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► **To cite this version:**

Victoria Orlando, Enrique Baquela, Neila Bhourri, Pablo Lotito. Analytical and simulation-based estimation of public transport demand. *Transportation Research Procedia*, 2022, 62, pp.727-734. 10.1016/j.trpro.2022.02.090 . hal-04273444

HAL Id: hal-04273444

<https://hal.science/hal-04273444v1>

Submitted on 7 Nov 2023

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Analytical and simulation-based estimation of public transport demand

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Abstract

We consider the problem of public transport demand estimation. For the practical use of many models of transit assignment in any planning study, the knowledge of the transport demand systematized as an origin-destination matrix is required. Generally, this matrix is very expensive and difficult to obtain. In [Bhourid et al. \(2021\)](#) we proposed to estimate the OD-matrix through the numerical solution of a bilevel optimization problem. One disadvantage of this formulation is the difficulty of obtaining descent directions; therefore, we proposed a derivative-free method for the resolution of the optimization problem. Our goal in this work is to extend the study to larger networks, combining analytical modeling and simulation-optimization methodology. We use both approaches to perform the demand estimation over a section of a real network, and compare the obtained results for different scenarios and parameters.

Keywords: Public transport demand; Transit assignment; Bi-level optimization; Simulation; Equilibrium model

1. Introduction

Given an origin-destination matrix representing the public transport demand, the distribution of flow among different lines can be obtained, assuming that it corresponds to a certain equilibrium characterized by an optimization problem. There are numerous works concerning this area [Spiess and Florian \(1989\)](#); [De Cea and Fernández \(1993\)](#); [Cominetti and Correa \(2001\)](#); [Cepeda et al. \(2006\)](#), each of them assuming different passenger behavior. Some authors consider that passengers select the best route to make their trip with the objective of minimizing their total expected travel time, which includes access, wait, and in-vehicle time. Others consider that passengers seek to minimize their

generalized cost, which can include not only the total travel time but also in-vehicle crowding and fares, among others. Another aspect in which transit assignment models differ is in how they handle the frequencies and in-vehicle travel times. Some works address the uncongested case considering times and frequencies independent of the passenger flow, others propose the semi congested case where only frequencies are fixed, and finally, in some cases, fixed travel times but frequencies dependent on the flow are considered.

Regardless of the aforementioned approaches, transit assignment models have something in common and is that all of them assume that the public transport demand is known, something that could be very expensive and even impossible to obtain in practice. Based on this, it is interesting to study how this demand can be estimated. Some works address this problem using statistical or econometric considerations [Ortuzar and Willumsen \(2001\)](#); [García-Ferrer et al. \(2006\)](#); [Cascetta \(2009\)](#); [Dike et al. \(2018\)](#). Others use the advantages of Information and Communication Technologies (ICT) to perform the OD-matrix estimation [Barcelo et al. \(2012\)](#); [Montero et al. \(2012\)](#). In [Bhouri et al. \(2021\)](#) we explore the estimation through the numerical solution of a bilevel optimization problem, using some directly measurable quantities like the real frequencies of the buses. The disadvantage of the proposed formulation is the difficulty of obtaining descent directions. Therefore we proposed a derivative-free method for the resolution of the optimization problem. As can be seen in [Bhouri et al. \(2021\)](#) the method was tested on small networks getting good results.

The use of a derivative-free method allowed us to model traffic flows using agent-based simulations. This work is intended to continue the task started previously, combining analytical modeling and a simulation-based approach to solve the estimation problem over bigger (and real) networks. Particularly we study a section of the bus network of Tandil (Buenos Aires, Argentina). Real frequencies and waiting times are sampled, and a transit simulation is built, in order to reproduce the transit system behavior accurately. Analytical and simulation-based approaches are performed, supposing different parameters and scenarios and comparing the results.

The paper is organized as follows: in Section 2 we describe the chosen assignment model; in Section 3 we present the demand estimation problem; Section 4 contains the numerical experiments and Section 5 presents the conclusions.

2. Traffic equilibrium model

We consider a directed graph $G = (N, A)$ where N is the set of nodes and A the set of arcs. The set N contains the bus-stop nodes N_s and the line nodes N_l . The arc set contains the alighting and boarding arcs connecting the bus-stop nodes with the line nodes and the on-board arcs connecting line nodes. For each node $i \in N$ we call A_i^- the set of incoming arcs, and A_i^+ the set of outgoing arcs. Finally, g_i^d represents the transport demand for some origin-destination pair $(i, d) \subset N \times N$ and D is the set of all nodes d that are destinations of some origin-destination pair.

Before tackling the demand estimation problem, we need to choose an assignment model that represents the dynamic of passengers. Between all works concerning this area, we choose the traffic assignment model proposed by [Cepeda et al. \(2006\)](#). This model is characterized by considering constant in-vehicle travel times, but frequencies dependent of the flow. For this purpose they define an effective frequency $f_a : \mathcal{V} \rightarrow [0, \infty]$ associated with each arc $a \in A$ which is strictly decreasing, so when the flow through arc a grows, $f_a(v) \rightarrow 0$, reflecting the congestion effect on the vehicles.

The idea behind this assignment model is that passengers seek to minimize their total travel time and for this, at each node they choose an arc to continue their trip. In this way, a Common Line Problem must be solved at each node $i \in N$: passengers select a nonempty subset $s \subseteq A_i^+$ and board the first vehicle that arrives at the stop and belongs to this set, in such a way that the chosen strategy minimizes their total expected travel time. Considering $V(g)$ the set of feasible flows for the demand g , composed by all flows $v_a^d \geq 0$ such that $v_a^d = 0$ for all $a \in A_i^+$ and satisfying the flow conservation constraints:

$$g_i^d + \sum_{a \in A_i^-} v_a^d = \sum_{a \in A_i^+} v_a^d, \quad \forall i \neq d. \quad (1)$$

is shown in [Cepeda et al. \(2006\)](#) that the equilibrium flow $v \in V^*(g)$ is the global minimizer of the following gap function, whose optimal value is 0:

$$G(v, g) = \sum_{d \in D} \left[\sum_{a \in A} t_a v_a^d + \sum_{i \neq d} \max_{a \in A_i^+} \frac{v_a^d}{f_a(v)} - g_i^d \tau_i^d(v) \right] \quad (2)$$

where t_a is the travel time through the arc a , v_a^d is the on-board flow with destination d right after the stop, v_a^d is the flow boarding at the stop with destination d and $\tau_i^d(v)$ is the total expected travel time from i to d .

3. Demand estimation problem

Following the assignment model previously mentioned, if demand were known, flows and frequencies for each lines could be computed. Based on this we proposed an inverse problem whose solution is an estimation of the actual demand. We were focus on correcting the given demand, that is, given a nominal demand (that may correspond, for example, to previous measurements or to the historical demand) and observed (measured) frequencies, we look for the demand that satisfy the observed frequencies. In other words, we look for the demand g that minimizes:

$$\min_{g, v} \sum_{a \in A_{obs}} \left(\frac{\bar{f}_a - f_a}{\bar{f}_a} \right)^2 + \gamma \sum_{(i, d)} \left(\frac{\bar{g}_i^d - g_i^d}{\bar{g}_i^d} \right)^2 \quad (3)$$

s.t.

$$v \in V(g), \quad (4)$$

$$G(v, g) = 0. \quad (5)$$

where \bar{f} represents the observed frequencies, \bar{g} is the nominal demand and $G(v, g)$ is the gap function described above.

The γ constant is a regularization parameter whose value can be freely chosen. However it should be noted that while a large value of γ has a beneficial effect on the convexity of the problem and on the uniqueness of its solution, large values of γ make the problem ignore the observations (in this case, the measured frequencies). This can be seen in [Figure 1](#) where they are shown the distances between nominal and estimated demand (blue line) and observed and obtained frequencies (orange line) for different estimations using different values of γ . It can be seen how as γ grows,

the distance $d(\bar{g}_i^d, g_i^d) = \sqrt{\sum_{(i, d)} \left(\frac{\bar{g}_i^d - g_i^d}{\bar{g}_i^d} \right)^2}$ decreases, but $d(\bar{f}_a, f_a) = \sqrt{\sum_{a \in A_{obs}} \left(\frac{\bar{f}_a - f_a}{\bar{f}_a} \right)^2}$ increases.

The disadvantage of the proposed formulation is the difficulty of obtaining descent directions, so we proposed a derivative-free method for the resolution of the optimization problem. In [Bhouri et al. \(2021\)](#) the method was tested on small networks getting good results. Now, since the use of a derivative-free method allowed us to model traffic flows using agent-based simulations, we propose to combine analytical modeling and a simulation-based approach to solve the estimation problem. This allows us to model larger and more complex scenarios than if we use only analytical modeling.

4. Numerical experiments

In order to continue the task started in [Bhouri et al. \(2021\)](#) we apply our method over a biggest (and real) network. We select a part of the bus network of Tandil (Buenos Aires, Argentina) and use analytical modeling and a simulation-based approach to solve the estimation problem. The bus network of Tandil is served by 6 bus lines connecting different areas of the city (see [Figure 2](#)). For a first study, we select a part of this network included in the downtown area of the city, considering 8 bus stops and 4 bus lines serving the area, each of them with a specific route (see [Figure 3](#)). The first representation of this sub-network can be seen in [Figure 4\(a\)](#), where boarding, alighting, and on-board arcs are represented. We consider boarding and alighting arcs with travel time equal to 0, while on-board arcs have associated

Table 1. Arcs travel times

Arc	Line	Travel time (min)	Arc	Line	Travel time (min)
$a = (1, 9)$	L_1	$t_a = 1.5$	$a = (1, 15)$	L_3	$t_a = 1.44$
$a = (9, 10)$	L_1	$t_a = 0.54$	$a = (15, 16)$	L_3	$t_a = 1.1$
$a = (10, 8)$	L_1	$t_a = 0.9$	$a = (16, 17)$	L_3	$t_a = 2.3$
$a = (1, 11)$	L_2	$t_a = 1.5$	$a = (17, 8)$	L_3	$t_a = 0.85$
$a = (11, 12)$	L_2	$t_a = 1.6$	$a = (2, 18)$	L_4	$t_a = 1.1$
$a = (12, 13)$	L_2	$t_a = 0.8$	$a = (18, 19)$	L_4	$t_a = 0.9$
$a = (13, 14)$	L_2	$t_a = 0.7$	$a = (19, 20)$	L_4	$t_a = 0.6$
$a = (14, 8)$	L_2	$t_a = 0.55$	$a = (20, 7)$	L_4	$t_a = 2.6$

Table 2. Bus lines data

Line	Route	Frequencies (veh/h)	Capacity (pas/bus)
L_1	$1 \rightarrow 4 \rightarrow 5 \rightarrow 8$	5	32
L_2	$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 6 \rightarrow 8$	8	32
L_3	$1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 8$	10	32
L_4	$2 \rightarrow 3 \rightarrow 4 \rightarrow 6 \rightarrow 7$	12	32

a positive travel time. Regarding the frequencies, on-board and alighting arcs are considered with infinite frequency, while boarding arcs have a finite frequency that depends on the nominal frequency of the line and the flow on the bus. Following the idea of Spiess and Florian (1989), if a node i have only one incoming link $a_1 \in A_i^-$ and one outgoing link $a_2 \in A_i^+$ with travel times t_{a_1} and t_{a_2} and frequencies f_{a_1} and f_{a_2} respectively, they can be replaced with a single link a with $t_a = t_{a_1} + t_{a_2}$ and $f_a = f_{a_1}$. The network obtained after these considerations can be seen in Figure 4(b).

4.1. Optimization approach

We consider the network shown in Figure 4(b) with the data service present in Table 1. Skipping boarding and alighting arcs, we can represent the network as in Figure 5. Taking this into account, we can describe the route of each bus line as shown in Table 2. Considering these data, we solve the estimation problem 3 - 5 for different cases.

We perform the first estimation considering effective frequencies that we obtained previously assigning demands $g_1^5 = 100$, $g_1^8 = 150$ and $g_2^8 = 200$. To solve the estimation problem we used the Nelder-Mead method (see Lagarias et al. (1998)), included in Python through the command `optimize.fmin` (Python v. 3.8.5). We consider $\gamma = 1/50$ and nominal demands $\bar{g}_1^5 = 90$, $\bar{g}_1^8 = 130$ and $\bar{g}_2^8 = 225$. The precision of assignments performing during the estimation was $\varepsilon = 0.00001$. The obtained results are shown in Table 3, and indicates a very good estimate.

Thinking about the possibility of nominal demands are not precise enough (for example, if they correspond to very old data or are measured in a very different context) we performed the estimation using nominal demands very far

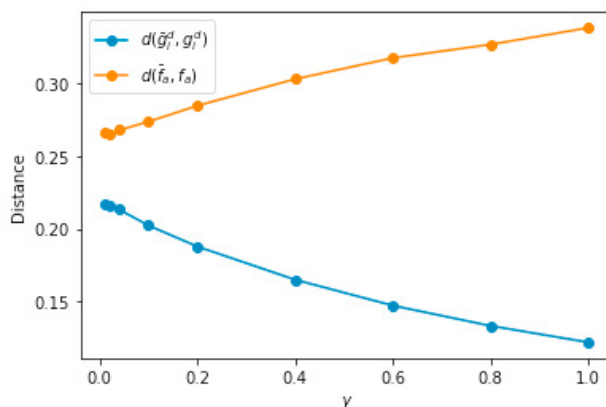


Fig. 1. Distances between nominal and estimated demand and observed and obtained frequencies

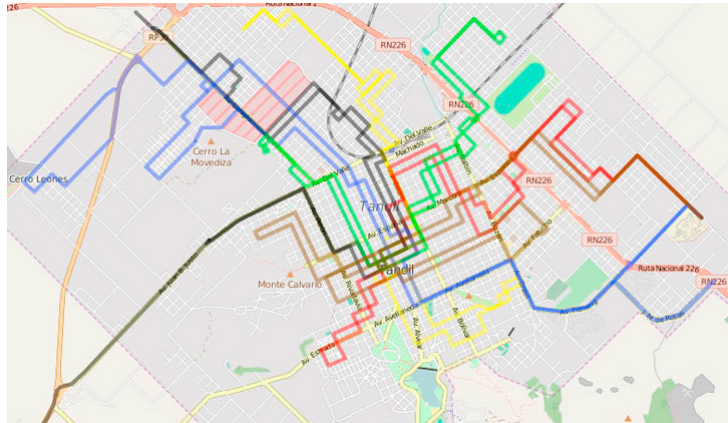


Fig. 2. Tandil bus network

from those we seek to estimate, like $\bar{g}_1^5 = 70$, $\bar{g}_1^8 = 110$ and $\bar{g}_2^8 = 170$. The estimation was performed for different values of γ , and the results are shown in Table 3. While all results could be considered acceptable, it can be seen how the estimate is better as γ decreases, which is logical considering that a lower value of γ indicates a greater consideration of the observed frequencies instead of the nominal demands.

We repeat the same examples reducing the precision value of assignments performing during the estimation. In Table 4 it can be seen how as ε value decreases, the computational time (measured in seconds) decreases too, while the estimated demands continue to be good in many cases.



Fig. 3. Routes of 4 lines serving the selected area

Table 3. Demand estimation results for optimization and simulation-optimization approach

Nominal demand	γ	Estimation by optimization	Simulation time	Estimation by simulation
$\bar{g}_1^5 = 90, \bar{g}_1^8 = 130, \bar{g}_2^8 = 225$	1/50	$g_1^5 = 99.46, g_1^8 = 149.97, g_2^8 = 200.17$	3548.14	$g_1^5 = 101.43, g_1^8 = 151.72, g_2^8 = 201.28$
$\bar{g}_1^5 = 70, \bar{g}_1^8 = 110, \bar{g}_2^8 = 170$	1/10	$g_1^5 = 89.22, g_1^8 = 152.99, g_2^8 = 197.95$	2394.80	$g_1^5 = 88.47, g_1^8 = 151.45, g_2^8 = 193.43$
	1/50	$g_1^5 = 93.83, g_1^8 = 154.32, g_2^8 = 199.92$	2190.05	$g_1^5 = 92.45, g_1^8 = 155.03, g_2^8 = 202.71$
	1/100	$g_1^5 = 95.78, g_1^8 = 153.15, g_2^8 = 199.94$	2839.97	$g_1^5 = 96.25, g_1^8 = 151.40, g_2^8 = 197.07$

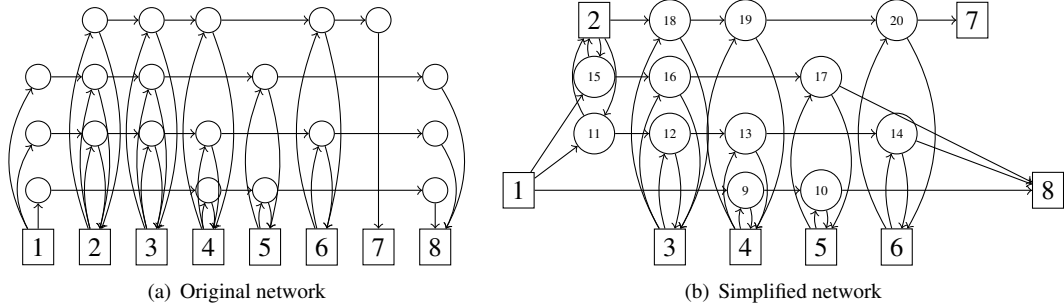


Fig. 4. Link representation of bus network

Table 4. Demand estimation results for different precision values ϵ in optimization approach

Nominal demand	γ	ϵ	Estimated demand	Computational time (sec)
$\bar{g}_1^5 = 90, \bar{g}_1^8 = 130, \bar{g}_2^8 = 225$	1/50	0.00001	$g_1^5 = 99.46, g_1^8 = 149.97, g_2^8 = 200.17$	13066.73
	1/50	0.00005	$g_1^5 = 99.52, g_1^8 = 149.29, g_2^8 = 200.51$	5598.87
	1/50	0.0001	$g_1^5 = 101.28, g_1^8 = 147.01, g_2^8 = 199.05$	5186.8
$\bar{g}_1^5 = 70, \bar{g}_1^8 = 110, \bar{g}_2^8 = 170$	1/10	0.00001	$g_1^5 = 89.22, g_1^8 = 152.99, g_2^8 = 197.95$	19743.81
	1/10	0.00005	$g_1^5 = 89.28, g_1^8 = 153.51, g_2^8 = 198.71$	6319.99
	1/10	0.0001	$g_1^5 = 89.36, g_1^8 = 152.32, g_2^8 = 199.03$	3976.06
$\bar{g}_1^5 = 70, \bar{g}_1^8 = 110, \bar{g}_2^8 = 170$	1/50	0.00001	$g_1^5 = 93.83, g_1^8 = 154.32, g_2^8 = 199.92$	24336.01
	1/50	0.00005	$g_1^5 = 92.76, g_1^8 = 154.87, g_2^8 = 199.29$	13886.32
	1/50	0.0001	$g_1^5 = 78.25, g_1^8 = 167.79, g_2^8 = 201.46$	2872.51
$\bar{g}_1^5 = 70, \bar{g}_1^8 = 110, \bar{g}_2^8 = 170$	1/100	0.00001	$g_1^5 = 95.78, g_1^8 = 153.15, g_2^8 = 199.94$	22567.9
	1/100	0.00005	$g_1^5 = 83.27, g_1^8 = 166.58, g_2^8 = 199.82$	5007.03
	1/100	0.0001	$g_1^5 = 78.30, g_1^8 = 169.63, g_2^8 = 200.94$	2594.98

4.2. Simulation approach

We reproduced the same examples using an approach based on simulation. The simulation-optimization methodology Fu et al. (2005); Nguyen et al. (2014); Bartz-Beielstein et al. (2018); Tian et al. (2018) consists of using a

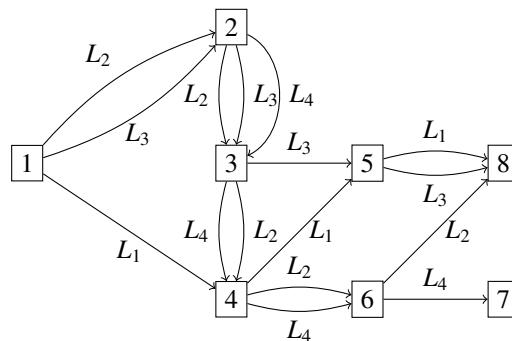


Fig. 5. Network without boarding and alighting arcs

simulator as the objective function of the problem (or a part of the objective function). The simulation works as a black-box, in which the decision variables are matching with the simulation parameters and the objective function value with the simulation output. If an optimization algorithm does not need the mathematical formulation of the objective function, but only the tuples $(x, f(x))$, then this algorithm can be coupled with the simulation. The main benefit of this approach is that complicated systems can be modeled in an easier way with respect to the purely mathematical approach.

In the context of the problem analyzed in this work, all buses trips and passengers allocations can be modeled as a simulation. We use the Simulation of Urban Mobility (SUMO, Lopez et al. (2018)), an Agent-based simulator that can simulate the dynamic of all buses going from stop to stop, and all passengers going to stops, selecting a bus and travel to the destination. SUMO allows to define several alternative trips schemes for each passenger, i.e., different ways to travel to the desired destination (including walking). So, in simulation time, due to the status of the system (number of passenger waiting at each stop, waiting time for each passenger, number of passenger in-travel, buses frequencies, among others), each passenger decide which scheme follow to minimize its total travel time (travel time plus waiting time). SUMO can model passenger loading times, traffic jams, congestions at bus-stops, traffic light, etc. Also, the traffic net can be downloaded from a service like Open-Street map. So, these features leveraging the work of building the graph, calculating distances and maximum travel times and adding operative constraints to the model, and allow to obtain a more realistic model. Additionally to this, SUMO offers a viewer that allows debugging of the model (Figures 6(a) and 6(b)). The main drawback of simulation-optimization is the computational cost associated with simulations, but there are many approaches that try to deal with this problem using meta-models Barton and State (2010); Bittner and Hahn (2013); Cristescu and Knowles (2015); Baquela and Olivera (2019).

SUMO is a microscopic-level space-continuous time-discrete simulator. It works under the agent-based simulation paradigm, modeling and simulating each vehicle and person, and updating their behavior time-step by time-step in base of the state of all closest vehicles and persons. For simulating vehicle movement, a Car-Following model is used Ravi Shankar and Mathew (2010); Aghabayk et al. (2015).

To model the traffic network, we replicated the bus-stops interconnection structure, so the same paths as in the analytical model can be reproduced. Edges lengths, travel speeds, and buses and passengers emission frequencies were tuned by a Simulated Annealing algorithm in order to match the frequencies of the test cases Chu et al. (2003); Cheng et al. (2014); Paz et al. (2015). No other agent except buses and passengers were included in the simulation.

In order to perform the simulation-optimization with this problem, we set as simulation inputs the number of passengers that want to travel from node i to node j (this is, the demand g_i^j) and, as output, the total travel time for each passenger. Having this output, mean frequencies can be calculated as the reciprocal of the sum travel times.

As in Section 4.1, the Nelder-Mead was used as the optimizer. Simulation time was set to 1 hour, and the simulated optimization process was aborted after a calculation time of 10 minutes. Estimated demands and simulation times are shown in Table 3. The estimates are compatible with those obtained by the optimization approach, but it can be seen how the execution times are reduced if the simulation times of Table 3 are compared with computational times for the optimization approach exposed in Table 4 for $\varepsilon = 0.00001$.

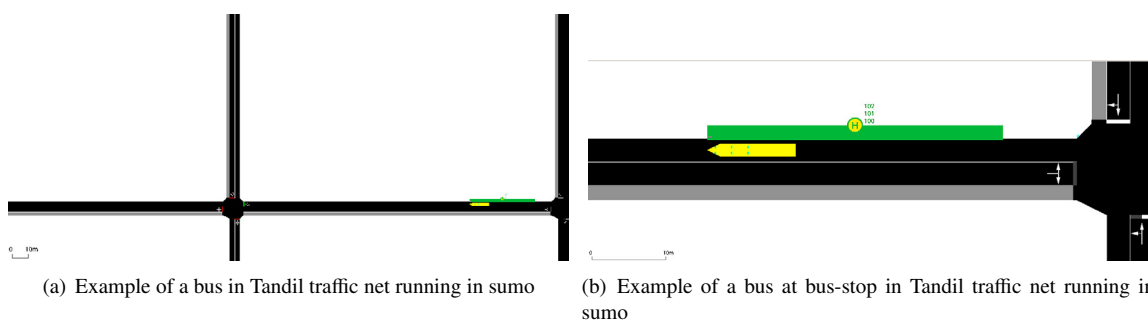


Fig. 6. Bus representation in SUMO

5. Conclusions

In this work, we extended the study started earlier, applying our demand estimation problem over bigger networks. We tested our model for some cases considering different nominal demands, obtaining good estimations even when nominal demands are less accurate. These results are good not only using the analytical method but also using simulations, something that is encouraging for the future since they allow us to address the demand estimation problem considering the entire public transport network and not just a part of it, using the advantages that simulation tools offer. Using this method and collecting data about real frequencies of the buses and information captured by transport services, we look forward to further extending our study for the whole public transport of the city. As we could see in these examples, the main drawback that we could experience when considering the entire network will be the computational effort to solve the estimation problem. However, the results showed that the execution times can be considerably reduced if we reduce the precision of assignments performing during the estimation or if we adopt the simulation-optimization approach.

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