



**HAL**  
open science

# Optimization of a smart beam for monitoring a connected inaccessible mechanical system: Application to bone-implant coupling

Anne-Sophie Poudrel, Vu-Hieu Nguyen, Guillaume Haiat, Giuseppe Rosi

## ► To cite this version:

Anne-Sophie Poudrel, Vu-Hieu Nguyen, Guillaume Haiat, Giuseppe Rosi. Optimization of a smart beam for monitoring a connected inaccessible mechanical system: Application to bone-implant coupling. *Mechanical Systems and Signal Processing*, 2023, 192, pp.110188. 10.1016/j.ymsp.2023.110188 . hal-04276181

**HAL Id: hal-04276181**

**<https://hal.science/hal-04276181>**

Submitted on 8 Nov 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Optimization of a smart beam for monitoring a connected inaccessible mechanical system : application to bone-implant coupling

Anne-Sophie Poudrel<sup>a</sup>, Vu-Hieu Nguyen<sup>b</sup>, Guillaume Haiat<sup>a</sup>, Giuseppe Rosi<sup>b</sup>

<sup>a</sup>CNRS, Univ Paris Est Creteil, Univ Gustave Eiffel, UMR 8208, MSME, F-94010 Créteil, France

<sup>b</sup>Univ Paris Est Creteil, Univ Gustave Eiffel, CNRS, UMR 8208, MSME, F-94010 Créteil, France

---

## Abstract

This paper deals with the optimization of piezoelectric patches positioning on a beam attached to an inaccessible system. Based on the electro-mechanical coupling coefficients, which are calculated from the mode shapes curvatures, the problem is to provide an optimal positioning of the piezoelectric patches in order to target the modes sensitive to beam boundary conditions. Following the theoretical description of a beam instrumented with collocated piezoelectric patches, a placement optimization strategy is proposed. This strategy lies on the definition of utility functions, based on modal weights and modal sensitivities, calculated from the coupling coefficient. An experimental validation of the method is performed on a concrete case study corresponding to real-time implant stability monitoring. A beam is temporarily rigidly fixed to the implant during its insertion into the bone cavity to allow impaction by the surgeon. The positions of the sensors on the beam are optimized to focus on the beam's modes carrying information on the bone-implant interface, the biomechanical issue being the real-time maximization of implant stability. The originality of the approach lies on the sensor placement optimization on the beam connected to the implant to maximize or minimize simultaneously the amplitude of several modes on the frequency response function at resonances, depending on their sensitivity to the bone-implant interface. The results show very good performance of the piezoelectric placement strategy proposed in this paper, paving the way for new applications of piezoelectric patches design and placement on structures.

*Keywords: electro-mechanical coupling, piezoelectric patch optimization, mode observability, smart device, boundary conditions monitoring, bone-implant coupling*

---

## 1. Introduction

Piezoelectric ceramic transducers bonded to flexible structures are widely employed in the context of dynamic measurements. They offer the advantage to be used as sensors or actuators due to their property to convert mechanical strains into electrical voltage and *vice versa*. The modal electro-mechanical coupling

*Preprint submitted to Mechanical Systems and Signal Processing*

11 factors (MEMCF) [1, 2] associated to one piezoelectric patch and one mode of vibration of the structure is  
12 determinant for vibration reduction control [3].

13 In the literature, there are several examples of multi-modal control, either active or passive with shunted  
14 piezoelectric patches on beams [4, 5, 6] and plates [7, 8, 9]. For these applications, the piezoelectric patches  
15 are generally uniformly distributed on the structure, considering that every mode has the same weight  
16 and the optimization concerns the associated electric circuit. However, in specific applications, the design  
17 constraints limit the number of sensor/actuator patches over the structure. Therefore, another strategy for  
18 vibration reduction lies in the optimization of the placement and the design of the piezoelectric patch on  
19 the structure, as presented in the exhaustive review of Gupta et al. [10]. Among the different criteria used  
20 in the literature, maximizing the degree of mode controllability and observability are of particular interest.  
21 For instance, Ducarne et al. [11] proposed a parametric study on the placement and geometry of the patches  
22 in order to maximize the electro-mechanical coupling coefficient of one specific mechanical vibration mode,  
23 to which the shunt is tuned. Others authors studied the same problematic of placement optimization, by  
24 considering active control [12, 13]. Moreover, Halim and Moheimani [14] addressed the notions of both  
25 spatial and modal controllability, to find the optimal locations of piezoelectric transducers on the plate. The  
26 procedure is based on the concept of the spatial  $\mathcal{H}$  norm, used to describe the spatial structural response  
27 of the system in an average sense. The optimization relies on the simultaneous vibration reduction over the  
28 entire structure, while maintaining modal controllability and observability of the selected vibration modes.

29 In this paper, the specific issue of optimizing the *observability* of a set of modes sensitive to the beam  
30 boundary conditions is addressed. Lots of vibro-acoustic applications are concerned by this issue, and among  
31 them, this paper focus on a biomedical application. In the orthopedic field, several methods are under  
32 development to assist and guide the surgeon during implant insertion into the bone, in order to achieve an  
33 optimal implant stability, corresponding to maximizing implant surrounding stresses while avoiding bone  
34 fracture risk. Among the different techniques, vibration measurement have proven to be efficient to monitor  
35 implant insertion into the bone [15, 16, 17]. Nevertheless, the fixation of sensors or probes on the implant  
36 and/or the bone is incompatible with a clinical application [18]. Therefore, several authors studied directly  
37 the vibrations of a square shape structure, named ancillary, and used by the surgeon to insert the implant into  
38 the bone [19, 20], which can easily be equipped with sensors and allow a good accessibility for measurement.  
39 This structure has one free extremity and another one bonded to the implant during its insertion. Due to  
40 this fixation, a previous study by our group showed that specific modes of vibration of this square beam  
41 structure carry information on the bone-implant system, and in particular on the system rigidity [20]. The

42 issue is then to propose an optimization approach for piezoelectric patch positioning on the square beam  
43 structure, in order to maximize or minimize the *observability* of the modes, depending on their sensitivity  
44 to the implant stability.

45 Several authors studied the piezoelectric placement optimization in order to target specific modes of  
46 interest. For instance, Rosi et al. [21] developed a similar transducer optimization approach for reducing  
47 the radiated sound power of thin plates produced by a set of modes. In the field of structural health  
48 monitoring, the position of the collocated piezoelectric transducers can be optimized in order to detect  
49 damage in thin plate-like structures based on the minimization of the Bayes risk [22]. Interestingly, not only  
50 the position but also the local beam stiffness could be adjusted by using the feedback voltage output from the  
51 piezoelectric sensors to tune the natural vibration of mode shape sensitive to the existence of a crack [23, 24].  
52 In these examples, the criteria of mode selection differs from the sensitivity to beam boundary conditions.  
53 Moreover, in our study, the piezoelectric patches are not shunted since they are used as sensors and not for  
54 control purpose. As only the position of the patch on the beam is studied, no optimization algorithm are  
55 needed, contrary to what is often done for passive control applications [25, 26, 27]. Eventually, methods for  
56 boundary force measurement in beam-like structures have been investigated in the literature by displacement  
57 measurement [28] or distributed sensor measurement [29]. However, the issue of our application is to monitor  
58 implant stability throughout specific resonance frequency rather than boundary forces measurement.

59 The originality of the present paper comes from the use of an optimization method, based on the modal  
60 electro-mechanical coupling factor (MEMCF), in order to maximize the *observability* of a subset of modes  
61 sensitive to the beam boundary conditions, related to the implant stability for the considered case study.  
62 The choice of this application comes from the need to maximize the initial fixation of orthopedic implants  
63 during their insertion into the bone, which is one of the main biomechanical issues related to this surgery.  
64 Since this fixation is tricky to maximize for the surgeon without creating a bone fracture, some quantitative  
65 methods are underdevelopment to measure in real time implant stability. Some of them are based on the  
66 vibro-acoustic characterization of the bone-implant system [17, 30, 20]. In particular, the results of [20],  
67 which showed that the implant stability is correlated to specific resonance frequencies of the surgical tool  
68 used to insert the implant, named “ancillary” and assimilated to a beam with one free extremity and the  
69 other one bounded to the implant being inserted. To improve the vibration measurement method for implant  
70 stability characterization by focusing on specific modes of interest, optimal patches positions are determined  
71 hereafter on the beam which has one free extremity and the other bounded to the implant being inserted.

72 The paper is organized in two main sections. Section 2 is dedicated to the description of the smart

73 beam electromechanical behavior and to the transducers position optimization strategy based on mode  
74 *observability*. In particular, the modal electromechanical coupling between the patch and the beam and the  
75 utility functions chosen for the optimization of a subset of modes are defined. In Section 3, an experimental  
76 validation of the optimization strategy for implant insertion monitoring is carried out.

## 77 2. Equations of the smart device

### 78 2.1. Geometry and kinematics

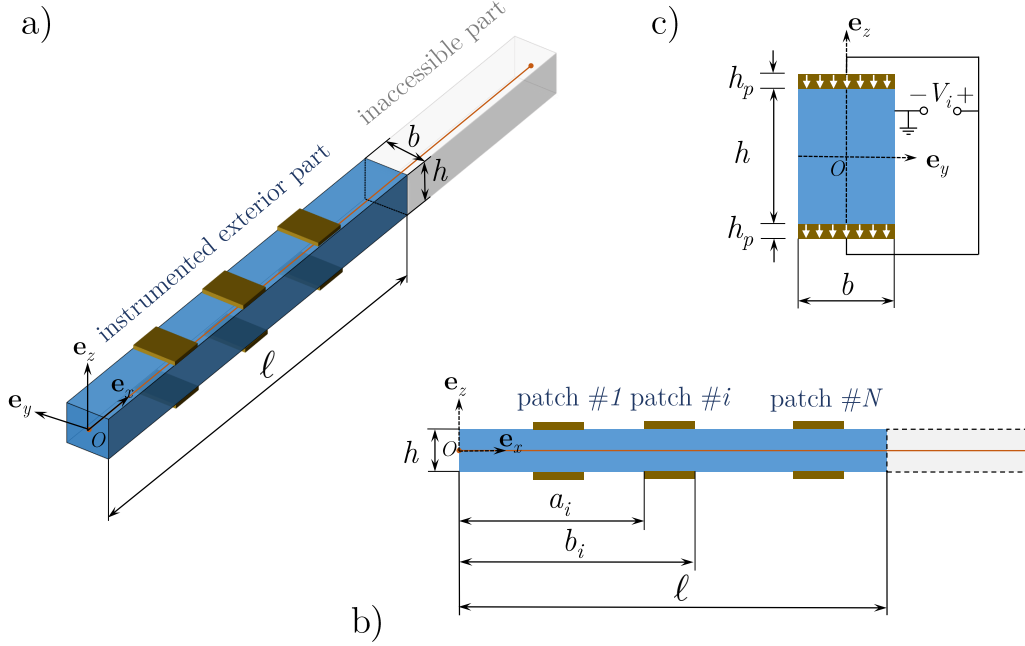


Figure 1: a) Geometry of the beam and connection with the inaccessible system b) Piezoelectric patches positions and numbering on the beam and c) A section containing  $i$ -th patches couple and the associated electric circuit with  $V_i$  the potential difference measured by each patches couple  $i$

79 Consider an elastic beam of length  $\ell$  with a rectangular section  $b \times h$  (Fig. 1a), on which piezoelectric  
80 layers are bonded at  $n_p$  segments along its longitudinal axis  $x$ . The beam is fixed at  $x_1 = \ell$  to an external  
81 mechanical system. At a segment  $x \in [a_i, b_i]$  ( $i = 1..n_p$ ), the beam is sandwiched between two identical  
82 piezoelectric patches of thickness  $h_p$  which are put symmetrically at the upper and the lower surfaces of the  
83 beam (i.e at  $z = h/2$  and  $z = -h/2$ ) (Fig. 1b). Each couple of piezoelectric layers is polarized along  $-e_z$   
84 (Fig. 1c) and thus form a pure bending sensor/actuator.

85 We assume the piezoelectric layers have the same width  $b$  with the beam's section. Each piezoelectric  
86 layer is covered on both its upper and lower surfaces by electrodes which are equipotential surfaces. The

87 electric field  $\mathbf{E}$  assumed to be constant, uniform and parallel to the transverse direction  $\mathbf{e}_z$ , is :  $E_z = -V/h_p$ ,  
 88 where  $V$  the voltage applied between the electrodes. The purely elastic part of the beam is grounded.

89 In this study, all forces acting along  $\mathbf{e}_y$  are neglected, the system may be described by a 1D Euler-  
 90 Bernoulli model in the plane  $(x, z)$ . Moreover, by assuming that the longitudinal and rotation of the beam  
 91 are negligible, only flexural movement is considered. All components of the displacement field at a point of  
 92 coordinates  $(x, y, z)$  may expressed in terms of the transverse displacement  $w(x)$  of the beam's neutral axis:

$$u_x(x, y, z, t) = -zw'(x, t) \quad (1)$$

$$u_y(x, y, z, t) = 0 \quad (2)$$

$$u_z(x, y, z, t) = w(x, t) \quad (3)$$

## 93 2.2. Constitutive law and governing equations

94 By using the Euler-Bernoulli beam model, the transverse stress  $\sigma_{zz}$  and the shear strain  $\epsilon_{xz}$  are negligible.  
 95 Consequently, the only non-zero strain component is:  $\epsilon_{xx} = u'_x = -zw''$  and the full piezoelectric constitutive  
 96 equation (see Appendix A) reduces to:

$$\sigma_{xx} = Y\epsilon_{xx} - e_{31}E_z, \quad (4)$$

$$D_z = e_{31}\epsilon_{xx} + \epsilon_{33}E_z. \quad (5)$$

97 At a point located in the elastic beam (i.e. for  $z \in [-h/2, h/2]$ ):  $Y = Y_b$ ,  $\rho = \rho_b$ ,  $e_{31} = 0$  and  $\epsilon_{33} = 0$ .  
 98 Otherwise, in the piezoelectric patches, the mechanical parameters are denoted by  $\rho = \rho_p$  and  $Y = Y_p$ . Since  
 99 piezoelectric ceramics are transversely isotropic, the elastic parameter  $Y_p$  is an in-plane Young modulus  
 100 obtained from the corresponding complete elasticity tensor  $\mathbf{c}$  using the following relation (see Appendix A:

$$Y_p = \frac{(c_{11} - c_{12})(c_{11} + c_{12})c_{33} - 2c_{13}^2}{c_{11}c_{33} - c_{13}^2} \quad (6)$$

101 In the following, the extended Hamilton principle will be used for obtaining the equations of motion and  
 102 continuity conditions [31]. See Appendix B for more details.

### 103 2.2.1. Equations in a segment containing $i$ -th patch ( $x \in [a_i, b_i]$ )

104 With the presence of the upper and lower piezoelectric patches, the beam has three layers and the kinetic  
 105 variation  $\delta T^*$  may be expressed by:

$$\delta T^* = \int_{a_i}^{b_i} \left( \int_{-b/2}^{b/2} \int_{-H/2}^{H/2} \rho(\delta \dot{w}) \dot{w} dy dz \right) dx = \int_{a_i}^{b_i} \bar{m} \delta \dot{w} \dot{w} dx, \quad (7)$$

106 where  $H = h/2 + h_p$  and  $\bar{m} = b(\rho_b h + 2\rho_p h_p)$  which is effective mass per unit length of the beam.

107 Similarly, by recalling that the electric potential is fixed ( $\delta \mathbf{E} = 0$ ) and by substituting Eqs. (1-5) into  
108 (B.10), the potential variation  $\delta W_e^*$  reads:

$$\delta W_e^* = \int_{a_i}^{b_i} \left( \int_{-b/2}^{b/2} \int_{-H/2}^{H/2} (-\delta \epsilon_{xx} \sigma_{xx}) dy dz \right) dx \quad (8)$$

$$= \int_{a_i}^{b_i} \delta w'' (-\mathcal{D} w'' + \mathcal{G}V) dx, \quad (9)$$

109 where

$$\mathcal{D} = \int_{-b/2}^{b/2} \int_{-H/2}^{H/2} Y z^2 dy dz = Y_b \frac{bh^3}{12} + Y_p \frac{bh_p(3h^2 + 6hh_p + 4h_p^2)}{6}, \quad (10)$$

$$\mathcal{G} = \int_{-b/2}^{b/2} \int_{-H/2}^{H/2} \frac{e_{31}z}{h_p} dy dz = e_{31} b(h + h_p) = b(h + h_p) d_{31}Y_p, \quad (11)$$

110 are a bending stiffness and a coupling coefficient, respectively.

111 Then, integrating  $\delta W_e^*$  by parts twice with respect to  $x$  gives:

$$\delta W_e^* = \int_{a_i}^{b_i} \delta w (-\mathcal{D} w'' + \mathcal{G}V)'' dx - [\delta w' (-\mathcal{D} w'' + \mathcal{G}V_i)]_{a_i}^{b_i} + [\delta w (-\mathcal{D} w'' + \mathcal{G}V_i)']_{a_i}^{b_i} \quad (12)$$

$$= \int_{a_i}^{b_i} \delta w M'' dx - [\delta w' M]_{a_i}^{b_i} + [\delta w M']_{a_i}^{b_i} \quad (13)$$

112 where  $M := -\mathcal{D} w'' + \mathcal{G}V$  is the bending moment in the beam.

113 If we only consider a distributed transverse load  $p(x, t)$  applied to the beam, the virtual work of external  
114 force is:

$$\delta W_{ext}^* = \int_{a_i}^{b_i} \delta w p dx. \quad (14)$$

115 In order to add the non conservative effects due to the internal damping of the beam, an extended Hamilton  
116 principle approach has to be used:

$$\int_{t_1}^{t_2} (\delta T^* + \delta W_e^* + \delta W_{ext} + \delta W_{nc}) = 0 \quad (15)$$

117 where  $\delta W_{nc}$  is the work expended by the the non conservative actions. In the case of internal damping of  
 118 the beam, we suppose the existence of a linear velocity dependent action and this term reads:

$$\delta W_{nc} = - \int_0^\ell \eta w \delta \dot{w} dx \quad (16)$$

119 Finally, by applying the extended Hamilton's principle, the equations of motion reads:

$$\bar{m} \ddot{w} + \eta \dot{w} - M'' = p \quad (17)$$

120 along with the following continuity conditions, where  $[[f(x)]] = f(x^+) - f(x^-)$  is a jump function:

$$[[w]] = 0, \quad (\text{continuity of displacement}), \quad (18)$$

$$[[w']] = 0, \quad (\text{continuity of rotation}), \quad (19)$$

$$[[\mathcal{D} w'' + \mathcal{G}V_i]] = 0, \quad (\text{continuity of moment}), \quad (20)$$

$$[[(\mathcal{D} w'' + \mathcal{G}V_i)']] = 0, \quad (\text{continuity of shear force}). \quad (21)$$

121 As the piezoelectric patch is very thin with respect to the beam, the flexural rigidity  $\mathcal{D}$  and the mass per  
 122 unit length  $\bar{m}$  of the beam, can be considered as uniform in  $[0, \ell]$ . In addition, the strain may be regarded  
 123 as uniform over its thickness  $\epsilon_{xx} = -z_m w''$  where  $z_m = (h + h_p)/2$  is the coordinate of center line of the  
 124 piezoelectric patch. The charge  $Q_i$  of the  $i$ -th patch is defined as the free electric contained electrode area  
 125 [11]:

$$Q_i = b \int_{a_i}^{b_i} -D_z dx = b \int_{a_i}^{b_i} (e_{31} z_m w'' - \epsilon_{33} E_z) dx \quad (22)$$

$$= \mathcal{G}(w'(b_i) - w'(a_i)) + C_i V_i \quad (23)$$

126 where  $C_i = \epsilon_{33} b(b_i - a_i)/h_p$ . If the piezoelectric segment is left open-circuited, then  $Q_i = 0$  and:



$$C_i V_i = -\mathcal{G}(w'(b_i) - w'(a_i)) \quad (24)$$

127 *2.2.2. Equation for the whole domain ( $x \in [0, \ell]$ )*

128 For describing the behavior of the beam with length  $\ell$  with  $n_p$  patches, one may use the same expression  
129 of the equation (17), with a more general formulation of bending moment:

$$M = -\mathcal{D}w'' + \sum_{i=1}^{n_p} \mathcal{G}V_i(H(x - a_i) + H(x - b_i)), \quad x \in [0, \ell]. \quad (25)$$

130 where  $H(\cdot)$  denotes the Heaviside function.

131 Moreover, considering the beam with one free extremity and the other mechanically connected to another  
132 mechanical system, the boundary conditions at  $x_1 = 0$  and  $x_1 = \ell$  are:

$$w''(0) = 0, \quad (\text{no bending moment on the free end}), \quad (26)$$

$$w'''(0) = 0, \quad (\text{no shear force on the free end}), \quad (27)$$

$$\mathcal{D} w''(\ell) = M_{ext}, \quad (\text{bending moment imposed by the external mechanical system}), \quad (28)$$

$$\mathcal{D} w'''(\ell) = T_{ext}, \quad (\text{shear force imposed by the external mechanical system}). \quad (29)$$

133 *2.3. Modal expansion*

134 The harmonic motion of the beam may be expressed by  $N$  short-circuit eigenmodes  $\omega_k, \phi_k(x)$  ( $i = 1..N$ ):

$$w(x, t) = \sum_{k=1}^N \phi_k(x) q_k(t) \quad (30)$$

135 where  $q_k(t)$  is a time-dependant function.  $\omega_k, \phi_k(x)$  may solved from by Eq. (17) with  $p = 0$  and  $V = 0$ :

$$-\omega_k^2 \bar{m} \phi_k + \mathcal{D} \phi_k^{(4)} = 0 \quad (31)$$

136 where  $^{(4)}$  represents the fourth derivative with respect to the coordinate  $x$ . The modes  $\phi_k$  are orthogonal  
137 and may be normalized by:

$$\int_0^\ell \phi_j(x)\phi_k(x) = \delta_{jk} \quad (32)$$

138 By substituting (30) into (25), (17) , and multiplying the results by  $\phi_j$ , one obtains  $N$  equations ( $j = 1..N$ ):

$$\bar{m}\ddot{q}_j(t) + \omega_j^2\bar{m}q_j(t) + \eta_j\dot{q}_j - \sum_{i=1}^{n_p} g_{ij}V_i = F_j \quad (33)$$

139 where  $F_j = \int_0^\ell \phi_j p(x, t) dx$ .

140 The value of the coupling coefficient  $g_{ij}$  of the patch  $i$  with respect to the mode  $j$  is defined by:

$$g_{ij}(a_i, b_i) = \mathcal{G} \int_{a_i}^{b_i} \phi_j''(x)dx = \mathcal{G} (\phi_j'(b_i) - \phi_j'(a_i)) \quad (34)$$

141 Similarly, substituting (30) into (24) leads to  $n_p$  equations ( $i = 1..n_p$ ):

$$C_i V_i + \sum_{j=1}^N g_{ij} q_j(t) = 0 \quad (35)$$

142 By seeking the harmonic solutions under the forms:

$$q_j(t) = \tilde{q}_j(\omega)e^{i\omega t}, \quad V_i(t) = \tilde{V}_i(\omega)e^{i\omega t}, \quad F_j(t) = \tilde{F}_j(\omega)e^{i\omega t}, \quad (36)$$

143 the final system of modal equations (Eqs. 37, 38) reads:

$$-\omega^2\bar{m}\tilde{q}_j + \omega_j^2\bar{m}\tilde{q}_j + i\omega\eta_j\tilde{q}_j - \sum_{i=1}^{n_p} g_{ij}\tilde{V}_i = \tilde{F}_j, \quad (j = 1..N) \quad (37)$$

$$C_i\tilde{V}_i + \sum_{j=1}^N g_{ij}\tilde{q}_j = 0, \quad (i = 1..n_p) \quad (38)$$

#### 144 2.4. Optimization of the transducers positions

145 In this section, an optimization procedure is described for positioning the piezoelectric patch on a struc-  
 146 ture so that the measurement of specific modes is maximized. The aim is to find the utility function that  
 147 represents the piezoelectric patch sensitivity as a function of the specific modes of interest. More specifically,  
 148 we intend to find the optimal placement  $a_i$  for the patch such as the coupling  $g_{ij}$  between the  $i$ -th patch  
 149 and the  $j$ -th mode is maximized.

150 *2.4.1. Modal sensitivity*

151 We compute  $\tilde{q}_j$  from Eq. (37), then inject the result into Eq. (38). By neglecting the quadratic terms  
 152 with respect to the coupling term  $g_{ij}$ , we obtain the so called sensor equation for each patch  $i$ :

$$\tilde{V}_i = - \sum_{j=1}^N \frac{g_{ij} \tilde{F}_j}{C_i \bar{m} (\omega_j^2 - \omega^2 + i\omega\eta_j/\bar{m})} \quad (39)$$

153 In our case the external force is a point force applied at a specific point of the beam  $x_f$ , with amplitude  
 154  $\tilde{F}_0(\omega)$ , i.e.  $\tilde{p}(x, \omega) = F_0(\omega)\delta(x - x_f)$ . This means that:

$$\tilde{F}_j = \int_0^\ell \tilde{p}(x, \omega) \phi_j(x) dx = \int_0^\ell \tilde{F}_0(\omega) \delta(x - x_f) \phi_j(x) dx = \tilde{F}_0 \phi_j(x_f) \quad (40)$$

155 then the voltage measured on each piezo is:

$$\tilde{V}_i = - \sum_{j=1}^N g_{ij} \frac{\tilde{F}_0(\omega) \phi_j(x_f)}{C_i \bar{m} (\omega_j^2 - \omega^2 + i\omega\eta_j/\bar{m})} \quad (41)$$

156 Finally, we can define the transfer function at the  $i$ -th patch as:

$$H_i(\omega) = \frac{\tilde{V}_i(\omega)}{\tilde{F}_0(\omega)} = - \sum_{j=1}^N \frac{g_{ij} \phi_j(x_f)}{C_i \bar{m} (\omega_j^2 - \omega^2 + i\omega\eta_j/\bar{m})} \quad (42)$$

157 This transfer function is at the core of the optimization process.

158 The contribution of the  $j$ -th mode to the frequency response function Eq. (42) is:

$$G_{ij}(\omega) = \left| \frac{g_{ij} \phi_j(x_f)}{C_i \bar{m} (\omega_j^2 - \omega^2 + i\omega\eta_j/\bar{m})} \right|$$

159 In order to estimate the sensitivity of each piezo patch  $i$  to mode  $j$ , with respect to the position  $a_i$ , we can  
 160 introduce the following norm  $\|\cdot\|_\infty$ , that in our case corresponds to evaluating the contribution of the  $j$ -th  
 161 mode at resonance frequency  $\omega_j$ :

$$\kappa_{ij} = \|G_{ij}(\omega_j)\|_\infty = \left| \frac{g_{ij} \phi_j(x_f)}{C_i i\omega_j \eta_j} \right| \quad (43)$$

162 For each given patch  $i$ , a normalised form of modal sensitivity to mode  $j$  is given by:

$$\mathcal{M}_{ij} = \frac{\kappa_{ij}}{\max_{j=1..N} \kappa_{ij}} \quad (44)$$

163 where  $\mathcal{M}_{ij}$  represents the control authority of the  $i$ -th patch fixed over the  $j$ -th mode. Since the aim is to  
 164 maximize the sensitivity of the piezoelectric sensor in regard to the detection of one or several mode(s) rather  
 165 than using the patch as an actuator to control the mode(s), in this paper, it is named *modal sensitivity* in  
 166 comparison with notion of *modal controllability* used in Halim and Moheimani [14].

#### 167 2.4.2. Definition of a new utility function

168 Different utility functions can be defined starting from the concept of modal sensitivity. The idea is to  
 169 obtain modal filtering, which is achieved by adding a weight in the optimization for specific modes. In this  
 170 way it is possible to “see” or “hide” a mode [21]. To this end we define a vector  $\xi$  of modal weights, whose  
 171 the component  $\xi_j$  corresponds to weight of the  $j$ -th mode:

$$\xi_j = \begin{cases} 1 & \text{if the } j\text{-th mode has to be seen} \\ -1 & \text{if the } j\text{-th mode has to be hidden} \\ 0 & \text{if the } j\text{-th mode is not relevant} \end{cases} \quad (45)$$

172 Then, for each value of  $a_i, b_i$  of the transducer and assuming that  $x_f$  is fixed, the utility function is  
 173 defined as:

$$\mathcal{F}(a_i, b_i, x_f) = \sum_{j=1}^N \xi_j \mathcal{M}_{ij}(a_i, b_i, x_f) \quad (46)$$

174 and the optimal positions  $\{\hat{a}_i, \hat{b}_i\}$  is defined by:

$$\{\hat{a}_i, \hat{b}_i\} = \arg \max(\mathcal{F}(a_i, b_i, x_f)) \quad (47)$$

175 Several transducers can be optimized at the same time, by choosing local maxima of the transfer function.  
 176 In this case, constraints on transducers sizes have to be considered to avoid the overlap of two patches. The  
 177 practical application of the method will be detailed in the case study presented in Section 3.

### 178 3. Case study : optimal transducers positioning for implant stability monitoring

179 The optimization method described in Section 2.4 is applied to a square beam named ancillary hereafter,  
 180 and connected to an inaccessible mechanical system corresponding to an orthopedic implant into a bone  
 181 mimicking phantom (Fig. 2). The ancillary is a part fixed to the implant during the surgery in order to

182 enable the surgeon to apply hammer impact at one of its extremity, leading to implant insertion. The aim  
 183 of this case study is to present and validate a procedure of piezoelectric patches placement optimization  
 184 for monitoring the stability of an implant which is not directly accessible for vibration measurement. The  
 185 optimization principle lies on focusing on specific modes of vibration of the ancillary, which are sensitive to  
 186 implant stability into the bone, and to hide modes which are not of biomechanical interest.

### 187 3.1. Geometry

188 A stainless steel beam of square section was designed with the geometrical and material properties given  
 189 in Table C.4 (see Appendix C.1). The bottom threaded extremity was rigidly fixed in the implant and the  
 190 top extremity was kept free. The implant was a cementless femoral stem made of titanium alloy (TiAl6Al4V)  
 191 and coated with hydroxyapatite (CERAFIT RMIS, Ceraver, Roissy, France). The implant was fully inserted  
 192 into a bone mimicking phantom (ORTHObones, 3B Scientific, Hamburg, Germany) embedded in polymer  
 193 resin in order to enable clamping to the vibration table, similarly as what was done in [20, 32].

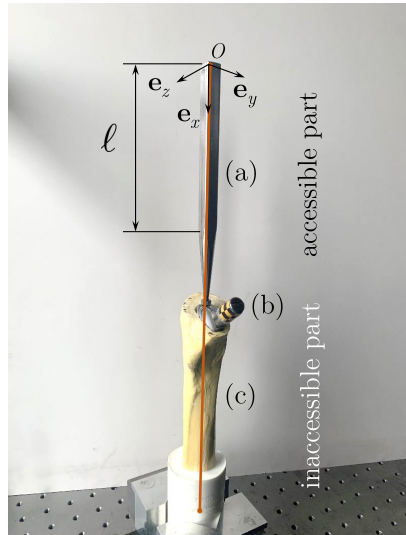


Figure 2: Experimental case study configuration. The same notations as the ones shown in Figure 1 are indicated : accessible instrumented length  $\ell$ , axis directions and coordinates origin  $O$ . The ancillary (a) corresponds to the accessible part, the implant (femoral stem) (b) and the bone (femur) (c) represent the inaccessible part.

### 194 3.2. Optimization algorithm

195 The experimental optimization approach consists in the following successive steps described in Fig. 3:

- 196 1. **Modal characterization of the ancillary without piezoelectric sensors.** The modes of vibration  
 197 of the ancillary are measured in the frequency range [0.4 12.8] kHz by Experimental Modal Analysis

198 (EMA). The experimental mode shapes  $\phi_j^e$ , the resonance frequencies  $f_j = 2\pi\omega_j$  and the damping  
 199 coefficients  $\beta_j = \eta_j/\bar{m}$  are retrieved.

200 **2. Piezoelectric patches placement optimization.** Different utility functions are defined from Eq.  
 201 (46) based on the modal sensitivity  $\mathcal{M}_{ij}$  calculated for each position  $a_i$  of piezo  $i$  and mode  $j$ . Con-  
 202 sidering the information carried by each mode  $j$  on the implant stability, a vector  $\xi$  is defined. For  
 203 each utility function  $\mathcal{F}$ , the optimal piezoelectric patch position  $\{\hat{a}_i, \hat{b}_i\}$  is determined by Eq. (47).  
 204 One patch is associated with one utility function and  $n_p$  patches are used.

205 **3. Piezoelectric patches placement validation:** The amplitude ratios at  $\omega_j$  are calculated between  
 206 the frequency response functions  $H_i$ , measured successively with each piezoelectric patch  $i$ . The results  
 207 are compared with what was expected from the definition of each utility function  $\mathcal{F}$ .

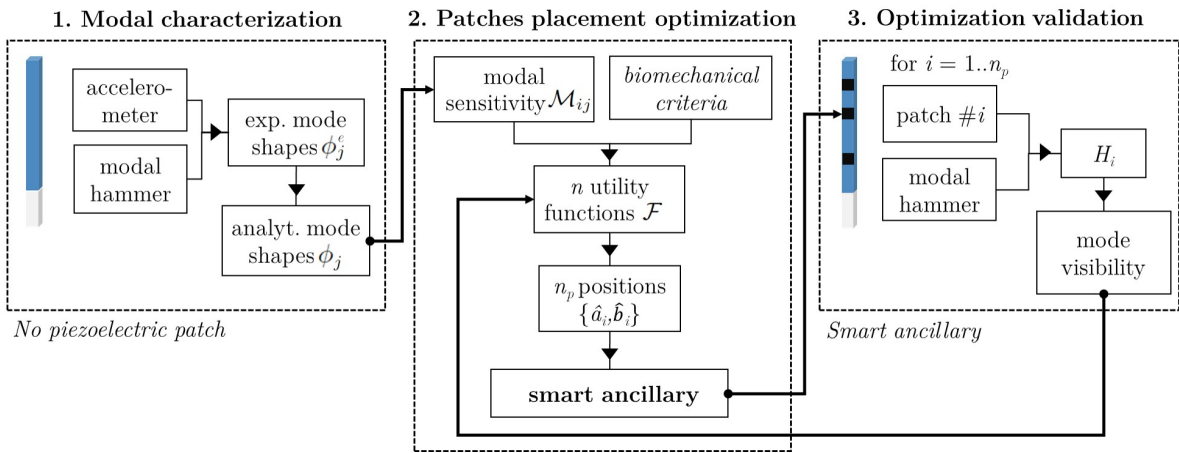


Figure 3: Experimental procedure for piezoelectric patches placement optimization and protocol validation.

### 208 3.3. Modal characterization of the ancillary

209 *Modal analysis measurements.* The ancillary modal features were determined by performing EMA following  
 210 the same protocol described in [20] and shown in Fig. 4. Only the mode shapes oscillating along  $e_z$ , were  
 211 measured, since the properties of the mode shapes along  $e_y$  were shown to not evolve with implant insertion  
 212 into the bone in a previous study [20]. The structure was excited by hammer impacts (modal hammer  
 213 8204, *Brüel & Kjaer, Naerum, Denmark*) at 13 locations  $x_f$  equally spaced of 10 mm beginning at 5 mm  
 214 from the ancillary top surface. At each location, the structure was excited 5 times along  $e_z$  (see Fig. 4).  
 215 The  $z$ -accelerations consecutive to each impact were recorded by a tri-axial accelerometer (365A01PCB  
 216 *Piezotronics, Depew, NY, USA*) glued at the ancillary top extremity, as shown in Fig. 4. The excitation  
 217 impact was lower than 50N to ensure it does not modify the implant position and consequently the boundary

218 conditions of the beam during the modal features measurement. A dedicated data acquisition module (BK  
 219 Connect, *Brüel & Kjaer, Naerum, Denmark*) with a sampling rate of 51.2 kHz was used to record the  
 220 acceleration signals during 0.25 s. We assume that the weight of the accelerometer (1 g) and the weight and  
 221 rigidity of the piezoelectric patches do not influence the structure dynamic behavior.

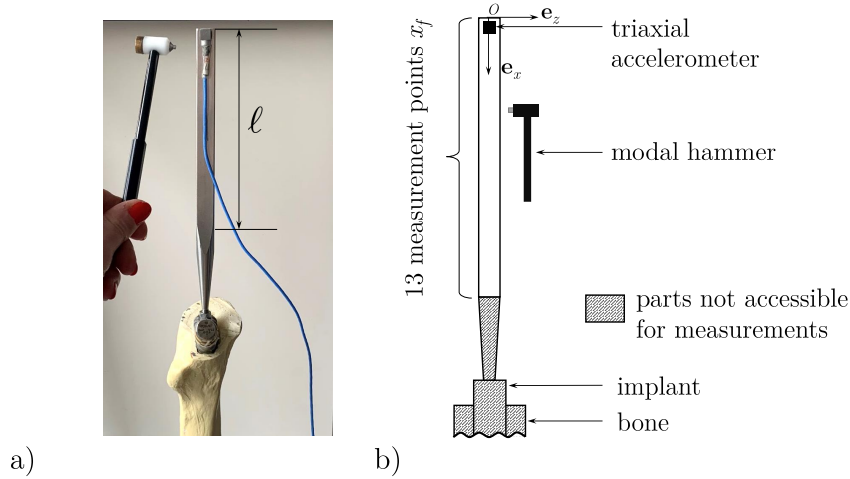


Figure 4: a) Picture and b) scheme of the Experimental Modal Analysis set-up with  $\ell = 125$  mm.

222 *Modes selection.* An example of two frequency response functions (*frf*) measured in the range [0.4 12.8]  
 223 kHz for an impact at  $x_f = 5$  mm along  $e_z$ , is presented in Fig. 5. The two *frf* correspond to two different  
 224 implant insertion stages, one at beginning of insertion (in grey) and one for the implant fully seated (in  
 225 black). The choice to show *frf* corresponding to an impact location at  $x_f = 5$  mm is explained because it is  
 226 a good compromise to see all the modes on the frequency range [0.4 12.8] kHz, due to the proximity to the  
 227 free extremity, where the mode shapes are close.

228 Five modes were selected from the *frf* of implant fully inserted, for the patch placement optimization  
 229 analysis, denoted  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  hereafter. Table 1 gathers the modal features of the five selected modes  
 230 obtained for the implant fully seated : resonance frequency  $f$  and damping factor  $\beta$ , calculated for each  
 231 resonance frequency with the 3 dB method. The resonance frequencies of modes  $A$  and  $B$  are in good  
 232 agreement with what was found in [20]. These two resonance frequencies increase during implant insertion  
 233 into the bone and reach values of 2.6 kHz and 3.3 kHz once the implant fully seated, which corresponds to  
 234 the behavior of modes  $2Y$  and  $2Y_b$ , identified in [20].

*Mode shapes.* The experimental modal shapes  $\phi_j^e$  of the five selected modes presented in Table 1 and in Fig.  
 5 are shown in Fig. 6. The five modes are of bending nature, with either two nodes (modes  $A$ ,  $B$  and  $C$ ) are

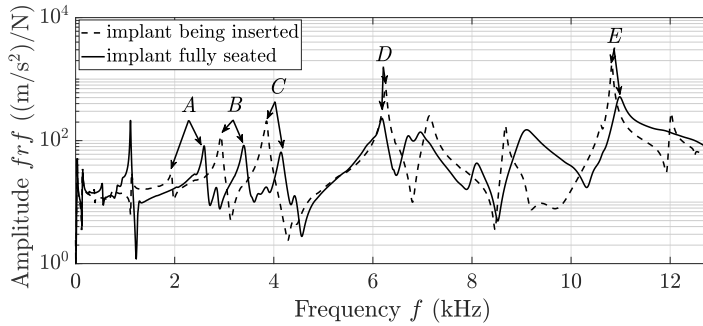


Figure 5: Frequency response functions,  $frf$ , measured by the accelerometer for an impact at  $x_f = 5$  mm along  $e_z$  at beginning (in grey) and end (in black) of implant insertion. The letters from  $A$  to  $E$  indicate the modes selected for the patch placement optimization analysis.

Mode #	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
$f$ (Hz)	2609	3391	4160	6157	10956
$\beta$ (-)	2.19	1.20	1.74	1.86	0.64

Table 1: Names and modal features (resonance frequency  $f$ , damping factor  $\beta$ ) of the modes selected for the piezoelectric patches placement optimization procedure.

three nodes for higher resonance frequencies (modes  $D$  and  $E$ ). This result confirms that mode  $A$  and  $B$  correspond to modes  $2Y$  and  $2Y_b$  in [20]. Each mode shape  $\phi_j^e$  was normalized by its Euclidean norm  $\|\cdot\|_2$ . Each discrete experimental mode shape  $\phi_j^e$ , was fitted using the general equation of normal beam shapes adapted from Eq. (31):

$$\phi_j^{(4)} = \frac{\omega_j^2 \bar{m}}{\mathcal{D}} \phi_j \quad (48)$$

where  $\lambda_j^{(4)} = \omega_j^2 \bar{m} / \mathcal{D}$ . Hence, the general solution of Eq. (48) is given by:

$$\phi_j = C_1 \sin(\lambda_j x) + C_2 \cos(\lambda_j x) + C_3 \sinh(\lambda_j x) + C_4 \cosh(\lambda_j x) \quad (49)$$

where the constant  $C_{1,2,3,4}$  are determined from the boundary conditions expressed by Eq. (26) to (29). In our case study, considering a free end at  $x = 0$ , the bending moment and shear force were both equal to zero at this end (Eq. (26) and Eq. (27)) which gives  $C_1 = C_3$  and  $C_2 = C_4$ . Then, for each mode  $j$ , the values of  $C_1$  and  $C_2$  were determined using an error minimization algorithm in Matlab. The mode shapes were fitted in order to further calculate the coupling coefficient  $g_{ij}$  from the mode shape curvature  $\phi_j''$  (Eq. 34), which requires a continuous shape.

#### 3.4. Piezoelectric patches placement optimization

This section is dedicated to the definition of one or more utility function(s)  $\mathcal{F}$  aiming at finding patches positions  $\hat{a}_i$  ( $i = 1..n_p$ ) on the ancillary such as a combination of modes  $\phi_j$  ( $j = 1..N$ ) are visible or hidden



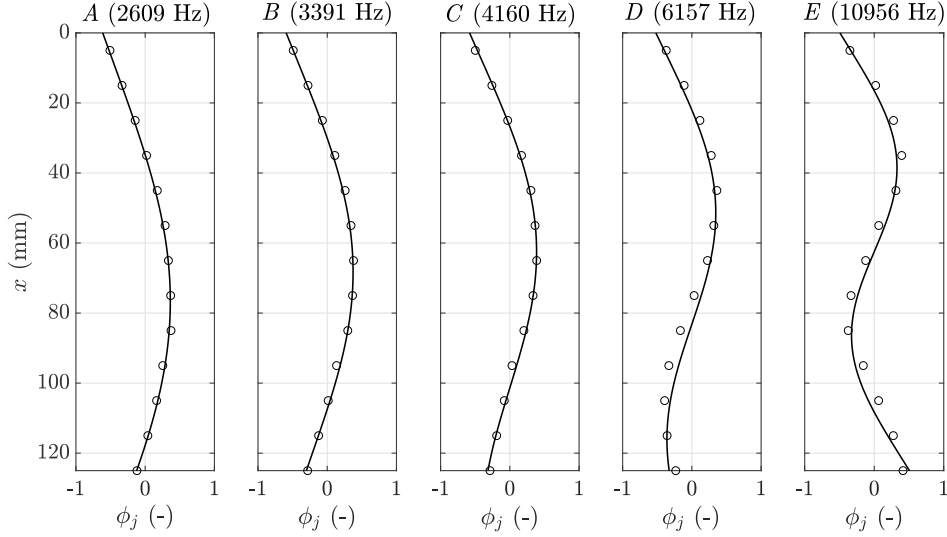


Figure 6: Experimental (o) and fitted (-) mode shapes  $\phi_j$  of modes  $A$  to  $E$ , vibrating along  $\mathbf{e}_z$ .

245 when measuring the frequency response function  $H_i$  by the piezoelectric patch  $i$ . The choice to see or hide  
 246 one or several mode(s) is based on the mode resonance frequency sensitivity to the implant position into  
 247 the bone, corresponding to the so-called "inaccessible part". The modes  $\phi_j$  considered for the optimization  
 248 analysis are modes # $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$ .

249 *Piezoelectric patch properties.* The piezoelectric transducers that are used for resonance frequency functions  
 250 measurement are square shaped with a length  $b = h = 10$  mm and a thickness  $h_p = 0.2$  mm. The geometry is  
 251 fixed and constant over the patches. The detailed properties are given in Table C.5 in the Appendix C.2. The  
 252 same configuration as presented in Section 2.1 and Fig. 1 with two piezoelectric patches put symmetrically  
 253 at the upper and lower surfaces of the beam is considered hereafter.

254 *Modal sensitivity.* As described in Section 2.4, the normalized modal sensitivity  $\mathcal{M}_{ij}$  defined by Eq. (44)  
 255 is calculated for each mode  $j$  and patch  $i$  by varying the position  $a_i$  of the patch. As the length  $\ell$  of the  
 256 instrumented beam is 125 mm and the patches are 10 mm length,  $a_i \in [0-115]$  mm, in order to avoid the  
 257 patch to exceed the accessible part of the structure. The spatial resolution for varying the position  $a_i$  of the  
 258 patch and calculating  $\mathcal{M}_{ij}$  is 1 mm.

259 For each mode  $j$ , the results show at least one minimum and maximum on the modal sensitivity function  
 260  $\mathcal{M}_{ij}$  (see Fig. 7) depending on the patch position  $a_i$ . The higher frequency modes  $D$  and  $E$  (6157 Hz and  
 261 10956 Hz, respectively) have two local maxima which offer two possibilities for patch placement whereas  
 262 lower frequency modes  $A$ ,  $B$  and  $C$  (2609 Hz, 3391 Hz and 4160 Hz, respectively) only hold one maximum.

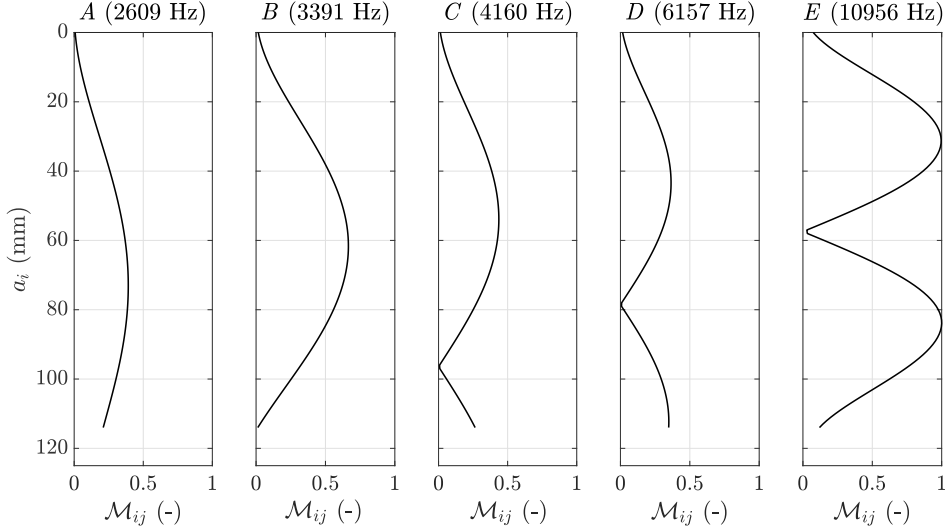


Figure 7: Modal sensitivities  $\mathcal{M}_{ij}$  depending on the patch position  $a_i$  on the ancillary calculated for each mode  $A$  to  $E$ .

263 *Choice of transducers positions.* The strategy employed for the definition of a utility function  $\mathcal{F}$  is to use  
264 one transducer associated to multiple modes, as explained in the section 2.4.2. Different utility functions  
265 can be defined, each function being associated with an optimal patch position  $\{\hat{a}_i, \hat{b}_i\}$ , defined by Eq. (47).  
266 The comparison of the *frf* measured at the beginning of implant insertion and once the implant is fully  
267 seated (see Fig. 5) shows that 4 modes (namely, modes # $A$ ,  $B$ ,  $C$  and  $E$ ) over the 5 selected for the study,  
268 are sensitive to implant stability with a frequency shift of more than 150 Hz to higher frequencies between  
269 the beginning and end of implant insertion. The sensitivity of the modes  $A$  and  $B$  is in good agreement  
270 with what was obtained in [20].

271 From these results and in order to demonstrate the performance of the optimization method, three  
272 examples of utility functions  $\mathcal{F}$  are defined with Eq. (46), for which the modal weights  $\xi_j$  associated to each  
273 mode  $j$  are shown in Table 2. Considering these examples, each utility function  $\mathcal{F}$  is associated with one  
274 position  $\hat{a}_i$  of the patch  $i$  such as:

- 275 •  $\mathcal{F}_1$  : The resulting position  $\hat{a}_1$  of the patch assumes to be the best compromise to see all the modes  
276  $A$  to  $E$  on the frequency response function ( $\xi_j=1$ ). This configuration acts as a *reference case*, which  
277 will allow the comparison with other configurations where modes should be filtered ( $\mathcal{F}_2, \mathcal{F}_3$ ), in order  
278 to evaluate the efficiency of the optimization method.
- 279 •  $\mathcal{F}_2$  : The resulting position  $\hat{a}_2$  of the patch assumes to be the best compromise to hide mode  $D$  ( $\xi_4=-1$ )  
280 while letting the other modes visible ( $\xi_{j \neq 4}=1$ ). This configuration enables to focus only on the modes

281 sensitive to implant insertion (namely, modes  $A$ ,  $B$ ,  $C$  and  $E$ ), since the resonance frequency of mode  
 282  $D$  has shown to not significantly change during implant insertion (see Fig. 5).

- 283 •  $\mathcal{F}_3$  : The resulting position  $\hat{a}_3$  of the patch assumes to be the best compromise to filter mode  $C$  ( $\xi_3=-1$ )  
 284 while ensuring visibility of mode  $B$  ( $\xi_2=1$ ). The other modes are not concerned by the optimization  
 285 configuration ( $\xi_{1,4,5}=0$ ). This configuration enables to focus on the tracking of the resonance frequency  
 286 of mode  $B$  for implant insertion monitoring, by avoiding confusing with mode  $C$ , due to their close  
 287 resonance frequencies.

Modes	$A$	$B$	$C$	$D$	$E$	$\{\hat{a}_i, \hat{b}_i\}$ (mm)
$\mathcal{F}_1$	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>{40, 50}</b>
$\mathcal{F}_2$	1	1	1	-1	1	{78, 88}
$\mathcal{F}_3$	1	1	-1	1	1	{91, 101}

Table 2: Modal weights  $\xi_j$  defined for the utility functions  $\mathcal{F}_{1,2,3}$  and resulting optimal piezoelectric positions  $\hat{a}_1$ ,  $\hat{a}_2$ ,  $\hat{a}_3$ . The reference case corresponding to  $\mathcal{F}_1$  is highlighted in bold.

288 Figure 8b shows the three utility functions  $\mathcal{F}_1$ ,  $\mathcal{F}_2$ ,  $\mathcal{F}_3$  calculated for the weight coefficients  $\xi_j$  defined in  
 289 Table 2. The modal sensitivities  $\mathcal{M}_{ij}$  are represented in Fig. 8a for  $j = 1..5$ . For each utility function, only  
 290 one maximum is identified, corresponding to the position  $\hat{a}_i$  of the piezoelectric patch defined by Eq. (47).  
 291 Note that to avoid overlap, for each patch  $i$ , we verified that  $|\hat{a}_i - \hat{a}_{i+1}| > b$ , in order to allow the bounding  
 292 of the 3 transducers on the same beam.

### 293 3.5. Validation of optimization placement protocol

294 The structure has been assembled following the optimization procedure described in the previous section.  
 295 The piezoelectric patches have been bonded on the surface of the beam using a one-component silver-filled  
 296 epoxy resin (Elecolit 3043, *Panacol, Germany*) cured at 160°C during 30 min. The resulting smart ancillary  
 297 is shown in Fig. 9.

298 To assess the validity of the patches placement optimization procedure, a frequency analysis was per-  
 299 formed in order to compare the frequency response functions  $H_i$  measured by each patch  $i$ . To do so, the  
 300 ancillary was excited by a modal hammer, impacting the structure at  $x_f = 5$  mm along  $e_z$ . The choice  
 301 of the position of the source  $x_f$  is explained by the similarity of the mode shapes near the free extremity  
 302 (see Fig. 6), which provides optimization results independent of  $x_f$ . However, as shown in Eq. (42), it  
 303 would have been possible to study and optimize the position of the excitation source  $x_f$  in the calculation  
 304 of the utility functions  $\mathcal{F}$ . The system dynamic response was measured successively by one of the three  
 305 piezoelectric patches, the two others being short-cut during the acquisition time. The sampling frequency

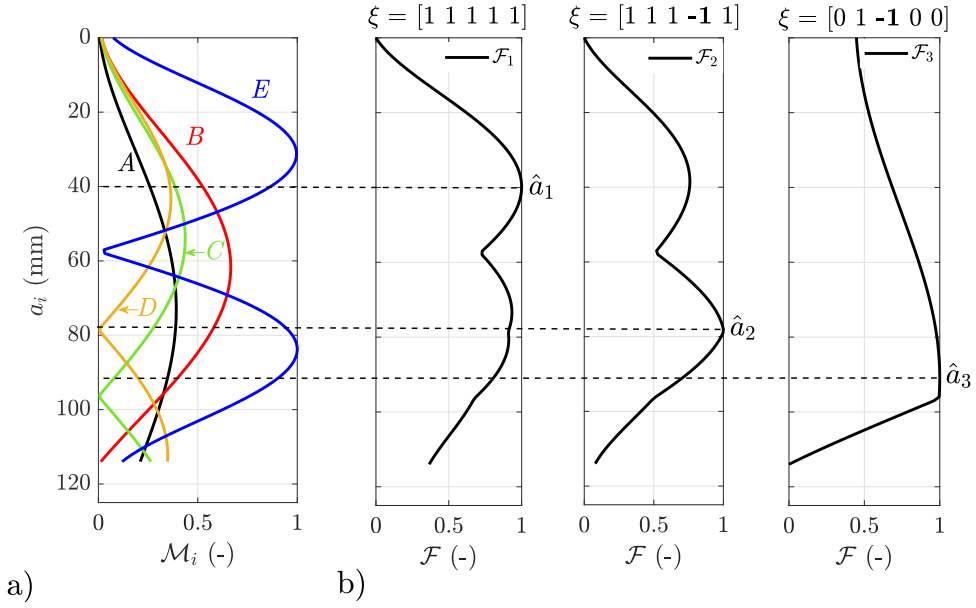


Figure 8: a) Modal sensitivities  $\mathcal{M}_{ij}$  as a function of the patch position  $a_i$  calculated for  $j = 1..5$  (namely, modes  $A$  to  $E$ ) and b) corresponding utility functions  $\mathcal{F}_{1,2,3}$  calculated for  $\xi = [1 \ 1 \ 1 \ 1 \ 1]$ ,  $\xi = [1 \ 1 \ 1 \ -1 \ 1]$  and  $\xi = [0 \ 1 \ -1 \ 0 \ 0]$ , respectively, with the optimal positions  $\hat{a}_i$  corresponding to the maximum of each utility function  $\mathcal{F}$ .

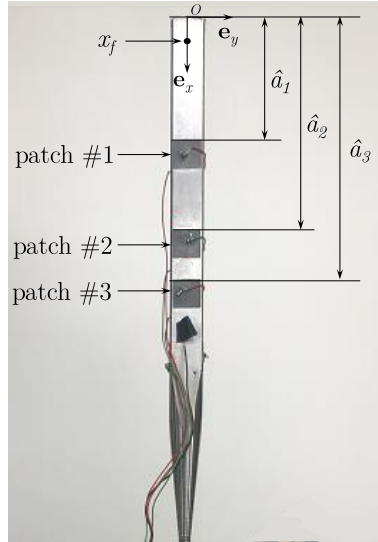


Figure 9: Image of the instrumented ancillary equipped with piezoelectric patches #1, #2, #3 bonded at locations  $\hat{a}_1 = 40$  mm,  $\hat{a}_2 = 78$  mm and  $\hat{a}_3 = 91$  mm, calculated from the optimization procedure described in Section 3.4

306 and the acquisition time were the same as for the modal characterization, leading to a frequency resolution  
 307  $df = 4$  Hz.

308 The frequency response function  $H_1$  obtained by the piezoelectric patch #1, placed such as the utility  
 309 function allow the visibility of all the set of modes  $A$  to  $E$ , is compared with  $H_2$  measured by the piezoelectric

310 patch #2 in Fig. 10a and with  $H_2$  measured by the piezoelectric patch #3 in Fig. 10b. The amplitudes at  
 311 resonances measured by the three piezoelectric patches are given in Table 3 together with the amplitude's  
 312 variation at resonance obtained for patch #2 and patch #3 with respect to the reference patch #1.

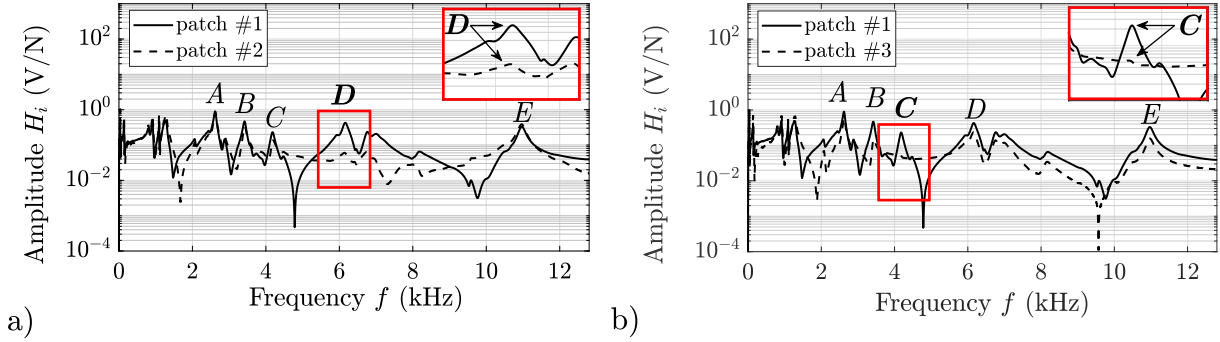


Figure 10: a) Comparison of the frequency response functions  $H_1$  and  $H_2$  measured by the piezoelectric patch #1 and the piezoelectric patch #2 and b) comparison of  $H_1$  and  $H_3$  measured by the piezoelectric patch #1 and the piezoelectric patch #3. A zoom on the target modes by the utility functions  $\mathcal{F}_2$  and  $\mathcal{F}_3$  is included in the graphs.

Modes	A	B	C	D	E
patch #1 (V/N)	<b>0.898</b>	<b>0.467</b>	<b>0.231</b>	<b>0.427</b>	<b>0.330</b>
patch #2 (V/N)	0.851 (-5.24%)	0.337 (-27.8%)	0.136 (-41.3%)	0.060 (-85.9%)	0.386 (+17.0%)
patch #3 (V/N)	0.600 (-33.2%)	0.152 (-67.4%)	0.054 (-76.8%)	0.234 (-45.2%)	0.156 (-52.7%)

Table 3: Amplitudes at resonances of modes A to E measured by patches #1, #2, #3 together with the percentage of amplitude's decrease at resonance indicated in brackets, with regard to measurements from patch #1. The patch #1 corresponds to the reference case and is highlighted in bold.

313 The amplitude decrease of 85.9% of mode D measured by the piezoelectric patch #2 with regard to  
 314 patch #1, validates the position optimization procedure since the utility function  $\mathcal{F}_2$  aimed to hide mode  
 315 D. Similarly, the frequency measurement with the patch #3 leads to a decrease of 76.9% of the amplitude  
 316 of the mode C with regard to patch #1, which was expected by the utility function  $\mathcal{F}_3$ . Nevertheless,  
 317 the unintended associated decrease of the amplitude of mode B show the limitation of the utility function,  
 318 defined by weight coefficient  $\xi_j$  and patch position  $a_i$ . This unintended decrease is explained by the strong  
 319 similarity between the shapes of mode B and C (see Fig. 6). Therefore, another condition based on a  
 320 minimum visibility of the modes associated with  $\xi_j=1$  should be implemented in the utility function to  
 321 ensure mode visibility. It could also be relevant to study the effect of the geometrical parameters of the  
 322 piezoelectric patch as the length  $b$  or the thickness  $h_p$ , both implied in the calculation of the coupling  
 323 coefficient  $g_{ij}$  and therefore  $\mathcal{M}_{ij}$ .

#### 324 4. Conclusion and discussions

325 This paper proposes a method for optimizing the positioning of piezoelectric patches on a beam attached  
326 to an unreachable structure. The issue of maximizing the observability on the frequency response function  
327 of a set of modes, which is sensitive to the beam boundary conditions, was addressed. For that purpose,  
328 an optimization strategy was developed, based on the definition of utility functions, expressed in terms of  
329 modal weight and in terms of the so-called modal observability, for which the expression has been exhibited.  
330 It has been shown that the modal observability depends on the electro-mechanical coupling coefficient,  
331 calculated from the patches geometrical and material properties, and the beam mode shapes curvatures.  
332 The originality of the paper comes from the use of the modal electro-mechanical coupling factor (MEMCF)  
333 to optimize sensors placement on the beam in order to improve beam boundary condition monitoring. An  
334 original application for implant stability measurement was proposed.

335 In the particular case developed in this paper, one piezoelectric patch is associated to several modes,  
336 whose the observability has to be improved. Therefore, the optimal piezoelectric patches positions are the  
337 result of a compromise obtained from the utility functions, aiming at improving the observability of a set  
338 of modes sensitive to beam boundary conditions, while minimizing the observability of the modes which are  
339 not sensitive to them. In the experimental case study, the particular issue of monitoring implant stability  
340 by means of a smart beam was investigated. The smart beam, which is basically a tool used by the surgeon  
341 to insert the implant, was equipped with patches for which the placement was determined to optimize the  
342 mode observability on the FRF, depending on the sensitivity to the stability, that was demonstrated in  
343 a previous study [20]. The mode observability measured by the position-optimized patches was in good  
344 agreement with what was expected from the utility functions definition. The amplitude of the modes that  
345 should be hidden on the FRF because they are not sensitive to implant stability decreased up to 85% with  
346 regard to the reference position. The results of this study open the path towards the conception of a smart  
347 tool optimized for the real-time monitoring of implant stability during the insertion of an implant by the  
348 surgeon.

349 However, since the optimization strategy is based on the modal observability and in particular the  
350 coupling coefficient, the positions strongly depend on the mode shapes curvatures. For low frequency  
351 modes, the placement possibilities are limited because of the few number of nodes, and therefore close mode  
352 shapes will tend for close piezoelectric patches positions. Moreover, another drawback of the study is that  
353 optimization of the dimensions of the piezoelectric patches was not studied, whereas the electro-mechanical  
354 coefficient also depends on the thickness or the length of the patch. This aspect was studied in [11] and

355 could be easily implemented in the strategy proposed in this paper. Another way of optimization, would be  
356 to use specific forms of piezoelectric patches, enabling accurate boundary force measurement, as proposed in  
357 [33]. Nevertheless, this method is more complex to implement regarding both the piezoelectric patch design  
358 and the post-processing analysis.

359 The method proposed in this paper is particularly adapted for applications where mode shapes change  
360 with boundary conditions, as it relies on mode shape curvatures. It could be applied for other configurations  
361 of beam-like structures with bending modes that should be tuned, based on the modal observability concept.  
362 Also, a relevant perspective will be to investigate the placement optimization of the excitation source, and  
363 in particular, by using one of the piezoelectric patch as an actuator. This outlook is of particular interest  
364 concerning the biomedical application presented in the case study, since it would lead to an automation  
365 of the measurement by preventing from the use of a modal hammer as a source of excitation that is not  
366 convenient in a clinical environment. Then, as investigated by several authors [23, 24, 6], another strategy  
367 approach could be to use of a network of sensors along the beam with different control signals to tune specific  
368 modes. The main advantage of this approach would be to conceive one unique beam for several applications.

## 369 **5. Acknowledgments**

370 This project has received funding from the European Research Council (ERC) under the European  
371 Union's Horizon 2020 research and innovation program (grant agreement No 682001, project ERC Consol-  
372 idator Grant 2015 BoneImplant), from the project OrthAncil (ANR-21-CE19-0035-03) and from the project  
373 OrthoMat ( ANR-21-CE17-0004).

374 **Appendix A. Constitutive equations of piezoelectric materials**

*Constitutive equations.* Assuming a linear piezoelectric material, the 3D constitutive equations may be written in matrix form as follows [34]:

$$\begin{pmatrix} \boldsymbol{\sigma} \\ \mathbf{D} \end{pmatrix} = \begin{bmatrix} \mathbf{c} & -\mathbf{e}^T \\ \mathbf{e} & \boldsymbol{\varepsilon} \end{bmatrix} \begin{pmatrix} \boldsymbol{\epsilon} \\ \mathbf{E} \end{pmatrix} \quad (\text{A.1})$$

where  $\boldsymbol{\sigma} = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy})^T$  is the stress vector;  $\mathbf{D} = (D_x, D_y, D_z)^T$  is the electric displacement vector;  $\boldsymbol{\epsilon} = (\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, 2\epsilon_{yz}, 2\epsilon_{xz}, 2\epsilon_{xy})^T$  is the strain vector; and  $\mathbf{E} = (E_x, E_y, E_z)^T$  is the electric field vector. The matrices  $\mathbf{c}$ ,  $\mathbf{e}$ ,  $\boldsymbol{\varepsilon}$  are the matrices with elastic, electro-mechanic and dielectric constant entries.

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}, \quad (\text{A.2})$$

Alternatively, the equation (A.1) may be written by:

$$\begin{pmatrix} \boldsymbol{\epsilon} \\ \mathbf{D} \end{pmatrix} = \begin{bmatrix} \mathbf{s} & \mathbf{d} \\ \mathbf{d}^T & \boldsymbol{\varepsilon} \end{bmatrix} \begin{pmatrix} \boldsymbol{\sigma} \\ \mathbf{E} \end{pmatrix} \quad (\text{A.3})$$

375 where  $\mathbf{s} := \mathbf{c}^{-1}$  is the compliance matrix and  $\mathbf{d} := \mathbf{se}^T$  contains the piezoelectric constants. Note that  
 376  $Y_b = 1/s_{11}$ .

377 **Appendix B. Extended Hamilton principle**

*Energy density function.* The total stored energy density in a unit volume of material is the sum of the mechanical work and of the electrical work [31]:

$$dW_e(\boldsymbol{\epsilon}, \mathbf{D}) = (d\boldsymbol{\epsilon})^T \boldsymbol{\sigma} + (d\mathbf{D})^T \mathbf{E} \quad (\text{B.1})$$

Note that for a conservative system,  $W_e = \int_{(0,0)}^{(\boldsymbol{\epsilon}, \mathbf{D})} dW_e$  which leads to the constitutive equation:

$$\boldsymbol{\sigma} = \frac{\partial W_e}{\partial \boldsymbol{\epsilon}}, \quad \text{and} \quad \mathbf{E} = \frac{\partial W_e}{\partial \mathbf{D}} \quad (\text{B.2})$$



The coenergy density function is defined by:

$$W_e^*(\boldsymbol{\epsilon}, \mathbf{E}) = \mathbf{E}^T \mathbf{D} - W_e(\boldsymbol{\epsilon}, \mathbf{D}) \quad (\text{B.3})$$

By calculating the total differential of  $dW_e^*$  and by using the (A.1), we can express  $W_e^*$  as:

$$W_e^*(\boldsymbol{\epsilon}, \mathbf{E}) = \frac{1}{2} \mathbf{E}^T \boldsymbol{\epsilon} \mathbf{E} + \boldsymbol{\epsilon}^T \mathbf{e} \mathbf{E} - \frac{1}{2} \boldsymbol{\epsilon}^T \mathbf{c} \boldsymbol{\epsilon} \quad (\text{B.4})$$

It follows that:

$$\boldsymbol{\epsilon} = \frac{\partial W_e^*}{\partial \boldsymbol{\sigma}}, \quad \text{and} \quad \mathbf{S} = \frac{\partial W_e^*}{\partial \mathbf{E}} \quad (\text{B.5})$$

*Hamilton's principle.* We assume the admissibility requirements which impose that the virtual displacements  $\delta \mathbf{u}$  must be compatible with the kinematic of the system, and the admissible flux linkage variation  $\delta \lambda$  must be compatible with Kirchhoff's voltage rule. The Hamilton's principle reads [2]:

$$\int_{t_1}^{t_2} (\delta T^* + \delta W_e^* + \delta W_{ext}) = 0 \quad (\text{B.6})$$

where

$$T^* = \frac{1}{2} \int_{\Omega} \rho \dot{\mathbf{u}}^T \dot{\mathbf{u}} dV \quad (\text{B.7})$$

$$W_e^* = \frac{1}{2} \int_{\Omega} \left( \mathbf{E}^T \boldsymbol{\epsilon} \mathbf{E} + 2 \boldsymbol{\epsilon}^T \mathbf{e} \mathbf{E} - \boldsymbol{\epsilon}^T \mathbf{c} \boldsymbol{\epsilon} \right) dV \quad (\text{B.8})$$

and which follows that:

$$\delta T^* = \int_{\Omega} \rho \delta \dot{\mathbf{u}}^T \dot{\mathbf{u}} dV \quad (\text{B.9})$$

$$\delta W_e^* = \int_{\Omega} \left( \delta \mathbf{E}^T \mathbf{D} - \delta \boldsymbol{\epsilon}^T \boldsymbol{\sigma} \right) dV \quad (\text{B.10})$$

The term  $\delta W_{ext}$  represents the virtual work of the external forces and of the external applied currents:

$$\delta W_{ext} = \int_{\Omega} \delta \mathbf{u}^T \mathbf{f}_v dV + \int_{S_f} \delta \mathbf{u}^T \mathbf{f}_s dS - \int_{S_q} \delta \phi^T \bar{q} dS, \quad (\text{B.11})$$

378 where  $\mathbf{f}_v$  is the volume force,  $\mathbf{f}_s$  is the surface forces applied on  $S_f$ ,  $\bar{q}$  is surface charge density applied on  
379  $S_q$ .

## 380 **Appendix C. Properties of the ancillary and the piezoelectric patches**

### 381 *Appendix C.1. Ancillary*

<b>Properties of the ancillary</b>		
Coefficient	Value	Description
$A$	$10 \times 10 \text{ mm}^2$	Section of the ancillary
$h$	10 mm	Thickness
$\ell$	125 mm	Length of the ancillary (squared part only)
$Y_b$	193 GPa	Young's modulus
$\rho$	$8000 \text{ kg m}^{-3}$	Density
$\mu$	0.3	Poisson's ratio

Table C.4: Dimensions and mechanical properties of the ancillary

382 *Appendix C.2. Piezoelectric patches*

<b>Properties of the piezoelectric ceramic</b>		
Coefficient	Value	Description
$h_p$	0.2 mm	Thickness
$b$	10 mm	Length
$d_{31}$	$-180 \times 10^{-12} \text{ C/N}$	Coupling coefficient
$Y_p$	62.3 GPa	In-plane Young's Modulus
$\rho_p$	$7800 \text{ kg m}^{-3}$	Density
$\mu_p$	0.35	Poisson's ratio

Table C.5: Dimensions and mechanical properties of the piezoelectric ceramic element

- [1] N. W. Hagood, A. von Flotow, Damping of structural vibrations with piezoelectric materials and passive electrical networks, *Journal of Sound and Vibration* 146 (2) (1991) 243–268. doi:10.1016/0022-460X(91)90762-9.
- [2] M. Porfiri, C. Maurini, J. Pouget, Identification of electromechanical modal parameters of linear piezoelectric structures, *Smart Materials and Structures* 16 (2007) 323–331. doi:10.1088/0964-1726/16/2/010.
- [3] O. Thomas, J. Ducarne, J.-F. Deü, Performance of piezoelectric shunts for vibration reduction, *Smart Materials and Structures* 21 (1) (2012) 015008. doi:10.1088/0964-1726/21/1/015008.
- [4] C. H. Park, Dynamics modelling of beams with shunted piezoelectric elements, *Journal of Sound and Vibration* 268 (1) (2003) 115–129. doi:10.1016/S0022-460X(02)01491-8.
- [5] U. Andreaus, F. Dell’Isola, M. Porfiri, Piezoelectric Passive Distributed Controllers for Beam Flexural Vibrations, *Journal of Vibration and Control* (2004) 625–657doi:10.1177/1077546304038224.
- [6] F. Dell’Isola, C. Maurini, M. Porfiri, Passive damping of beam vibrations through distributed electric networks and piezoelectric transducers: prototype design and experimental validation, *Smart Materials and Structures* (2004) 299–308doi:10.1088/0964-1726/13/2/008.
- [7] S. Vidoli, F. dell’Isola, Vibration control in plates by uniformly distributed PZT actuators interconnected via electric networks, *European Journal of Mechanics - A/Solids* 20 (3) (2001) 435–456. doi:10.1016/S0997-7538(01)01144-5.
- [8] S. Alessandrini, U. Andreaus, F. Dell’Isola, M. Porfiri, A passive electric controller for multimodal vibrations of thin plates, *Computers and Structures* (2005) 15doi:10.1016/j.compstruc.2004.08.028.
- [9] G. Rosi, J. Pouget, F. dell’Isola, Control of sound radiation and transmission by a piezoelectric plate with an optimized resistive electrode, *European Journal of Mechanics - A/Solids* 29 (5) (2010) 859–870. doi:10.1016/j.euromechsol.2010.02.014.
- [10] V. Gupta, M. Sharma, N. Thakur, Optimization Criteria for Optimal Placement of Piezoelectric Sensors and Actuators on a Smart Structure: A Technical Review, *Journal of Intelligent Material Systems and Structures* 21 (12) (2010) 1227–1243. doi:10.1177/1045389X10381659.
- [11] J. Ducarne, O. Thomas, J. F. Deü, Placement and dimension optimization of shunted piezoelectric patches for vibration reduction, *Journal of Sound and Vibration* 331 (14) (2012) 3286–3303. doi:10.1016/j.jsv.2012.03.002.
- [12] I. Bruant, L. Gallimard, S. Nikoukar, Optimal piezoelectric actuator and sensor location for active vibration control, using genetic algorithm, *Journal of Sound and Vibration* 329 (10) (2010) 1615–1635. doi:10.1016/j.jsv.2009.12.001.
- [13] M. Biglar, M. Gromada, F. Stachowicz, T. Trzpieciński, Optimal configuration of piezoelectric sensors and actuators for active vibration control of a plate using a genetic algorithm, *Acta Mechanica* 226 (2015) 3451–3462. doi:10.1007/s00707-015-1388-1.
- [14] D. Halim, S. O. Reza Moheimani, An optimization approach to optimal placement of collocated piezoelectric actuators and sensors on a thin plate, *Mechatronics* 13 (1) (2003) 27–47. doi:10.1016/S0957-4158(01)00079-4.
- [15] G. Qi, W. P. Mouchon, T. E. Tan, How much can a vibrational diagnostic tool reveal in total hip arthroplasty loosening?, *Clinical Biomechanics* 18 (5) (2003) 444–458. doi:10.1016/s0268-0033(03)00051-2.
- [16] P. Henys, L. Capek, J. Fencl, E. Prochazka, Evaluation of acetabular cup initial fixation by using resonance frequency principle, *Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine* 229 (1) (2015) 3–8. doi:10.1177/0954411914561485.
- [17] K. Denis, L. C. Pastrav, S. Leuridan, Vibration Analysis of the Biomechanical Stability of Total Hip Replacements, in: *Experimental Methods in Orthopaedic Biomechanics*, Elsevier, 2017, pp. 313–328.
- [18] S. A. Yousefsani, H. Dejnabadi, O. Guyen, K. Aminian, A Vibrational Technique for In Vitro Intraoperative Prosthesis Fixation Monitoring, *IEEE transactions on bio-medical engineering* 67 (10) (2020) 2953–2964. doi:10.1109/TBME.2020.2974380.
- [19] S. Leuridan, Q. Goossens, L. C. Pastrav, M. Mulier, W. Desmet, J. Vander Sloten, K. Denis, Development of an Instrument to Assess the Stability of Cementless Femoral Implants Using Vibration Analysis During Total Hip Arthroplasty, *IEEE Journal of Translational Engineering in Health and Medicine* 9 (2021) 1–10. doi:10.1109/JTEHM.2021.3128276.
- [20] A.-S. Poudrel, G. Rosi, V.-H. Nguyen, G. Haiat, Modal Analysis of the Ancillary During Femoral Stem Insertion: A Study on Bone Mimicking Phantoms, *Annals of Biomedical Engineering* 50 (1) (2022) 16–28. doi:10.1007/s10439-021-02887-9.
- [21] G. Rosi, R. Paccapeli, F. Ollivier, J. Pouget, Optimization of piezoelectric patch positioning for passive sound radiation control of plates, *Journal of Vibration and Control* 19 (5) (2012) 658 – 673. doi:10.1177/1077546312437236.
- [22] E. B. Flynn, M. D. Todd, Optimal Placement of Piezoelectric Actuators and Sensors for Detecting Damage in Plate Structures, *Journal of Intelligent Material Systems and Structures* 21 (3) (2010) 265–274. doi:10.1177/1045389X09338080.
- [23] S. Zhao, N. Wu, Q. Wang, Crack identification through scan-tuning of vibration characteristics using piezoelectric materials, *Smart Materials and Structures* 26 (2) (2016) 025005. doi:10.1088/1361-665X/aa520a.
- [24] S. Zhao, N. Wu, Q. Wang, Damage Detection of Beams by a Vibration Characteristic Tuning Technique Through an Optimal Design of Piezoelectric Layers, *International Journal of Structural Stability and Dynamics* 16 (10) (2016) 1550070. doi:10.1142/S0219455415500704.
- [25] A. M. Sadri, J. R. Wright, R. J. Wynne, Modelling and optimal placement of piezoelectric actuators in isotropic plates using genetic algorithms, *Smart Materials and Structures* 8 (4) (1999) 490. doi:10.1088/0964-1726/8/4/306.
- [26] A. R. Mehrabian, A. Yousefi-Koma, A novel technique for optimal placement of piezoelectric actuators on smart structures, *Journal of the Franklin Institute* 348 (1) (2011) 12–23. doi:10.1016/j.jfranklin.2009.02.006.
- [27] I. Bruant, L. Gallimard, S. Nikoukar, Optimization of Piezoelectric Sensors Location and Number Using a Genetic Algorithm, *Mechanics of Advanced Materials and Structures* 18 (7) (2011) 469–475. doi:10.1080/15376494.2011.604600.
- [28] S. Chesné, Indirect boundary force measurements in beam-like structures using a derivative estimator, *Journal of Sound and Vibration* 333 (24) (2014) 6438–6452. doi:10.1016/j.jsv.2014.07.026.

- 447 [29] S. Chesne, C. Pezerat, Distributed piezoelectric sensors for boundary force measurements in Euler–Bernoulli beams, *Smart*  
448 *Materials and Structures* 20 (7) (2011) 075009. doi:10.1088/0964-1726/20/7/075009.
- 449 [30] L. C. Pastrav, S. V. Jaecques, I. Jonkers, G. V. d. Perre, M. Mulier, In vivo evaluation of a vibration analysis technique  
450 for the per-operative monitoring of the fixation of hip prostheses, *Journal of Orthopaedic Surgery and Research* 4 (2009)  
451 10. doi:10.1186/1749-799X-4-10.
- 452 [31] A. Preumont, *Mechatronics: Dynamics of Electromechanical and Piezoelectric Systems*, softcover reprint of hardcover 1st  
453 ed. 2006 édition Edition, Springer, 2011.
- 454 [32] A. Tijou, G. Rosi, R. Vayron, H. A. Lomami, P. Hernigou, C.-H. Flouzat-Lachaniette, G. Haïat, Monitoring cementless  
455 femoral stem insertion by impact analyses: An in vitro study, *Journal of the Mechanical Behavior of Biomedical Materials*  
456 88 (2018) 102–108. doi:10.1016/j.jmbbm.2018.08.009.
- 457 [33] S. Chesne, C. Pezerat, Force identification by using specific forms of PVDF patches, *Smart Structures and Systems* 15  
458 (2015) 1203–1214. doi:10.12989/sss.2015.15.5.1203.
- 459 [34] ANSI/IEEE, IEEE Standard on Piezoelectricity, ANSI/IEEE Std 176-1987 (1988). doi:10.1109/IEEESTD.1988.79638.