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## 25th Euro Working Group on Transportation Meeting (EWGT 2023)

# Connectivity Reliability For Assessing Public Transport Networks

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### Abstract

This paper addresses the issue of dynamic connectivity reliability in a public transportation network, which is based on bus routes and schedules. A dynamic k-shortest path algorithm is used to find up to "k" bus paths connecting each Origin-Destination pair in {time x space}. The connectivity reliability is determined by the basic reliability "p" of each path element, and standardized reliability indicators have been defined. Equations have been developed that relate the reliability of a specific Origin-Destination pair or the entire network to various factors such as the number of path elements, the number of paths (taking into account their degree of overlap), the straightness of the routes, and bus frequencies. This enables operators to assess and compare solutions that can enhance the quality of service by improving the basic reliability, increasing the number of alternative bus lines, straightening the routes, and increasing bus frequencies.

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**Keywords:** Bus transportation network; "k" shortest paths algorithm; {time x space}; Connectivity Reliability; Geometric Average; Logarithm

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### 1. Introduction

For public transportation systems like bus networks, connectivity reliability refers to the likelihood of maintaining connectivity between bus stops, while taking into account the existence of links between each origin-destination (OD) pair. Connectivity reliability is often linked with the reliability of delivering passenger flows (Kim et al., 2016) and studying the dynamic resilience of the network following disruptions (Mahdavi et al., 2020). However, actual bus schedules may pose constraints that are not considered.

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In our research, we only focus on connectivity reliability and compute the OD paths based on the bus schedules.

A shortest path algorithm (Sakarovitch, 1968), implemented in space-time, provides the paths, within a specified user departure time and a maximum allowed waiting time, while discarding some irrelevant paths.

## 2. Research Contribution

Vulnerability, defined as one minus reliability, is expressed on a network as the product of the vulnerabilities of each space-time path that connects any OD, taking into account path overlap. This product demonstrates how the existence of alternative paths and the frequency of buses impact reliability.

By studying the reciprocal function of reliability, we observe that there is a trade-off, from a reliability standpoint, between the number of elements on a path and the number of alternative paths available.

Our application allows operators to evaluate and improve the service quality by exploring different possibilities.

## 3. Methodology

Let  $\omega$  be an OD and  $\phi$  be a path connecting  $\omega$ . The description of  $\phi$  includes three sets of elements: the sets of buses, bus stops, or links between bus stops. In this case, we selected the set of links, based on two metrics. In the first metric, each link is weighted by its length. In the second metric, which is called "standard", the length of each link is divided by the distance between the origin and the destination, as the crow flies. This ratio would be equal to 1 for perfect networks, and enables comparisons between ODs, regardless of their distance as the crow flies.

Let  $N(\phi)$  be the number of elements of the path  $\phi$ . Even if the reliabilities of the elements differ, we can retrieve, without loss of generality, the case where all elements have the same reliability  $p$  of operation. Since the elements are disjoint, the events "element  $i$  is operating" ( $1 \leq i \leq N(\phi)$ ) are assumed to be independent. The reliability of a path  $\phi$  is the product of the reliabilities of its elements:

$$R(\phi) = p^{N(\phi)} \text{ or reciprocally: } N(\phi) = \frac{\text{Log}(R(\phi))}{\text{Log}(p)} \quad (1)$$

Let  $M$  be the number of elements in the network. A state of the network is modeled by a vector of dimension  $M$ , with components of either 1 or 0 based on whether the corresponding element operates or not. An event of "path  $j$  is operating" (or not) corresponds to a subset  $S_j$  of vectors (or  $\bar{S}_j$ ).

When several paths connect an OD  $\omega$ , the reliability  $R(\omega)$  of  $\omega$  is the probability that at least one path  $j$  is operating, which is the probability of the union of subsets  $S_j$ . The de Moivre equation (also known as the inclusion-exclusion principle) gives the probability of the union of  $k$  subsets, without any double counting:

$$R(\omega) = \text{Proba}(US_j) = \sum_{j=1}^k \text{proba}(S_j) - \sum_{j,j'} \text{proba}(S_j \cap S_{j'}) + \sum_{j,j',j''} \text{proba}(S_j \cap S_{j'} \cap S_{j''}) \dots + (-1)^{k-1} \prod_{j=1}^k S_j$$

Applying Eq. 1 to OD  $\omega$  yields a reduced number  $N'(\omega)$ , which is smaller than  $N(\phi)$ . This reduced number indicates that  $k$  paths are equivalent, from a reliability point of view, to a single shorter path. This information is useful for the operator to optimize the bus network and improve the reliability of the service.

$$N'(\omega) = \frac{\text{Log}(R(\omega))}{\text{Log}(p)} \text{ or equivalently } \text{Log}(R(\omega)) = N'(\omega) \text{Log}(p) \quad (2)$$

An OD is not connected when the 1st, 2nd, ...  $k$ th paths are not operating simultaneously. The probability of this event, which is called "vulnerability" and is equal to 1 minus reliability, also equal to  $\text{oba}(\bar{S}_1 \cap \bar{S}_2 \cap \dots \cap \bar{S}_k)$ .

Using the chain rule of probability (also known as the multiplication rule of probability) which combines conditional probabilities, this expression can be factorized as:

$$1 - R(\omega) = \text{Proba}(\bar{S}_1 \cap \bar{S}_2 \cap \dots \cap \bar{S}_k) = \text{proba}(\bar{S}_1) \cdot (\prod_{j=2}^k \text{proba}(\bar{S}_j | \bar{S}_1 \cap \bar{S}_2 \cap \dots \cap \bar{S}_{j-1})) \quad (3)$$

The  $j^{\text{th}}$  factor of the product  $\text{proba}(\overline{S}_j | \overline{S}_1 \cap \overline{S}_2 \cap \dots \cap \overline{S}_{j-1})$  gives the (multiplicative) contribution of path  $j$  on vulnerability. This factor is written in a “canonic” form as  $(1 - R(\varphi_j(\omega)))^{\beta_j(\omega)}$  with  $\beta_j(\omega)$  satisfying :

$$\beta_j(\omega) = \frac{\text{Log}(\text{proba}(\overline{S}_j | \overline{S}_1 \cap \overline{S}_2 \cap \dots \cap \overline{S}_{j-1}))}{\text{Log}(\text{proba}(S_j))}$$

When path  $j$  is disjoint from the previous ones, the event  $S_j$  is independent of previous ones, the condition “ $\overline{S}_1 \cap \overline{S}_2 \cap \dots \cap \overline{S}_{j-1}$ ” has no impact on  $S_j$  nor  $\overline{S}_j$  then  $\beta_j(\omega)=1$ . Otherwise  $0 < \beta_j(\omega) < 1$ .

The impact of any path  $\varphi_j(\omega)$  on vulnerability appears in Equation 4:

$$1-R(\omega) = \prod_{j=1}^k (1 - R(\varphi_j(\omega))^{\beta_j(\omega)}) \text{ or } \text{Log}(1-R(\omega)) = \sum_{j=1}^k \beta_j(\omega) \cdot \text{Log}(1 - R(\varphi_j(\omega))) \quad (4)$$

$\beta_j(\omega)$  measures the level of overlap between path  $j$  and the previous paths. It indicates to the operator that, when  $k$  paths connect  $\omega$ , there are only  $\sum_{j=1}^k \beta_j(\omega)$  equivalent independent paths from the reliability point of view.

The geometric average of vulnerabilities (and the arithmetic average of their logarithms) are also factored by extending the product in Equation 4 to  $\prod_{\omega} \prod_{j=1}^k$ , or the sum to  $\sum_{\omega} \sum_{j=1}^k \beta_j(\omega)$ . This sum, divided by the number of connected ODs, represents the number of equivalent independent paths per OD.

#### 4. Results

The Rennes bus network in Brittany includes 707 bus stops, among which 680 have been considered, resulting in 680\*679 ODs (considering each bus stop as an origin and destination). The algorithm was looking for a maximum of  $k=5$  paths per OD; the maximum allowed waiting time was one hour, and the time of departure for users was fixed at 8:00 am. It was found that 90% of the ODs were connected, for which, on average, there were 2.48 paths - discarding identical paths that just shifted in time. The average length of the shortest paths per OD was 13.86 km, with an average "sinuosity" of 1.55, where sinuosity is the ratio of the actual path length to the distance as the crow flies of the OD. The reliability improvement provided by the alternative paths is equivalent to reducing the length to 10.91 km (assuming a basic reliability  $p$  per km of 96%) or reducing sinuosity to 1.22 (with  $p=70\%$ , a value chosen for comparisons between length and standard reliabilities).

Table 1. Average reliability, for the Rennes network

	Reliability		Average Number of Elements		Equivalent Number of Independent Paths per OD
	First Path	All Paths	Real (N)	Reduced (N')	
Length (p=96%)	0.596	0.674	13.86(km)	10.91 (km)	1.387
Standard(p=70%)	0.587	0.671	1.555(sinuosity)	1.224(sinuosity)	1.122

#### 5. Discussion

This study represents a valuable attempt to link reliability and basic probability value  $p$ , frequencies, alternative routes, and their sinuosity, in order to provide insights into the performance of public transportation networks. While the results shed light on reliability, future research should aim to incorporate demand and capacity constraints.

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