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# Interval parameter sensitivity analysis based on interval perturbation propagation and interval similarity operator

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## Abstract

An interval parameters sensitivity analysis is developed to quantify the impact of simulation model parameters on the model outputs. This sensitivity analysis contains two main steps the interval uncertainty propagation and the interval sensitivity index. The interval perturbation method is introduced to estimate the extreme bounds of model outputs according to the interval input parameters, which significantly reduces the computation cost of extensive Monte Carlo simulations. Since the output of the interval model are interval quantities, the traditional probabilistic sensitivity method and its sensitivity index is inappropriate as we only have the bounds of samples without inner data points. Hence, this work proposes an interval similarity operator based on the relative interval position operator, which is applicable to measure the variation of interval outputs. This interval sensitivity operator mainly quantifies the discrepancy between intervals based on six typical cases of the interval relative position. Finally, an academic case and a satellite structure case are analyzed to verify the feasibility and efficiency of the proposed method.

**Keywords:** Sensitivity analysis; interval uncertainty; interval sensitivity operator; interval relative position operator; interval perturbation method.

## 1 Introduction

A practical engineering model has to cope with various uncertainties existing in systems and structures. Uncertainties generally arise from the observed scattering of environmental conditions, lack of knowledge, inhomogeneity of materials, and measurement uncertainty. Those uncertainties are typically divided into two distinct forms, i.e., aleatory uncertainty and epistemic uncertainty (Kitahara et al. 2022). Meanwhile, non-deterministic analysis (Wang et al. 2011) has gained wide interest, and elaborate literature is available in this field. Uncertainty analysis can be generally classified into two categories of probabilistic and non-probabilistic

1 techniques (QI et al. 2021; Singh and Bhushan 2020). The interval model is one of the most  
2 representative non-probabilistic approaches, which quantifies the uncertainties by the bounds  
3 of datapoints. In this work, we mainly focus on the sensitivity analysis with interval  
4 uncertainties.

5 Sensitivity analysis (SA) has rapidly developed to quantify the impact of model parameters  
6 on outputs due to the growing complexity of mathematical models. One classical definition of  
7 sensitivity is “the study of how the uncertainty in the output of a model (numerical or otherwise)  
8 can be apportioned to different sources of uncertainty in the model inputs” (Andrea 2002),  
9 which is typically distinct from the uncertainty analysis. The importance of parameters is  
10 compared through ranked sensitivity indexes corresponding to each input parameter. The  
11 growth of uncertainty analysis has greatly promoted the development of sensitivity analysis.

12 The importance of sensitivity analysis is widely acknowledged. One object of sensitivity  
13 analysis is to identify the contributions of model inputs to the variation of the outputs, which is  
14 used as evidence for significant parameter selection before model calibration. For example,  
15 sensitivity analysis is generally distinguished between local methods (Ha 2018) and global  
16 methods (Sobol 2001). In the context of local methods, the changes in outputs are analyzed  
17 while one input parameter is changed, with the rest kept at reference values. Jacomel et al.  
18 (2021) presented a priori error estimates for local reliability-based sensitivity analysis. Achyut  
19 et al. (2022) proposed local sensitivity analysis by using an efficient approach called modified  
20 forward finite difference. Global sensitivity (Cheng et al. 2019) analysis captures the interaction  
21 effects among parameters when exploring the responses of the model by varying all inputs at  
22 the same time. Examples of global methods include the first-order sensitivity index of Sobol’s  
23 method (Liu et al. 2019; Sobol 1993), the extended Fourier amplitude sensitivity test (Saltelli  
24 et al. 1999), the Morris screening method (Shin et al. 2013), the Multi-output support vector  
25 regression (M-SVR) (Cheng et al. 2019), and the distribution-based global sensitivity analysis  
26 (Lukáš 2022). When structures with large-scale parameters, it leads to the expensive  
27 computational cost issue due to quantifying the effects of inputs on the output response globally.  
28 A hybrid metamodel using the orthogonal constraints of radial basis function and sparse  
29 polynomial chaos expansions for the global sensitivity analysis of time-consuming models was  
30 developed (Wu et al. 2020).

31 Sensitivity analysis has been implemented in various areas, such as model Verification and  
32 Validation (V&V) (Eamon and Rais-Rohani 2008; Ehre et al. 2020; Papaioannou and Straub  
33 2021; Suzana et al. 2022), structural optimization design (Eamon and Rais-Rohani 2008; Liu  
34 et al. 2019), structural reliability analysis (Ehre et al. 2020; Papaioannou and Straub 2021),  
35 mechanical property analysis of laminated plates (Longfei et al. 2012), and robust design in  
36 aerospace engineering (Dasari et al. 2020). However, most variance-based or moment-  
37 independent sensitivity analyses (Zhang et al. 2015; Zhou et al. 2014) involve evaluating partial

1 derivatives of probabilistic model outputs at the nominal values of the input parameters, which  
2 should combine the sampling-based probabilistic methods and mathematical models to quantify  
3 uncertainties. In the case of a complex model with massive numbers of input variables, the  
4 sensitivity analysis from the probabilistic view is highly time-consuming.

5 In the context of non-probabilistic uncertainty quantification, the input parameters and output  
6 responses are both non-probabilistic. Traditional variance-based sensitivity analysis is  
7 inapplicable to the interval model due to the lack of probabilistic information on inputs and  
8 outputs. Recently, a prediction on the static response of structures with uncertain-but-bounded  
9 parameters based on the adjoint sensitivity analysis was developed, where the sensitivity  
10 analysis is implemented without considering the interval characteristics of uncertain parameters  
11 (Luo et al. 2020). It is necessary to consider interval uncertainties when sensitivity and  
12 uncertainty analysis is performed. Up to now, however, most sensitivity analysis methods and  
13 sensitivity coefficients are mainly established for probabilistic parameter selection, such as the  
14 Sobol sensitivity index (Sobol 1993), the total-effect index (Homma and Saltelli 1996), the  
15 Morris sensitivity index (Morris 1991), and FAST sensitivity index (Mcrae et al. 1982).  
16 Sensitivity analysis is developed to provide information for the reliability-based design. Xiao  
17 and Huang et al. (2011) proposed a reliability sensitivity analysis method for the model with  
18 both epistemic and aleatory uncertainties using P-boxed. Bi et al. (2019) developed a stochastic  
19 sensitivity analysis with a novel sensitivity index based on the Bhattacharyya distance. Those  
20 research efforts have been made on sensitivity analysis when both hybrid epistemic and aleatory  
21 uncertainties. However, with the limitation of samples, the stochastic characteristics of  
22 parameters cannot be precisely determined. Besides, those mentioned probabilistic sensitivity  
23 coefficients for the stochastic models or models with hybrid uncertainties are not applicable to  
24 the model with purely non-probabilistic uncertainties. Hence, it is necessary to extend the  
25 sensitivity analysis to a wider application with only interval uncertainties. A novel sensitive  
26 coefficient based on the geometric interval quantification method is presented to quantify the  
27 parameter sensitivity in this paper.

28 The interval sensitivity analysis process relies on the accurate propagation of uncertainties  
29 in the form of intervals. However, the interval arithmetic operations are difficult to implement  
30 directly in uncertainty propagation. Interval analysis is introduced to estimate the interval  
31 outputs according to the interval inputs, which is named interval uncertainty propagation.  
32 Interval analysis is the basis of interval sensitivity, which predicts the interval output for  
33 estimating the sensitivity indices. Monte-Carlo simulation (Callens et al. 2022) is one of the  
34 typical uncertainty propagation methods which has been introduced into Sobol's sensitivity  
35 analysis. Interval analysis typically requires a global optimization procedure with Monte Carlo  
36 simulation to determine the interval bounds on the output side of a computational model.  
37 However, in the context of complex models in practice, massive Monte Carlo simulation brings

1 excessive computation, sharply increasing the computation cost of sensitivity analysis. Some  
 2 efficient interval propagation methods, such as advanced interval analysis (Fujita and Takewaki  
 3 2011), multivariate interval quantification approach based on the concept of the convex hull  
 4 (Faes et al. 2019; Faes et al. 2017), Infor-gap uncertainty quantification models (Ben-Haim  
 5 2004), and some interval surrogate models (Fang et al. 2015; Khodaparast et al. 2011) are  
 6 rapidly developed to reduce the computation cost. The interval perturbation methodology (Li  
 7 et al. 2018; Wang and Qiu 2014), a representative interval propagation method, has some  
 8 advantages over the Monte Carlo simulation, including lower computation cost because of  
 9 calculating only by the information of a single point that allows the consideration of the  
 10 complexity of structure. Therefore, this work introduces the interval perturbation propagation  
 11 method to effectively estimate the output interval according to interval inputs.

12 A parameter sensitivity analysis method with a novel interval-based sensitivity metric is  
 13 developed by introducing the interval propagation methodology in this work. The *interval*  
 14 *similarity operator (ISO)* is employed as a sensitivity metric to measure the discrepancy  
 15 between two interval model outputs corresponding to initial and changed interval parameters,  
 16 respectively. This metric is developed for interval uncertainty quantification as it is computed  
 17 only based on the extreme bounds of the interval without their inner data points. The interval  
 18 perturbation method is adopted to estimate the bounds of model outputs to improve the  
 19 computation effectiveness. The feasibility and accuracy of the proposed method is verified by  
 20 two typical academic cases.

21 This work is organized as follows. Section 2 presents an overview of sensitivity analysis with  
 22 interval uncertainties. Section 3 presents how to calculate the proposed interval-based  
 23 sensitivity index. Section 4 illustrates the comprehensive framework of sensitivity analysis with  
 24 interval uncertainties. Section 5 gives two study cases, i.e., an academic case and a more  
 25 complex satellite case, to investigate the proposed method.

## 26 **2 Background of sensitivity analysis for interval parameters**

27 A finite element model or other complex black box models can be expressed as  $F(\cdot)$  as  
 28 follows

$$29 \quad \mathbf{f} = F(\boldsymbol{\theta}) \quad (1)$$

30 where  $\mathbf{f}$  is the model output.  $F$  represents a propagation function of the model system.  $\boldsymbol{\theta}$  is  
 31 the model parameters, where  $\boldsymbol{\theta} = \{\theta_i\}, i = 1, 2, 3, \dots, n$ , and  $n$  is the number of model  
 32 parameters.

33 The sensitivity index  $\mathbf{S}=\{S_i\}$  of model parameter  $\boldsymbol{\theta}$  is simply expressed as:

$$34 \quad \mathbf{S} = \frac{\Delta \mathbf{f}}{\Delta \boldsymbol{\theta}} = \frac{F(\boldsymbol{\theta} + \Delta \boldsymbol{\theta}) - F(\boldsymbol{\theta})}{\Delta \boldsymbol{\theta}} \quad (2)$$

35 where  $\Delta \boldsymbol{\theta} = \{\Delta \theta_i\}$  represents the variation of parameter  $\boldsymbol{\theta}$ .  $\Delta \mathbf{f} = \{\Delta f_j\}$  represents the  
 36 variation of  $\mathbf{f}$ , where  $j=1, 2, \dots, m$ ,  $m$  represents the dimensions of model output features.

1 In the context of a model with interval parameters  $\theta^I$ , Eq. (1) can be given as

$$2 \quad \mathbf{f}^I = \mathbf{F}(\theta^I) \quad (3)$$

3 where  $\theta^I = \{\theta_i^I\}$  represents the interval parameters of model  $\mathbf{F}$ , and  $\mathbf{f}^I = \{f_j^I\}$  are interval  
4 uncertainties. Then, the sensitivity index for interval parameters is accordingly converted as:

$$5 \quad \mathbf{S}_{Int} = \frac{\partial \mathbf{f}^I}{\partial \Delta \theta^I} = \frac{F(\theta^C + \Delta \theta^I) - F(\theta^C + \Delta \hat{\theta}^I)}{\Delta \theta^I - \Delta \hat{\theta}^I} \quad (4)$$

6 where  $\Delta \hat{\theta}^I = \{\Delta \hat{\theta}_i^I\}$  means the changed interval radius. It can be seen that  $\Delta \theta^I$  and  $\Delta \hat{\theta}^I$  is  
7 independent of the interval midpoints  $\theta^C$ . We mainly focus on identifying the contributions of  
8 the slight change in model interval inputs to the variation of uncertainty degree of the outputs  
9 in this work.

10 For an interval variable  $\theta^I$ , it can be determined by the extreme bounds  $\underline{\theta}$  and  $\bar{\theta}$  or by the  
11 inter center  $\theta^C$  and the interval radius  $\Delta \theta$ , as follows:

$$12 \quad \theta^I = [\theta^C - \Delta \theta, \theta^C + \Delta \theta]$$

$$13 \quad = [\underline{\theta}, \bar{\theta}] \quad (5)$$

14 The fundamental operations of arithmetic for intervals (Moore 1996) are interpreted by two  
15 interval variables  $\theta_1^I$  and  $\theta_2^I$ , which is given as

$$16 \quad \left\{ \begin{array}{l} \theta_1^I + \theta_2^I = [\theta_1^l + \theta_2^l, \theta_1^u + \theta_2^u] \\ \theta_1^I - \theta_2^I = [\theta_1^l - \theta_2^u, \theta_1^u - \theta_2^l] \\ \theta_1^I \times \theta_2^I = [\min\{\theta_1^l \theta_2^l, \theta_1^l \theta_2^u, \theta_1^u \theta_2^l, \theta_1^u \theta_2^u\}, \max\{\theta_1^l \theta_2^l, \theta_1^l \theta_2^u, \theta_1^u \theta_2^l, \theta_1^u \theta_2^u\}] \\ \frac{\theta_1^I}{\theta_2^I} = \theta_1^I \times \left[ \frac{1}{\theta_2^u}, \frac{1}{\theta_2^l} \right] \end{array} \right. \quad (6)$$

17 This work mainly focuses on the problem of local sensitivity analysis, which ignores the  
18 relationship between input parameters. The sensitivity analysis index  $\mathbf{S}_{Int}$  is to quantify the  
19 impact of the change of interval parameters from  $\theta^I$  into  $\theta^I + \Delta \theta^I$  on the model uncertainty  
20 output  $\mathbf{f}^I$ . Interval sensitivity analysis is simply illustrated in Fig. 1. We can find that for the  
21 sensitive interval parameters, once there is some variation in their boundaries, both the interval  
22 center and the interval radius of outputs significantly change. However, on the contrary, even  
23 if the insensitive interval parameters encounter large perturbations, it may cause a small impact  
24 on model outputs.

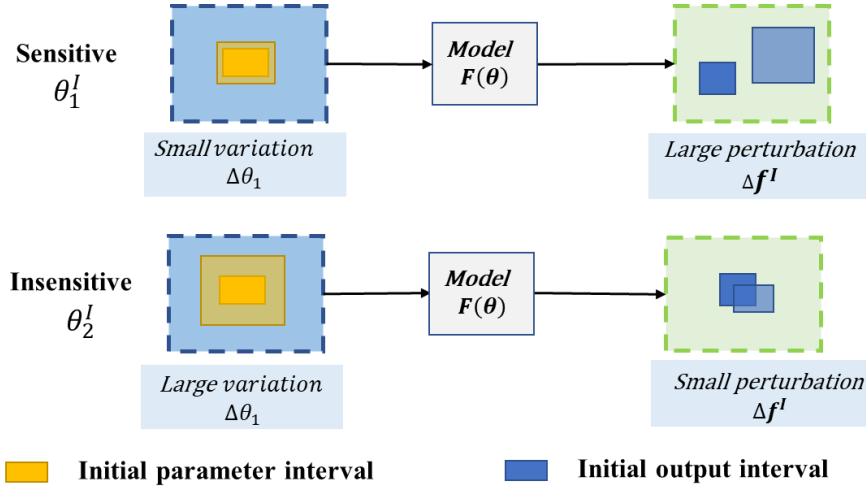


Fig. 1 Diagrammatic sketch for sensitivity analysis of interval parameters.

### 3 Interval perturbation FE method

In the context of a model containing interval uncertainties, Fig. 2 depicts the relationships among interval propagation, sensitivity analysis, and model updating, also called model calibration. The interval uncertainty propagation approach is investigated to determine the bounds of model outputs. The relative significance of interval parameters is assessed using sensitivity analysis. The model updating for sensitive inputs is applied to improve the accuracy of the simulation model. Interval propagation is regarded as a critical operator for both the sensitivity analysis and the model updating. The interval perturbation method (Zhao et al. 2018; Zhao et al. 2020), a typical interval propagation method, is investigated in this work.

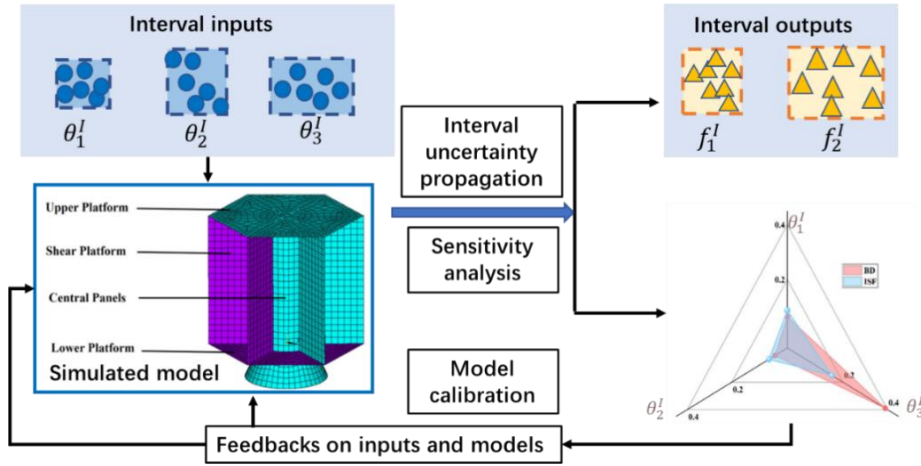


Fig. 2 The relationship between the sensitivity analysis, the interval uncertainty propagation, and the model calibration.

Interval uncertainties can be rewritten as

$$\boldsymbol{\theta}^I = \{\theta_i^I\} = \{\theta^c + \Delta\theta^I\}_i, i = 1, 2, \dots, N \quad (7)$$

where  $\theta_i^I = \theta_i^c + \Delta\theta_i^I = \theta_i^c + \Delta\theta_i \cdot \varepsilon_i^I$ , and  $\varepsilon_i^I = [-1, 1]$ .

According to the perturbation method, the interval output  $F(\boldsymbol{\theta}^I)$  can be given in a series mode

1 as

$$2 \quad \mathbf{F}(\boldsymbol{\theta}^I) = \overline{\mathbf{F}(\varepsilon_i^I)} = f_0 + \varepsilon_i^I f_1 + (\varepsilon_i^I)^2 f_1 + \dots + (\varepsilon_i^I)^m f_m, \quad i = 1, 2, \dots, N \quad (8)$$

3 where  $m$  is the truncation order of the series. Based on the Taylor series expansion expanded  
4 at the middle point of the interval vector  $\boldsymbol{\theta}^I$ , Eq. (8) can be transformed as

$$5 \quad \mathbf{F}(\boldsymbol{\theta}^I) = \mathbf{F}(\boldsymbol{\theta}^C) + \sum_{p_1=1}^N \left. \frac{\partial \mathbf{F}(\boldsymbol{\theta})}{\partial \theta_{p_1}} \right|_{\theta_i^I = \theta^C, i \neq p_1} \cdot \Delta \theta_{p_1} \\ 6 \quad + \frac{1}{2} \sum_{p_1=1}^N \sum_{p_2=1}^N \left. \frac{\partial^2 \mathbf{F}(\boldsymbol{\theta})}{\partial \theta_{p_1} \partial \theta_{p_2}} \right|_{\theta_i^I = \theta^C, i \neq p_1, p_2} \cdot \Delta \theta_{p_1} \Delta \theta_{p_2} + \dots \\ 7 \quad + \frac{1}{m} \sum_{p_1=1}^N \dots \sum_{p_m=1}^N \left. \frac{\partial^m \mathbf{F}(\boldsymbol{\theta})}{\partial \theta_{p_1} \dots \partial \theta_{p_m}} \right|_{\theta_i^I = \theta^C, i \neq p_1, \dots, p_m} \cdot \Delta \theta_{p_1} \dots \Delta \theta_{p_m} + R \quad (9)$$

8 Since the values of  $\Delta \theta_{p_1} \cdot \Delta \theta_{p_2}, \dots, \Delta \theta_{p_1} \cdot \dots \cdot \Delta \theta_{p_m}$  in more than second-order form is very  
9 small, Eq.(9) can be approximated to a first-order form based on the interval algorithm, which  
10 is given as

$$11 \quad \mathbf{F}(\boldsymbol{\theta}^I) \cong \widehat{\mathbf{F}}(\boldsymbol{\theta}^I) = \mathbf{F}(\boldsymbol{\theta}^C) + \sum_{j=1}^N \left. \frac{\partial \mathbf{F}(\boldsymbol{\theta})}{\partial \theta_j} \right|_{\theta_i^I = \theta^C, i \neq j} \cdot \Delta \theta_j, \quad j = 1, 2, \dots, N \quad (10)$$

12 Then, we can obtain the following equations that

$$13 \quad \widehat{\mathbf{F}}(\boldsymbol{\theta}^I) = \sum_{j=1}^n (\theta_j^c + \Delta \theta_j^I) F_i = \mathbf{F}(\boldsymbol{\theta}^C) + \sum_{j=1}^n F_i \cdot \Delta \theta_j^I \quad (11)$$

14 where  $F_i = \sum_{j=1}^N \left. \frac{\partial \mathbf{F}(\boldsymbol{\theta})}{\partial \theta_j} \right|_{\theta_i^I = \theta^C, i \neq j}$ .

15 From Eq.(12), we can find that once we have the uncertain part of  $\sum_{j=1}^n F_i \cdot \Delta \theta_j^I$ , we can  
16 calculate the approximate bounds of the model output  $\widehat{\mathbf{f}}^I = \widehat{\mathbf{F}}(\boldsymbol{\theta}^I)$ . The differential method is  
17 introduced to calculate the lower and upper bounds of the model output  $\widehat{\mathbf{f}}^I$ , which is  
18 shown as

$$19 \quad \begin{cases} \overline{\widehat{\mathbf{f}}} = \mathbf{F}(\boldsymbol{\theta}^C) + \sum_{j=1}^N \frac{\mathbf{F}(\theta_j^c + \delta \theta_j) - \mathbf{F}(\theta_j^c)}{\delta \theta_j} \Delta \theta_j \\ \underline{\widehat{\mathbf{f}}} = \mathbf{F}(\boldsymbol{\theta}^C) - \sum_{j=1}^N \frac{\mathbf{F}(\theta_j^c + \delta \theta_j) - \mathbf{F}(\theta_j^c)}{\delta \theta_j} \Delta \theta_j \end{cases} \quad (12)$$

20 where  $\delta \theta_j$  is the minor variable of the interval variable  $\theta_j$ .

## 21 4 Interval sensitivity analysis with *Interval Similarity Operator*

### 22 4.1 Interval Similarity Operator

23 In the context of local sensitivity analysis, for example, when the parameters are changed  
24 from  $\boldsymbol{\theta}^I = \{\theta_1^I, \theta_2^I, \dots, \theta_n^I\}$  to  $\widehat{\boldsymbol{\theta}}^I = \{\widehat{\theta}_1^I, \theta_2^I, \dots, \theta_n^I\}$ , namely  $\theta_1^I$  becomes  $\widehat{\theta}_1^I$ , the outputs are  
25 accordingly changed from the initial value of  $\mathbf{f}^I = \{f_j^I\}$  to the perturbed value of  $\widehat{\mathbf{f}}^I = \{\widehat{f}_j^I\}$ .

26 It should be noted that although only one input parameter interval changes, all the output  
27 intervals are changed simultaneously. Zhao et al. (2022) proposed an uncertainty quantification  
28 metric of *interval similarity operator (ISO)* to address the issue of structural model updating.  
29 In this work, this metric is introduced to propose a novel sensitivity index to quantify the

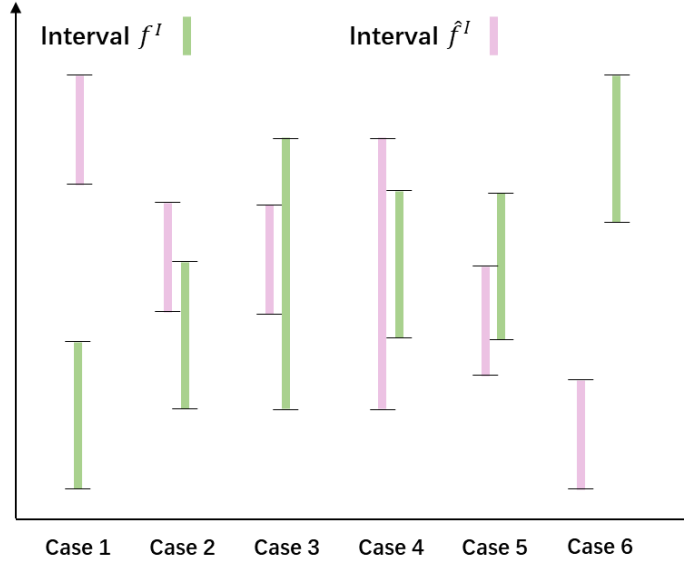


1 discrepancy between one-dimensional intervals  $f^I$  and  $\hat{f}^I$ , reflecting the sensitivity of each  
 2 input interval parameter.

3 Firstly, the *Interval Relative Position Operator (IRPO)* is utilized to measure the difference  
 4 between two intervals based on the mathematical rule of interval length  $L(\cdot)$ , defined as follows:

$$5 \quad L(f^I) = \bar{f} - \underline{f} \quad (15)$$

6 Two intervals  $f^I = [\underline{f}, \bar{f}]$  and  $\hat{f}^I = [\underline{\hat{f}}, \bar{\hat{f}}]$  are utilized to explain the proposed *Interval*  
 7 *Similarity Operator*. Six typical positional relationships between intervals  $f^I$  and  $\hat{f}^I$  are  
 8 presented in Figure 3.



9

Fig. 3 Six interval relative positions.

10  
 11 The *IRPO* is calculated according to different overlap cases in Fig. 3, and its calculation rules  
 12 are given as

$$13 \quad IRPO(f^I, \hat{f}^I) = \begin{cases} \frac{(\bar{f} - \underline{\hat{f}})}{\max\{L(f^I), L(\hat{f}^I)\}} & \text{Case 1,2} \\ \frac{(\bar{f} - \underline{f})}{\max\{L(f^I), L(\hat{f}^I)\}} & \text{Case 3} \\ \frac{(\bar{\hat{f}} - \underline{f})}{\max\{L(f^I), L(\hat{f}^I)\}} & \text{Case 4} \\ \frac{(\bar{\hat{f}} - \underline{\hat{f}})}{\max\{L(f^I), L(\hat{f}^I)\}} & \text{Case 5,6} \end{cases} \quad (16)$$

14 where  $\max\{L(f^I), L(\hat{f}^I)\}$  represents the maximum interval length between  $f^I$  and  $\hat{f}^I$ .

15 For the cases 1 and 6,  $f^I$  and  $\hat{f}^I$  have an overlapping space, and then the *IRPO* is negative.  
 16 When the length of intervals  $f^I$  and  $\hat{f}^I$  are infinite, the denominator of *IRPO* tends to zero.  
 17 In cases 2-5, there is an overlap between  $f^I$  and  $\hat{f}^I$ , and the value of the *IRPO* is clearly  
 18 positive and restrained to the range of (0,1). If both the position and the length of  $f^I$  is  
 19 consistent with that of  $\hat{f}^I$ , *IRPO* achieves its maximum value of 1. Hence the range of *IRPO*

1 is

$$2 \quad IRPO(f^I, \hat{f}^I) \in (-\infty, 1] \quad (17)$$

3 Next, impose that the *IRPO* has a high gradient as the value moves close to one; we develop  
4 an *Interval Sensitivity Operator* based on the *IRPO* to quantify the similarity between two  
5 interval vectors concerning their geometric position and shape. The fundamental calculation  
6 rule of *ISO* is given by

$$7 \quad ISO(f^I, \hat{f}^I) = mean\left(1 - \frac{1}{1 + \exp\{-IRPO(f_j^I, \hat{f}_j^I)\}}\right), j = 1, 2, \dots, m \quad (18)$$

8 where  $mean(\cdot)$  represents the mean value of  $(\cdot)$ .

9 From Eq. (18), we can find that the value of *ISO* is limited within  $(0, +\infty)$ . A high gradient  
10 of Eq. (18) reflects the similarity between  $f^I$  and  $\hat{f}^I$ . On the contrary, when *ISO* moves to  
11 the infinite positive, this means  $f^I$  is significantly different from  $\hat{f}^I$ . In the context of  
12 sensitivity analysis, this implies that variations in parameter  $\theta_i^I$  have a strong influence on the  
13 outputs  $f^I$ .

14 According to Eq. (4), the proposed interval sensitivity index  $\mathbf{S}$  based on *ISO* can be  
15 expressed as Eqs. (19)-(20). This index vector consists of a series of sensitive index variables  
16  $\{S_i, i = 1, \dots, n\}$ , and each variable  $S_i$  quantifies the sensitivity of parameter  $\theta_i^I$ .

$$17 \quad \mathbf{S} = \{S_i\}, i = 1, \dots, n \quad (19)$$

$$18 \quad S_i = \frac{\Delta f^I}{\Delta \theta^I} = \frac{ISO(f^I, \hat{f}^I)}{\Delta \theta_i^I} \quad (20)$$

19 We can rank the sensitivity index  $S_i$  in descending order to select the sensitive parameters.  
20 It should be noted that this proposed interval sensitivity index is calculated based on the  
21 boundaries of model outputs without any inner data points, which is especially appropriate for  
22 the model with pure interval uncertainties. Meanwhile, when there are stochastic uncertainties  
23 or hybrid stochastic and interval uncertainties in the model since the geometric position and  
24 range of model outputs can be measured through *ISO*, this index can be adopted for models  
25 with complicated uncertainties as well.

## 26 4.2 Framework of sensitivity analysis with interval uncertainties

27 This work belongs to the One-at-a-Time method (OAAT), so the problem of coupling  
28 between model parameters is not considered here. Besides, in contrast to the sensitivity analysis  
29 with stochastic uncertainties, this interval sensitivity analysis is performed with no hypothesis  
30 of probabilistic distributions. The objective is to quantify the variation of interval outputs  $f^I \rightarrow$   
31  $\hat{f}^I$  caused by the change in the interval parameters  $\theta_i^I \rightarrow \hat{\theta}_i^I$ , such that the sequence of  $S_i$   
32 represents the sensitivity of inputs  $\theta^I$ .

33 The framework of the proposed sensitivity analysis method is illustrated in Fig. 4, which  
34 consists of three parts: (1) the major body, (2) the interval uncertainty propagation, and (3) the  
35 sensitivity index calculation. As mentioned above, the interval perturbation method is

1 introduced to estimate model output intervals effectively. Then, the sensitivity index  $S_i$   
 2 corresponding to each parameter  $\theta_i^I$  is accordingly calculated to measure the discrepancy  
 3 between the initial interval output  $f^I$  and the perturbed output  $\hat{f}^I$ . The major body contains a  
 4 pre-determined initial and perturbed range of parameters, namely  $\theta_i^I$  and  $\hat{\theta}_i^I$ . Finally, we rank  
 5 the sensitivity index  $S_i$  in descending order. The detailed steps are illustrated as follows:

6 Step 1: The framework starts from a pre-determined range of the model parameters  $\theta^I =$   
 7  $\{\theta_1^I, \dots, \theta_n^I\}$  with its initial interval center  $\theta^C = \{\theta_i^C\}$  and corresponding interval radius  $\Delta\theta^I =$   
 8  $\{\Delta\theta_i^I\}$ . The initial interval  $\theta^I$  represents the gross knowledge from engineering judgments.

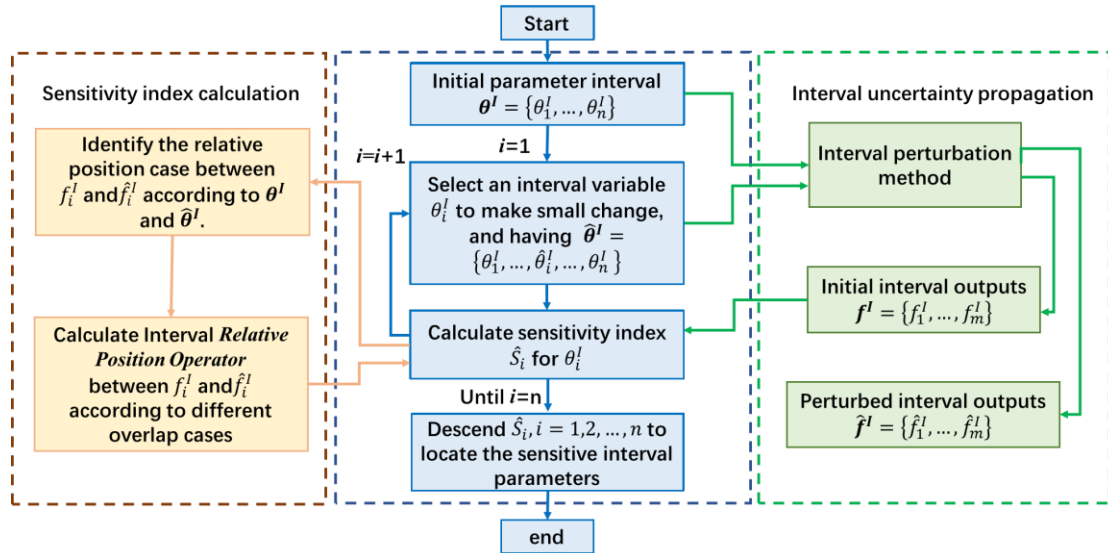
9 Step 2: Select an interval parameter component  $\theta_i^I$  and change its interval radius  
 10 proportionally to model the small change of  $\theta_i^I$ . Keep other interval parameter components  
 11 unchanged. We finally obtain the interval perturbed parameter  $\hat{\theta}^I = \{\theta_1^I, \dots, \hat{\theta}_i^I, \dots, \theta_n^I\}$ .

12 Step 3: Estimate the initial model output interval  $f^I = \{f_1^I, \dots, f_m^I\}$  and the perturbed output  
 13 interval  $\hat{f}^I = \{\hat{f}_1^I, \dots, \hat{f}_m^I\}$  through the interval perturbation method, which concerns the initial  
 14 interval parameter  $\theta^I = \{\theta_1^I, \dots, \theta_n^I\}$  and the perturbed interval parameter  $\hat{\theta}^I =$   
 15  $\{\hat{\theta}_1^I, \theta_2^I, \dots, \theta_n^I\}$ , respectively.

16 Step 4: Calculate the *IRPO* and the *ISO* of  $f^I$  and  $\hat{f}^I$ , and calculate the sensitivity value  $\hat{S}_1$   
 17 corresponding to  $\hat{\theta}_1^I$ .

18 Step 5: Repeat Step 2-Step 4  $n$  times to obtain a series of sensitivity indexes  $\hat{S}_i, i = 1, 2, \dots, n$   
 19 corresponding to interval parameters  $\{\theta_1^I, \dots, \theta_n^I\}$ .

20 Step 6: Sequence the value of  $\hat{S}_i, i = 1, 2, \dots, n$  in descending order to present the sensitivity  
 21 of interval parameters  $\{\theta_1^I, \dots, \theta_n^I\}$ .



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Fig. 4 Flowchart of sensitivity analysis for interval parameters.

## 6. Case studies

### 6.1 Case 1: Ishigami function

#### 6.1.1 Problem description

A tutorial case study of the Ishigami function is presented in this section, which originated from Ref. (Ishigami and Homma 1990) and is analyzed by Ref. (Bi et al. 2019). The Ishigami function is a general example of sensitivity analysis, which is given as follows:

$$y(P) = \sin(p_1) + a \sin(p_2)^2 + bp_3^4 \sin(p_1) \quad (21)$$

where  $P = \{p_1, p_2, p_3\}$  is the input parameters;  $y$  is the output feature;  $a$  and  $b$  are constant coefficients with pre-determined values as Ref. (Marrel et al. 2008),  $a=7$  and  $b=0.1$ .

Since Bi et al. (2019) mainly focus on the stochastic sensitivity analysis with both aleatory and epistemic uncertainties, the uncertainties  $p_{1-3}$  in Ref. (Bi et al. 2019) assumed that  $p_1$  and  $p_2$  are prescribed to follow the uniform distribution, and  $p_3$  follows the Gaussian distribution. However, this uncertainty characteristic is inappropriate for the application of the proposed method because the determined distribution types of  $p_{1-3}$  belong to probabilistic uncertainties without any interval uncertainties.

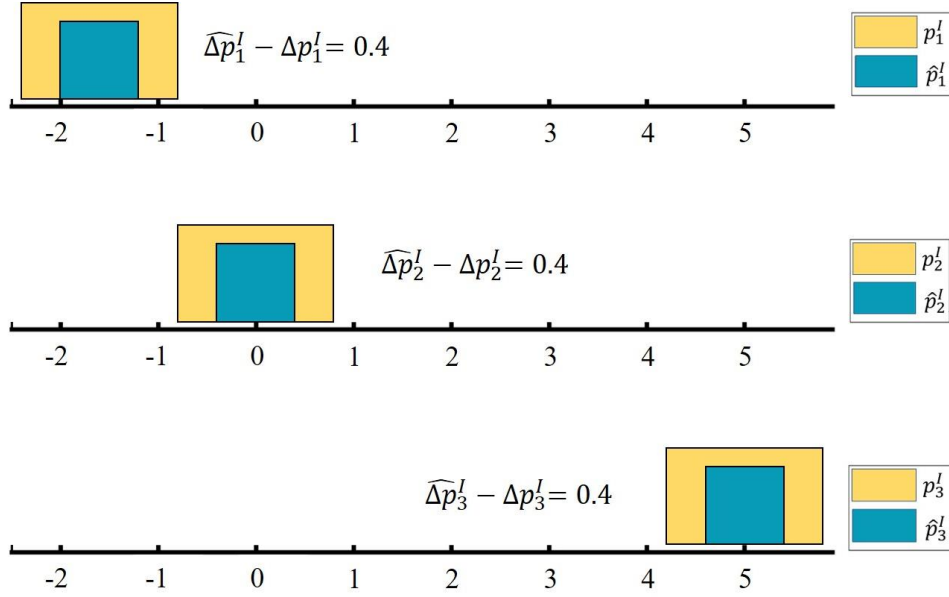
In this case,  $p_{1-3}$  are defined as interval parameters, and according to Eq. (4), we assume that the lengths of interval radius  $\Delta P$  are changed 50% as presented in Table 1. For example, the initial length of  $\Delta p_1$  is 0.8, which is changed to 0.4 in the perturbed interval  $\hat{p}_1^I$ , namely the interval of  $p_1^I$  is changed from  $[-2.4, -0.8]$  to  $[-2, -1.2]$ . Since this work is mainly about local sensitivity analysis, the other variables  $p_2^I$  and  $p_3^I$  of  $P^I$  are unchanged. By comparing the variation of the outputs caused by the initial and perturbed intervals of parameters, the sensitivity of each input can be identified.

Table 1 The initial and perturbed parameter intervals.

Parameters	$p_1^I$	$p_2^I$	$p_3^I$	Variation proportion ratio
Initial intervals $P^I$	$[-2.4, -0.8]$	$[-0.8, 0.8]$	$[4.2, 5.8]$	50%
Perturbed intervals $\hat{P}^I$	$[-2, -1.2]$	$[-0.4, 0.4]$	$[4.6, 5.4]$	50%

Since this work belongs to the problem of local sensitivity analysis, this interval sensitivity analysis aims to quantify the importance of each input according to how many intervals of uncertainty space of all outputs can be changed when the interval length of this initial input is proportionally increased, where the changed inputs are called perturbed inputs here. For this case, the perturbed interval parameters  $p_{1-3}$  are presented in Table 1. The objective of proportionally changing the interval radius  $\Delta p^I = \{\Delta p_1^I, \Delta p_2^I, \Delta p_3^I\}$  to generate perturbed inputs with a radius of  $\widehat{\Delta p}^I = \{\widehat{\Delta p}_1^I, \widehat{\Delta p}_2^I, \widehat{\Delta p}_3^I\}$  is illustrated in Figure 5, where the interval centers are completely fixed. The most intuitive manner is to measure how much the output interval space is changed to reflect the sensitivity of parameters. Hence the following operator to calculate the

1 interval outputs according to  $p^l$  and  $\hat{p}^l$ . As the Ishigami function is simple, 5000 Monte Carlo  
 2 simulations are conducted to estimate the initial interval outputs  $y^l$  and perturbed output  $\hat{y}^l$ .



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Fig. 5 Initial and perturbed input intervals.

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Table 2 presents the initial output interval  $y^l$  calculated based on the initial parameter intervals  $p_1^l$ ,  $p_2^l$ , and  $p_3^l$ . In the context of local sensitivity analysis, there are three perturbed interval output spaces corresponding to variations sequentially occurring in  $p_1^l$ ,  $p_2^l$ , and  $p_3^l$ . The three perturbed interval outputs are given in Table 2. The output variability significantly reflects the degree of dispersion in the output uncertainty space, which is investigated in Figure 6. For example, the perturbed procedure is executed for an interval of  $\hat{p}_1^l$  meanwhile, keeping the interval bounds of  $p_2^l$ , and  $p_3^l$ . Accordingly, the perturbed output  $\hat{y}_{p_1}^l$  is available. From Figure 6, it can be seen that the perturbation in  $p_3^l$  results in the most obvious changes in output, i.e.  $y^l$  to  $\hat{y}_{p_3}^l$ , implying the  $p_3^l$  is sensitive to the outputs  $y^l$ .

Table 2 Initial and perturbed input intervals.

Initial parameters	Initial output $y^l$	Perturbed parameter orders	Perturbed output $\hat{y}^l$
$p_1^l, p_2^l, p_3^l$	$y^l = [-111.55, -21.52]$	$\hat{p}_1^l, p_2^l, p_3^l$	$\hat{y}_{p_1}^l = [-113.07, -27.78]$
		$p_1^l, \hat{p}_2^l, p_3^l$	$\hat{y}_{p_2}^l = [-111.82, -22.04]$
		$p_1^l, p_2^l, \hat{p}_3^l$	$\hat{y}_{p_3}^l = [-85.30, -29.65]$

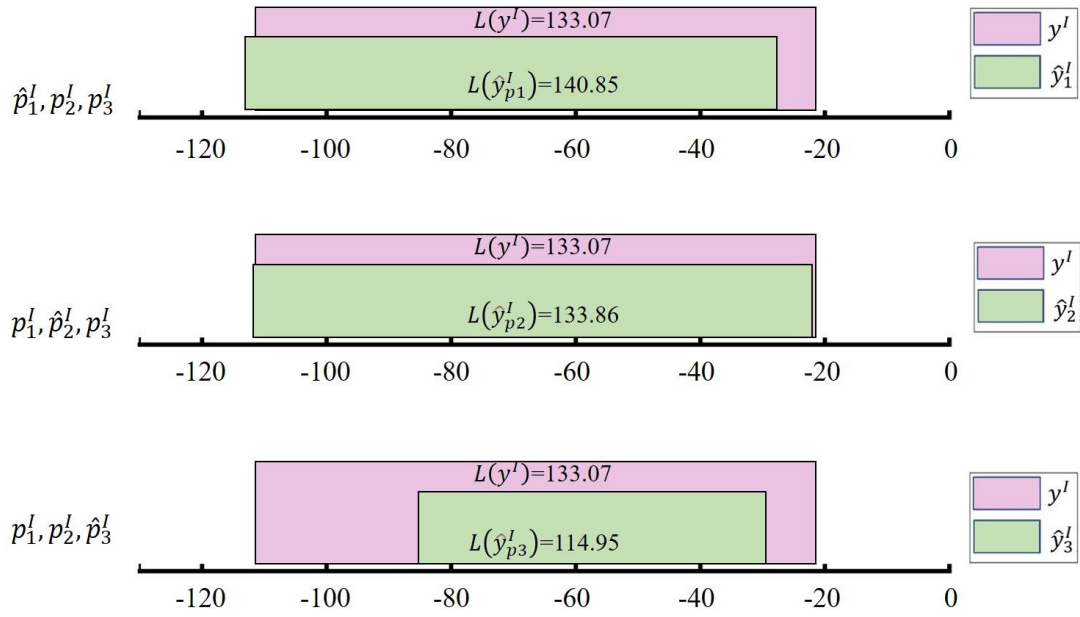
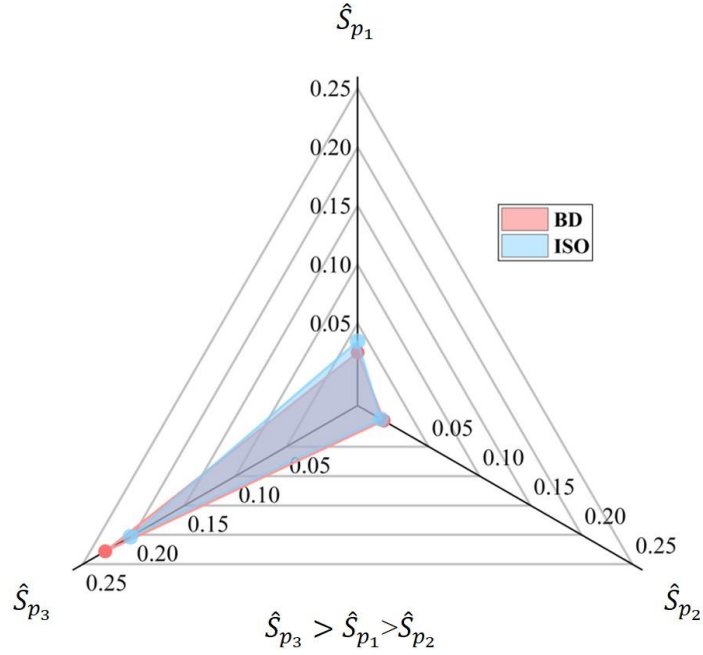


Fig. 6 Comparison between the initial and perturbed outputs intervals.

The *IRPO*, *ISO*, and comprehensive sensitivity index of  $p_1^I$  are evaluated according to Section 3. The same strategy is performed for  $p_2^I$ , and  $p_3^I$ , and their corresponding sensitivity indices are presented in Table 3. According to the determined input parameters, 5000 times Monte Carlo samples with the assumption of parameters following uniform distribution are performed to propagate the uncertainty from the inputs to the outputs, yielding 5000 data points bounded by the output intervals. Hence the sensitivity indices of Bhattacharyya distance in Ref. (Bi et al. 2019) can be adopted in this case, whose results are similar to those calculated by the proposed indices as shown in Figure 7.

Table 3 Sensitivity analysis of  $p_{1-3}$ .

Sensitivity index	$\hat{S}_{p_1}$	$\hat{S}_{p_2}$	$\hat{S}_{p_3}$	Sensitivity rank
Bhattacharyya distance Ref. (Bi et al. 2019)	0.02525	0.0055	0.228	$\hat{S}_{p_3} > \hat{S}_{p_1} > \hat{S}_{p_2}$
<i>ISO</i>	0.03475	0.003	0.20325	$\hat{S}_{p_3} > \hat{S}_{p_1} > \hat{S}_{p_2}$

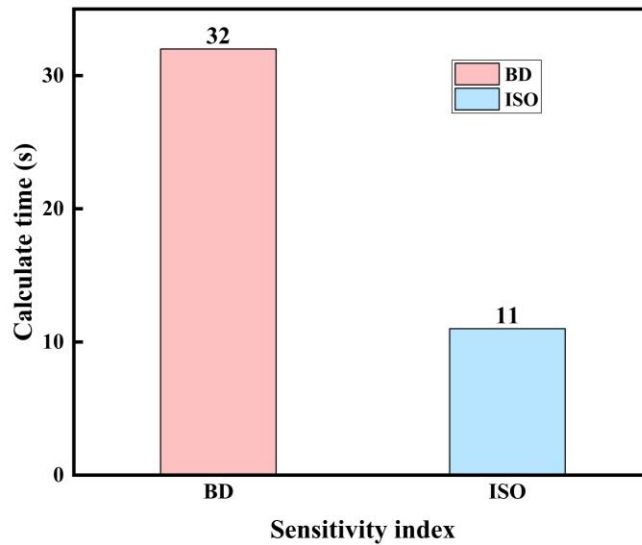


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Fig. 7 Sensitivity analysis of  $p_1^I$ ,  $p_2^I$ , and  $p_3^I$

The sensitivity ranking is  $\hat{S}_{p_3} > \hat{S}_{p_1} > \hat{S}_{p_2}$ , and  $\hat{S}_{p_1}$  and  $\hat{S}_{p_2}$  are significantly smaller than  $\hat{S}_{p_3}$ , indicating that  $\hat{S}_{p_3}$  is more sensitive than  $\hat{S}_{p_1}$  and  $\hat{S}_{p_2}$  from the point of view of interval uncertainty. Meanwhile,  $ISO_{p_1}$  and  $ISO_{p_2}$  are close to 0, implying a high geometric similarity between intervals  $y^I$  and  $\hat{y}^I$ .

It should be noted that the Bhattacharyya distance is calculated based on the distribution function of data, implying it is unable to deal with interval uncertainties. The proposed  $ISO$  is calculated only by the bounds of output intervals, which is reliable in engineering. Besides, the calculation time of the  $ISO$  is much less than that of the Bhattacharyya distance, which is proved by Fig. 8.



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Fig. 8 Comparison of calculation time of the  $ISO$  and Bhattacharyya distance metrics.

## 6.1.2 Parameters with hybrid probabilistic and interval uncertainties

To assess the effectiveness of the proposed sensitivity analysis method, a published work on this Ishigami function, namely Ref. (Bi et al. 2019) is utilized herein as a reference to compare with the results of the current work. In Ref. (Bi et al. 2019), the parameters of the Ishigami function contain both hybrid probabilistic and interval uncertainties, which are expressed by the P-box technique. More complex uncertainty characteristics of the parameters are assigned as presented in Table 4, and correspondingly, the outputs  $y^{IR}$  are with hybrid stochastic and interval uncertainties. The P-box of an imprecise uniform distribution can be easily determined, i.e., the P-box of  $p_1$  is enveloped by the CDFs of  $\underline{p}_1 \sim U(-4, 2)$  and  $\bar{p}_1 \sim U(-3, 3)$  as shown in Figure 9. The uncertainty of  $p_1$  is controlled by the coefficients of  $a_1$  and  $b_1$  in 错误!未找到引用源。 , and the P-boxes of  $p_{2-3}$  can be determined by the uncertain coefficients of  $a_2$ ,  $b_2$ ,  $\mu_3$ , and  $\sigma_3$ . A two-level procedure is proposed in Ref. (Bi et al. 2019) to calculate the P-boxes of outputs according to the input P-boxes. The Bhattacharyya distance is utilized to measure the discrepancy between the bounded CDF of the output P-box. For comparison, the proposed *ISO* is adopted to replace the Bhattacharyya distance for sensitivity analysis

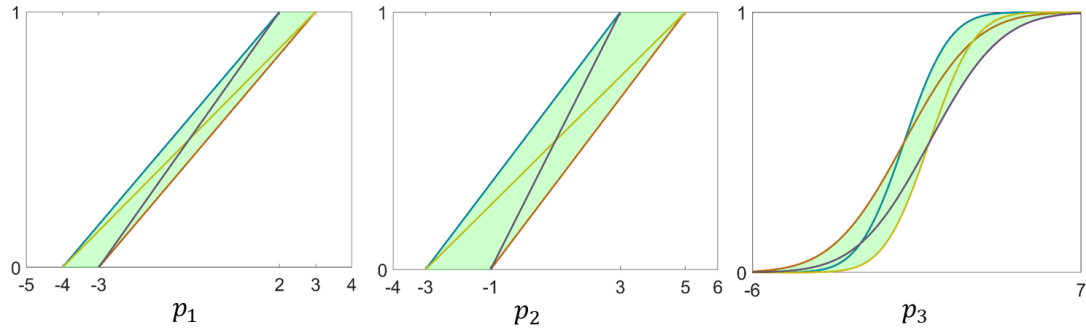


Fig. 9 The P-box of  $p_{1-3}$  in Ref. (Bi et al. 2019).

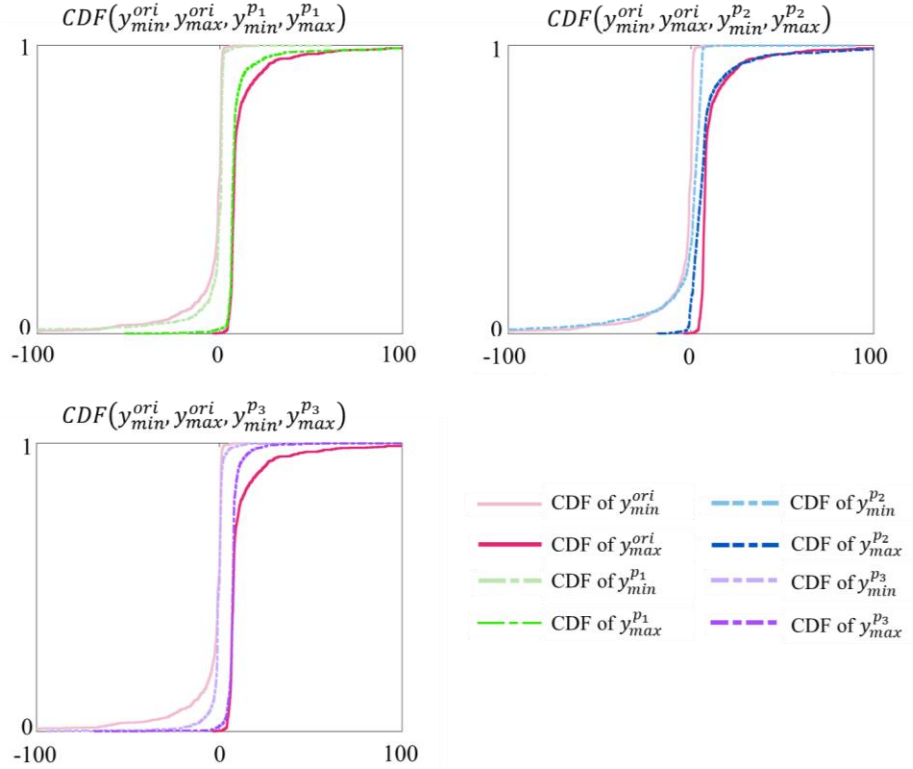
Table 4 Uncertainty characteristics of  $p_{1-3}$  in Ref. (Bi et al. 2019).

Parameters	Parameter probabilistic distribution	Uncertain coefficient
Ref. (Ishigami and Homma 1990)		
$p_1$	$p_1 \sim U(a_1^l, b_1)$	$a_1^l \in [-4.0, -3.0]; b_1^l \in [2.0, 3.0]$
$p_2$	$p_2 \sim U(a_2^l, b_2)$	$a_2^l \in [-3.0, -1.0]; b_2^l \in [3.0, 5.0]$
$p_3$	$p_3 \sim U(\mu_3^l, \sigma_3^2)$	$\mu_3^l \in [0.0, 1.0]; \sigma_3^l \in [\sqrt{5}, \sqrt{2}]$

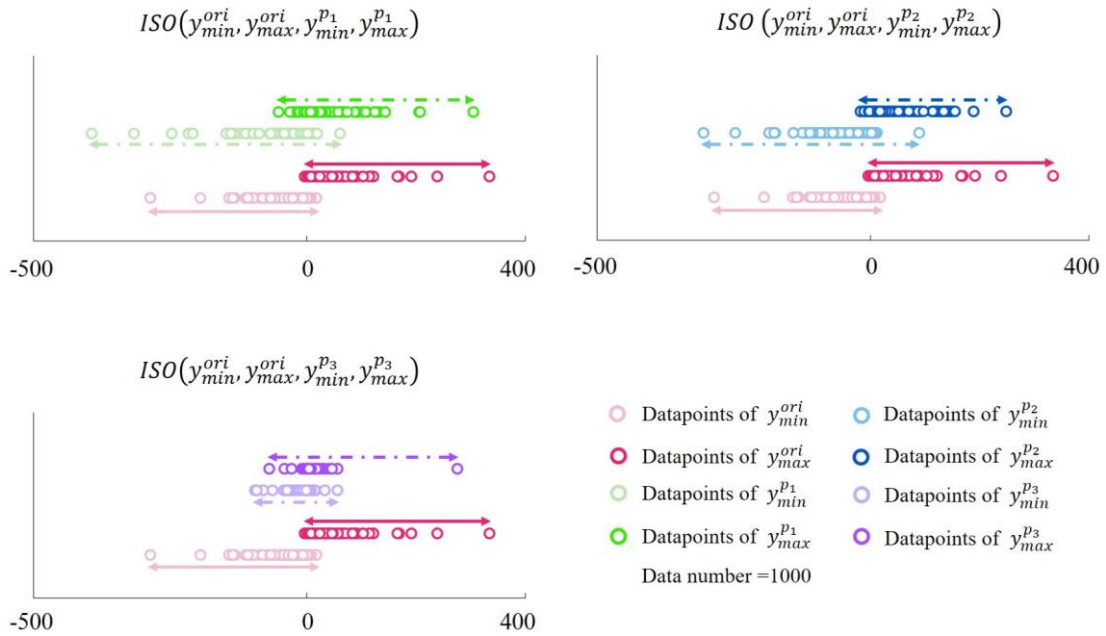
To calculate the sensitivity of the parameter  $p_1$ , ten levels of interval  $a_1$  and  $b_1$  are investigated by assigning ten equidistant values within the intervals. The full factorial design results in 100 configurations of  $a_1$  and  $b_1$ , and correspondingly 100 perturbed P-boxes of  $p_1$  are obtained. Then 100 groups of perturbed P-boxes of outputs are simulated according to 100 groups of an interval variable  $\{a_1^l, b_1^l\}_{j=1, \dots, 100}$ , respectively. To compare the uncertain space to give an explicit sensitivity ranking of  $p_{1-3}$ , the metrics of Bhattacharyya distance and



1 ISO are adopted to quantify the uncertainty space of the output P-box. Figure 10 presents one  
 2 of the 100 perturbed P-boxes according to  $p_{1-3}$  in the form of CDFs, and Figure 11 gives  
 3 specific data points of perturbed outputs. The sensitivity rank of parameters is evaluated based  
 4 on the difference between the perturbed and original outputs concerning  $p_{1-3}$ .



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 6 Fig. 10 Reduced output stochastic space when epistemic uncertainties of input parameters are  
 7 reduced compared with the original P-box, which is calculated according to Ref. (Bi et al.  
 8 2019).



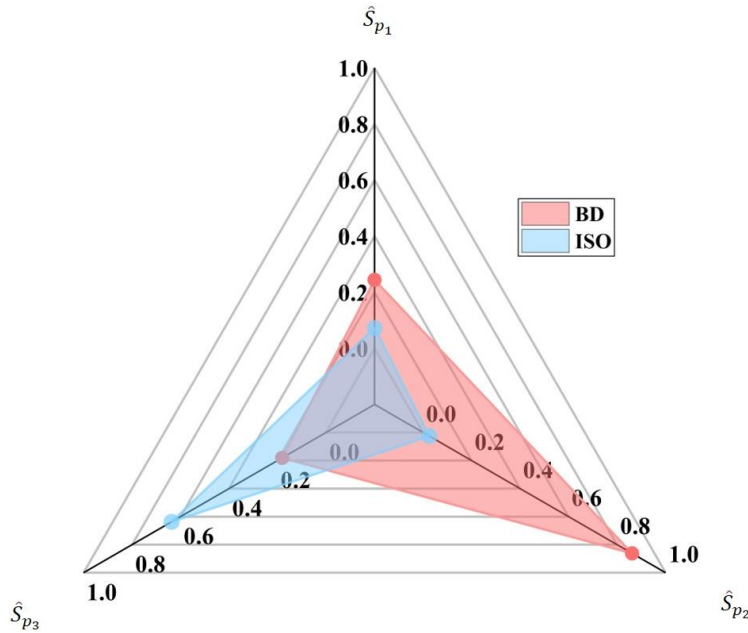
9  
 10 Fig. 11 Reduced output interval space when the epistemic uncertainties of input parameters

1 are reduced compared with the original P-box.

2 Figure 10 illustrates the sensitivity rank rule of Ref. (Bi et al. 2019), where the reduced  
 3 uncertainty space of output is measured from the view of CDF. Figure 11 investigates the  
 4 variation between the original and perturbed output uncertainty space by applying the interval  
 5 concept. It is difficult to rank the parameter sensitivity with manual observation, but it is easy  
 6 to directly tell the sensitivity rank through an interval observation mode, as shown in Figs 10-  
 7 11. Since those uncertain data points are quantified by different uncertainty quantification tools,  
 8 the results of sensitivity rank are inconsistent, as shown in Table 5 and Fig.12.

9 Table 5 Uncertainty characteristics of  $p_{1-3}$ .

Rank	Results according to different indices	
	$\hat{S}_{ISO}$	$\hat{S}_{BD}$ in Ref. (Bi et al. 2019)
1	$\hat{S}_{p_3}$	$\hat{S}_{p_2}$
2	$\hat{S}_{p_1}$	$\hat{S}_{p_1}$
3	$\hat{S}_{p_2}$	$\hat{S}_{p_3}$



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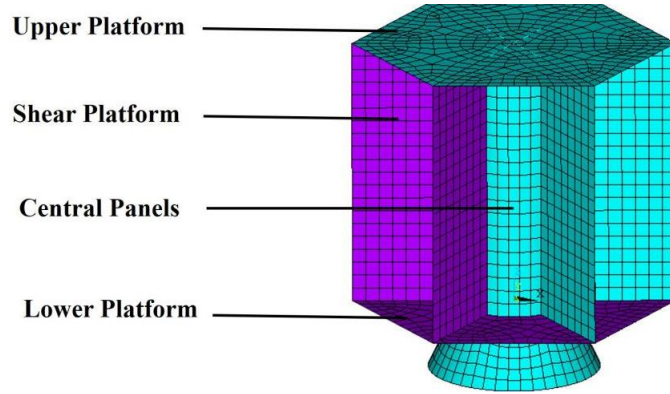
11 Fig. 12 Sensitivity analysis comparison with respect to the metrics of *ISO* and Bhattacharyya  
 12 distance.

13 From the point of view of the interval concept, the uncertain output space of  $p_3$  is changed  
 14 most obviously compared with the original one, where the subjective judgment consists of the  
 15 objective calculation results calculated by *ISO*. The ranks calculated by *ISO* differ from that of  
 16 Ref. (Bi et al. 2019). This is because the interval is mainly affected by the extreme data points,  
 17 while the CDF is calculated according to the dispersion of the datapoints. The sensitivity rank  
 18 computed by interval uncertainty quantification of *ISO* is credible. This is because we mainly  
 19 focus on the low-probability tail risk, which occurs when the model outputs are extremely large

1 or small in engineering practice. It can be found that the change in data from the probabilistic  
 2 point of view cannot imply a significant change in data from the interval point of view.

### 3 **6.2 Case 2: Satellite FE model**

4 This case of a satellite model is derived from Ref. (Zhang et al, 2019), which is utilized for  
 5 model calibration analysis. Here, this model is analyzed to demonstrate the performance of the  
 6 *ISO* metric within the proposed sensitivity analysis. The FE model of the satellite is presented  
 7 in Figure 13, and this model consists of the upper platform, the shear platform, the central  
 8 panels, and the lower platform.



9  
 10 Fig. 13 Finite element model of the satellite.

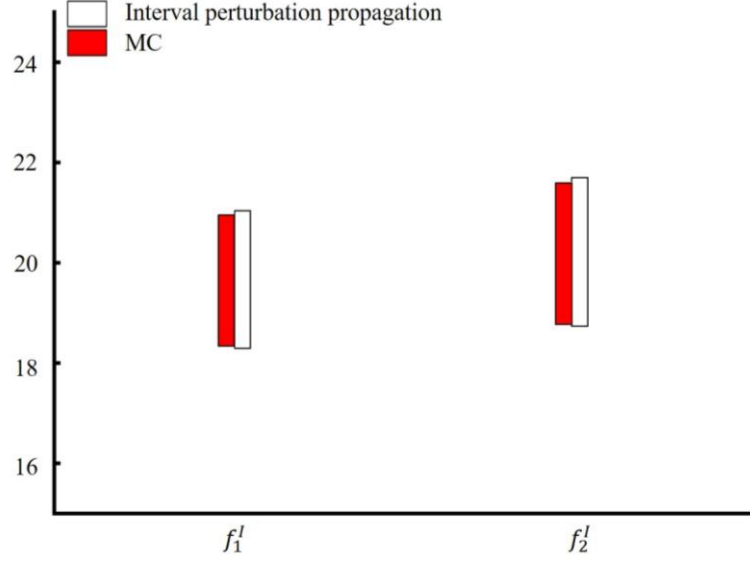
#### 11 **6.2.1 Interval uncertainties propagation**

12 In this satellite FE model, the Elastic modulus of the FE model  $\theta_1$  is  $7.0 \times 10^{10}$  pa, the density  
 13  $\theta_2$  is  $2.7 \times 10^3 \text{ kg} \cdot \text{m}^{-3}$ , and the thickness of the lower platform  $\theta_4$  is 1mm. Some parameters of  
 14 this FE model are assumed to be interval as given in Table 6. The first two eigenfrequency  
 15 intervals  $f_1^I$  and  $f_2^I$  are regarded as the model output features.

16 Table 6 Satellite parameter table

Parameters		Interval centers	Interval radius
$\theta_3^I$	The thickness of the central panel	2 (mm)	0.2 (mm)
$\theta_5^I$	The thickness of the shear platform	3 (mm)	0.2 (mm)
$\theta_6^I$	The thickness of the upper platform	2 (mm)	0.2 (mm)

17 Since it is time-consuming to calculate the FE model of the satellite structure, the interval  
 18 perturbation method is adopted to estimate the interval outputs efficiently. 10000 Monte Carlo  
 19 simulations are conducted to estimate  $f_1^I$  and  $f_2^I$ . The results of the two methods are presented  
 20 in Fig. 14 and Table 7. It can be found that the bounds calculated by the perturbed method are  
 21 consistent with that of MC simulation, demonstrating the accuracy of the interval perturbation  
 22 method. Besides, the calculation time of MC simulations is 85960s, while that of the interval  
 23 perturbation method is about 240s, as shown in Fig. 15. This illustrates the effectiveness of  
 24 interval perturbation method when coping with the problem of interval propagation.

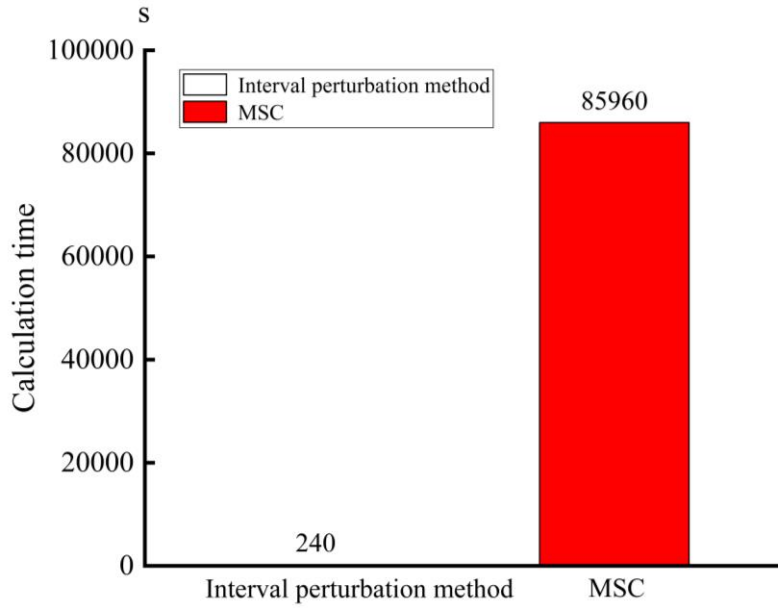


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Fig. 14 Interval of  $f_1^I$  and  $f_2^I$  calculated by MCS and the interval perturbation method.

Table 7 Interval perturbation method

Parameters	MC simulation	Interval perturbation method	Relative error
$f_1^I$	[ 18.33,20.94]	[18.29, 21.04]	[ 0.21%, 0.47%]
$f_2^I$	[ 18.78, 21.6]	[18.74, 21.7]	[0.21%, 0.46% ]



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Fig. 15 Comparison of calculation time between MCS and the interval perturbation method.

### 6.2.2 Sensitivity analysis of $\theta_3^I$ , $\theta_5^I$ and $\theta_6^I$

7 A sensitivity analysis is executed for some interval parameters of the  $\theta_3^I$ ,  $\theta_5^I$  and  $\theta_6^I$ .  
8 Proportional changes in the interval radius  $\Delta\theta^I = \{\Delta\theta_3^I, \Delta\theta_5^I, \Delta\theta_6^I\}$  generate input intervals with  
9 the perturbed radius of  $\widehat{\Delta\theta}^I = \{\widehat{\Delta\theta}_3^I, \widehat{\Delta\theta}_5^I, \widehat{\Delta\theta}_6^I\}$ , where the interval centers are completely fixed.  
10 Figure 16 intuitively presents the changed interval inputs, and Table 8 lists the initial parameter  
11 interval  $\theta^I$  and the perturbed parameter interval  $\widehat{\theta}^I$ .

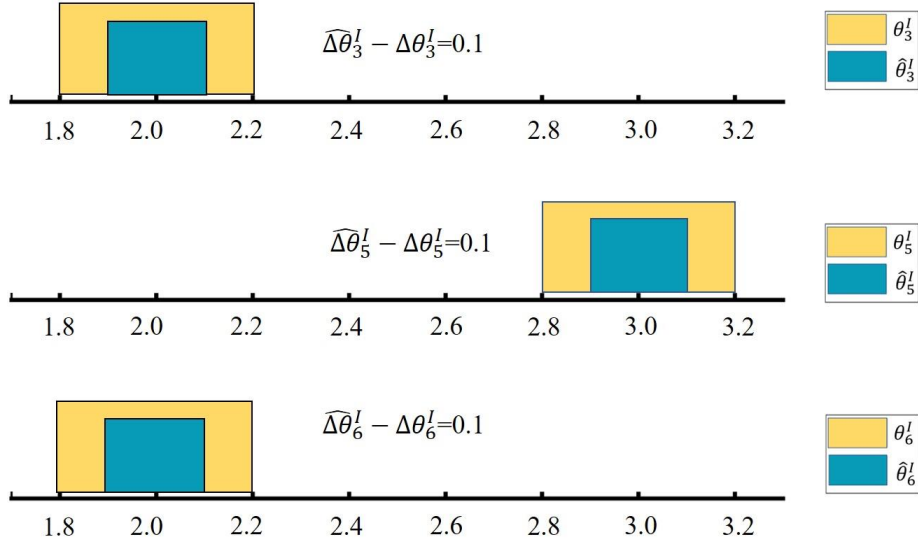


Fig. 16 Comparison between initial and perturbed input intervals.

Table 8 The initial and perturbed intervals of  $\theta_3^I$ ,  $\theta_5^I$  and  $\theta_6^I$

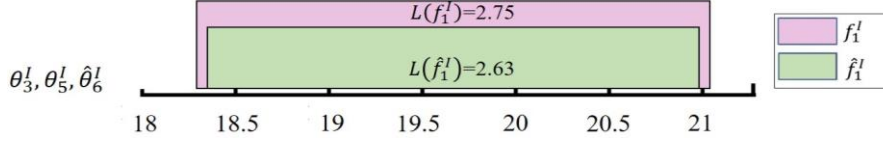
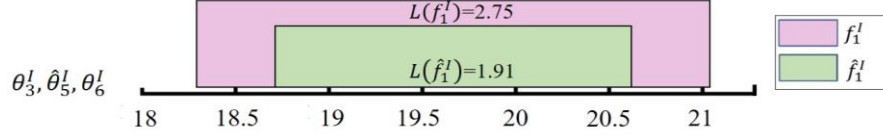
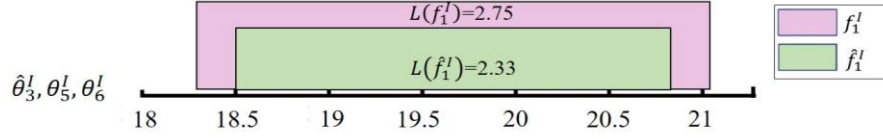
Parameters	$\theta^I$	$\hat{\theta}^I$	Variation proportion ratio of interval radius
$\theta_3^I$	[1.8,2.2]	[1.9,2.1]	50%
$\theta_5^I$	[2.8,3.2]	[2.9,3.1]	50%
$\theta_6^I$	[1.8,2.2]	[1.9,2.1]	50%

The initial output intervals  $f_1^I$  and  $f_2^I$  are calculated according to the initial parameters  $\theta_3^I$ ,  $\theta_5^I$  and  $\theta_6^I$ . Next, the sensitivity framework in Section 4 is implemented to rank the sensitivity of input parameters. Three perturbed interval output spaces  $\hat{\theta}^I$  are listed in Table 9 respective to the parameter perturbed sequentially. The initial and perturbed output intervals of  $f_1^I$  and  $f_2^I$  are shown in Figs. 17 -18, respectively.

Table 9 Sensitivity analysis for  $\theta_3^I$ ,  $\theta_5^I$  and  $\theta_6^I$

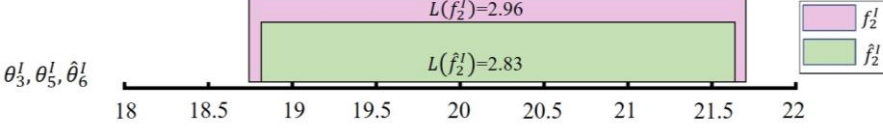
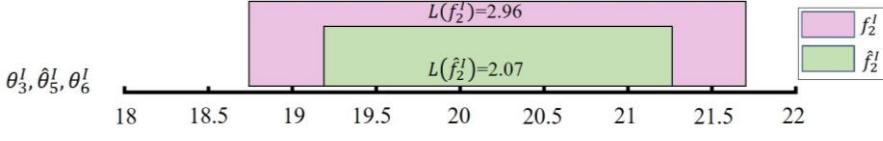
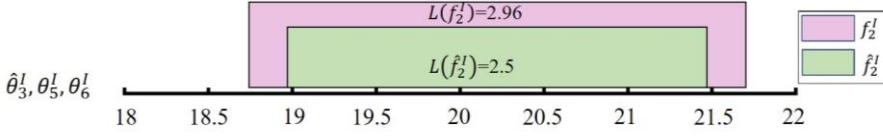
Parameters	Perturbated $\hat{f}^I$	Initial $f^I$	$\hat{S}_{\theta_i}$	Sensitivity rank
$\hat{\theta}_3^I, \theta_5^I, \theta_6^I$	$\hat{f}_1^I = [18.50, 20.83]$ $\hat{f}_2^I = [18.97, 21.47]$		3.004	
$\theta_3^I, \hat{\theta}_5^I, \theta_6^I$	$\hat{f}_1^I = [18.71, 20.62]$ $\hat{f}_2^I = [19.19, 21.26]$	$f_1^I = [18.29, 21.04]$ $f_2^I = [18.74, 21.70]$	3.326	$\hat{S}_{\theta_5} > \hat{S}_{\theta_3} > \hat{S}_{\theta_6}$
$\theta_3^I, \theta_5^I, \hat{\theta}_6^I$	$\hat{f}_1^I = [18.35, 20.98]$ $\hat{f}_2^I = [18.81, 21.64]$		2.772	

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Fig. 17 Initial and perturbed output intervals of  $f_1^I$



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Fig. 18 Initial and perturbed output intervals of  $f_2^I$ .

5 From Figs. 17-18, we can find that the variations between the initial and perturbed outputs  
6 space of  $\hat{\theta}_5^I$  are the largest, which reveals the impact of the model input interval parameter  
7  $\theta_5^I$ , namely the thickness of the shear platform on the model output intervals  $f_1^I$  and  $f_2^I$  is  
8 significant. On the contrary,  $\theta_6^I$  is the least important parameter. The last column of Table 9  
9 presents the ranking results of the current work, namely  $\theta_5 > \theta_3 > \theta_6$ , according to the  
10 proposed sensitivity index, which is visualized in Fig. 19. The sensitivity ranks imply that the  
11 interval uncertainty of shear platform thickness  $\theta_5$  significantly impacts the primary model  
12 natural frequencies of the satellite model.

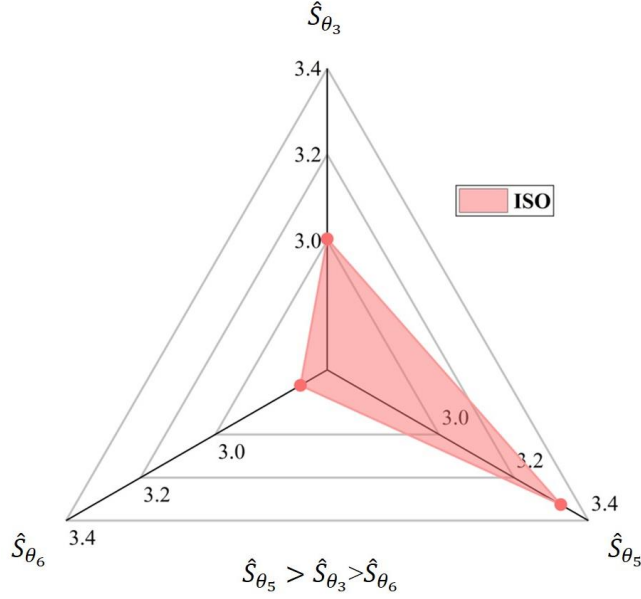


Fig. 19 Sensitivity order of  $\theta_3^I$ ,  $\theta_5^I$ , and  $\theta_6^I$

### 6.2.3 Sensitivity analysis for model with multiple parameters and multiple outputs

In this section, the parameters  $\theta_1, \theta_2$ , and  $\theta_4$  are assumed to be intervals, and the first eigenfrequency to the eighth eigenfrequency  $f_{1-8}^I$  are investigated as multiple output features. The proposed sensitivity analysis is performed for six parameters  $\theta_{1-6}^I$ , and the initial and perturbed intervals of parameters are presented in Table 10.

Table 10 The initial and perturbed intervals of  $\theta_1^I$  to  $\theta_6^I$

Parameters	$\theta^I$	$\hat{\theta}^I$	Variation proportion ratio of interval radius
$\theta_1^I$	[6.8,7.2]	[6.9,7.1]	50%
$\theta_2^I$	[2.5,2.9]	[2.6,2.8]	50%
$\theta_3^I$	[1.8,2.2]	[1.9,2.1]	50%
$\theta_4^I$	[0.8,0.2]	[0.9,1.1]	50%
$\theta_5^I$	[2.8,3.2]	[2.9,3.1]	50%
$\theta_6^I$	[1.8,2.2]	[1.9,2.1]	50%

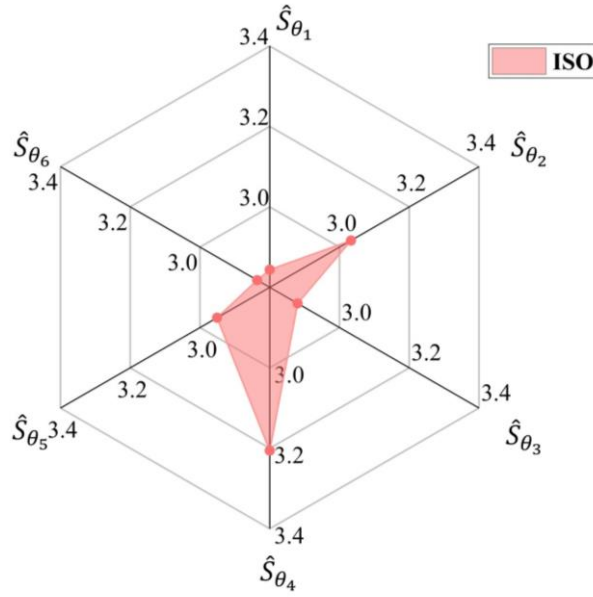
We calculate the sensitivity indexes with respect to each parameter, and finally rank them as given in Table 11 and Fig 20.  $\theta_4^I$ , the lower platform of the satellite model, is the most sensitive parameter among those six interval parameters, and  $\theta_6^I$ , the thickness of the upper platform is the most not sensitive parameter. Besides, it should be noted that the rank of  $\theta_3^I, \theta_5^I$ , and  $\theta_6^I$  in this case is consistent with that in Section 6.2.2, while with the different sensitivity index value. This is because the dimensions of outputs are increased from 3 to 8. For multiple outputs, the sensitivity of parameters is not changed in this Satellite FE model, which

1 illustrates the stability of the proposed sensitivity analysis method.

2

Table 11 Sensitivity analysis for  $\theta_{1-6}^I$ .

Parameters	$\hat{S}_{\theta_i}$ with <i>ISO</i>	Sensitivity rank
$\theta_1^I$	2.844	$\hat{S}_{\theta_4} > \hat{S}_{\theta_2} > \hat{S}_{\theta_5} > \hat{S}_{\theta_3} > \hat{S}_{\theta_1} > \hat{S}_{\theta_6}$
$\theta_2^I$	3.033	
$\theta_3^I$	2.880	
$\theta_4^I$	3.206	
$\theta_5^I$	2.951	
$\theta_6^I$	2.836	



$$\hat{S}_{\theta_4} > \hat{S}_{\theta_2} > \hat{S}_{\theta_5} > \hat{S}_{\theta_3} > \hat{S}_{\theta_1} > \hat{S}_{\theta_6}$$

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Fig. 20 Sensitivity order of  $\theta_{1-6}^I$

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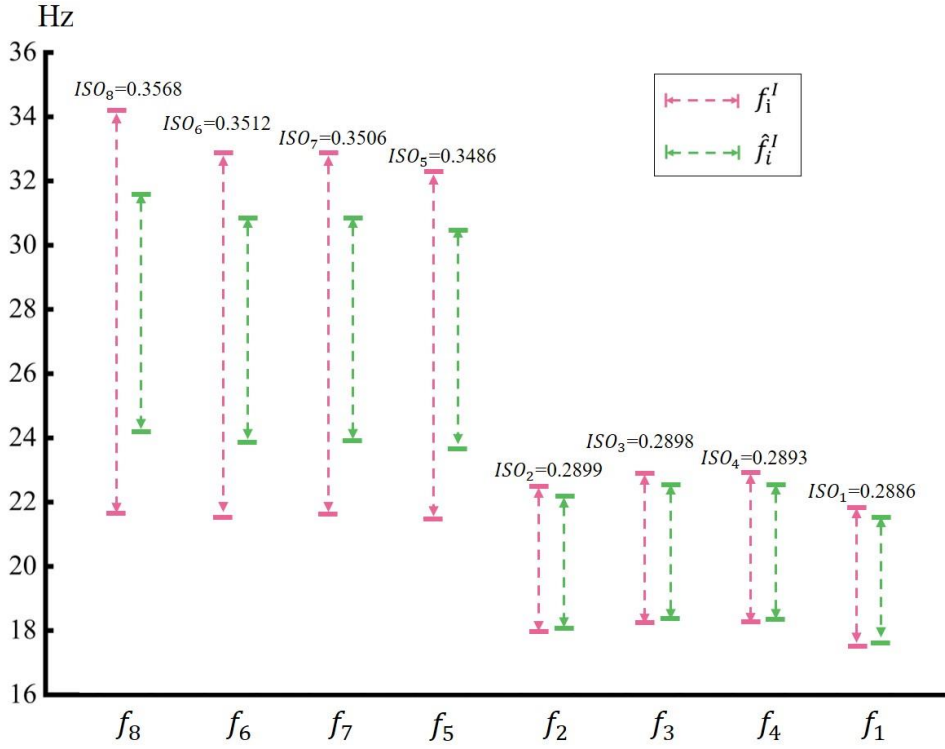
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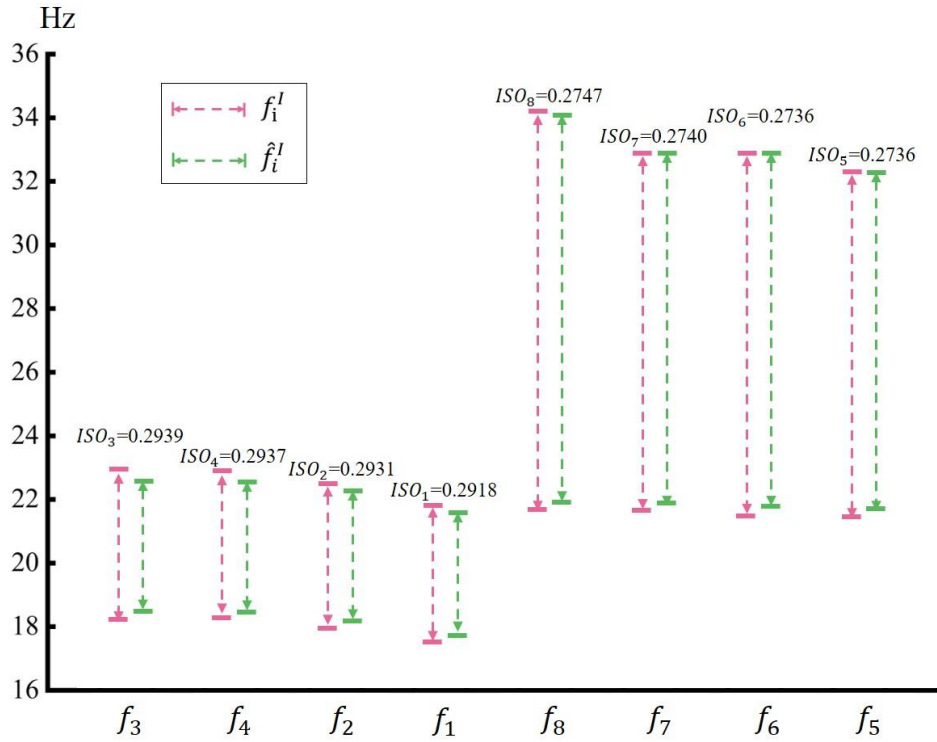
Since the proposed method is a local sensitivity analysis method, we can determine the sensitivity of each output feature of  $f_1^I$  to  $f_8^I$  to each parameter based on the sensitivity index with *ISO*. For example, Fig. 21 presents the initial and perturbed intervals of  $f_1^I$  to  $f_8^I$  concerning the most sensitive parameter  $\theta_4^I$ . We can observe that the variations between the  $f_8^I$  and  $\hat{f}_8^I$  caused by the changes of  $\theta_4^I$  are apparent, and the value of *ISO* between  $f_8^I$  and  $\hat{f}_8^I$  is 0.3568. Meanwhile, the most stable parameter of  $\theta_6^I$ , the changes in  $\theta_6^I$  lead to slight changes in outputs  $f_1^I$  to  $f_8^I$  as shown in Fig. 22. The proposed method not only can order the sensitivity of interval parameters but also can measure the changes of each interval output features.





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Fig. 21 The variation of  $f_1^I$  to  $f_8^I$  results from the changes of  $\theta_4^I$



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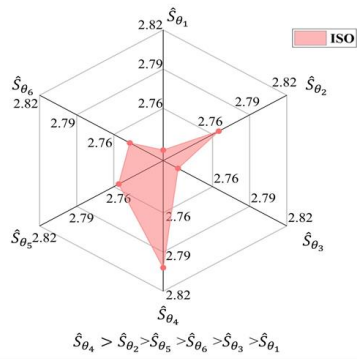
Fig. 22 The variation of  $f_1^I$  to  $f_8^I$  results from the changes of  $\theta_6^I$

5 In order to examine the potential impact of variation proportion ratio of parameter interval  
6 radius on the sensitivity of parameters, the sensitivity ranks for  $\theta_1^I$  to  $\theta_6^I$  corresponding to  
7 different variation of the parameter interval radius are shown in Table 12 and Fig. 23.

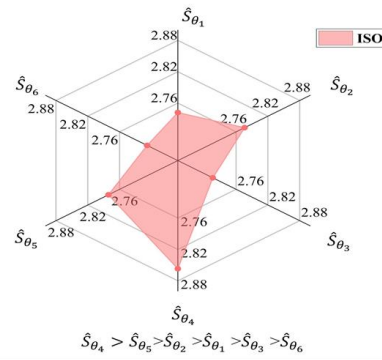
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Table 12 TSensitivity ranks according to different variation proportion ratio of interval radius.

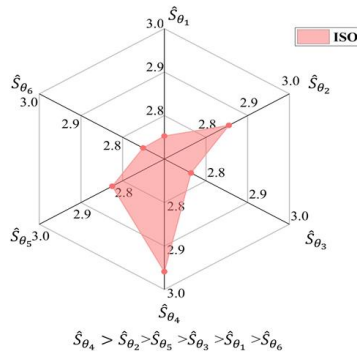
Variation proportion ratio of interval radius	Sensitivity ranks of $\theta_1^I - \theta_6^I$
10%	$\hat{S}_{\theta_4} > \hat{S}_{\theta_2} > \hat{S}_{\theta_5} > \hat{S}_{\theta_6} > \hat{S}_{\theta_3} > \hat{S}_{\theta_1}$
20%	$\hat{S}_{\theta_4} > \hat{S}_{\theta_5} > \hat{S}_{\theta_2} > \hat{S}_{\theta_1} > \hat{S}_{\theta_3} > \hat{S}_{\theta_6}$
30%	$\hat{S}_{\theta_4} > \hat{S}_{\theta_2} > \hat{S}_{\theta_5} > \hat{S}_{\theta_3} > \hat{S}_{\theta_1} > \hat{S}_{\theta_6}$
40%	$\hat{S}_{\theta_4} > \hat{S}_{\theta_5} > \hat{S}_{\theta_2} > \hat{S}_{\theta_6} > \hat{S}_{\theta_3} > \hat{S}_{\theta_1}$
50%	$\hat{S}_{\theta_4} > \hat{S}_{\theta_2} > \hat{S}_{\theta_5} > \hat{S}_{\theta_3} > \hat{S}_{\theta_1} > \hat{S}_{\theta_6}$
60%	$\hat{S}_{\theta_4} > \hat{S}_{\theta_5} > \hat{S}_{\theta_2} > \hat{S}_{\theta_3} > \hat{S}_{\theta_1} > \hat{S}_{\theta_6}$



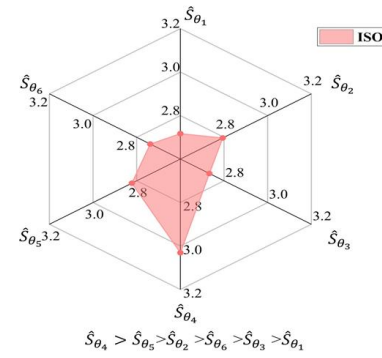
Variation proportion ratio=10%



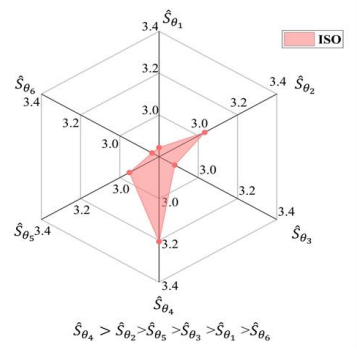
Variation proportion ratio=20%



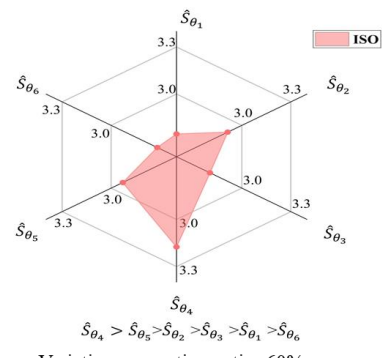
Variation proportion ratio=30%



Variation proportion ratio=40%



Variation proportion ratio=50%



Variation proportion ratio=60%

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Fig. 23 Sensitivity ranks of  $\theta_1^I - \theta_6^I$  corresponding to different variation proportion ratio of interval radius.

1 We can find that as the variation proportion ratio of the interval radius increases from 10%  
2 to 60%, the sensitivity ranks of parameters experience slight changes. Of notable significance  
3 is parameter  $\theta_4^I$ , which represents the thickness of the lower platform, as it consistently  
4 maintains the first order rank in influencing output features, regardless of the variation of the  
5 parameter interval radius. Conversely, uncertainties surrounding parameter  $\theta_6^I$ , which denotes  
6 the thickness of the upper platform, have minimal impact on output uncertainty and rank  
7 towards the end of the sensitivity order. Hence, it is recommended to pay more attention to the  
8 thickness of the lower platform, and it may be more beneficial to focus on adjusting and  
9 optimizing parameter  $\theta_4^I$  in order to achieve desired output results.

## 10 **7 Conclusion**

11 An exhaustive interval sensitivity analysis method based on the interval perturbation method  
12 and *interval similarity operator* is developed. In this interval sensitivity analysis framework,  
13 the *ISO* metric is adopted to quantify the discrepancy between two intervals based on the  
14 interval geometric position and the interval bounds without requiring inner interval samples.  
15 This metric can rank different sensitivity analysis frameworks, e.g. interval sensitivity analysis  
16 and sensitivity analysis with hybrid stochastic and interval uncertainties, which is illustrated by  
17 the tutorial case of the Ishigami function. The interval perturbation method is introduced for  
18 interval uncertainty propagation, which has the advantage of not requiring abundant FE  
19 simulation to estimate precise extreme bounds of interval outputs. It is a significant benefit for  
20 sensitivity analysis in the presence of practical engineering structures. The feasibility and  
21 effectiveness of this proposed interval sensitivity analysis algorithm are verified by two  
22 numerical examples of the classical Ishigami function and the satellite example. Further  
23 development for interval analysis includes the applications for nonlinearity systems, the  
24 consideration of the inner relationship between multi-dimensional outputs, and the hybrid  
25 stochastic and interval uncertainties propagation.

## 26 **Declaration of Competing Interest**

27 The authors declare that they have no known competing financial interests or personal  
28 relationships that could have appeared to influence the work reported in this paper.

29 **Replication of results:** All modeling parameters are given in the case, and the corresponding  
30 finite element model can be obtained according to the case modeling. The presented results are  
31 produced using our in-house code surrogate-based optimization and sensitivity analysis. The  
32 code and data for producing the presented results will be made available by request. The  
33 relevant codes for the algorithms could be available on request by emailing the first author. The  
34 authors wish to withhold the source code for commercialization purposes. This includes the  
35 finite strain elastoplastic analysis code implementing the finite strain elastoplastic analysis and

1 adjoint sensitivity analysis.

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