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# Overview of theoretical research in France on ultrafast processes in molecules

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## Abstract

Ultrafast processes initiated in molecules by light or collision are extremely widespread. The intrinsic timescale of nuclear motion is the femtosecond ( $1 \text{ fs} = 10^{-15} \text{ s}$ ) and the one of electrons, lighter particles, is the attosecond ( $1 \text{ as} = 10^{-18} \text{ s}$ ). Dynamics simulations is essential for understanding the mechanism, rate and yield of ultrafast processes. In this article, we review recent theoretical works, performed in France, to describe photon-induced or collision-induced ultrafast processes in molecules. In particular, we discuss recent studies on quantum dynamics of small molecules or Hamiltonian models, and works focused on “on-the-fly” mixed quantum-classical dynamics of molecules. Both state-of-the-art applications and method developments towards overcoming current bottlenecks are presented.

**Keywords:** Photochemistry, Non-adiabatic dynamics, Quantum dynamics, Mixed quantum-classical dynamics, On-the-fly dynamics

## 1 Introduction

Ones of the most fundamental and widespread processes in chemistry and physics are the absorption of light and the collision with atomic or molecular systems to excite electrons of matter and potentially induce a rearrangement of the nuclei [1]. The intrinsic timescale of nuclear motion is the femtosecond ( $1 \text{ fs} = 10^{-15} \text{ s}$ ) while the intrinsic timescale of the motion of electrons, lighter

particles, is the attosecond ( $1 \text{ as} = 10^{-18} \text{ s}$ ). As a result of excitation into an electronic excited state, the distribution of electrons and thus the reactivity of a molecule differ significantly from the ones in the ground state. We distinguish photochemistry - when the absorption of light causes a chemical reaction - from photophysics - otherwise. The most common photon-induced or collision-induced chemical reactions are isomerization, pericyclic reaction, dissociation and electron transfer. Although the applications of these processes cover today many fields, their practical use is limited by the quantum yield of the desired process, the latter almost always competing with other processes. A challenge for chemists is therefore to design more efficient molecular systems and optical control methods. The idea of understanding the behaviour of a molecule upon light absorption or molecular collision has thus motivated, and continues to motivate researchers in the field of physical chemistry.

From the late 1980s, the development of femtosecond lasers has allowed the experimental study of the structure and dynamics of atoms in molecules [2]. By using a light pulse that is shorter than the characteristic time for nuclear motion, it is possible to promote the system in a localised region on an excited electronic potential energy landscape. The subsequent nuclear motion could then be probed as a function of the time delay between pump and probe pulses. In 2001, the “femtosecond barrier” was broken with the first generation of attosecond pulses [3, 4]. This generation relies on the time-energy uncertainty principle:  $\Delta E \Delta t \geq \hbar$ . By collecting together coherent light sources with a large energy bandwidth  $\Delta E$  of several eV, one can obtain an ultrashort pulse with duration  $\tau \approx \hbar/\Delta E$  of few attoseconds. The emergence of attosecond measurement tools opened up the possibility to experimentally observe and control electrons, on their intrinsic timescale. Attosecond pulses were first applied to atoms [5] and then condensed-matter [6] and polyatomic molecules [7], the latter giving birth to the field of “attochemistry”. One could finally investigate changes in the electronic distribution during a chemical reaction, until now considered as instantaneous.

From a theoretical perspective, in order to understand the behaviour of a molecule upon photon- or collision-induced excitation, one needs to evaluate the importance of the different relaxation pathways. Only a dynamic strategy, which simulates the evolution of the molecular wavepacket over time, provides a complete and intuitive image of the electronic states and nuclear structures visited by the system after excitation, and of the different relaxation pathways. Describing the time evolution of an electronically-excited molecule implies going beyond the Born-Oppenheimer approximation [8]. Huge advances have been made and different flavours of non-adiabatic dynamics methods have been developed over the last decades to model, from first principles, the chemical properties and dynamics of molecules in their electronic excited states [9, 10].

The scope of the present article is to review and illustrate the recent advances made in France in theoretical chemistry and physics, to describe ultrafast processes in molecules, both in terms of applications and method developments. We chose to limit ourselves to several French theoretical

studies published in the last few years, investigating the ultrafast coupled electron-nuclear dynamics in molecules (two atoms or more). In section 2, we present recent studies of quantum dynamics on small molecules or Hamiltonian models. In section 3, works focused on state-of-art “on-the-fly” mixed quantum-classical dynamics are reported. Section 4 concludes this article with a summary of the recent advances and some perspectives for the future of this field.

## 2 Quantum dynamics of small molecules and model systems

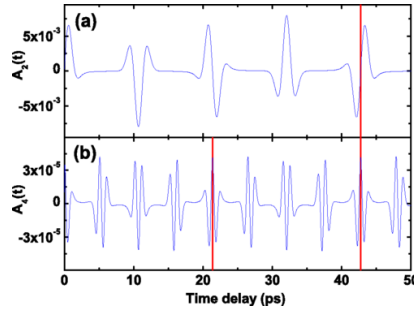
The most accurate quantum nuclear dynamics methods are either grid-based simulations or using the MCTDH (Multi-Configuration Time-Dependent Hartree) method [11]. These imply a fully quantum treatment of the molecular dynamics. As will be seen from the cited examples, grid-based quantum dynamics simulations are often limited to few nuclear coordinates. Using the MCTDH method and extensions of it, one can afford more nuclear coordinates. In the following, we first detail several state-of-the-art applications of such methods. We then present works focused on developments to improve the practical use of these methods.

### 2.1 From applications...

Using grid-based calculations with the resolvent-operator method [12], Catoire *et al.* studied the dynamics of the  $\text{He}_2^{2+}$  diatomic ion induced by a strong IR laser pulse [13]. They investigated in particular the role of electronic excited states in the dissociation dynamics. Cornaggia and co-workers studied nuclear dynamics of  $\text{H}_2$  laser-induced double ionization using one-dimensional grid-based simulations [14].

A typical illustration of theoretical work of nuclear quantum dynamics by the same group is the study of molecular reorientation during photo-ionization using a two-dimensional model [15, 16]. Alignment of linear molecules induced by light leads to time-dependent observables. The latter can be analysed in terms of spectral periodic components  $A_{2l}$ , with  $l$  being a positive integer. Using available calculated ionization yields of fixed-in-space molecules of  $\text{CO}_2$  – a symmetric linear molecule, they performed numerical calculations of laser-induced impulsive alignment and tunnel ionization. Figure 1 shows, for  $l = 1$  and  $l = 2$ , four fractional revivals within each rotational period  $\tau_{2l}$ . The revivals exhibit constant successive phase shifts that depend on the parity of the initial rotational states. This analysis provides an alternative explanation for the so-called high-order revivals.

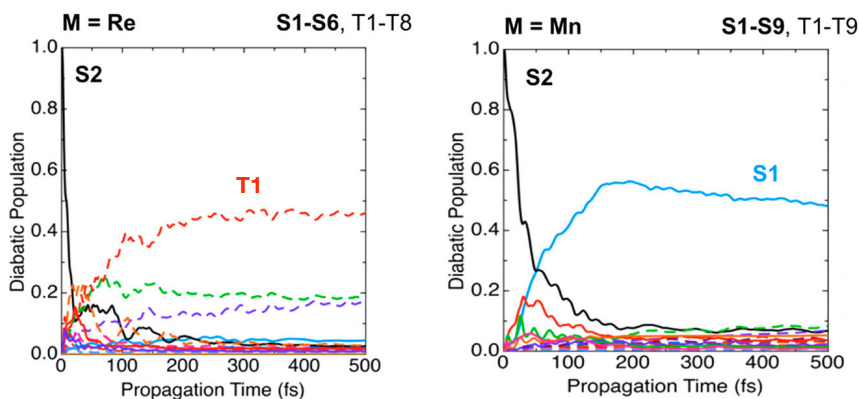
Using the MCTDH method, Lasorne, Lauvergnat and co-workers studied the ozone photolysis induced by deep ultraviolet radiation [17]. Upon excitation to the  $B$  state of ozone by a 10-fs pulse, the quantum dynamics simulations show an early splitting of the wavepacket at an inflection point of the potential energy surface between a path leading to dissociation and an oscillating



**Fig. 1**  $A_{2l}$  components of the ion signal  $\text{CO}_2^+$  as a function of time. (a):  $l = 1$  and (b):  $l = 2$ . For each  $l$  value, the period  $\tau_{2l}$  and its multiples are represented by vertical red lines. Reprinted figure with permission from [15]. Copyright (2015) by the American Physical Society.

path revolving around the Franck-Condon region at around 20 fs. As the non-dissociative part of the wavepacket oscillates between the Franck-Condon region and the inflection point, both revivals around the Franck-Condon region and dissociation occur periodically, approximately every 20 fs. The same collaboration also studied the ultrafast internal conversion in 4-aminobenzonitrile upon UV light absorption, from a charge-transfer state to a locally-excited state [18]. Using the multi-layer (ML) version of the MCTDH method, the Hamiltonian model constructed included three diabatic electronic states and all nuclear coordinates, i.e 39 dimensions. They showed that the relaxation occurs sequentially along an extended seam of conical intersections, with planar geometries dominating at early times ( $t < 20$  fs). Using two singlet, 4 triplet electronic states and 63 vibrational modes, Huix-Rotllant and co-workers studied the ultrafast intersystem crossing in xanthone [19]. Their ML-MCTDH quantum dynamics simulations provided a sequential mechanism, consistent with the El Sayed's rule for intersystem crossing: from the second singlet excited state, (i) an internal conversion between singlets  $^1\pi\pi^* \rightarrow ^1n\pi^*$  occurs on a timescale of 85 fs, (ii) an intersystem crossing  $^1n\pi^* \rightarrow ^3\pi\pi^*$  on a 2.0 ps timescale, and (iii) an internal conversion between triplets  $^3\pi\pi^* \rightarrow ^3n\pi^*$  on a timescale of 602 fs. The unusually fast transition was explained by the absence of barriers in the paths to the conical intersections and the proximity of the latter with the minimum energy structures. With a similar approach (but in reduced dimensionality), the same group simulated, for a heme-CO complex, the photolysis and spin-crosser reactions happening concurrently on a femtosecond timescale [20]. Using the MCTDH method as well, Gatti and co-workers simulated the collision reactions between an atom and small molecules [21–23]. Using a two-level spin-boson Hamiltonian model, the same group studied the role of initial bath conditions in the photoinduced electron transfer and electronic coherence in plant cryptochromes [24]. A spin-boson model has also been used by Mangaud *et al.* with the hierarchical equations of motion method, to describe the time evolution of an open quantum system strongly coupled to a bath [25].

Another typical example of MCTDH quantum dynamics study is the work on transition metal carbonyl  $\alpha$ -diimine complexes by Fumanal *et al.* [26]. Using a reduced-dimensional Hamiltonian model (15 normal modes) of  $[\text{M}(\text{imidazole})(\text{CO})_3(\text{phenanthroline})]^+$  ( $\text{M} = \text{Mn}, \text{Re}$ ), they studied the competition between luminescence, electron transfer, and photoinduced CO release, with a focus on the role of the metal atom on the branching ratio between these paths. Figure 2 shows the diabatic populations of  $[\text{Re}(\text{imidazole})(\text{CO})_3(\text{phenanthroline})]^+$  (left) and  $[\text{Mn}(\text{imidazole})(\text{CO})_3(\text{phenanthroline})]^+$  (right). In the rhenium complex, the intersystem crossing occurs readily in the S2 state to the long-lived lowest triplet state T1 within 100 fs while, in contradiction with the Kasha's rule, the lowest singlet state S1 is not populated. In contrast, in the manganese complex, the lowest S1 state is efficiently populated within 50 fs from S2, and the low-lying triplet states remain marginally populated because of modest spin-orbit coupling strength. In other works, the same group studied in detail the CO release pathway [27] and the effects of ligand substitution and conformers [28]. For a review on ultrafast processes in transition metal complexes, the reader is referred to the review by Daniel [29].



**Fig. 2** Time evolution of the diabatic electronic populations of (left)  $[\text{Re}(\text{imidazole})(\text{CO})_3(\text{phenanthroline})]^+$  and (right)  $[\text{Mn}(\text{imidazole})(\text{CO})_3(\text{phenanthroline})]^+$ . Adapted with permission from [26]. Copyright 2018 American Chemical Society.

## 2.2 ...to method developments

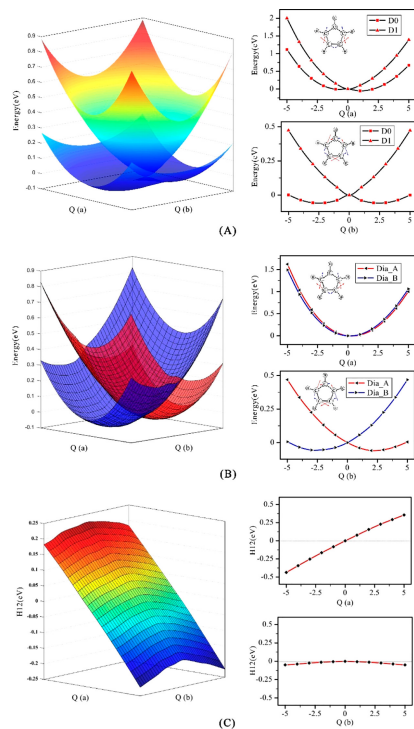
A key ingredient to quantum dynamics simulations is the choice of system of nuclear coordinates. Physically well-adapted curvilinear coordinates have the advantage to lead to a Hamiltonian operator as separable as possible and thus simplifies the resolution of the time-dependent Schrödinger equation. The corresponding curvilinear expression of the kinetic energy operator can be used either analytically or numerically, the memory requirement for the latter

approach becoming often extremely demanding. Nauts *et al.* develop an on-the-fly algorithm to reduce the memory storage bottleneck [30]. They tested it recently on a three-dimensional model of the *cis-trans* photoisomerisation of part of the retinal chromophore [31].

The MCTDH method involves the development of a model Hamiltonian often based on harmonic potentials and, typically, a reduced number of nuclear coordinates. The development of a model Hamiltonian implies a heavy work of calculating the potential energy surfaces and relevant couplings on a grid and fitting them, prior to any dynamics simulations. Due to computational cost, the coupling constants are sometimes approximated from excited-state energies. Fumanal *et al.* developed a protocol to calculate more accurate interstate vibronic coupling constants at the time-dependent density functional theory level through the overlap integrals between excited-state adiabatic auxiliary wavefunctions [32]. To address the same problem, Yalouz *et al.* work on a variational quantum eigensolver to estimate analytically non-adiabatic couplings between electronic states (and gradients for excited states) [33].

Another important point is that electronic structure quantities needed to construct the model are calculated in the adiabatic basis, while the quantum dynamics is simulated in a (quasi-)diabatic basis. The diabatisation (procedure to convert the adiabatic data into diabatic ones) is not straightforward. Lasorne and co-workers develop strategies to perform this step. They for instance showed how a pseudofragmentation scheme can be used to define relevant diabatic representations, and illustrated it with small oligomers [34]. They also develop a “black-box” valence-bond-based automatic diabaticization method [35]. The idea is to first reduce the full Hamiltonian to a low rank matrix that preserves the target eigensubspace. This is followed by a sequence of Householder transformations. The diabaticity criterion is implemented in a way that maximizes the diversity of valence bond structure weights between different diabatic states. They also proposed a pointwise phase correction scheme [36]. They illustrated their approach on several prototypical examples. For instance, Figure 3 shows a two-dimensional description of the cyclopentadienyl radical Jahn-Teller conical intersection spanned by 2-fold degenerate displacements Q(a) and Q(b). Figure 3A displays the adiabatic potential energy surfaces of the ground and first excited states, with a typical Mexican hat shape. Figures 3B and C display the diabatic potential energy surfaces and the phase-corrected diabatic coupling, respectively: the developed approach produces a smooth crossing, with no cusp in the diabatic coupling. Along Q(a), the two diabatic states appear degenerate and the diabatic coupling is a nonzero quasi-linear function; along Q(b), the diabatic potentials are close to the adiabatic ones and the diabatic coupling is near zero.

To overcome the bottleneck of constructing and fitting the model before simulating the dynamics, Panadés-Barrueta *et al.* develop an automated methodology for the generation of global potential energy surfaces and molecular properties surfaces [37]. Their approach includes, in particular, a tensor decomposition algorithm to turn an automatically generated surface into sum



**Fig. 3** Potential energy surfaces of the cyclopentadienyl radical around the conical intersection along Q(a) and Q(b) distortions: (A) Adiabatic energies. (B) Diabatic diagonal energies (H11 and H22). (C) Phase-corrected diabatic coupling (H12). Reproduced with permission from [36]. Copyright 2021 American Chemical Society.

of products (Tucker) form, as required in the MCTDH method. They tested their methodology on the *cis-trans* isomerisation reactions in HONO in full dimensionality (6 dimensions).

### 3 “On-the-fly” mixed quantum-classical dynamics of molecules

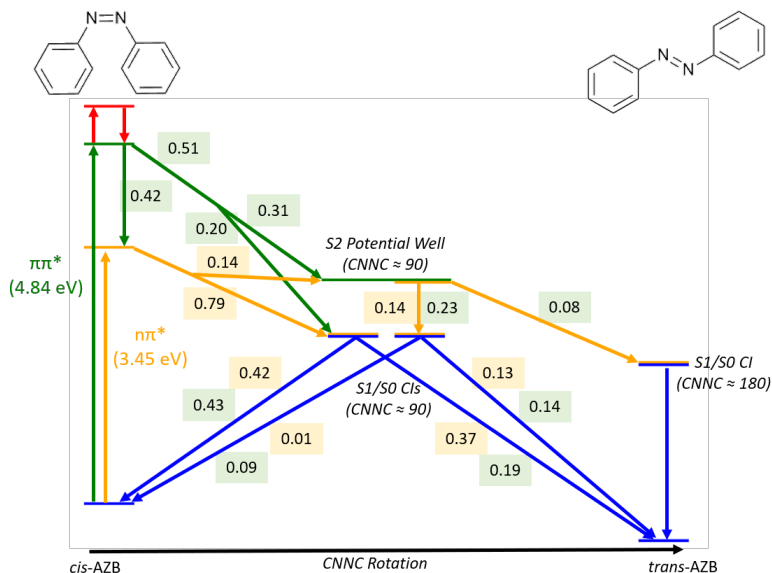
To address the different issues of quantum dynamics, “on-the-fly” dynamics methods have been developed. In these, the potential energy surfaces are calculated as needed along nuclear trajectories, the latter serving as a basis to describe the nuclear wavepacket motion. The nuclear trajectories are most of time moving classically in all nuclear dimensions, according to Newton’s equations of motion. Similarly to the previous section, we first detail several state-of-the-art applications of such mixed quantum-classical dynamics methods. We then present works focused on the impact of the approximations of these methods and on developments to improve their accuracy.



### 3.1 From applications...

The most popular on-the-fly mixed quantum-classical method is surface hopping: it uses an ensemble of independent classical trajectories, and a quantum propagation of the electronic wavepacket inducing stochastically jumps between electronic states. To illustrate such a method, one can present the work of Vacher and co-workers on the *cis*-to-*trans* photoisomerisation of azobenzene [38]. One of the most interesting properties of azobenzene is the dependence of the measured quantum yield on the wavelength of the incident light used to trigger the photo-isomerisation. While there have been numerous works focusing on the *trans*-to-*cis* isomerisation of azobenzene, the *cis*-to-*trans* photoisomerisation has been much less investigated. Their surface hopping simulations of the dynamics in full dimensionality (33 dimensions) provided the quantum yields of *cis*-to-*trans* isomerisation upon excitation to the  $n\pi^*$  ( $S_1$ ) and  $\pi\pi^*$  ( $S_2$ ) states, which occur on the timescale of several hundred of fs. They reproduced the decrease in quantum yield of 0.10 measured recently when exciting to the higher energy state and suggested a mechanism for it, different from the one accepted for the *trans*-to-*cis* direction. Figure 4 summarises the major relaxation pathways accessible to *cis*-azobenzene, and the fraction of trajectories which follow each pathway in both excitation cases. Two factors explain the quantum yield reduction: (i) a potential well that, after  $\pi\pi^*$  excitation, traps photochromes which under  $n\pi^*$  excitation would have exclusively formed *trans*-azobenzene and (ii) photochromes in this well excited initially to the  $\pi\pi^*$  state are more likely to reform *cis*-azobenzene upon leaving the well than those excited to the  $n\pi^*$  state.

Vacher and co-workers also simulated with the surface hopping method the competition between several relaxation pathways in furanone and derivatives [39]: ring-puckering spreads the excited state wavepacket quickly enough to influence the outcome of an otherwise expectedly direct ring-opening reaction in less than 50 fs. The same group also simulated the intersystem crossing that occurs within 1 ps in thiopyridone [40], and studied the role of conical intersection topography in the non-adiabatic transition probability [41]. Using the surface hopping method as well, Huix-Rotllant and co-workers simulated the behaviour of photoexcited thymine [42]: their dynamics explained the unusual long excited-state lifetime by an ultrafast ( $\approx 30$  fs)  $S_2 \rightarrow S_1$  internal conversion, followed by a trapping in  $S_1$  state from which the decay to the ground state is slow ( $\approx 6.1$  ps). The same group simulated the ultrafast internal conversion on porphine [43]: the dominant mechanism demonstrated by the dynamics is sequential involving a set of dark states populated within  $\approx 20$  fs from the initially excited B band, and which then populate the Q band within  $\approx 100$  fs. Barbatti and co-workers studied the photodynamics of coumaryl Meldrum and sinapoyl Meldrum [44, 45]. The fast ( $<1$  ps) and efficient non-radiative decay to repopulate the electronic ground state upon UV absorption makes these molecules good candidates for sunscreen applications. The same group also simulated the dynamics of benzene excimer formation in 2 ps after  $S_1$  excitation [46], the photoinduced formation of hydrogen-bonded ion pair in



**Fig. 4** Major pathways, and the fraction of trajectories which follow each pathway, accessible to *cis*-azobenzene upon excitation to the  $n\pi^*$  (numbers in yellow) and  $\pi\pi^*$  (numbers in green) states. Reproduced from [38] with permission from the PCCP Owner Societies.

hydrochlorofluorocarbon-133a in the sub-ps timescale [47], and the photoisomerization of a macrocyclic cyclobiazobenzene molecule in less than 100 fs [48]. Monari and co-workers studied and compared the intersystem crossing rate for formyluracil and formylcytosine, i.e. oxidative lesions and epigenetic intermediates possibly acting as intrinsic DNA photosensitizers [49]. The same group investigated the importance of hydrogen transfer in photoexcited curcumin which happens in the sub-ps regime and compete with the intersystem crossing channel [50], and the photophysics of phenanthrene [51].

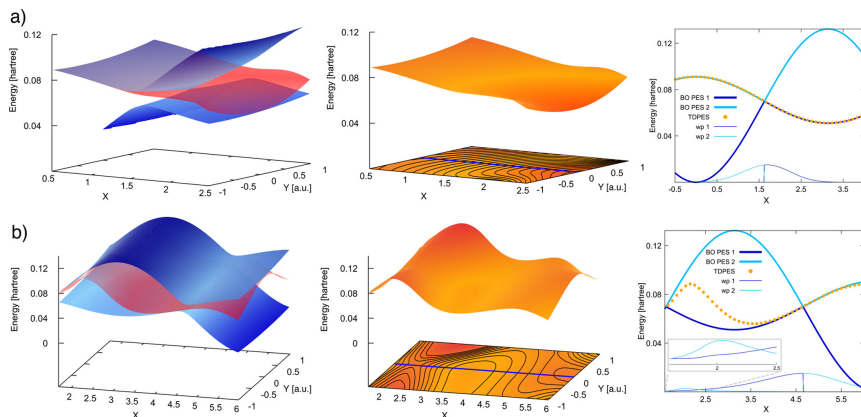
### 3.2 ...to method developments

A key ingredient to the mixed quantum-classical dynamics simulations are the initial conditions. Barbatti and co-workers worked on their definition and impact on the subsequent dynamics, in particular trying to take into account the effect of temperature [52], or when a reaction is initiated by thermal light [53]. Another important choice in the surface hopping method is how to adjust the velocity after a jump between electronic states in order to conserve total energy. The several options are for instance the nonadiabatic coupling vector, the momentum, and the energy gradient difference. Barbatti studied the impact of this choice on time constants and structural evolution of photoexcited ethylene, and the comparison did not show any significant differences [54]. The same group extended the surface hopping method to describe irreversible decay mechanism using an imaginary potential [55]. They employed to survey the relaxation mechanisms of the shape resonant anions of iodoethene.

The main bottleneck of such mixed quantum-classical dynamics simulations remain the electronic structure calculations. Barbatti and co-workers develop machine-learned potentials to be used in dynamics [56] to drastically reduce the cost of the simulations and allow to reach longer timescales [57]. The aim is to generate accurate machine-learned models of potential energy surfaces with no more than 10,000 points while conventional on-the-fly mixed quantum-classical simulations would typically require 100 trajectories with a time step of 0.5 fs for 1 ps, i.e. 200,000 electronic structure calculations.

The limits of these on-the-fly mixed quantum-classical methods like surface hopping are the classical treatment of the motion of nuclei and the non-exact description of electronic coherence. To address these points, the strategy is to couple the trajectories to recover some “quantumness”. Along those lines, Agostini and co-workers develop a coupled-trajectory mixed quantum-classical algorithm, based on the exact factorization equations [58]. In the latter approach, instead of using the Born-Huang expansion, the total molecular wavefunction is exactly expressed as a single product of an electronic wavefunction and a nuclear wavefunction [59]. There, the electronic wavefunction is of course not an eigenfunction (nor a superposition of eigenfunctions) of the traditional Born–Oppenheimer electronic Hamiltonian, but that of a somewhat more involved electronic Hamiltonian which depends on the nuclear wavefunction itself. An important feature of the exact factorization formalism is that dynamics occurs on a time-dependent potential energy surface, as in the Ehrenfest method. But, in contrast to Ehrenfest, an additional “coupled-trajectory” term appears in both the electronic and nuclear equations [60]. To illustrate the fundamental difference between Born-Huang and exact factorization formalisms, in particular in the vicinity of a conical intersection, dynamics was simulated on a two-state two-dimensional model [61]. Initial conditions were chosen so that once promoted from state 1 to state 2, the molecular wavepacket rapidly approaches a first conical intersection region ( $t = 295$  au), and almost fully transfers back to state 1 non-radiatively with a small portion remaining in state 2 after passing through the conical intersection ( $t = 675$  au). The molecular wavepacket enters then a second coupling region. Figure 5a compares the adiabatic and exact factorization representations of the dynamics at  $t = 295$  au. The conical shape formed by the adiabatic surfaces is absent from the time-dependent potential energy surface. The latter shows a smooth character bridging the two adiabatic surfaces diabatically, as confirmed by the initially pure diabatic behaviour of the molecular wavepacket. However, at  $t = 675$  au (Figure 5b), the time-dependent potential energy surface continues to follow the adiabatic surfaces diabatically at large  $X$  on either side of the second conical intersection but it also exhibits additional features at smaller  $X$ . This is a signature of part of the molecular wavepacket that has remained in state 2 after the first conical intersection. In other words, the splitting of the molecular wavepacket enforces a step in the time-dependent potential energy surface. The exact factorization method has recently been extended to describe for instance spin-forbidden intersystem crossings [62, 63], quantum nuclear

effects [64], mass and inertia effects of all molecular particles (i.e. electrons as well) [65], and electron dynamics in the Floquet formalism [66].



**Fig. 5** At two different times of the simulation (a)  $t = 295$  au and (b)  $t = 675$  au: comparison of the adiabatic potential energy surfaces (left and right, in blue) with the time-dependent one (left in red, middle and right in orange) and (right) nuclear densities in states 1 and 2 along a cut in the X direction. Reproduced with permission from [61]. Copyright 2017 American Chemical Society.

Another promising on-the-fly method with quantum accuracy is the direct-dynamics variational multi-configurational Gaussian (DD-vMCG) method, based on the propagation of variationally coupled Gaussian wavepackets following quantum trajectories [67]. Vacher and co-workers applied it to simulate the very early nuclear dynamics induced in benzene induced upon tunnel ionization and measured using high harmonic spectroscopy [68].

## 4 Summary and perspectives

In this article, we reviewed and illustrated several recent studies, performed in France, dedicated to the theoretical description of ultrafast processes in molecules induced by light or collision, both in terms of state-of-the-art applications and method development works.

We first highlighted studies simulating quantum dynamics of systems ranging from diatomic molecules using grid-based methods, to transition metal complexes using the MCTDH method and Hamiltonian models. We then presented works focused on developments to improve the practical use of fully quantum dynamics methods, including for instance on-the-fly algorithms to overcome memory storage bottleneck, diabatisation strategies, automated methodologies to generate global potential energy surfaces.

We then turned our attention toward studies focusing on on-the-fly mixed quantum-classical dynamics of molecules. In particular, we illustrated applications of the popular surface hopping method on several medium-sized organic

molecules. Moreover, we presented works focused on the impact of the approximations intrinsic to these methods (initial conditions, velocity rescaling, etc.) and we highlighted the recent efforts made to go beyond the classical treatment of the nuclear motion in on-the-fly dynamics methods.

A common bottleneck of all dynamics methods is the electronic structure calculations, either for the generation and fitting of global potential energy surfaces for quantum dynamics or in an on-the-fly manner for direct dynamics methods. Not only potential energy but also quantities such as energy derivatives and non-adiabatic couplings are required. To overcome this bottleneck, novel technologies such as machine learning [37, 56, 57] or quantum computing [33] are being developed and could be adapted to the theoretical description of ultrafast processes in molecules. These would really revolutionize the application of the current state-of-the-art dynamics methods. This would allow describing more complex and realistic systems, treating for instance the environment of molecules in solvent or in biological medium. This would also allow the simulation of dynamics over longer times bridging the processes happening on ultrafast timescales to those happening on longer ones. All of this will help us pushing the current limits of the fundamental understanding about the behaviour of molecules upon light absorption.

## Data Availability Statement

No data associated in the manuscript.

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