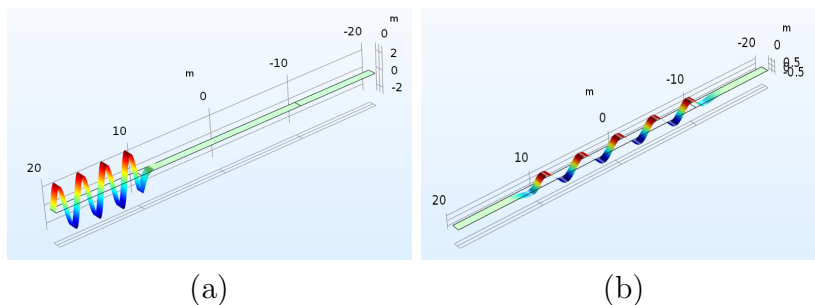


Figure 4: Two types of modes in a homogeneous waveguide modeled using PMLs. (a) Propagative mode contained in PMLs; (b) Propagative mode in the useful area without reflection or attenuation.

281

282 2.4. Parameters setup of waveguide models used for the characterization of
 283 bounded periodic media

284 First, it is useful to get the dispersion curves for an infinite periodic
 285 medium in this 1D case using the F-B approach. The geometry parameters
 286 and the material properties of the unit cell defined in Table I are used. As
 287 the periodicity is only in the \mathbf{e}_1 direction and free edge conditions are applied
 288 on the horizontal boundaries, the Brillouin irreducible zone is reduced to a
 289 segment: $-X \rightarrow \Gamma \rightarrow X$ (Figure 5(left)). The dispersion curves are obtained
 290 from the numerical solution of the eigenvalue problem (3), for $\mathbf{k} = k\mathbf{e}_1$ with k
 291 evolving between $-\frac{\pi}{c}$ and $\frac{\pi}{c}$. Figure 5(right) thus presents the eigenvalues
 292 $\omega(\mathbf{k})$ calculated numerically on the unit cell. For each value of the Bloch
 293 wave vector \mathbf{k} , the first six modes are calculated. It is noted the existence of
 294 frequency bands (in purple), for which there is no corresponding real wave
 295 vector \mathbf{k} . These are frequency bandgaps of the periodic medium. The first
 296 one appears around 300 Hz in particular.

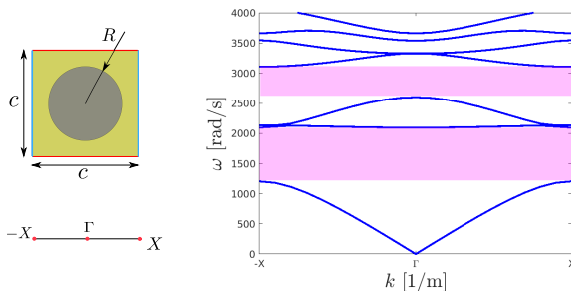


Figure 5: Unit cell Q_0 and associated Brillouin zone $Q^0 = -X \rightarrow \Gamma \rightarrow X$ (left) used to obtain the dispersion curves (right). The periodicity conditions are drawn in blue, and the edges in red are free edges.

297 Now, bounded periodic domains are considered, and it is expected that
 298 the presence of a finite number of periodically placed matrix-inclusion cells
 299 in the useful area (L_u) of the waveguide modifies the spectrum. Thus, to
 300 properly compare such a waveguide with the reference homogeneous waveguide,
 301 the parameters introduced previously must be chosen in an appropriate way.
 302 A key point here is to assess the effect of the marginal zone (L_m) on the
 303 obtained spectrum and characteristic modes of the waveguide.

304 To study the influence of L_u , (i.e., the size of the useful area) on the
 305 evolution of the valid frequency domain, an example is first studied with
 306 a single unit cell to observe the spectrum and characteristic modes of the

307 waveguide. For this example, the parameters are $L_u = c$, and $L_m = 0$ m
 308 (Figure 6(a)). The parameters used for the geometry and properties are
 309 those of Table I. The eigenvalue solutions (ω) of the problem (Eq.(1)) for
 310 the waveguide are shown in the complex plane in the Figure 6, with the
 311 dispersion curves of the infinite periodic medium (*cf.* Figure 5) shown in red
 312 for comparison.

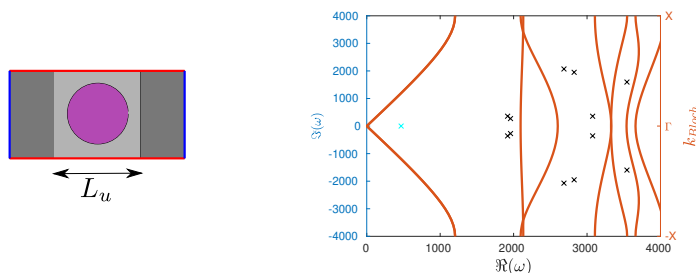


Figure 6: Study of a 1D waveguide model with $L_u = c = 1$ m and $L_m = 0$ m. (left) Model definition; (right) Eigenvalue solutions showing lack of information on the low-frequency propagative modes, and compared with the behaviour of the infinite periodic medium solved with F-B approach (presented in red).

313 According to the obtained results, a first problem appears: due to the
 314 very short length of the model, (i.e., L_u is very small), the modal analysis of
 315 the waveguide does not give enough information in the low-frequency domain.
 316 Thus, the density of available modes in $[0, 4500]$ rad/s the frequency range
 317 of interest is low compared to the homogeneous solution (Figure 3). Indeed,
 318 the longest wavelength taken into account by the waveguide model is of the
 319 same order of magnitude as L_u , which, in the current case, corresponds to
 320 the frequency of about 4000 rad/s. So the frequencies taken into account by
 321 this model are higher than the first two bandgaps considered here.

322 It is, therefore, necessary to increase $L_{tot} = 2 \cdot L_u$ the total length of the
 323 waveguide to reach the low-frequency range. For that, one of the solutions is
 324 to increase the length of the margins L_m between the PMLs and the bounded
 325 periodic medium so that L_u is close to the longest wavelength of interest. In
 326 the case of the waveguide with a single cell, this choice results in a value of
 327 L_m almost equal to the longest wavelength considered.

328 However, when the number of cells increases (n_{cells}), if it is chosen to
 329 keep L_m unchanged, the waveguide becomes longer and longer. This choice
 330 leads to obtaining many modes, which are not interesting because they evolve
 331 outside the bounded periodic medium and pollute the interpretation of the

332 results. It is, therefore, necessary to decrease L_m so that the size of the
 333 waveguide does not increase with the number of cells.

334 Other ways around this problem are out of scope for this paper but in
 335 summary, for the setup of the numerical waveguide model, the final solution
 336 adopted is to keep the length $L_{tot} = 2 \cdot L_u$ constant, large enough to analyze
 337 up to a large number of cells, while to adapt the length L_m according to the
 338 number of cells. Another advantage of such a setup is that the frequency
 339 domain in which all waveguides evolve (with different numbers of inclusions)
 340 is identical, so the comparison is easier. Thus in the following, it is proposed
 341 to use a waveguide of the same length, the one already defined for the study of
 342 the homogeneous waveguide in the previous section, whatever the considered
 343 number of cells.

344 Then the case of a single cell is studied again with this setup, and the
 345 obtained spectrum is represented in the Figure 7. As expected, the same
 346 essential spectrum, now represented by magenta dots, is obtained (Figure 3).
 347 Indeed, the existence of a single cell far away from the PMLs does not signif-
 348 icantly disturb the essential spectrum. As the essential spectrum represents
 349 the behavior of the waves in the PMLs (i.e., at infinity), they are not taken
 350 into account in the following analysis of the attenuated frequency bands. It
 351 is rather the analysis of the other modes, associated with the discrete spec-
 352 trum and concentrated around the real axis, will highlight the attenuation
 353 and the convergence to the bandgaps predicted by the F-B theory for the
 infinite periodic network.

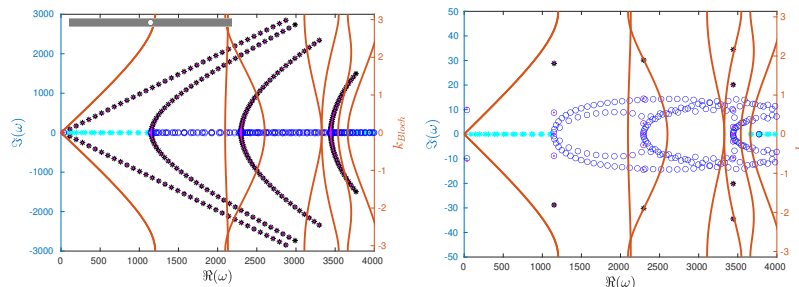


Figure 7: Spectrum (left) and its zoom around the real axis (right) of the studied waveguide with a single unit cell, compared with the dispersion curves (in red) of the corresponding infinite periodic medium obtained by F-B analysis.

354

355 Within the discrete spectrum, the modes associated with a complex eigen-
 356 value with a nonzero imaginary part, displayed in blue or black in the Fig-
 357 ure 7, are attenuated and show two types of behavior, illustrated in the

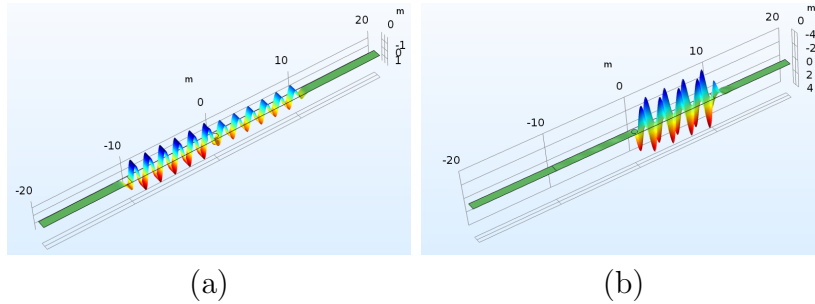


Figure 8: Two types of modes in the discrete spectrum of a waveguide with a single inclusion. (a) Mode attenuated by the presence of an inclusion; (b) Mode reflected by the presence of the inclusion.

358 Figure 8. The discrimination between these two types of modes is set to a
 359 value of the imaginary part equal to 20 rad/s. This value corresponds to
 360 95% attenuation of signal over a distance of $4c$. The Figure 8(a) presents
 361 a mode attenuated by the presence of the matrix-inclusion that still propa-
 362 gates in the medium (displayed in blue in the Figure 7). The eigenfrequency
 363 of this mode is $\omega = 1338 + 5.45i$ rad/s. The Figure 8(b) shows a mode not
 364 transmitted after the inclusion, as its eigenvalue has a rather large imaginary
 365 part (displayed in black in the Figure 7) The eigenfrequency corresponding
 366 to this mode is $\omega = 1497 + 10.5i$ rad/s. Such a mode can be considered as a
 367 mode reflected by the matrix-inclusion cell.

368 A comparison between the discrete spectrum and the dispersion curves of
 369 the infinite periodic medium shows that the study of a bounded medium with
 370 only one cell does not agree at all with the modeling of an infinite medium
 371 (Figure 7), which is quite normal. Indeed, only the first passband, between
 372 0 and 1200 rad/s, is completely recovered by the real discrete eigenvalues.
 373 Otherwise, nearly all the other eigenvalues in σ_{disc} are complex numbers and
 374 do not give rise to well-distinguished bandgaps. Now the main idea is to use
 375 this waveguide model to determine the minimum number of matrix-inclusion
 376 cells needed to recover the bandgaps behavior of the infinite medium within
 377 the frequency range of interest considered.

378 In practice, the number of periodic matrix-inclusion cells is limited to
 379 a finite number. For instance, modifying soils in a dense urban area can
 380 be costly due to various constraints. It is thus of significant interest to
 381 evaluate with precision for a bounded periodic metamaterial the effectiveness
 382 of the analytically predicted frequency bandgaps using Floquet-Bloch theory

383 regarding their position and bandwidth.

384 **3. Quantitative analysis of effective properties of bandgaps of 1D** 385 **bounded periodic media**

386 The parametric study using numerical models based on the above-mentioned
387 principle is now presented. The influence of the number of periodically placed
388 cells on the wave modes is analyzed. The results are compared with those of
389 the corresponding infinite domain obtained by considering only the unit cell
390 using the F-B transform.

391 *3.1. Parametric study setup, analysis of discrete spectra, and characteriza-* 392 *tion of modes*

393 As explained in the previous section, in order to consider identical fre-
394 quency ranges (i.e. corresponding to comparable wavelengths), the length of
395 the waveguide L_u is determined by the largest number of cells considered in
396 this study. Thus, with the choice of a periodic cell of side $c = 1$ m and a
397 maximum number of eighteen inclusions, the useful zone is chosen of length
398 $L_u = 20$ m. The length of each PML is chosen to equal half the total length
399 of the useful area L_u , $L_{PML} = 10$ m, in all models in order to limit the
400 number of modes of the essential spectrum calculated. A consequence of
401 keeping the waveguide length constant is that the size of the margins varies
402 according to the number of cells to be considered. The material properties
403 and parameters of the elementary cells are defined in Table I.

404 In the following, the modal analysis of the waveguide, including a bounded
405 periodic medium and, in particular, its discrete spectrum, is compared to
406 the dispersion curves obtained theoretically for the corresponding infinite
407 periodic medium (Figure 5). The results for four values of the number of
408 cells, 3, 9, 15, and 18, are presented in Figure 9.

409 As for the 1-cell case (Figure 7), the computed eigenvalues are repre-
410 sented in the complex plane: the real eigenvalues are shown in cyan, the
411 complex ones with an imaginary part less than 20 rad/s in blue, and the
412 other complex ones in black. The essential spectrum of the model without
413 inclusions is represented by magenta dots. Systematically, the figures on the
414 right are enlargements of those on the left of the area around the axis of the
415 reals, and so, in particular, they do not display most of the modes of the
416 essential spectrum. The dispersion curves predicted by the F-B theory of the
417 corresponding infinite periodic medium are plotted here in red.

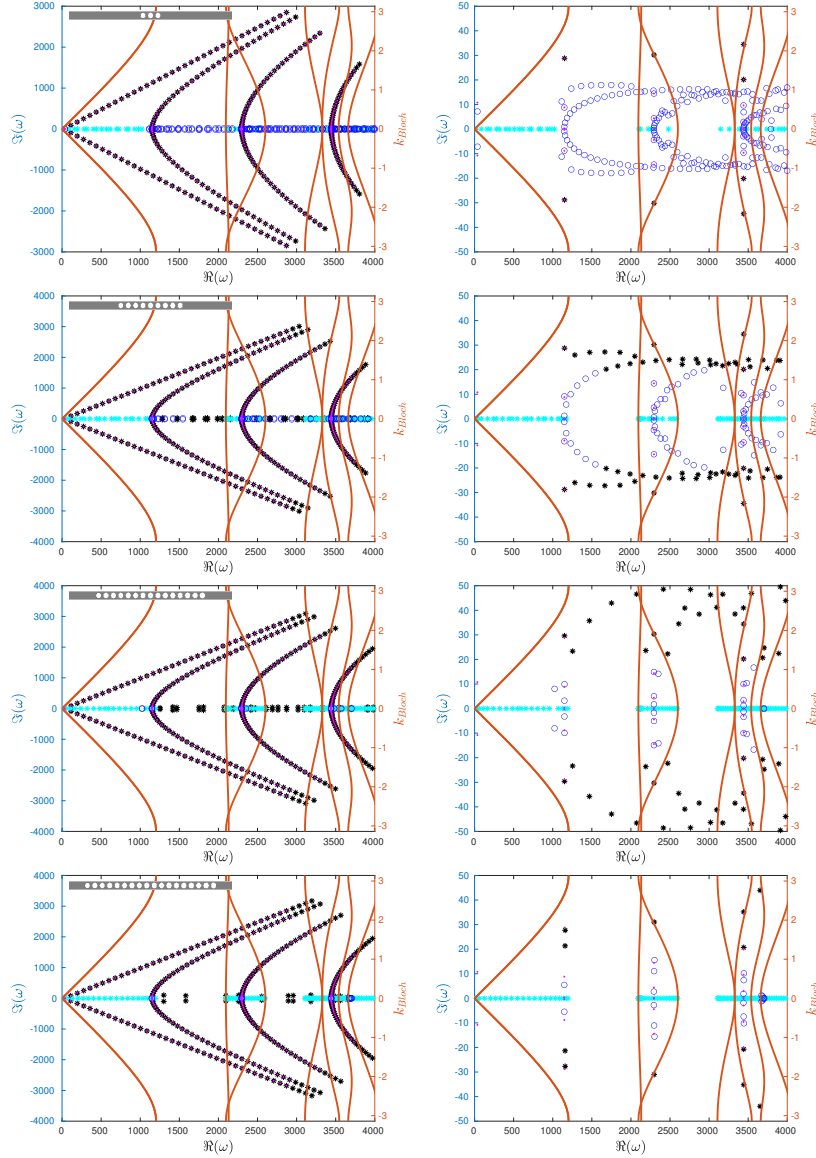


Figure 9: Parametric study on the influence of the number of cells on the eigenvalue spectrum and comparison with the dispersion curves (in red) obtained with the F-B theory for the corresponding infinite periodic medium. Spectrum (left) and its zoom around the real axis (right) of the studied waveguides with 3, 9, 15, and 18 unit cells. The real eigenvalues of the discrete spectrum are displayed in cyan. The complex eigenvalues with an imaginary part smaller than 20 rad/s are displayed in blue, and the others in black. The essential spectrum of the model without inclusions is displayed with magenta dots.

418 Again from Figure 9, as expected, the larger the number of cells, the
 419 fewer the modes with real eigenvalues (cyan) or complex eigenvalues with a
 420 small imaginary part (blue) in the frequency bandgaps predicted by the F-B
 421 theory. The modes associated with a complex eigenvalue that remain in the
 422 bandgaps are either a reflected mode concentrated in the margins between the
 423 PML and the edge of the bounded periodic medium (Figure 8(b)) or a mode
 424 with a decreasing amplitude into the periodic media like the one displayed
 425 on the Figure 10. Identifying the type between both of these modes cannot
 be done without analyzing their shapes.

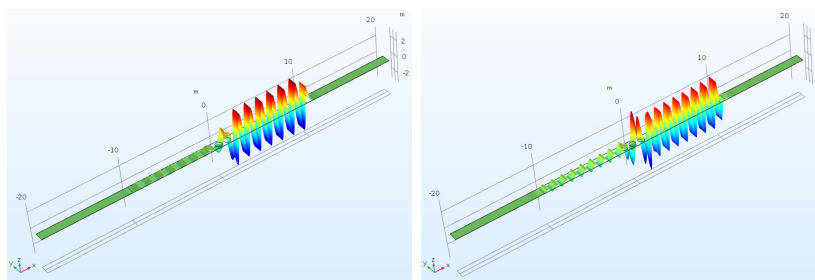


Figure 10: Mode attenuated into the periodic media (case with 3 inclusions). The imaginary part is equal to 17.9 rad/s (left) and to 15.1 rad/s (right).

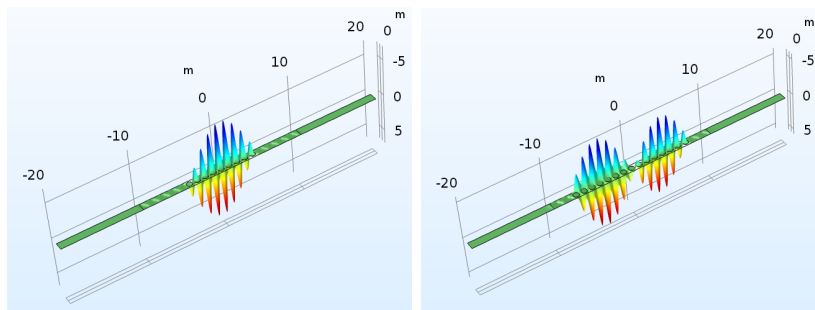


Figure 11: Trapped modes in bounded periodic media with respectively (left) 9 and (right) 15 inclusions.

426
 427 Moreover, the imaginary parts of the complex modes of the discrete spec-
 428 trum increase with the number of cells. Thus, the type of modes that are
 429 only attenuated but still able to propagate to the other side of the bounded
 430 periodic medium, observed in the case with a small number of cells (Figure 8
 431 (a)), is less present in the case with a larger number of cells. For instance,
 432 in the case of five cells, such modes are no longer visible among the calcu-
 433 lated ones. Thus, the complex modes of the discrete spectrum within the

434 bandgaps are all like the modes shown in (Figure 8(b)) and in Figure 10,
 435 i.e., “*stopped*” or “*reflected*” by the bounded periodic medium. Hence, the
 436 waveguide’s behavior approaches that of an infinite medium in terms of band
 437 gaps when the number of inclusions increases. For the two modes shown in
 438 Figure 10, the values of the eigenfrequencies are $\omega = 1749 + 17.9i$ rad/s and
 439 $\omega = 2128 + 15.1i$ rad/s.

440 Furthermore, the increase in the number of cells also makes it possible to
 441 observe the appearance of trapped modes in the bounded periodic medium.
 442 Two examples in the cases with respectively nine and fifteen cells are given
 443 in Figure 11. These modes are trapped in the bounded periodic medium;
 444 in other words, they are propagative modes in the corresponding infinite
 445 periodic medium. The corresponding eigenfrequencies are $\omega = 2096$ rad/s
 446 (9 cells) and $\omega = 2097$ rad/s (15 cells). In the spectrum, the eigenvalues
 447 of these trapped modes are always real, whatever the model of PML used
 448 in the numerical model, conjugate or classical. Hence, the analysis of the
 449 spectrum obtained with the classical PMLs makes it possible to differenti-
 450 ate them from the propagative modes obtained outside frequency bandgaps
 451 because these bandgaps are associated with complex eigenvalues. The Fig-
 452 ure 12 presents the spectrum obtained for the case with nine cells using
 453 the model with the classical PMLs. Thanks to this change in the defini-
 454 tion of the PML, some trapped modes (cyan) are distinguished close to the
 455 border of frequency bandgaps (especially several modes that appear around
 456 2100 rad/s). However, it is no longer possible with this definition to dis-
 457 tinguish the propagative modes outside the frequency bandgaps from the
 458 attenuated and reflected modes by the periodic medium. In other words,
 459 using conjugate PLMs in these models highlights the complex modes that
 460 are interesting because they are in the forbidden frequency bands.

461 *3.2. Convergence and effectiveness analysis: Number of cells necessary to*
 462 *validate the periodicity hypothesis*

463 The Figure 13(left) summarizes the previous results by representing only
 464 the real modes as a function of the number of inclusions. The curve corre-
 465 sponding to the infinite case shows the dispersion curves obtained using the
 466 F-B transform, the red curves shown in any previous spectrum figure and
 467 projected onto the horizontal line when the k_{Bloch} -axis is compressed to one
 468 point. It should be noted that when the number of cells is small, the large
 469 frequency bands with no real modes cannot be “identified” as a frequency

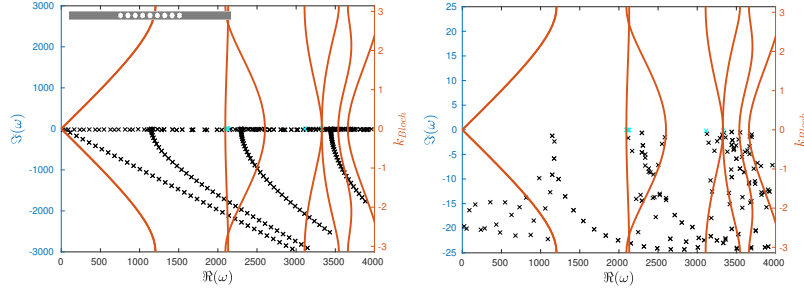


Figure 12: Spectrum obtained with non-conjugated PMLs (left) and a zoom (right) around the real axis to highlight the trapped modes for the case of 9 inclusions.

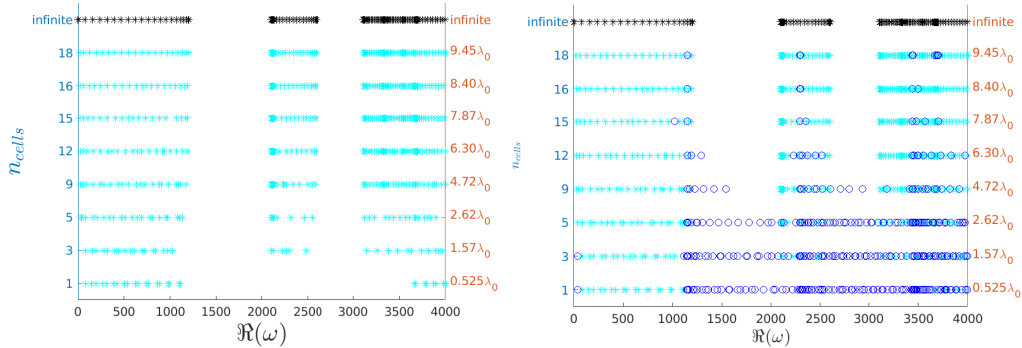


Figure 13: Real modes (cyan) and weakly attenuated modes (blue) as a function of the number of cells with (left) real modes only and (right) real and weakly attenuated modes

470 bandgap. Indeed, it will show that complex modes with a small attenua-
 471 tion coefficient (the imaginary part of the eigenvalue) exist in these bands
 472 (Figure 13(right)).

473 In Figure 13(left), the y-axis on the right shows the equivalence of the
 474 size of the periodic medium in terms of wavelength. The chosen reference
 475 wavelength, $\lambda_0 = 0.525$ m, corresponds to the wave propagating in the matrix
 476 at the lowest frequency of the first frequency bandgap of the infinite periodic
 477 medium, which is $\omega_0 = 1204$ rad/s.

478 For the numerical models considered here, two frequency bands are pre-
 479 dicted in the infinite case. For the i -th bandgap ($i = 1, 2$), its position is
 480 defined by ω_i^{inf} the frequency at its middle and W_i^{inf} its width (*cf.* Fig-
 481 ure 5). Then the values obtained numerically on a bounded medium, ω_i
 482 and W_i , are compared to these reference values ($\omega_i^{inf}, W_i^{inf}$) by using two

483 “effectiveness” indicators I_ω and I_W defined as follows:

$$I_\omega = \frac{1}{N} \sum_{i=1}^N \epsilon_{\omega_i}, \text{ with } \epsilon_{\omega_i} = \frac{|\omega_i - \omega_i^{inf}|}{\omega_i^{inf}} \quad (10a)$$

484

$$I_W = \frac{1}{N} \sum_{i=1}^N \epsilon_{W_i}, \text{ with } \epsilon_{W_i} = \frac{|W_i - W_i^{inf}|}{W_i^{inf}} \quad (10b)$$

485 The evolution of these effectiveness indicators as a function of the number
 486 of cells, by counting only the real modes, is given in the Figure 14. For
 487 these examples, $N = 2$. Thus, it could be concluded that when the size
 488 of the bounded periodic medium is greater than four times the reference
 489 wavelength, i.e., containing about nine cells, its behavior regarding the first
 490 two frequency bandgaps begins to reach that of the infinite medium. In
 491 other words, the effectiveness in terms of bandgaps predicted for the infinite
 492 medium is almost recovered by such a bounded medium. These results are
 in agreement with previous works [20, 30, 31, among others].

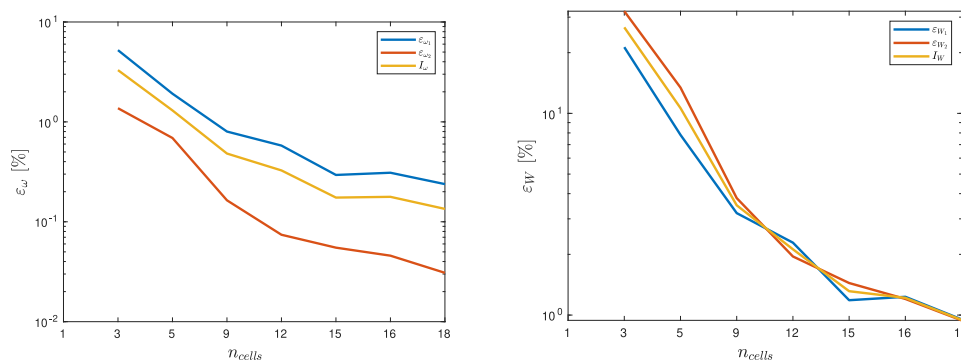


Figure 14: Evolution of the effectiveness indicators on (left) the position (average frequency) and (right) the width of frequency bandgaps by counting the real modes alone.

493

494 As already indicated, the previous analysis is not appropriate when the
 495 bounded domain contains only very few cells because the observed large
 496 frequency bands with no real mode cannot be considered as a frequency
 497 bandgap. Therefore, an improved approach is proposed below to perform a
 498 more qualitative analysis of effectiveness for bounded periodic media.

499 The idea consists of considering the complex modes with an imaginary
 500 part lower than a certain threshold in addition to the real ones. The Fig-
 501 ure 13(right) shows the real modes and the complex modes with an imaginary

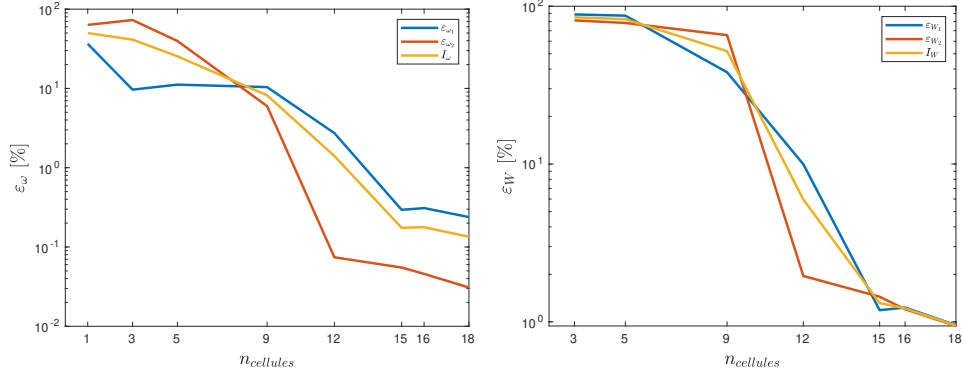


Figure 15: Evolution of the effectiveness indicator on (left) the average frequency and (right) the width of frequency bandgaps by counting real modes and weakly attenuated modes.

502 part smaller than 20 rad/s (displayed in blue); the latter lets part of the wave
 503 propagate through the bounded periodic medium. It can be seen that this
 504 analysis is more relevant because large frequency bands without modes are
 505 no longer observed in cases with very few cells. Hence, it allows us to better
 506 compare the effectiveness of bounded periodic media to the infinite one by
 507 taking into account the weakly attenuated modes.

508 Concerning the calculation of the effectiveness indicators, the width W_i
 509 is obtained by looking for the maximum difference between two successive
 510 weakly attenuated modes, the blue points, and the identified interval also
 511 gives the average frequency ω_i .

512 The evolution of the effectiveness indicators by counting both real and
 513 weakly attenuated complex modes as a function of the number of cells is
 514 given on the Figure 15. It highlights the fact that it converges to the values
 515 obtained for the infinite medium when the number of cells increases. The
 516 indicators on the average position (ε_{ω_i} and I_ω) are smaller than the ones
 517 on the width (ε_{W_i} and I_W) of the bandgaps. Thus, from twelve cells, the
 518 indicator on the position is lower than 5% while that on the width is around
 519 15%. For the position, the indicator goes down to 0.2% for fifteen cells if we
 520 consider each band separately, and the indicator on the width is lower than
 521 2% for this number of cells.

522 **4. Quantitative analysis of effective properties of bandgaps of 2D**
 523 **bounded periodic media**

524 The waveguide modal analysis presented above allows the study of bounded
 525 periodic media only in the \mathbf{e}_1 direction but not the consideration of 2D peri-
 526 odicity. In order to analyze 2D-periodic barriers, infinite in the \mathbf{e}_2 direction
 527 and bounded in the \mathbf{e}_1 direction, it is possible nevertheless to prescribe the
 528 F-B conditions on the horizontal boundaries, which were assumed to be free
 529 of stresses in the previous modeling (Figure 5). Thus, the previously imple-
 530 mented waveguide models are extended to 2D-periodic media by adding a
 531 second direction of periodicity along \mathbf{e}_2 . By adding periodicity conditions on
 532 the horizontal boundaries, the new waveguide models contain infinite cells
 533 in the \mathbf{e}_2 direction while remaining with a finite number of cells in the \mathbf{e}_1
 direction. The Figure 16 presents the problem thus defined.

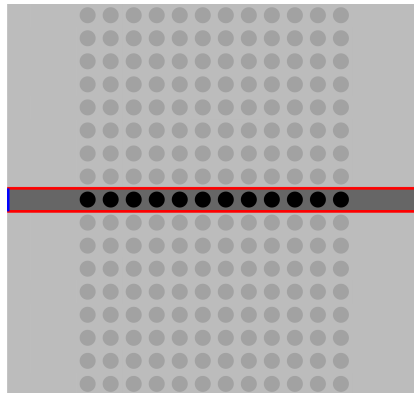


Figure 16: 2D-periodic barrier and its waveguide model with a single row of cells shown in dark gray (with inclusions in black), which is the elementary cell used for the F-B transform Equation (11) and the periodicity conditions are applied on its horizontal boundaries (red lines).

534 In this framework, the behavior of the waveguide is then governed by the
 535 following F-B equation:
 536

$$(\nabla_{\mathbf{x}} + i\mathbf{k}) \cdot (\mu (\nabla_{\mathbf{x}} + i\mathbf{k}) u) = -\rho\omega^2 u. \quad (11)$$

537 with $\mathbf{k} = k_2 \mathbf{e}_2$.

538 For the new waveguide models, the definition of the PMLs remains un-
 539 changed (see Eq.(9)) because it is not affected by the F-B transform and the
 540 change of variable in the PMLs is only on the x_1 variable.

541 In the following, the results obtained with the waveguide models are an-
542 alyzed and compared with the frequency bandgaps of the corresponding infi-
543 nite 2D-periodic medium, predicted by the F-B theory (Figure 17(c)). In this
544 Figure, the Bloch wave number is presented in abscissa following the contour
545 of the Brillouin irreducible zone $M \rightarrow \Gamma \rightarrow X \rightarrow M$. In the same way as
546 before, the dispersion curves are obtained from the numerical solution of the
547 eigenvalue problem (Eq.(3)). It is remarked that, compared to the 1D peri-
548 odic case (Figure 5), the first two bandgaps are narrower and move towards
the higher frequency range.

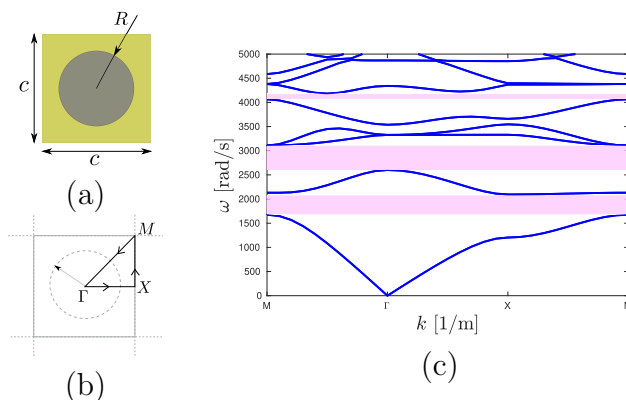


Figure 17: Unit cell Q_0 of the periodic medium in physical space and corresponding first Brillouin zone Q^0 in reciprocal space. (a) A unit cell corresponding to an infinite medium in two directions; (b) A reciprocal cell on which the contour of the Brillouin irreducible zone, $\Gamma \rightarrow X \rightarrow M \rightarrow \Gamma$, is represented; (c) Dispersion curves highlighting bandgaps.

549

550 4.1. Analysis of discrete spectra, characterization of modes, and ability to 551 reconstruct the 2D periodicity behavior

552 Following the proposed approach, the combined waveguide and F-B modal
553 analysis are performed for the set of k values sweeping the entire Brillouin
554 zone $\left[-\frac{\pi}{c}, \frac{\pi}{c}\right]$ in the \mathbf{e}_y direction in which the F-B conditions are applied.
555 The obtained modes are then plotted to show the spectrum of each F-B
556 waveguide.

557 First, as presented previously for the case of 1D propagation, Figure 18
558 shows the spectrum of a homogeneous waveguide without any inclusion. It
559 can be seen that, compared to the previous 1D-periodic case, many more
560 modes are obtained for the essential (black) and discrete (cyan) spectra.
561 Indeed, each value of k in the Brillouin zone gives rise to a different spectrum.

562 Thus, the modes obtained should be more complex to analyse than in the
 1D case.

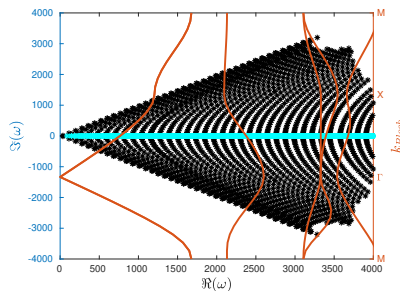


Figure 18: Spectrum of the homogeneous waveguide in the complex plane: σ_{disc} with real eigenvalues (cyan) and σ_{ess} (black); Dispersion curves (in red) obtained with the F-B theory.

563

564 Moreover, contrary to the essential spectrum obtained in the 1D-periodic
 565 case (Figure 3), many modes belonging to the essential spectrum are very
 566 close to the real axis. Therefore, it would not be easy to distinguish them
 567 from the complex modes belonging to the discrete spectrum of waveguide
 568 models with inclusions.

569 A parametric study on the number of cells similar to that carried out
 570 previously for the 1D case is set up. The number of cells ranges from one
 571 to eighteen cells. The spectrum results are given in Figure 19. It is noted
 572 immediately that the frequency bands defined by the real modes (in cyan)
 573 of the waveguide models coincide with the passbands defined by the propa-
 574 gating modes of the corresponding 2D infinite medium calculated with the
 575 F-B transform (in red). Therefore, it can be concluded that our waveguide
 576 models that consider only one of the infinite directions can be used to gradu-
 577 ally approach the bandgap solution of the corresponding 2D infinite periodic
 578 medium.

579 As for the previously presented 1D-periodic waveguide models, the den-
 580 sity of real modes increases with the number of cells within theoretically
 581 predicted passbands.

582 Inside the theoretically predicted bandgaps, as before, there are non-real
 583 modes, and their number depends on the number of cells. They are atten-
 584 uated modes (since they have a non-zero imaginary part). However, unlike
 585 the periodic 1D case, some complex modes close to the real axis always ap-
 586 pear, i.e., with a relatively small imaginary part, even when the number of
 587 cells increases. This is due to the periodicity in the \mathbf{e}_2 direction conserved

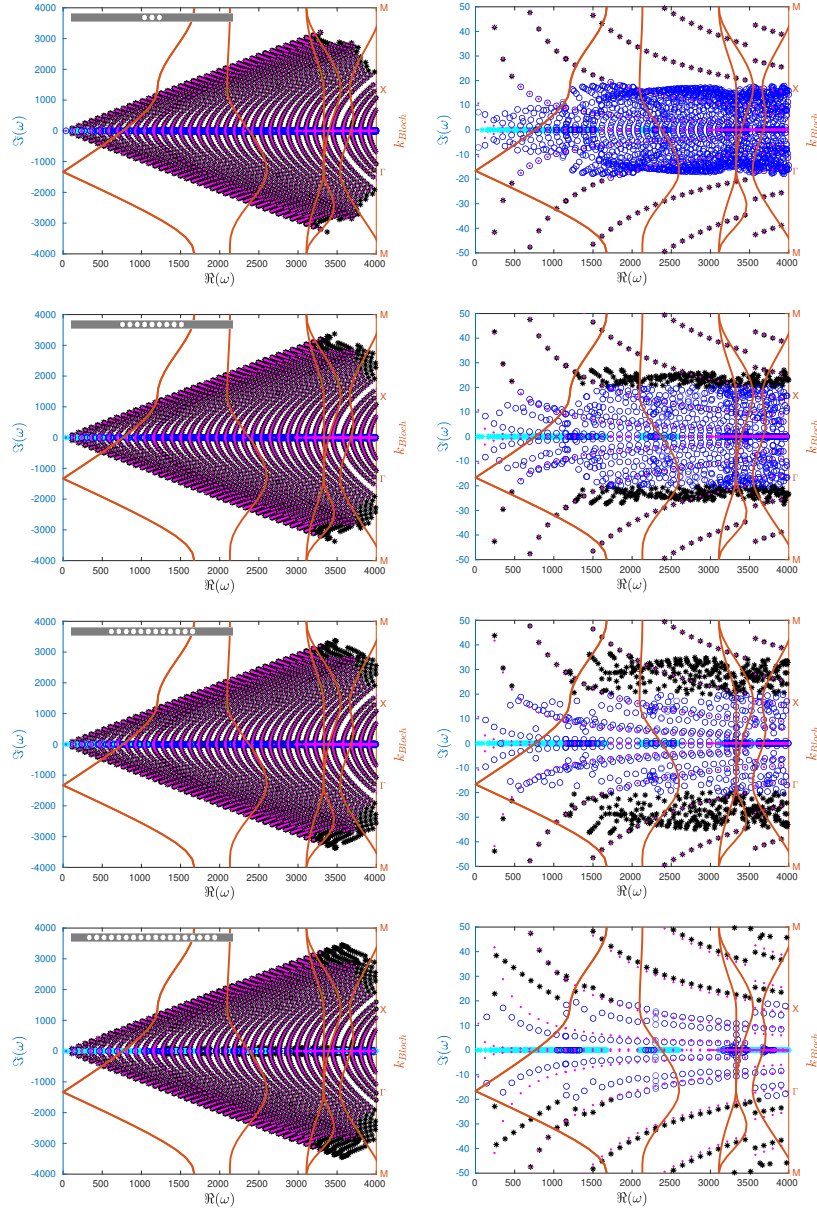


Figure 19: Parametric study on the influence of the number of cells on the eigenvalue spectrum and comparisons with the dispersion curves (in red) obtained with the F-B theory for the corresponding infinite 2D periodic medium. Spectrum (left) and its zoom around the real axis (right) of the studied waveguides with respectively 3, 9, 12, and 18 cells. The real eigenvalues of the discrete spectrum are in cyan. The complex eigenvalues with an imaginary part smaller than 20 rad/s are in blue, and the others are in black. The essential spectrum without inclusions is displayed with magenta dots.

588 in our F-B waveguide models. An example of such a mode is given in Fig-
 589 ure 20(left). The real part of its eigenvalue is 1828.3 rad/s, and the imaginary
 590 part is equal to 15.2 rad/s for a value of k equal to 1.57. It is a mode of the
 591 essential spectrum. The behavior of this mode looks like a reflected mode
 592 despite its low imaginary part. Because of the periodicity in the e_2 direction,
 593 and so the fact that the wave also propagates in the transverse direction e_2 ,
 594 this mode is weakly influenced by the periodic medium's size. Hence, it is a
 595 mode propagating in the direction e_2 and localized in the margins between
 596 the bounded periodic medium and the PML. As the size of the margins de-
 597 creases when the number of cells increases, these modes move away from
 598 the real axis, but not as quickly as the other complex modes, "stopped" or
 599 attenuated by the bounded periodic medium. Note that the thinner the mar-
 600 gins, the more the essential spectrum is modified compared to the one of the
 601 homogeneous waveguide (represented by the pink points). However, it keeps
 602 the same smooth shape.

603 To compare, Figure 20(right) represents a complex mode of the model
 604 with nine cells obtained for $k = 2.51 \text{ m}^{-1}$, for which the imaginary part of its
 605 eigenvalue is equal to about 23 rad/s and a real part equal to 1903 rad/s. In
 606 this case, the wavefront propagates along the e_1 direction and is attenuated
 607 by the bounded periodic medium. This type of mode rapidly moves away
 608 from the real axis when the number of cells increases. It can be seen that,
 609 for waveguide models with a large number of cells, the only modes that stay
 610 close to the real axis in the theoretically predicted frequency bandgaps are
 those from the essential spectrum.

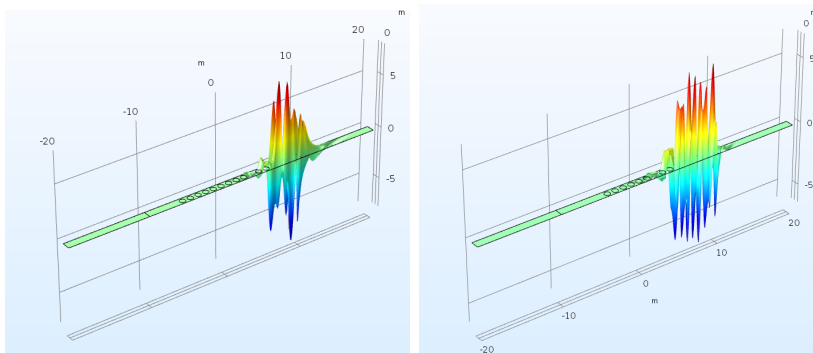


Figure 20: (left) Propagative mode in the direction e_2 (case of 12 inclusions) and explaining the modes inside the bandgaps; (right) Attenuated mode in the direction e_1 (case of 9 inclusions).

611

612 4.2. Convergence and effectiveness analysis: Number of cells necessary to
 613 validate the periodicity hypothesis

614 As in the 1D analysis, Figure 21 displays the real modes as a function
 615 of the number of cells. The conclusions previously made for the 1D periodic
 616 waveguide models apply. For a small number of cells, the density of real
 617 modes is very low, which does not mean that large frequency bandgaps are
 618 obtained. When the number of cells increases, the analysis becomes relevant;
 619 thus a convergence study on the effectiveness of the considered bounded
 periodic media could be carried out.

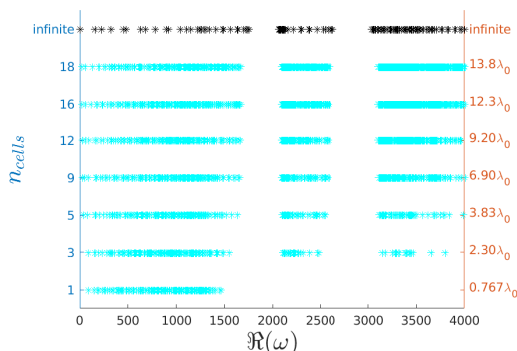


Figure 21: Real modes versus the number of inclusions (2D model).

620
 621 In the same way as before, Figure 22 presents the evolution of the effec-
 622 tiveness indicators given by the equations (10a) and (10b) as a function of
 623 the number of cells. For the model with only one cell, the indicators are not
 624 computed for the obvious reason that only one large “attenuated” frequency
 625 band, instead of two, is obtained in the ω interval considered. These figures
 626 show the decrease of the error committed by adding inclusions in the \mathbf{e}_1 di-
 627 rection. It is important to remark that the effectiveness indicators are better
 628 regarding the position of the frequency bandgaps (less than 5% from three
 629 cells in the \mathbf{e}_1 direction) than their width.

630 It is also noted that the evolution of effectiveness indicators by counting
 631 the real and weakly attenuated complex modes like that done for the 1D
 632 periodic waveguide models is not presented here. Indeed, it is more compli-
 633 cated to distinguish the weakly attenuated complex modes from the complex
 634 modes of the essential spectrum, as already explained above. Indeed, the
 635 removal of the modes of the essential spectrum close to the real axis to take
 636 into account only the other complex modes linked to the useful zone (the

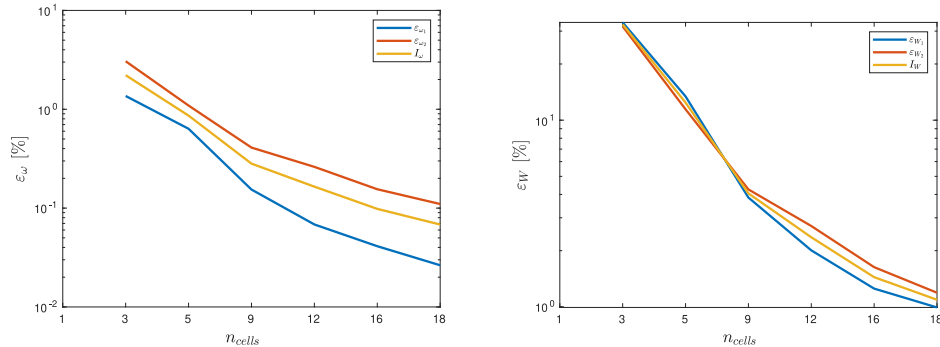


Figure 22: Evolution of the effectiveness indicators on (left) the position (average frequency) and (right) the width of frequency bandgaps, by counting the real modes alone.

637 bounded periodic medium), i.e., those of the discrete spectrum, is for the
 638 moment possible but at the price of manual sorting, which is laborious. Nev-
 639 ertheless, such a study is possible if more quantitative characterization is
 640 required with the development of more automatic tools.

641 5. Application to locally resonant metamaterials

642 In this section, the previously proposed method is applied to analyze a
 643 bounded 1D- and 2D-periodic matrix-inclusion medium with local resonators
 644 or resonant metabarriers.

645 In order to obtain frequency bandgaps at lower frequencies while keep-
 646 ing a reasonably small size of the periodic cells, as it is expected for seismic
 647 purposes, one of the main solutions is to use locally resonant metamaterials
 648 [9, 12, 17, 43, 47, 48, among others]. The local resonator conception cho-
 649 sen for this study was proposed in Liu et al.’s work [43] by using Helmholtz
 650 resonators made by a heavy mass inside a very soft intermediate layer (Fig-
 651 ure 23(a)). The dimensions and the materials properties for the soft interme-
 652 diate layer (μ_s, ρ_s) and for the heavy mass inside (μ_i, ρ_i) are given in Table
 653 II, the materials properties for the matrix (μ_m, ρ_m) being unchanged and
 654 already given in Table I.

655 As in the previous cases, the dispersion curves for the infinite medium are
 656 obtained by applying the F-B theory to an FE model of the unit cell. They
 657 are represented in Figure 23(b) for the 1D-periodic model and in Figure 23(c)
 658 for the 2D-periodic one. It is important to note that, for the same cell size,
 659 the first frequency bandgap appears in a very low-frequency range, around

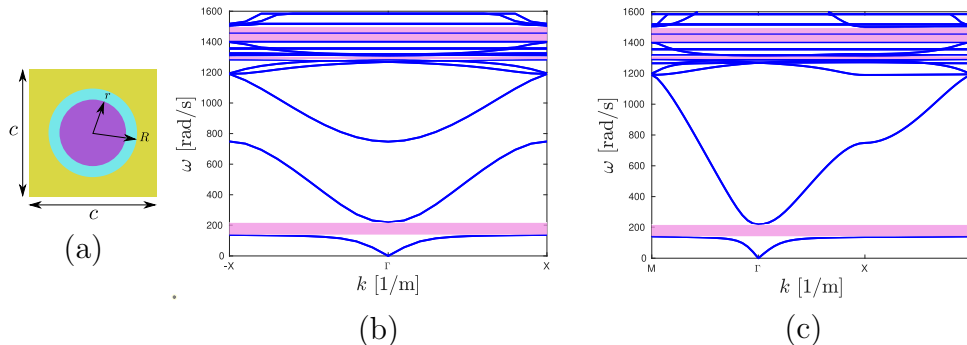


Figure 23: Locally resonant material made of Helmholtz resonator. (a) Unit cell (left), dispersion curves obtained by the F-B theory in the infinite case (b) with 1D-periodicity and (c) 2D-periodicity.

c (m)	R (m)	r (m)	μ_s (GPa)	μ_i (GPa)	ρ_s (kg/m ³)	ρ_i (kg/m ³)
1	0.346	0.260	0.0015	210	1200	7850

Table II: Summary of the data used for the simulation of the 2D-periodic medium with local resonators, where subscripts “s” and “i” represent the intermediate soft layer and the central heavy mass, respectively.

660 200 rad/s, instead of 2000 rad/s in the case considered in the previous sections
 661 (i.e. without local resonators).

662 A parametric study is performed by modifying, as previously, the number
 663 of cells, going from three to eighteen cells, in the waveguide. The results of
 664 the 1D-periodic case are presented in Figure 24. As in the heretofore studied
 665 case, the real modes do not appear in the theoretically predicted bandgaps
 666 for the infinite reference case and some attenuated modes exist inside those
 667 bandgaps for the smaller number of cells. However, it is important to note
 668 that there are less attenuated modes than in the case of Bragg scattering
 669 models.

670 The Figure 25 summarizes the analysis of bandgaps by representing only
 671 the real modes and weakly attenuated modes as a function of the number
 672 of cells, and the same conclusions as for the Bragg scattering are obtained.
 673 The higher the number of cells, the higher the effectiveness of the waveguide
 674 with periodic media, and also it could be considered that the first frequency
 675 bandgap predicted for the infinite case is recovered with 18 cells.

676 As the frequency of the first bandgap appears at a much lower frequency
 677 range, the density of points used to characterize the effectiveness of such

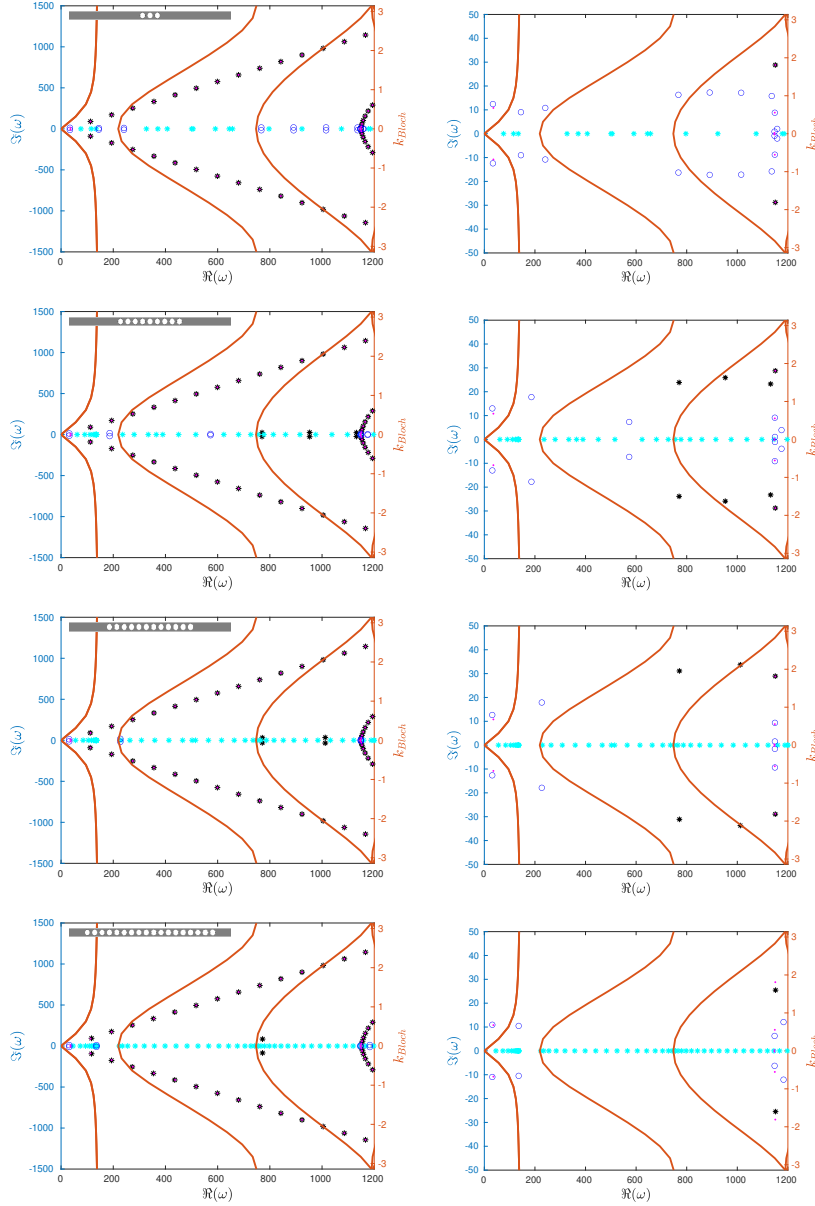


Figure 24: Parametric study on the influence of the number of cells on the eigenvalue spectrum and comparisons with the dispersion curves (in red) obtained with the F-B theory for the corresponding infinite 1D-periodic medium. Spectrum (left) and its zoom around the real axis (right) of the studied waveguides with respectively 3, 9, 12, and 18 cells. The real eigenvalues of the discrete spectrum are in cyan. The complex eigenvalues with an imaginary part smaller than 20rad/s are in blue, and the others in black. The essential spectrum without inclusions is displayed with magenta dots.

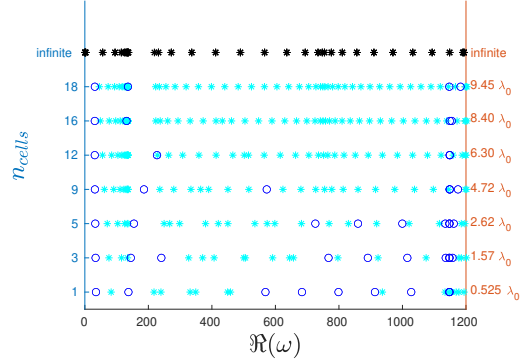


Figure 25: Real modes (cyan) and weakly attenuated modes (blue) as a function of the number of cells.

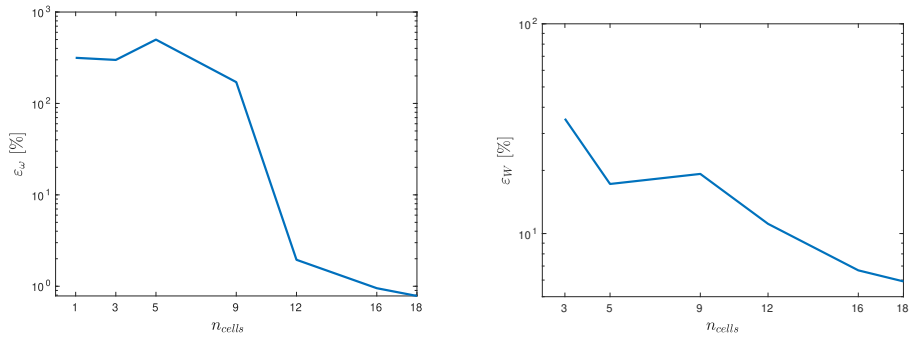


Figure 26: Evolution of the effectiveness indicator on (left) the average frequency and (right) the width of frequency bandgaps by counting real modes and weakly attenuated modes.

678 a bounded periodic media decreases. It would be useful to take a longer
 679 waveguide. However the same trend as in the Bragg scattering is visible: the
 680 more cells, the smaller the difference between infinite and finite cases. Thus,
 681 efficiency indicators are plotted on Figure 26 for the unique bandgap visible
 682 around 200 rad/s. It appears that the central frequencies are well evaluated
 683 for a number of cells greater than 12. The indicator is there around 2% and
 684 it is remarked that the effectiveness of the width of the bandgap also follows
 685 the same trend. The indicator only goes down to 10% for the highest number
 686 of cells (i.e. 18 cells).

687 As in the Section 4, the model 1D-periodic is then studied with a 2D-
 688 periodicity by applying F-B conditions on the horizontal boundaries of the

689 model. The spectrum of this model is shown on Figure 27 for a different
690 number of unit cells. The red curves represent the 2D-periodic dispersion
691 curves (Figure 23(c)). These results allow to recover the same behaviour as
692 in the previous studies. The weakly attenuated modes inside the frequency
693 bandgap disappear when the number of unit cells increases. Nevertheless,
694 this tool (especially the way the modes are sorted) must be improved in order
695 to have a way to quantify the effectiveness in the case of 2D-periodicity.

696 6. Conclusions

697 In this article, a method to quantitatively characterize the behaviours re-
698 garding acoustic waves of bounded 1D- and 2D-periodic metamaterials made
699 with a matrix and circular inclusions have been presented. This approach
700 allows a complementary analysis to the classical Floquet-Bloch approach ded-
701 icated to infinite periodic media. The proposed method is based on wave-
702 guide modal analysis. By using PML, it is possible to isolate the effects of
703 the bounded metamaterial without dealing with boundary conditions. Two
704 different types of metabarriers (i.e. with and without local resonators) were
705 numerically analysed so as to assess the induced wave amplitude attenuation.

706 The findings reveal that a minimum number of cells is necessary to fit
707 the theoretically predicted bandgaps obtained with the Floquet-Bloch theory.
708 This minimum number of cells depends on the criteria defined in terms of
709 wave attenuation by the bounded periodic medium. The waveguide modal
710 analysis confirm that depending on the size of the bounded periodic medium,
711 more or less attenuated complex modes can exist in the frequency bandgaps
712 theoretically predicted.

713 Finally, the obtained results also indicate that in the locally resonant
714 systems the first frequency bandgap appears in a very low-frequency range
715 compared to the same configuration but without local resonators.

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719 CRediT authorship contribution statement

720 **Michaël Darche:** Conceptualization, Methodology, Software, Valid-
721 ation, Formal analysis, Writing - Original Draft, Writing - Review & Editing,

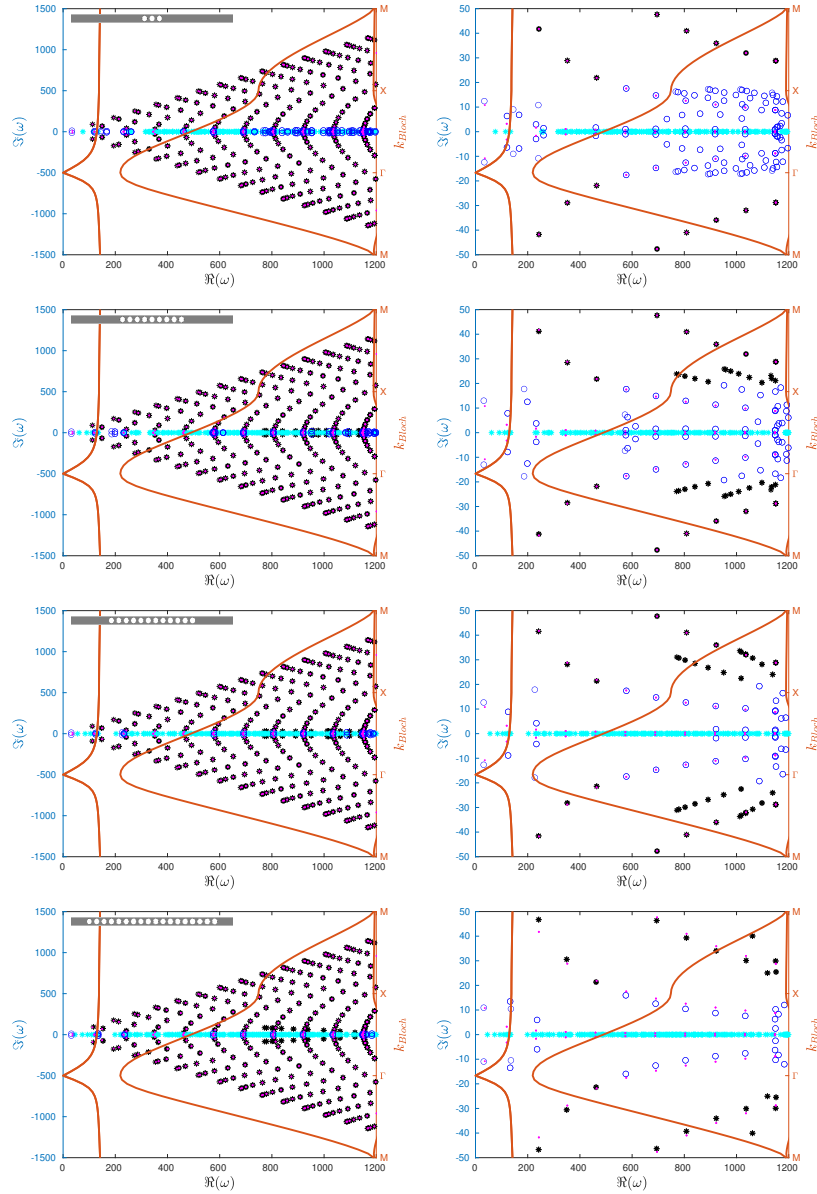


Figure 27: Parametric study on the influence of the number of cells on the eigenvalue spectrum and comparisons with the dispersion curves (in red) obtained with the F-B theory for the corresponding infinite 2D periodic medium. Spectrum (left) and its zoom around the real axis (right) of the studied waveguides with respectively 3, 9, 12, and 18 cells. The real eigenvalues of the discrete spectrum are in cyan. The complex eigenvalues with an imaginary part smaller than 20rad/s are in blue, and the others in black. The essential spectrum without inclusions is displayed with magenta dots.

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725 Formal analysis, Writing - Original Draft, Writing - Review & Editing, Vi-
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