



HAL
open science

Isotacticity in chiral phononic crystals for low-frequency bandgap

Wei Ding, Tianning Chen, Dewen Yu, Chen Chen, Rui Zhang, Jian Zhu,
Badreddine Assouar

► **To cite this version:**

Wei Ding, Tianning Chen, Dewen Yu, Chen Chen, Rui Zhang, et al.. Isotacticity in chiral phononic crystals for low-frequency bandgap. *International Journal of Mechanical Sciences*, In press, 261, pp.108678. 10.1016/j.ijmecsci.2023.108678 . hal-04271863

HAL Id: hal-04271863

<https://hal.science/hal-04271863>

Submitted on 6 Nov 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0 International License

Isotacticity in chiral phononic crystals for low-frequency bandgap

Wei Ding^a, Tianning Chen^a, Dewen Yu^{b, c}, Chen Chen^a, Rui Zhang^a, Jian Zhu^{a, *}, Badreddine Assouar^{d, *}

^aSchool of Mechanical Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, PR China

^bKey Laboratory of Ministry of Education for Modern Design and Rotor-Bearing System, School of Mechanical Engineering, Xi'an Jiaotong University, Xianning West Road, Xi'an 710049, China

^cSchool of Civil and Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore, Singapore.

^dUniversité de Lorraine, CNRS, Institut Jean Lamour, F-54000 Nancy, France

*Corresponding authors. E-mail addresses: jianzhuxj@xjtu.edu.cn (J. Zhu), badreddine.assouar@univ-lorraine.fr (B. Assouar).

Abstract

Realizing a low-frequency and broad bandgap is extremely challenging because the lattice constant, equivalent stiffness, and equivalent density must be taken into account in practice. Despite the advances of inertial amplification which is one of the ideal strategies to address this issue, the design ideas are mainly limited to the classic geometries featured by mirror symmetry (syndiotacticity), confining then progress in this topic. Here, we report numerical and experimental investigation of a chiral phononic crystal without mirror symmetry (isotacticity) according to the analogous Thomson scattering. Numerical analysis shows that due to the lack of one degree of freedom in isotactic lattice (ISL) compared to syndiotactic lattice (SYL), the bandgap starting-frequency of the ISL is significantly lower than that the one of the SYL by up to 66%. The bandgap properties are further demonstrated by simulation and experimental measurements. This research reveals that the condition of having alternating arrangement orientations between neighboring atoms is not indispensable for this kind of

bandgap, and a rational geometry satisfying the Thomson scattering-based bandgap conditions can further reduce the bandgap frequency without affecting the stiffness and lightweight of the structure.

1 **1. Introduction**

2 The bandgap in phononic crystals provides a new path to high device performance in vibration
3 isolation [1, 2], thermal insulation [3, 4], and robust topological wave transmission [5, 6]. In terms of
4 the low-frequency and broadband isolation for elastic waves, the great efforts inspired by Bragg
5 scattering [7] and local resonance [8] have been done, such as rainbow trapping [9-11], gradient
6 distribution [11-15], coupled Bragg scattering and local resonance [16-19], acoustic black holes [20,
7 21], or nonlinear local resonance [22-25]. Even, the quasi zero stiffness, another one of appealing
8 strategies, might be a solution of this dilemma because it can lower the frequency of the Bragg
9 scattering bandgap [26-28], nevertheless the robustness and stability of the system is debatable when
10 the metastructures are at the quasi-zero state. In general, these strategies are based on sacrificing the
11 stiffness or (and) the lightweight of the system. It is indisputable that absolute ultra-low and ultra-wide
12 bandgaps can be achieved as long as discounting the static properties such as lattice constants,
13 equivalent stiffness, and static equivalent density [29-34]. However, these static properties are critical
14 to ensure superior stability, reliability, and maneuverability of the high-performance devices in
15 engineering practice. Therefore, overcoming the conflict between low-frequency and broad bandgaps
16 and those three properties still remains great challenges.

17 In 2007, Yilmaz et al. proposed the inertial amplification to solve the above challenge [35]. In the
18 inertial amplification system, the static inertia can be amplified many times, as a result, it can break
19 the dependency of the local resonance and Bragg scattering on the stiffness and mass [35, 36]. The
20 anti-resonance frequency in this structure can enhance the attenuation in low frequency [37-39].
21 Inspired by the inertial amplification, more and more novel studies are proposed. For instance, an
22 octahedron structure was built to isolate the special omnidirectional elastic waves [40]; incorporating
23 inertial amplification structures in continuous beams to manipulate the transverse wave [41, 42]; a 2D
24 thin plates embedded the classical inertial amplifier structures was proposed to modulate low-
25 frequency and broad lamb waves [43, 44]; the low-frequency vibration isolation of the corrugated-core

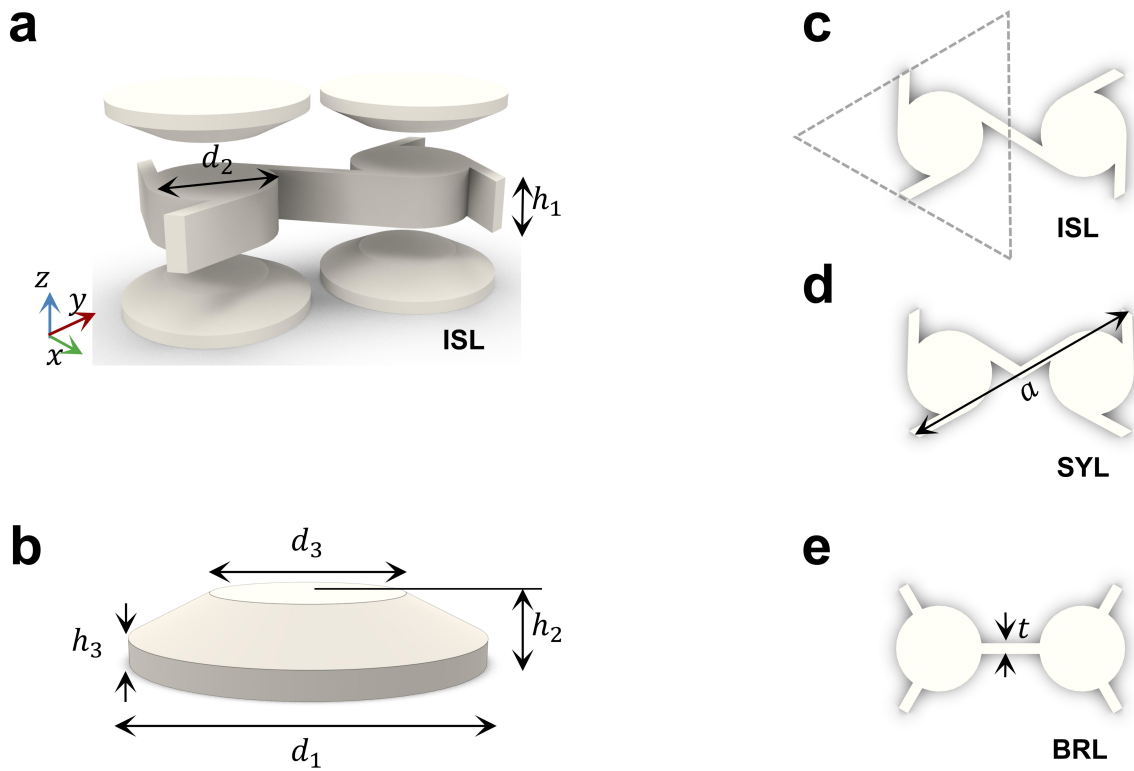
26 sandwich panels can be enhanced through embedding the inertial amplification system [45]; a lever-
27 type inertial amplification system can achieve the broadband isolation for the surface wave which is
28 induced by the seismic [46, 47]; the combination of the inertial amplification and local resonance can
29 improve the extremely narrow effective attenuation of the classical local resonance bandgap [48] and
30 release the dependence on lowering stiffness and increasing mass for low-frequency local resonance
31 bandgap [49]; the multiple, broadband and highly attenuative bandgaps can be obtained by a system
32 with inertial amplification, local resonance, and Bragg scattering affect simultaneously [50]. However,
33 in the last two decades, the design strategies [42, 45, 51-55] have been limited to the form of the
34 classical inertial amplification sub-structure.

35 In recent years, the inertial amplification was demonstrated in the axis twist metamaterials as well [56,
36 57]. Subsequently, an analogue of Thomson scattering was proposed theoretically, numerically, and
37 experimentally to describe its underlying physics [58]. From the physics point of view, the coupling
38 of orthogonal polarizations is a means to amplify the inertia [56, 57, 59], while the destructive
39 interference is another key factor in opening the bandgap [58]. Achieving destructive interference
40 relies on secondary scattered waves of the same polarization having opposite motion directions [58].
41 Based on this, a planar orthorhombic chiral lattice based on Thomson scattering was reported recently
42 [60], in which an ultra-broad bandgap was achieved. It's worth noting that, an identical geometric
43 feature can be observed from these studies [35, 57, 61-63], i.e., these geometries have a distinctive
44 feature of alternating movement orientations between adjacent atoms (described as syndiotacticity [57]
45 for convenience).

46 In fact, according to the three conditions for the bandgap generation [58], it is possible to achieve this
47 bandgap even if the adjacent atoms in this non-mirror chiral lattice do not have alternating arrangement
48 directions (isotacticity) as long as the horizontal tangential motion of the planar non-mirror chiral
49 lattice (three-triplet lattice) is suppressed. Since the ligaments between adjacent lumped masses are
50 continuous, theoretically, the number of wave reflections can be effectively reduced, which in turn
51 reduces the resonance peaks in the FRF. Eventually, a bandgap with lower frequency than the mirror
52 lattice will be produced.

53 Here, a planar chiral phononic crystal with non-mirror feature is proposed. To differentiate from the
 54 classic models, we name the phononic crystals with mirroring property which have Thomson
 55 scattering-induced bandgaps as syndiotacticity, and the other one with non-mirror property as
 56 isotacticity. To emphasis the novelty in this work, an isotactic lattice (ISL) and a syndiotactic lattice
 57 (SYL) are compared on the condition of the same lattice constant, equivalent stiffness, and density.
 58 We demonstrate that although both ISL and SYL satisfy the Thomson scattering conditions, i.e.,
 59 orthogonal motions coupled to the lumped masses and the opposite motion directions in the second
 60 scattering, and because of the lack of the symmetry, at the starting boundary of the attenuation, ISL
 61 has one less resonant peak than SYL, thus, inducing a lower-frequency bandgap than that of SYL.

62 2. Structures and results



63
 64 **Fig. 1. (a) Schematics of the arrayed lattice (ISL). (b) Dimensions of the lumped mass. (c)-(e) The**
 65 **various in ligaments between ISL, SYL, and BRL.**

66 Tacticity is a concept in polymer science to describe the relative orientation of adjacent chiral centers
 67 [64, 65]. This concept was firstly introduced in the chiral phononic crystal to describe the bandgap
 68 variances in syndiotactic lattice (SYL) and isotactic lattice [57]. To facilitate the distinction, we state

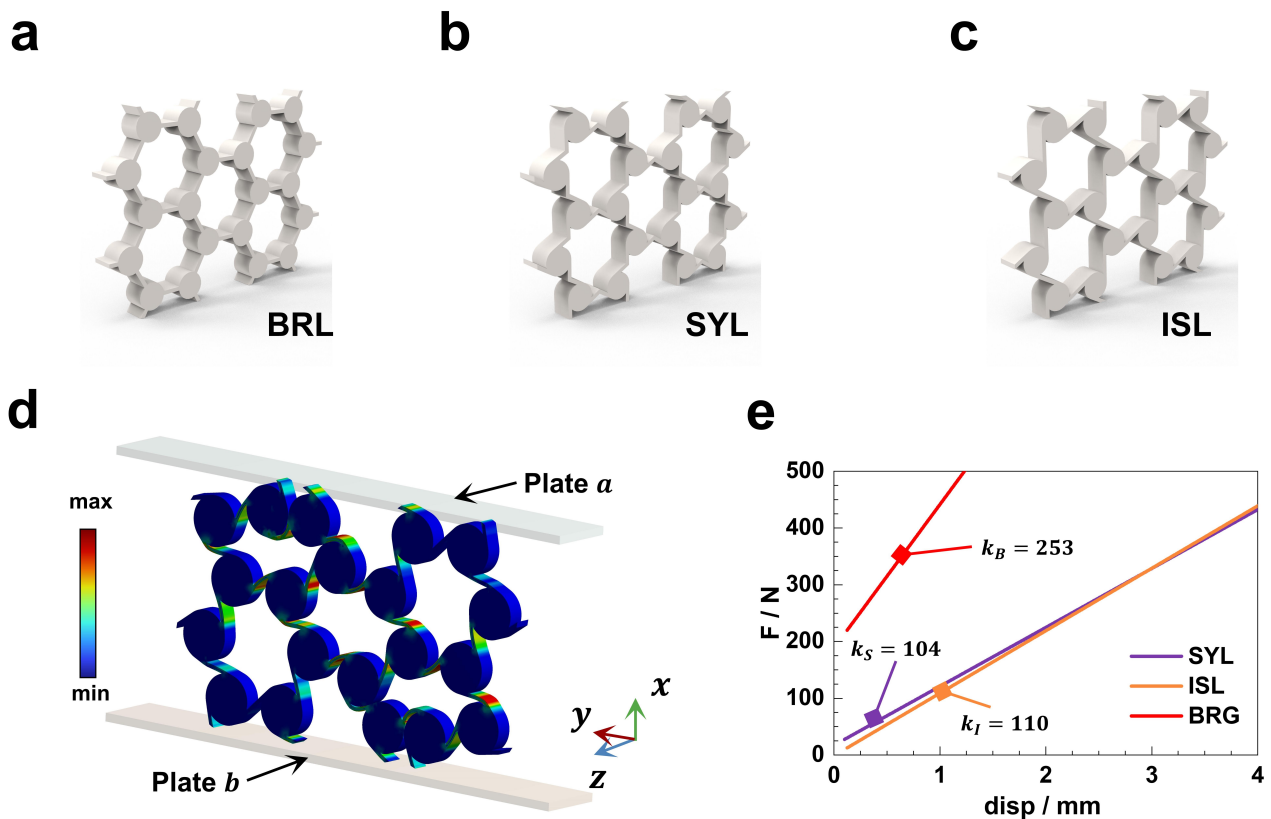
69 syndiotacticity and isotacticity to define the lattice with mirror and non-mirror features, respectively.
70 To clarify the novelty of this study, SYL, ISL and Bragg scattering lattice (BRL) are discussed together.
71 **Fig. 1(a)** shows the ISL schematic with lattice constant $a=80$ mm, $d_1=37$ mm, $d_2=30$ mm, $d_3=27$
72 mm, $h_1=10$, $h_2=8$ mm, $h_3=3$ mm, and $t=2.5$ mm. The details of the lumped mass can refer to **Fig.**
73 **1(b)**. It should be stated that the shape of the lumped mass is intended to obtain the inertia amplification
74 factor as larger as possible [66]. Meanwhile, it should be note that three sorts of lattice have the same
75 outline except the shape of the ligaments. As illustrated in **Fig. 1(c)**-1(e), the ligament in ISL is tangent
76 with the circle whose diameter is d_2 . SYL is by mirroring the sub unit cell of ISL (highlighted by gray
77 dotted line in **Fig. 1(c)**). In BRL, the centerline of the ligament is perpendicular to the tangent of the
78 circle with d_2 .

79 Traditional phononic crystals consisted of a homogeneous continuous matrix and periodic scatterers,
80 so the non-dimensional bandgap was evaluated using normalized criteria of the material acoustic speed
81 and lattice constants [17, 67]. Later, the quadratic tailoring matrix was proposed because it can
82 drastically enhance the impedance between the matrix and scatterers, thus realizing an absolute low-
83 frequency and broadband Bragg scattering bandgaps [29, 68]. Observing from kinetic equation, the
84 enhancement of the impedance reduces the equivalent stiffness or (and) equivalent density of the
85 system, i.e., these two metrics (equivalent stiffness and equivalent density) are inversely proportional
86 to the bandgap starting frequency [66, 69, 70]. Therefore, the bandgap evaluation should take into
87 account the equivalent stiffness and equivalent density in addition to the lattice constant. In order to
88 fairly evaluate these three bandgaps, we use Eq. (1) to normalize the band structure.

$$89 \quad f_n = f a \sqrt{\frac{m_e}{k_e}}, \quad (1)$$

90 where $m_e = 0.15$ kg in three lattices. To obtain the equivalent stiffness, using ANSYS workbench
91 LS-dyna simulates the compression for the geometric models shown in **Fig. 2(a-c)**. The material is one
92 kind of Nylon with the elastic modulus $E = 1.6 \times 10^9$ Pa, Poisson's ratio $\nu = 0.4$, and density $\rho =$
93 1000 kg/m³. h_1 equals 20 mm to avoid the z-axis deformation. We should note that the thickness
94 h_1 of the models shown in **Fig. 2(a-c)** is two times of that in bandgap calculation, therefore, the actual
95 stiffness of the models in the normalized bandgap is 0.5 times of the stiffness denoted in **Fig. 2(e)**.
96 However, in the normalized band structure, because the stiffness of BRG, ISL, and SYL is missing a

97 same coefficient which is 0.5 times, there is no influence on the fair comparison. Plate a is fixed and
 98 plate b has a velocity of 500 mm/s along the $-x$ axis. For convergence, the contact frictional
 99 coefficient between the sample and plates a and b , and between the sample and itself is 0.1 [71]. As
 100 shown in Fig. 2(d), the strain of the system focuses on the ligaments, that means the equivalent stiffness
 101 is mainly determined by the ligaments. This is the reason that we neglect the contribution of the lumped
 102 mass to the equivalent stiffness in order to speed up the calculation.



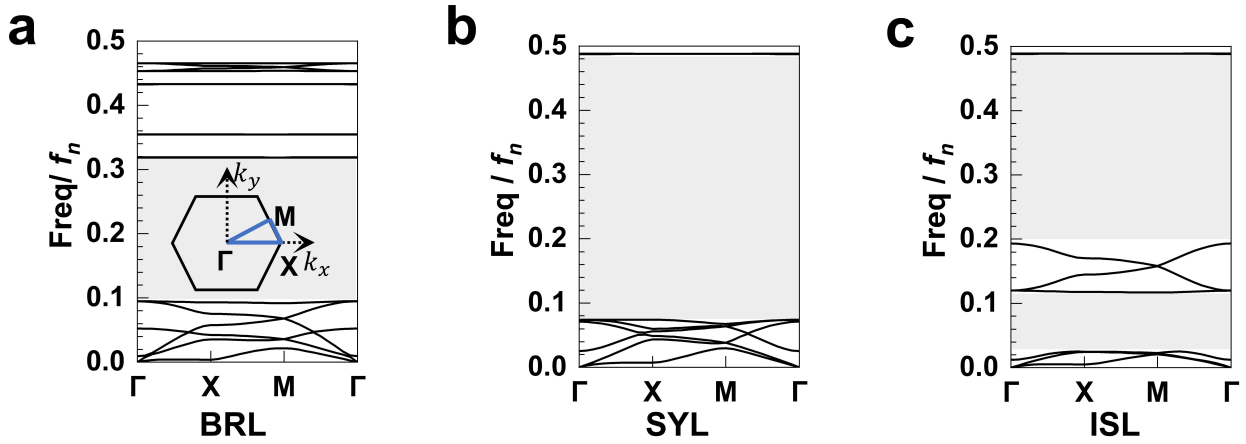
103
 104 **Fig. 2. (a) – (c) structure schematics of ISL, SYL, and BRL in compression simulation,**
 105 **respectively. (d) Equivalent stress contour of ISL when the displacement is 4 mm. (e) Fitted**
 106 **force-displacement results of ISL, SYL, and BRL, where k_B , k_S , and k_I means the stiffness of**
 107 **BRG, SYL, and ISL, respectively.**

108 **Fig. 2(e) shows the force-displacement results in linear elastic stage. Remarkably, although ISL and**
 109 **SYL have distinct configurations in ligaments, ISL ($k_I=110$ N/mm) and SYL ($k_S=104$ N/mm) have**
 110 **almost identical equivalent stiffness, which effect has been verified experimentally in our previous**
 111 **study [60]. Because of the distinct ligament configuration of the BRL, its equivalent stiffness is**
 112 **$k_B=253$ N/mm and significantly higher than the other two results. Because the stiffness is a key factor**

113 effecting on the frequency, as a result, it requires us to normalize the band structures (Fig. 3) using Eq.
 114 (1) for a fair appraisal.

115 Fig. 3 shows the band structures calculated in COMSOL Multiphysics. In simulation, Bloch period
 116 boundary is applied in the unit cell and the band structures can be obtained through sweeping along
 117 the wave vector Γ -X-M- Γ as shown in Fig. 3(a). Due to the fact that translation and rotation are coupled
 118 only in the xoy plane; the degree of freedom of the ligaments along the z-axis are constrained to
 119 avoid the interference from out-of-plane modes.

120 The band structures in non-dimensional range of 0.5 are presented Fig. 3(a-c). In classic BRL (Fig.
 121 3(a)), a broad bandgap extending from 0.0949 to 0.318 can be found. However, the bandgap of SYL
 122 (Fig. 3(b)), extending from 0.074 to 0.488, is broader than BRL. The most prominent is that ISL (Fig.
 123 3(c)) has two bandgaps. The first one extends from 0.0251 to 0.117 and the other one from 0.193 to
 124 0.488. When the bandgap width is evaluated at the center frequency of the bandgap [72], their relative
 125 widths are 1.08 (BRL), 1.47 (SYL), and 1.29 & 0.866 (ISL), respectively.



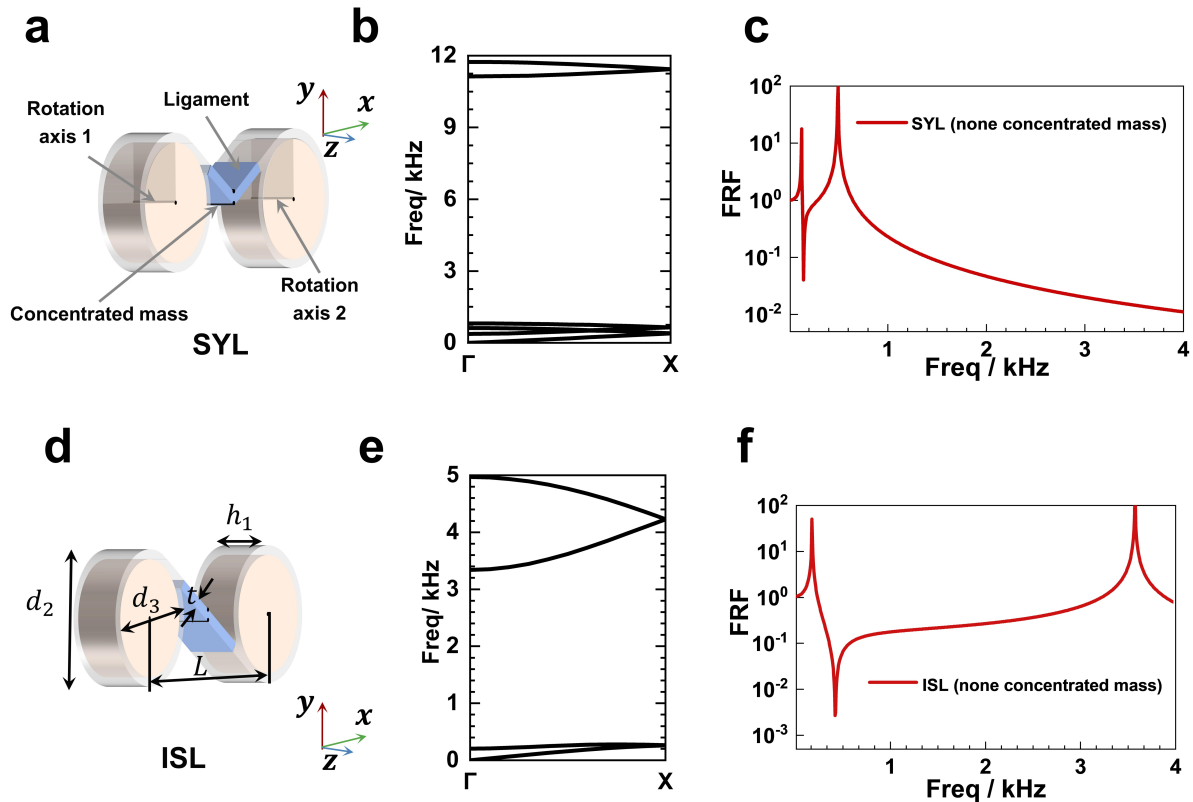
126
 127 **Fig. 3. Normalized band structures of BRL, SYL, and ISL. The shaded areas indicate the**
 128 **bandgaps.**

129 Comparing with BRL, one can see that ISL and SYL have significant advantages in width and bandgap
 130 starting frequency of the first bandgap. It cannot be denied that SYL has a wider first bandgap than
 131 ISL. Nevertheless, the bandgap frequency of ISL is 66% lower than that of SYL, which occurs at the
 132 same lattice constant, same static equivalent density and quasi-identical equivalent stiffness. This is an
 133 encouraging result as it practically verifies the argument [57] that the combination of inertial
 134 amplification and alternating arrangement orientations of adjacent atoms is merely one special case

135 for this bandgap formation. This indirectly demonstrates that the analogous Thomson scattering [58]
 136 has universality.

137 3. Bandgap mechanism

138 To illustrate the bandgap mechanism of ISL and the cause of the bandgap difference between ISL and
 139 SYL, we simplify ISL and SYL with 2D periodicity to a one-dimensional model and simulate the
 140 transmission of an ISL and an SYL. The models are presented in **Fig. 4(a)** and **Fig. 4(d)**.



141
 142 **Fig. 4. (a) Simplified ISL unit cell. Blue domain indicates the ligament and the remaining part is**
 143 **lumped masses. Two black lines in the ligament denote the location of the concentrated mass**
 144 **m_a , and the concentrated mass is not considered the shape in xoy plane and evenly distributed**
 145 **on both black lines if it is not 0. (b) Band structure of the simplified ISL unit cell, in which the**
 146 **density of the ligaments is 1000 kg/m^3 while m_a is 0. (c) FRF results of one simplified SYL**
 147 **unit cell. (d) Simplified ISL unit cell. In this simplified unit cell, $L = 60 \text{ mm}$, $d_2 = 30 \text{ mm}$, $d_3 = 26$**
 148 **mm , $t = 2 \text{ mm}$, $h_1 = 10 \text{ mm}$. (e) Band structure of simplified ISL unit cell in which the density of**
 149 **the ligaments is 1000 kg/m^3 while m_a is 0. (f) FRF results of one simplified ISL unit cell.**
 150 **Notable, the concentrated mass in the simulations of (e-f) is 0 and the ligaments are density-free.**

151 According to the conditions of elastic Thomson scattering [58], ISL and SYL have to own translation-
152 rotation coupling. It has been demonstrated that the chirality is an effective way to achieve the motion-
153 coupling condition, thus producing the inertial amplification effect [59, 61]. However, not all chiral
154 structures have this basic property [60]. The unit cell like SYL can guarantee this property due to its
155 symmetry [60]. However, due to the lumped masses in an asymmetric lattice like ISL do not have
156 symmetry, the entire finite-period structure will be a shear movement. In the absence of external forces
157 to counteract its shear movement along the y -axis (refer to coordinate in **Fig. 4(d)**), the lumped masses
158 like ISL shown in **Fig. 4(d)** do not stably produce the motion coupling and thus provide inertial
159 amplification [60].

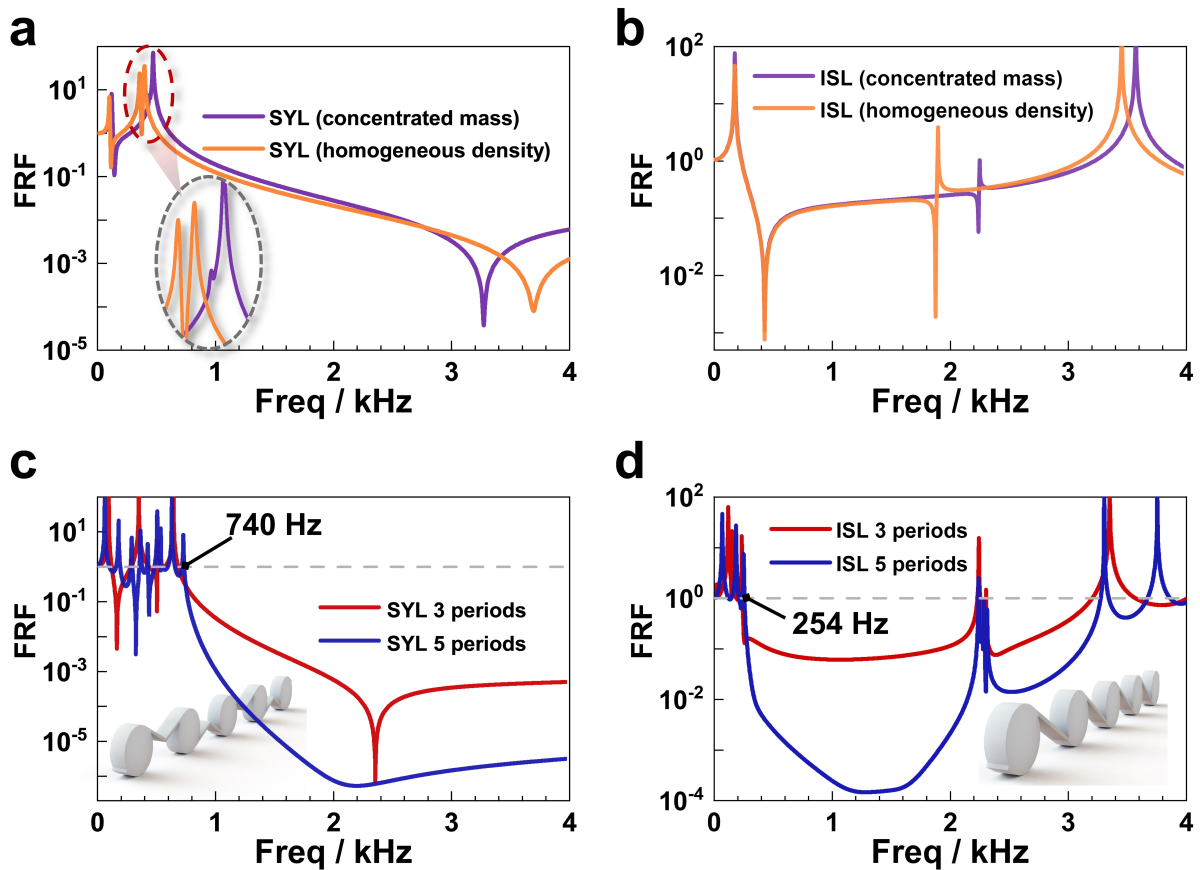
160 To manually avoid the shear movement along the y -axis, we allow rotation axes 1 and 2 to rotate
161 around the z -axis and move along the x -axis only. The motion of the entire system along the z -axis
162 direction is not allowed in order to avoid the interference of the z -axis modes. A harmonic uniform
163 line load along the x -axis is applied on the rotation axis 1. Two domain point probes are additional at
164 the centers of axes 1 and 2 to pick up the acceleration. The transmission is calculated as a_o/a_i .

165 In order to control variable of that avoiding the interference from the classical inertial amplification
166 form [35, 37] in the ligament domain of SYL and facilitate us to find the bandgap difference between
167 ISL and SYL, the case of non-concentrated mass at the center of the ligament is discussed first and the
168 FRFs are presented in **Fig. 4(c)** and **Fig. 4(f)**.

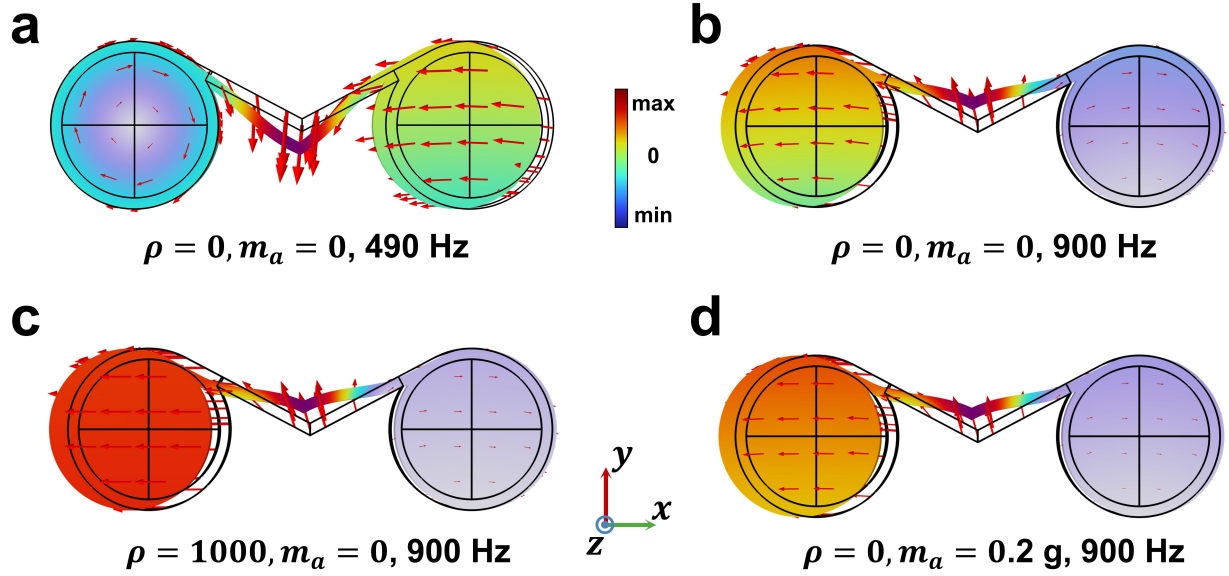
169 Observing from **Fig. 4(c)**, one can see that in SYL (**Fig. 4(c)**), the attenuation starts from the second
170 resonance peak (around 500 Hz); the attenuation curve from the second resonance peak to the 4 kHz
171 range is smooth. In terms of ISL (**Fig. 4 (f)**), there are some differences compared to SYL. The first
172 one is that there is only one resonance peak before the attenuation of ISL. The attenuation range is cut
173 off at around 3500 Hz. Besides, there is an anti-resonance notch at 400 Hz, thus significantly enhancing
174 the attenuation at low frequencies.

175 In practice, the ligaments cannot be density-free. Therefore, we compared the case of ligaments with
176 homogeneous density of 1000 kg/m^3 . For SYL, when the ligament has homogeneous density, an anti-
177 resonance notch can be found at 3700 Hz. If we equate the homogeneous density to a concentrated
178 mass ($m_a = 0.2 \text{ g}$), as shown in **Fig. 5(a)**, both results will be similar except for the slight shift of the

179 second resonance peak toward the lower frequency in case of homogeneous density. It is of interest
 180 that, in ISL shown in Fig. 5(b), two cases have no influence on the anti-resonance notch. Besides,
 181 regardless of homogeneous density or a concentrated mass, there is only one phenomenon that a new
 182 resonance peak in 1500-2500 Hz truncate the original attenuation from 200 – 3500 Hz. Eventually, the
 183 first bandgap of ISL covers from 254 Hz to 2240 Hz (Fig. 5(d)), but the bandgap of SYL starts from
 184 740 Hz (Fig. 5(c)) and extends beyond 4000 Hz.



185
 186 **Fig. 5. (a-b) FRF results for finite structures with different boundaries. The inset in (a) reveals**
 187 **the resonance-peak details of the blue line. (c-d) FRF results for finite structures with different**
 188 **periods of simplified ISL and SYL unit cell whose ligaments are density free and have a**
 189 **concentrated mass of 0.2g. The insets in (c) and (d) are three-period structures of SYL and ISL,**
 190 **respectively. The gray lines in (c) and (d) are the reference line of attenuation.**



191

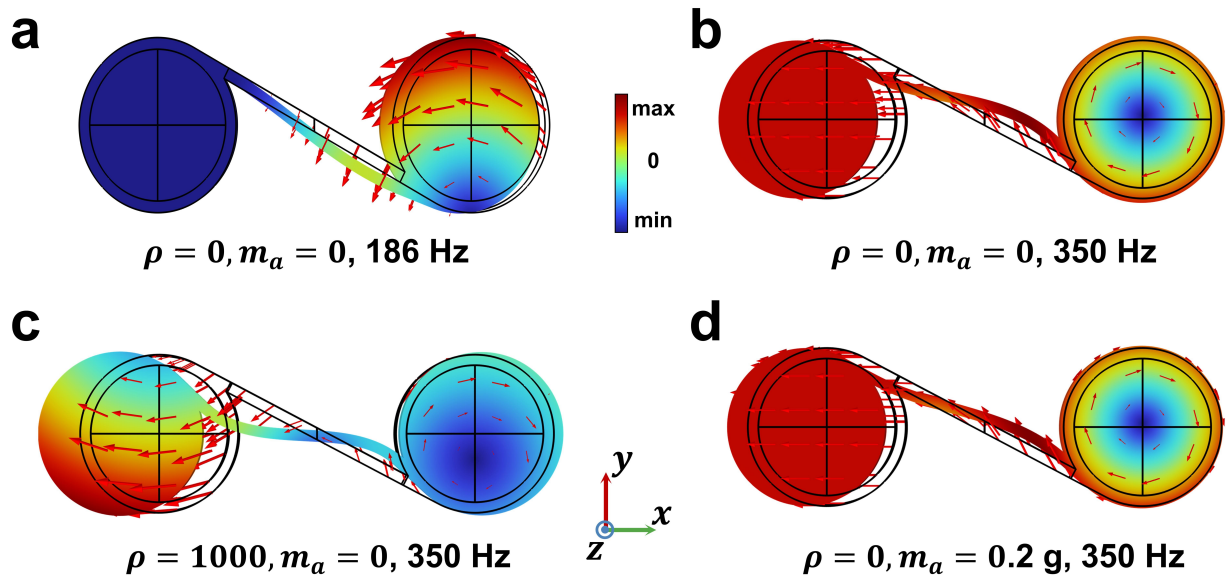
192 **Fig. 6.** The deformation contours of the simplified 1D SYL at the resonance peak and within the
 193 **bandgap. (a)** Deformation contour of the resonance peak when the ligament is density free and
 194 **concentrated mass m_a is 0. (b)** Deformation contour within the bandgap when the ligament is
 195 **density free and concentrated mass m_a is 0. (b)** Deformation contour within the bandgap when
 196 **the density of the ligament is 1000 kg/m^3 and concentrated mass m_a is 0. (c)** Deformation
 197 **contour within the bandgap when the density of the ligament is 0 and concentrated mass m_a is**
 198 **0.2 g.**

199 Furthermore, we analysis the differences of the bandgap difference between ISL and SYL on
 200 deformation contours. Because ISL and SYL have different bandgap starting frequency but each one
 201 has similar starting attenuation frequency under the three conditions (i.e., density free, concentrated
 202 mass, and homogeneous), we choose the deformation contours which is the nearest to their starting
 203 attenuation, as well as the ones at 350 Hz for ISL and 900 Hz for SYL which are within the bandgap.
 204 Fig. 6 is the deformation contours of SYL and Fig. 7 is that of SYL.

205 In terms do the SYL, there is a folded corner, which is similar with the form of the classical inertial
 206 amplification [35, 37, 73]. The classical inertial amplification requires a concentrated mass at the folder
 207 corner, in order to provide an amplified inertia for main structures. On the one hand, the amplified
 208 inertia can decrease the resonant peaks before the bandgap, at the same time, it can induce an anti-
 209 resonance peak to enhance the attenuation of the bandgap. If the sub-structure of the inertial
 210 amplification is non-mass, the inertial amplification effect will disappear [36]. Therefore, in order to

211 avoid the interference from the classical inertial amplification, we compared three cases of density free,
 212 concentrated mass, and homogenous.

213 Fig. 6(a) shows that even though the folded corner is density-free, the resonant peak of the SYL still
 214 contains the significant motion of the folded corner in addition to that of two lumped masses, which
 215 means the resonant peak is strongly related to the folded corner. If the folded mass has a small
 216 concentrated mass, i.e., $m_a = 0.2$ g, there will be the classical inertial amplification. In this case,
 217 there will be three resonance peaks before the bandgap as shown in Fig. 5(a); besides, the concentrated
 218 mass will induce an anti-resonant notch in the attenuation range. Comparing Fig. 6(b) - (d), regardless
 219 the presence or absence of the classical inertial amplification, the shapes of the deformation contours
 220 within the bandgap are similar. Notably, the entire mode includes the translation and rotation of other
 221 two lumped mass, and in addition, the folded corner shows the similar transverse as the classical
 222 inertial amplification [54, 72]. In other words, the geometrical form like the classical inertial
 223 amplification sub-structure is a key factor for the second resonant peak at 490 Hz; however, it does
 224 not determine the existence of the Thomson scattering-induced bandgap but can contribute to
 225 enhancing attenuation of the bandgap since its anti-resonant notch.



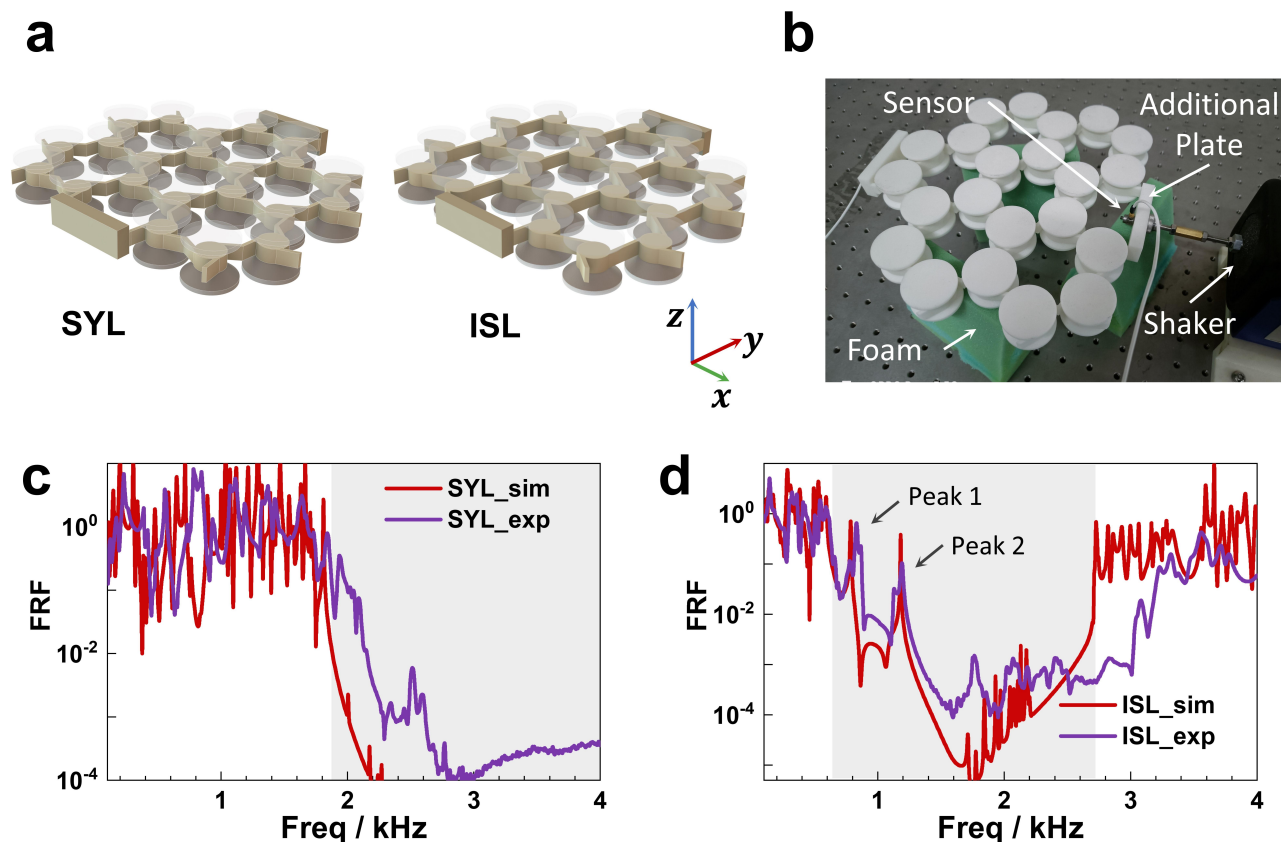
227 **Fig. 7. The deformation contours of the simplified 1D ISL at the resonance peak and within the**
 228 **bandgap. (a) Deformation contour of the resonance peak when the ligament is density free and**
 229 **concentrated mass m_a is 0. (b) Deformation contour within the bandgap when the ligament is**
 230 **density free and m_a is 0. (c) Deformation contour within the bandgap when the density of the**

231 **ligament is 1000 kg/m^3 and m_a is 0. (c) Deformation contour within the bandgap when the**
232 **density of the ligament is 0 and m_a is 0.2 g.**

233 For ISL, because the folded corner does not exist, the mode deformations become concise. Fig. 7(a)
234 shows that the resonance peak originates from the second lumped mass and hardly involves the first
235 concentrated mass and the significant transverse of the ligament. Even in the case of the concentrated
236 mass or the homogenous ligament, the ligament has no obvious transverse as in SYL. Variously, when
237 the ligament is homogenous, the two lumped masses will have rotation by deviating from their own
238 geometric central axis. Luckily, these deformation differences have no influence on the starting
239 frequency of the bandgap and the attenuation within the bandgap. Notably, as shown in **Fig. 5(b)**, the
240 concentrated mass and the densely ligament will bring about an extra resonance peak at high
241 frequencies (Refer to **Fig. 5(b)**), thus truncating this Thomson scattering-induced bandgap.

242 Overall, although SYL can make the realistic ligaments favorable to generate the anti-resonance peak,
243 it cannot compensate its natural disadvantage that the resonance peaks raise the attenuation starting
244 frequency. In particular, from the perspective in 500 Hz range, one can see that SYL has two or three
245 resonance peaks in three cases. In the case of non-mass, the second resonant frequency of SYL is 495
246 Hz. However, in the ISL, there is only one resonant frequency of 190 Hz. That is because the ligaments
247 of the SYL have a folded corner, resulting in additional degrees of freedom and thus an additional
248 resonance peak in the FRF. In contrast, the continuous ligament of ISL makes the lack of a resonance
249 frequency, hence the attenuation starting frequency (190 Hz) is 61.6% lower than the one of the SYL
250 (495 Hz). With increasing periodicity, the two lattices have a difference of about 65.7% in the starting
251 frequency, which is consistent with the prediction (the bandgap frequency of ISL is 66% lower than
252 that of SYL) of the band structures shown in Fig. 3, and thus verified the reasonableness of the model
253 simplification. Detailly, as shown in **Fig. 5(c) and Fig. 5(d)**, the effective attenuation of the ISL starts
254 from 254 Hz and that of the SYL starts from 740 Hz if we take $\text{FRF} = 10^0$ as reference line. This is
255 the reason why the bandgap of ISL predicted in band structure shown in **Figs. 3** is about 66% lower
256 than that of SYL.

4. Verification and Discussion



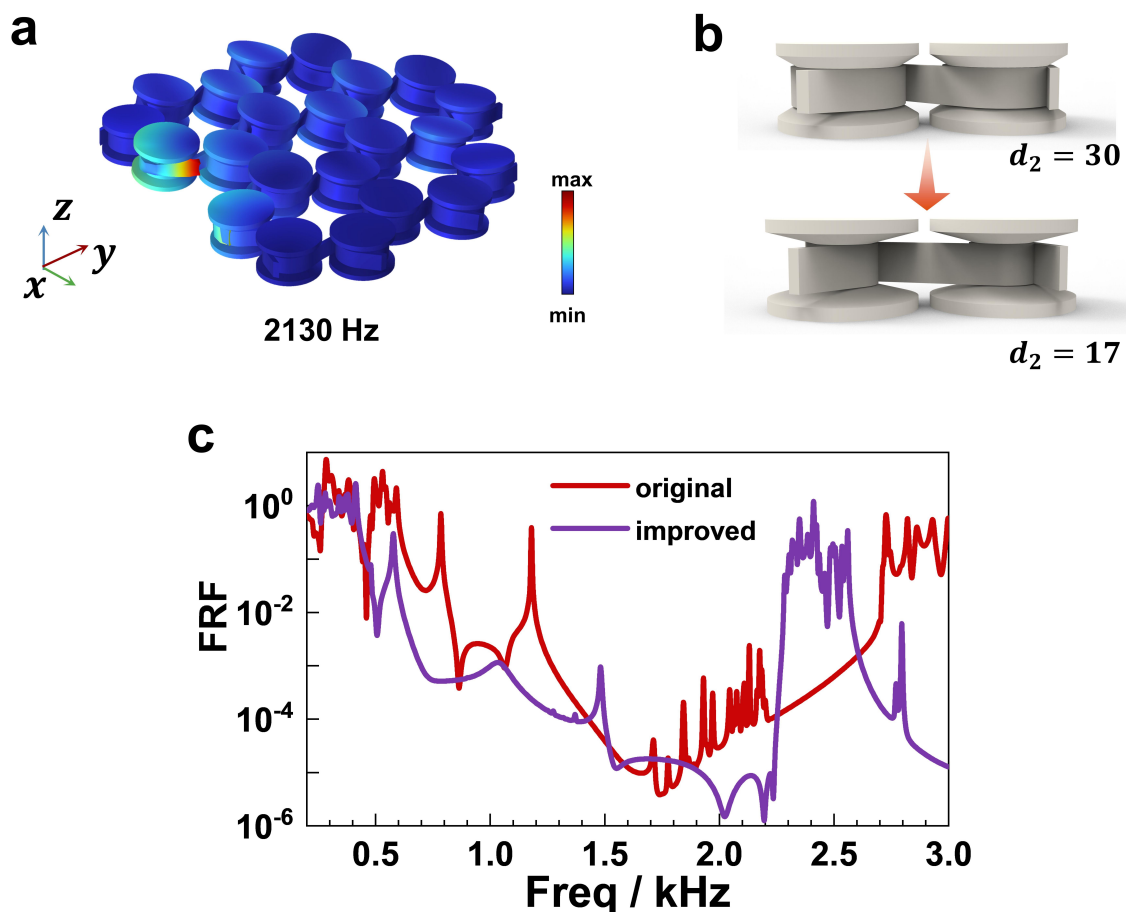
258

259 **Fig. 8. (a) Schematic of the finite SYL and ISL. In order to denote the details of the ligaments,**
 260 **the lumped masses of the top layer are set to be transparent. (b) Experimental configuration**
 261 **photograph. (c) numerical and experimental results of ISL. (d) Numerical and experimental**
 262 **results of SYL. The shaded areas indicate the numerical bandgaps.**

263 To validate the bandgaps predicted in the band structures, the finite-period structures of SYL (Fig.
 264 8(a)) and ISL (Fig. 8(b)) are investigated numerically and experimentally. In simulation, two domain
 265 point probes located at the center of the additional plates are applied to acquire the input and output
 266 acceleration. The entire structure is free except the input end where a harmonic force is applied. It is
 267 notable that the out-of-plane motion of the finite period structure is free in stand of restrained as in the
 268 calculation of the band structure, because we cannot realize this ideal condition in experiment. The
 269 harmonic excitation signal sweeps from 100 Hz to 4000 Hz with the resolution of 5 Hz. As illustrated
 270 in Fig. 8(a), the extra additional plates are designed to connect the shaker (Modelshop K2007E01) and

271 sensors (PCB 353B15). The foam is used to support and level samples, as well as isolate the vibration
 272 from the optical test platform.

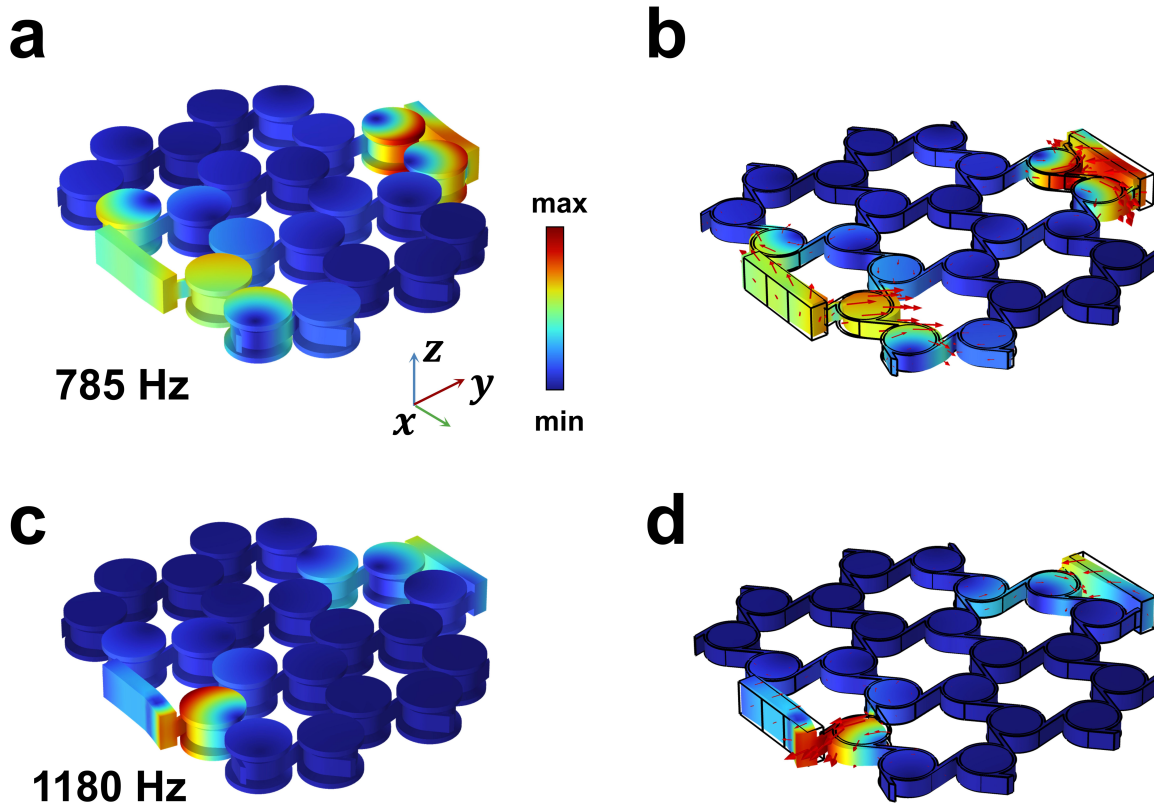
273 **Fig. 8(c)** and **Fig. 8(d)** show the numerical and experimental results of the ISL and SYL. For ISL,
 274 experiment and simulation denote that the bandgap appears from 600Hz to 2700Hz, while the one of
 275 the SYL starts at about 1900 Hz. Overall, the experimental measurements corroborate the numerical
 276 transmission results. Due to the manufacturing errors and the random factors in the experiment, the
 277 results shift towards high frequency compared to the simulation, such as the starting attenuation
 278 frequency in **Fig. 8(c)**.



279
 280 **Fig. 9. (a) Deformation contour of ISL at 2130 Hz. (b) The variation in geometry of ISL to**
 281 **eliminate resonant peaks within 1700 Hz - 2200 Hz. (c) FRFs of original (red line) and improved**
 282 **(blue line) ISL**

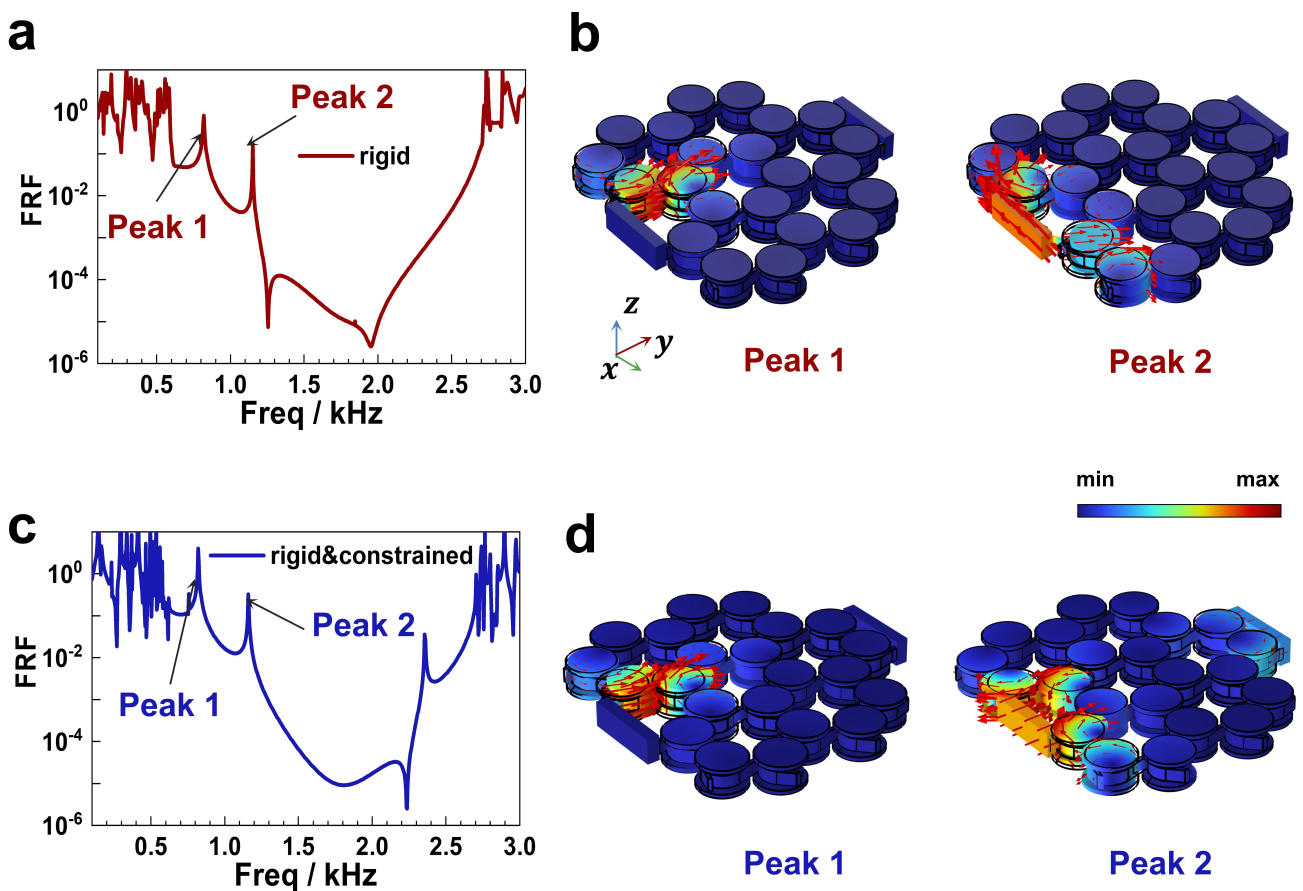
283 In **Fig. 8(d)**, there are two obvious peaks at 785 Hz and 1180 Hz and a bunch of weak resonance peaks
 284 in 1700 Hz - 2200 Hz, thus breaking the bandgap continuity. In order to find the cause of the resonance

285 peaks in 1700 Hz – 2200 Hz, **Fig. 9(c)** shows the deformation contour at 2130 Hz, which reveals an
 286 out-of-plane mode. In short, the excitation in xoy plane activates the out-of-plane mode. Theoretically,
 287 the out-of-plane mode should be suppressed to be consistent with the boundaries of band-structure
 288 calculation. However, it is easy to implement in simulation but hard in experiment. In fact, the out-of-
 289 plane modes can be avoided by the structure design even if in practice we do not provide extra force
 290 on the surface of the sample to prevent its out-of-plane motion. As in the gyroscope principle, the
 291 relative angular momentum (the ratio of the rotational inertia of lumped mass to that of the circle with
 292 d_2) is small due to the small dimensional ratio of d_1 to d_3 , and thus leads to the dynamic stability of
 293 the lumped mass in the xoy plane. Therefore, by increasing the difference between d_1 and d_3 (as
 294 shown in **Fig. 9(b)**), these unfavorable resonance peaks can be effectively inhibited such as illustrated
 295 in **Fig. 9(c)**. This is the reason for the shape of the additional lumped mass denoted in Fig. 1(b).
 296



297
 298 **Fig. 10. Deformation contour of ISL at (a)785 Hz, (c) 1180 Hz. The ligament deformation details**
 299 **at (b)785 Hz, (d) 1180 Hz.**

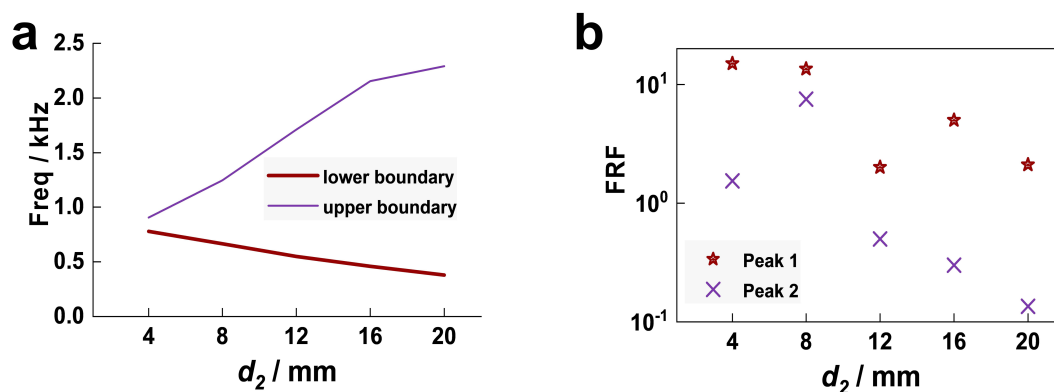
300 In Fig. 8(d), the two peaks at 785 Hz and 1180 Hz show almost no attenuation. To find the cause of
 301 these resonance peaks, the deformation contours of the entire finite structure at 785 Hz and 1180 Hz
 302 are drawn in Fig. 10(a) and Fig. 10(c); Fig. 10(b) and Fig. 10(d) show the deformations of the ligaments
 303 for more details. One can see the obvious local deformation of the additional plates and the global
 304 rotation in the lumped masses attached with the additional plates rather than the ligaments. It seems to
 305 indicate that these two peaks are strictly related to the additional plates and the lumped masses but it's
 306 not completely accurate. Therefore, we did the further discussion below.



307
 308 **Fig. 11. (a) FRF of finite structure with rigid additional plates. (b) Deformation contour of the**
 309 **peak 1 and peak 2 in Fig. 11(a). (c) FRF of finite structure with rigid additional plates whose**
 310 **movements along x-axis and z-axis are constrained. (d) Deformation contour of the peak 1 and**
 311 **peak 2 in Fig. 11(c). The arrows in the deformation contours mean the surface deformation**
 312 **direction.**

313 As illustrated in Fig. 11(a), when the additional plates are rigid to avoid the local deformation of the
 314 additional plates, the peaks still exist. In Fig. 11(b), the black frame is the original positions of the

315 period structure. Taking its original position as the reference, one can find that both deformations
 316 include the translation along the y -axis and rotation around z -axis of the lumped masses. Besides,
 317 peak 2 is not only related to the shear movement (the motion along x direction) of the additional
 318 plates, but also to the shear movement of the lumped masses. Therefore, we restrict additional plates
 319 from moving on y -axis and z -axis. Nevertheless, these peaks still exist and do not change in the
 320 frequency and value, as shown in Fig. 11(c). The arrows in Fig. 11(d) reveal that, these two peaks still
 321 mainly depend on the resonance modes of the lumped mass. The motion restriction for the additional
 322 plates does not change the resonance mode of the first peak, and different from the first resonance
 323 peak, the second peak involves the vibration of more lumped masses. In other words, the boundary
 324 conditions on the additional plates are not the key factor to determine these two resonance peaks.

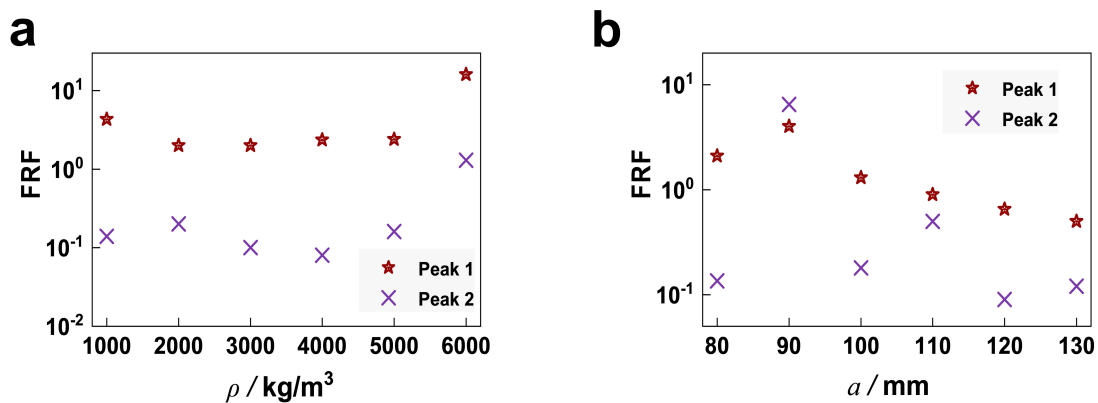


325
 326 **Fig. 12. Parameter discussion about geometrical dimension d_2 of ISL. (a) The influence of d_2**
 327 **on the first bandgap width of ISL. (b) The influence of d_2 on the peak 1 and peak 2 of ISL.**
 328 **Please refer to Appendix S1 to find the variation principle of d_2 .**

329 It is worth noting that the resonance modes corresponding to these two peaks are asymmetric in the
 330 xoy plane. This is due to the fact that ISL is asymmetry, which leads to unequal contributions of the
 331 input wave to the connected ligaments on both sides. It is to be expected that the asymmetry will get
 332 smaller and smaller with the decrease of d_2 . However, as illustrated by Fig. 12, when $d_2 = 4$ mm,
 333 the width of the bandgap drops rapidly to almost close. Nevertheless, these two peaks are still greater
 334 than 10^0 . Briefly, it is undesirable to completely eliminate this asymmetry, because the asymmetry is
 335 the critical condition to produce the motion coupling of the lumped mass and motion coupling is
 336 fundamental condition for Thomson scattering-induced bandgap. If decreasing d_2 continuously, the

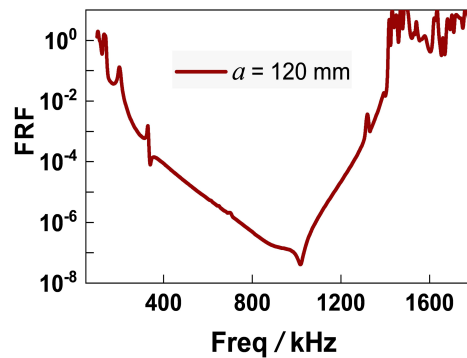
337 asymmetry will gradually go forward to symmetry, and then ISL will evolve to BRL. Consequently,
 338 the Thomson scattering-induced bandgap will disappear.

339 Generally, the equivalent stiffness and density of the system are two underlying factors on the
 340 resonance peaks. Within all geometrical parameters, increasing lattice constant can modify the length
 341 of the ligaments and thus control equivalent stiffness; varying the density of the lumped masses can
 342 manipulate the equivalent density of the system independently. In order to control a single variable,
 343 the lattice constant changes or the density changes while all other parameters remain constant (Please
 344 refer to Appendix S1 to find the variation principle of the lattice constant). Fig. 13 shows the results
 345 of the parameter discussion about density and lattice constant. From Fig. 13(a), one can see that the
 346 suppression for these two resonance peaks isn't absolutely better with the increase of the density. For
 347 peak 1, regardless the density variation, it still exceeds 10^0 ; but for peak 2, there seems to be a great
 348 value when density is 3000 to 5000 kg/m^3 . Fortunately, Fig. 13(b) reveals that both peaks will show
 349 attenuation when the lattice constant is larger than 100 mm. In short, the combination adjustment for
 350 equivalent stiffness and equivalent density can indeed be a strategy to suppress these two unfavorable
 351 resonance peaks. Therefore, as denoted by the case in Fig. 14, through increasing the lattice constant
 352 and the asymmetry of the ISL unit cell for modifying stiffness, and increasing the dimension of lumped
 353 masses for adjusting equivalent density of the unit cell, we can weaken the performance of these two
 354 peaks in the FRF. (Please refer to Appendix S2 to find the geometrical details).



355
 356 **Fig. 13. Parameter discussion about the influence of (a) the lattice constant variation and (b) the**
 357 **density of the lumped masses on peak 1 and peak 2. All other geometries keep constant when the**

358 lattice constant changes. Please refer to Appendix S1 to find the variation principle of the lattice
359 constant.



360
361 **Fig. 14. FRF of finite structure with lattice constant $\alpha = 120$ mm. In numerical calculation, the**
362 **additional plates have an elastic modulus of $1.6e13$ Pa and “density free” indicates that the**
363 **density of the additional plates is 0 while the one of the rest components in finite structure is 1000**
364 **kg/m^3 . Compared to the ISL mentioned in Fig. 1, the altered dimensions of the unit cell can be**
365 **found in table 1 attached to Appendix S2. Other conditions are keeping constant.**

366

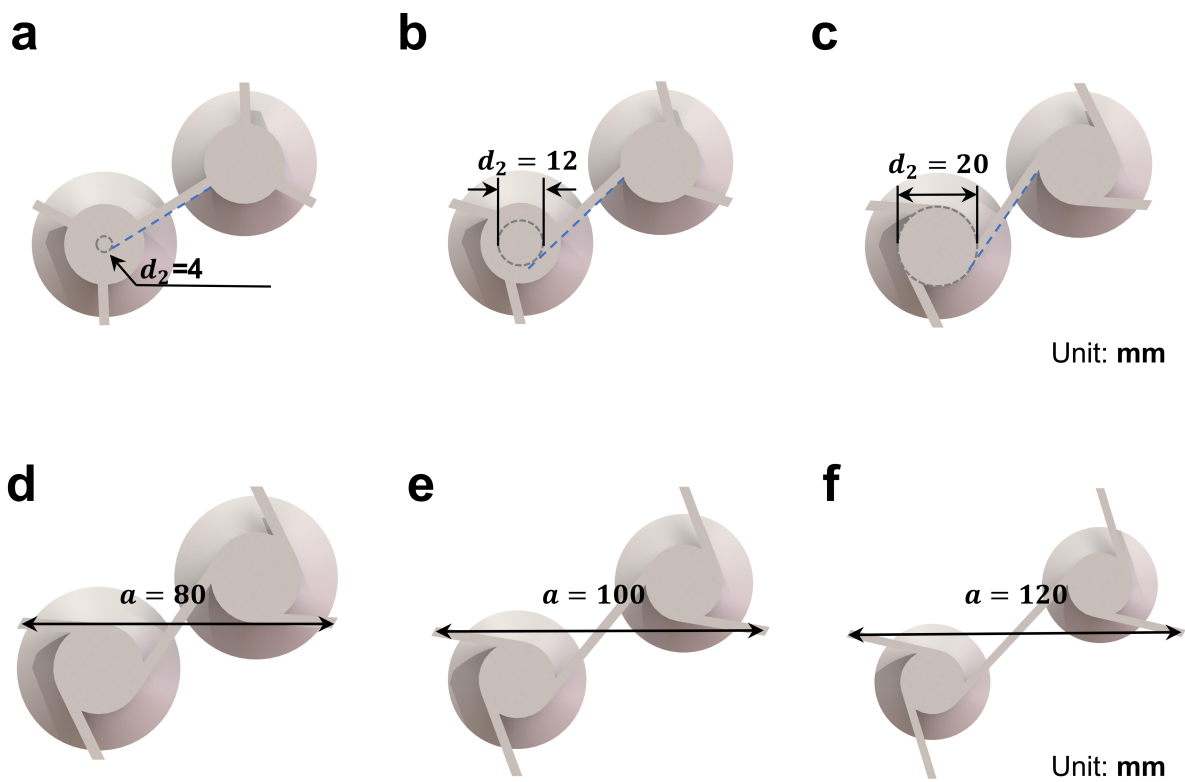
367 5. Conclusion

368 In summary, this study has reported an isotactic chiral phononic crystal with a Thomson scattering-
369 induced bandgap for a low-frequency and broad bandgap. The band structures of the classical Bragg
370 scattering, syndiotactic, and isotactic lattices have been numerically calculated and compared together
371 to elucidate the novelty of the isotactic chiral phononic crystals. The band structures and FRFs of
372 simplified one-dimensional ISL and SYL have been compared to find the cause of the bandgap
373 difference in ISL and SYL. Then the finite period structures have been designed, manufactured, and
374 tested for the transmission, to verify the bandgap prediction in the band structure. Based on above
375 study, the following conclusion can be drawn.

376 Distinct from previous studies in which the syndiotactic phononic crystals are considered as a more
377 superior way to realize low-frequency and broad bandgaps than isotactic phononic crystals [57, 61, 74,
378 75], we have verified that although both ISL and SYL have the motion coupling which is one of the
379 way to achieve the inertial amplification [60], the lack of the mirror symmetry makes the ISL having
380 one less degree of freedom than SYL and hence there is less a resonant peak than SYL. As a result,

381 the bandgap starting frequency of ISL is lower by 66% than that of SYL. In addition, the relative width
 382 of the bandgap of up to 129% can be guaranteed. This research has illustrated that the condition of
 383 having alternating arrangement orientations between neighboring atoms is not indispensable for
 384 bandgap formation. These variances between the SYL and ISL do not depend on altering the lattice
 385 constant, equivalent stiffness, and static equivalent density of the system. The consequent radically
 386 unusual behaviors may make this design strategy attractive in all the fields where vibrations play a
 387 crucial role, such as for instance civil, aerospace and mechanical engineering.

388 Appendix S1



389
 390 Fig. S1. The variation principle of (a-c) the diameter d_2 and (d-f) the lattice constant.

391 Appendix S2

392 Table 1. Dimensions of ISL in Fig. 13

Parameter	a	d_1	d_2	d_3
Dimension	120	50	30	25

393 **Acknowledgment**

394 This work was financially supported by the National Natural Science Foundation of China (No.
395 12002258), and the China Postdoctoral Science Foundation (No. 2022M712540). Wei Ding is grateful
396 to the support of China Scholarship Council (Grant No. 202206280170). Wei Ding thanks Huan Lu
397 (Xi'an Jiaotong University) for his help in the analysis of the study problem.

398

399 **References**

- 400 [1] Oudich M, Gerard NJRK, Deng Y, Jing Y. Tailoring Structure - Borne Sound through Bandgap
401 Engineering in Phononic Crystals and Metamaterials: A Comprehensive Review. *Adv Funct Mater.*
402 2022;33:2206309.
- 403 [2] Brule S, Javelaud EH, Enoch S, Guenneau S. Experiments on seismic metamaterials: molding
404 surface waves. *Phys Rev Lett.* 2014;112(13):133901.
- 405 [3] Nomura M, Laude V, Maldovan M. Phononic crystals at various frequencies. *APL Materials.*
406 2022;10(5):050401.
- 407 [4] Maldovan M. Narrow Low-Frequency Spectrum and Heat Management by Thermocrystals. *Phys*
408 *Rev Lett.* 2013;110(2):025902.
- 409 [5] Wang J-Q, Zhang Z-D, Yu S-Y, Ge H, Liu K-F, Wu T, et al. Extended topological valley-locked
410 surface acoustic waves. *Nat Commun.* 2022;13(1):1324.
- 411 [6] Zheng S, Man X, Kong Z-L, Lin Z-K, Duan G, Chen N, et al. Observation of fractal higher-order
412 topological states in acoustic metamaterials. *Science Bulletin.* 2022;67(20):2069-75.
- 413 [7] Kushwaha MS, Halevi P, Dobrzynski L, Djafari-Rouhani B. Acoustic band structure of periodic
414 elastic composites. *Phys Rev Lett.* 1993;71(13):2022-5.
- 415 [8] Liu Z, Zhang X, Mao Y, Zhu YY, Yang Z, Chan CT, et al. Locally Resonant Sonic Materials.
416 *Science.* 2000;289(5485):1734-6.
- 417 [9] Celli P, Yousefzadeh B, Daraio C, Gonella S. Bandgap widening by disorder in rainbow
418 metamaterials. *Appl Phys Lett.* 2019;114(9):091903.
- 419 [10] De Ponti JM, Iorio L, Riva E, Ardito R, Braghin F, Corigliano A. Selective Mode Conversion and
420 Rainbow Trapping via Graded Elastic Waveguides. *Phys Rev Appl.* 2021;16(3):034028.
- 421 [11] Meng H, Chronopoulos D, Fabro AT, Elmadih W, Maskery I. Rainbow metamaterials for
422 broadband multi-frequency vibration attenuation: Numerical analysis and experimental validation. *J*
423 *Sound Vib.* 2020;465:115005.
- 424 [12] Baravelli E, Ruzzene M. Internally resonating lattices for bandgap generation and low-frequency
425 vibration control. *J Sound Vib.* 2013;332(25):6562-79.
- 426 [13] De Ponti JM, Colombi A, Ardito R, Braghin F, Corigliano A, Craster RV. Graded elastic
427 metasurface for enhanced energy harvesting. *New J Phys.* 2020;22(1):013013.
- 428 [14] Li C, Jiang T, He Q, Peng Z. Stiffness-mass-coding metamaterial with broadband tunability for
429 low-frequency vibration isolation. *J Sound Vib.* 2020;489:115685.

430 [15] Hu G, C. M. Austin A, Sorokin V, Tang L. Metamaterial beam with graded local resonators for
431 broadband vibration suppression. *Mech Syst Sig Process.* 2021;146:106982.

432 [16] Liu L, Hussein MI. Wave Motion in Periodic Flexural Beams and Characterization of the
433 Transition Between Bragg Scattering and Local Resonance. *J Appl Mech.* 2012;79(1):011003.

434 [17] D'Alessandro L, Belloni E, Ardito R, Corigliano A, Braghin F. Modeling and experimental
435 verification of an ultra-wide bandgap in 3D phononic crystal. *Appl Phys Lett.* 2016;109(22):221907.

436 [18] Matlack KH, Bauhofer A, Krödel S, Palermo A, Daraio C. Composite 3D-printed metastructures
437 for low-frequency and broadband vibration absorption. *Proc Natl Acad Sci USA.* 2016;113(30):8386-
438 90.

439 [19] Tian X, Chen W, Gao R, Liu S. Merging Bragg and Local Resonance Bandgaps in Perforated
440 Elastic Metamaterials with Embedded Spiral Holes. *J Sound Vib.* 2021;500:116036.

441 [20] Park HW, Seung HM, Kim M, Choi W, Oh JH. Continuum Flexural Metamaterial for Broadband
442 Low-Frequency Band Gap. *Phys Rev Appl.* 2021;15(2):024008.

443 [21] Park S, Jeon W. Ultra-wide low-frequency band gap in a tapered phononic beam. *J Sound Vib.*
444 2021;499:115977.

445 [22] Fang X, Wen J, Bonello B, Yin J, Yu D. Ultra-low and ultra-broad-band nonlinear acoustic
446 metamaterials. *Nat Commun.* 2017;8(1):1288.

447 [23] Zhou J, Dou L, Wang K, Xu D, Ouyang H. A nonlinear resonator with inertial amplification for
448 very low-frequency flexural wave attenuations in beams. *Nonlinear Dyn.* 2019;96(1):647-65.

449 [24] Yang H, Xu J, Xiong Z, Lu X, Zhang R-Y, Li H, et al. Optically Reconfigurable Spin-Valley Hall
450 Effect of Light in Coupled Nonlinear Ring Resonator Lattice. *Phys Rev Lett.* 2021;127(4):043904.

451 [25] Yu J, Park S, Hwang I, Kim D, Demmerle F, Boehm G, et al. Electrically tunable nonlinear
452 polaritonic metasurface. *Nature Photonics.* 2022;16(1):72-8.

453 [26] Cai C, Zhou J, Wang K, Pan H, Tan D, Xu D, et al. Flexural wave attenuation by metamaterial
454 beam with compliant quasi-zero-stiffness resonators. *Mech Syst Sig Process.* 2022;174:109119.

455 [27] Zhang Q, Guo D, Hu G. Tailored Mechanical Metamaterials with Programmable Quasi - Zero -
456 Stiffness Features for Full - Band Vibration Isolation. *Adv Funct Mater.* 2021;31(33):2101428.

457 [28] Wang K, Zhou J, Ouyang H, Chang Y, Xu D. A dual quasi-zero-stiffness sliding-mode
458 triboelectric nanogenerator for harvesting ultralow-low frequency vibration energy. *Mech Syst Sig*
459 *Process.* 2021;151:107368.

460 [29] Jia Z, Chen Y, Yang H, Wang L. Designing Phononic Crystals with Wide and Robust Band Gaps.
461 *Phys Rev Appl.* 2018;9(4):044021.

462 [30] Lucklum F, Vellekoop MJ. Bandgap engineering of three-dimensional phononic crystals in a
463 simple cubic lattice. *Appl Phys Lett.* 2018;113(20):201902.

464 [31] Salari-Sharif L, Haghpanah B, Guell Izard A, Tootkaboni M, Valdevit L. Negative-Stiffness
465 Inclusions as a Platform for Real-Time Tunable Phononic Metamaterials. *Phys Rev Appl.*
466 2019;11(2):024061.

467 [32] Fei X, Jin L, Zhang X, Li X, Lu M. Three-dimensional anti-chiral auxetic metamaterial with
468 tunable phononic bandgap. *Appl Phys Lett.* 2020;116(2):021902.

469 [33] Gerard NJRK, Oudich M, Xu Z, Yao D, Cui H, Naify CJ, et al. Three-Dimensional
470 Trampolinelike Behavior in an Ultralight Elastic Metamaterial. *Phys Rev Appl.* 2021;16(2):024015.

471 [34] Iglesias Martínez JA, Moughames J, Ulliac G, Kadic M, Laude V. Three-dimensional phononic
472 crystal with ultra-wide bandgap at megahertz frequencies. *Appl Phys Lett*. 2021;118(6):063507.

473 [35] Yilmaz C, Hulbert GM, Kikuchi N. Phononic band gaps induced by inertial amplification in
474 periodic media. *Phys Rev B*. 2007;76(5):054309.

475 [36] Yilmaz C, Kikuchi N. Analysis and design of passive low-pass filter-type vibration isolators
476 considering stiffness and mass limitations. *J Sound Vib*. 2006;293(1-2):171-95.

477 [37] Yilmaz C, Hulbert GM. Theory of phononic gaps induced by inertial amplification in finite
478 structures. *Phys Lett A*. 2010;374(34):3576-84.

479 [38] Acar G, Yilmaz C. Experimental and numerical evidence for the existence of wide and deep
480 phononic gaps induced by inertial amplification in two-dimensional solid structures. *J Sound Vib*.
481 2013;332(24):6389-404.

482 [39] Xiao Y, Wen J, Yu D, Wen X. Flexural wave propagation in beams with periodically attached
483 vibration absorbers: Band-gap behavior and band formation mechanisms. *J Sound Vib*.
484 2013;332(4):867-93.

485 [40] Taniker S, Yilmaz C. Design, analysis and experimental investigation of three-dimensional
486 structures with inertial amplification induced vibration stop bands. *Int J Solids Struct*. 2015;72:88-97.

487 [41] Sun Y, Zheng H, Lee HP. Beam-type metastructure with X-shape inertial amplification
488 mechanisms for vibration suppression. *Thin Wall Struct*. 2023;189:110893.

489 [42] Li J, Yang P, Li S. Multiple band gaps for efficient wave attenuation by inertial amplification in
490 periodic functionally graded beams. *Compos Struct*. 2021;271:114130.

491 [43] Zeng Y, Cao L, Wan S, Guo T, Wang Y-F, Du Q-J, et al. Seismic metamaterials: Generating low-
492 frequency bandgaps induced by inertial amplification. *Intl J Mech Sci*. 2022;221:107224.

493 [44] Frandsen NMM, Bilal OR, Jensen JS, Hussein MI. Inertial amplification of continuous structures:
494 Large band gaps from small masses. *J Appl Phys*. 2016;119(12):124902.

495 [45] Xi C, Dou L, Mi Y, Zheng H. Inertial amplification induced band gaps in corrugated-core
496 sandwich panels. *Compos Struct*. 2021;267:113918.

497 [46] Zeng Y, Cao L, Wan S, Guo T, An S, Wang Y-F, et al. Inertially amplified seismic metamaterial
498 with an ultra-low-frequency bandgap. *Appl Phys Lett*. 2022;121(8):081701.

499 [47] Li L, Wang Q, Liu H, Li L, Yang Q, Zhu C. Seismic metamaterials based on coupling mechanism
500 of inertial amplification and local resonance. *Physica Scripta*. 2023;98(4):045024.

501 [48] Zhao C, Zhang K, Zhao P, Hong F, Deng Z. Bandgap merging and backward wave propagation
502 in inertial amplification metamaterials. *Intl J Mech Sci*. 2023;250:108319.

503 [49] Mu D, Wang K, Shu H, Lu J. Metamaterial beams with graded two-stage inertial amplification
504 and elastic foundation. *Intl J Mech Sci*. 2022;236:107761.

505 [50] Mazzotti M, Foehr A, Bilal OR, Bergamini A, Bosia F, Daraio C, et al. Bio-inspired non self-
506 similar hierarchical elastic metamaterials. *Intl J Mech Sci*. 2023;241:107915.

507 [51] Cai C, Zhou J, Wu L, Wang K, Xu D, Ouyang H. Design and numerical validation of quasi-zero-
508 stiffness metamaterials for very low-frequency band gaps. *Compos Struct*. 2020;236:111862.

509 [52] Li J, Yang P, Li S. Phononic band gaps by inertial amplification mechanisms in periodic
510 composite sandwich beam with lattice truss cores. *Compos Struct*. 2020;231:11458.

511 [53] Muhammad S, Wang S, Li F, Zhang C. Bandgap enhancement of periodic nonuniform
512 metamaterial beams with inertial amplification mechanisms. *J Vib Control*. 2020;26(15-16):1309-18.

513 [54] Yuksel O, Yilmaz C. Realization of an ultrawide stop band in a 2-D elastic metamaterial with
514 topologically optimized inertial amplification mechanisms. *Int J Solids Struct.* 2020;203:138-50.

515 [55] Banerjee A, Adhikari S, Hussein MI. Inertial amplification band-gap generation by coupling a
516 levered mass with a locally resonant mass. *Intl J Mech Sci.* 2021;207:106630.

517 [56] Orta AH, Yilmaz C. Inertial amplification induced phononic band gaps generated by a compliant
518 axial to rotary motion conversion mechanism. *J Sound Vib.* 2019;439:329-43.

519 [57] Bergamini A, Miniaci M, Delpero T, Tallarico D, Van Damme B, Hannema G, et al. Tacticity in
520 chiral phononic crystals. *Nat Commun.* 2019;10(1):4525.

521 [58] Ding W, Chen T, Chen C, Chronopoulos D, Assouar B, Zhu J. Experimental Observation and
522 Description of Bandgaps Opening in Chiral Phononic Crystals by Analogy with Thomson scattering.
523 Preprint doi:1048550/arxiv220704282. 2022.

524 [59] Park J, Lee D, Jang Y, Lee A, Rho J. Chiral trabeated metabeam for low-frequency multimode
525 wave mitigation via dual-bandgap mechanism. *Commun Phys.* 2022;5(1):194.

526 [60] Ding W, Chen T, Chen C, Chronopoulos D, Zhu J, Assouar B. Thomson scattering-induced
527 bandgap in planar chiral phononic crystals. *Mech Syst Sig Process.* 2023;186:109922.

528 [61] Zhao P, Zhang K, Hong F, Deng Z. Tacticity-based one-dimensional chiral equilateral lattice for
529 tailored wave propagation and design of elastic wave logic gate. *J Sound Vib.* 2022;521:116671.

530 [62] Zhao P, Zhang K, Qi L, Deng Z. 3D chiral mechanical metamaterial for tailored band gap and
531 manipulation of vibration isolation. *Mech Syst Sig Process.* 2022;180:109430.

532 [63] Barys M, Zalewski R. Analysis of inertial amplification mechanism with smart spring-damper for
533 attenuation of beam vibrations. *Matec Web of Conferences.* 2018;157:03002.

534 [64] Paukkeri R, Väänänen T, Lehtinen A. Microstructural analysis of polypropylenes produced with
535 heterogeneous Ziegler-Natta catalysts. *Polymer.* 1993;34(12):2488-94.

536 [65] Alamo RG, Kim M-H, Galante MJ, Isasi JR, Mandelkern L. Structural and kinetic factors
537 governing the formation of the γ polymorph of isotactic polypropylene. *Macromolecules.*
538 1999;32(12):4050-64.

539 [66] Ding W, Chen T, Chen C, Chronopoulos D, Zhu J. A three-dimensional twisted phononic crystal
540 with omnidirectional bandgap based on inertial amplification by utilizing translation-rotation coupling.
541 *J Sound Vib.* 2022;541:117307.

542 [67] Li H, Hu Y, Huang H, Chen J, Zhao M, Li B. Broadband low-frequency vibration attenuation in
543 3D printed composite meta-lattice sandwich structures. *Compos B Eng.* 2021;215:108772.

544 [68] Bilal OR, Ballagi D, Daraio C. Architected Lattices for Simultaneous Broadband Attenuation of
545 Airborne Sound and Mechanical Vibrations in All Directions. *Phys Rev Appl.* 2018;10(5):054060.

546 [69] Phani AS, Woodhouse J, Fleck NA. Wave propagation in two-dimensional periodic lattices. *J*
547 *Acous Soc Am.* 2006;119(4):1995-2005.

548 [70] Delpero T, Hannema G, Damme BV, Schoenwald S, Zemp A, Bergamini A. Inertia Amplification
549 in Phononic Crystals for Low Frequency Band Gaps. VIII ECCOMAS Thematic Conference on
550 Smart Structures and Materials SMART 20172017. p. 1657-68.

551 [71] Lu H, Wang X, Chen T. Design and quasi-static responses of a hierarchical negative Poisson's
552 ratio structure with three plateau stages and three-step deformation. *Compos Struct.* 2022;291:11591.

553 [72] Taniker S, Yilmaz C. Generating ultra wide vibration stop bands by a novel inertial amplification
554 mechanism topology with flexure hinges. *Int J Solids Struct.* 2017;106-107:129-38.

- 555 [73] Yuksel O, Yilmaz C. Shape optimization of phononic band gap structures incorporating inertial
556 amplification mechanisms. *J Sound Vib.* 2015;355:232-45.
- 557 [74] Ding W, Chen T, He Y, Chen C, Huang W, Jiang Z, et al. 3D syndiotactic elastic metastructure
558 with single-phase material. *Phys Lett A.* 2020;384(35):126892.
- 559 [75] Zhou Y, Ye L, Chen Y. Investigation of novel 3D-printed diatomic and local resonant
560 metamaterials with impact mitigation capacity. *Intl J Mech Sci.* 2021;206:106632.
- 561