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1	A NOVEL AND ENHANCED CALIBRATION OF THE TILTING WEIR
2	AS A FLOW MEASUREMENT STRUCTURE
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19 ABSTRACT

A large collection of laboratory measurements of piezometric head (*h*) and discharge (*Q*) are made over hydraulic models of a tilting weir at nine different angles, ranging from 25.7° to 90° within a 0.3-m wide flume. These measurements are corroborated with additional laboratory data taken within a 1-m wide flume across four inclination angles. The range of both inclination angle

 (θ) and flow scale examined in this study elucidate the nature of the head-discharge rating equation 24 beyond previous work. Results show that as θ decreases under a constant Q, the h over the weir 25 decreases in a monotonic fashion due to the shallower angle of attack of the flow, which results 26 in less curvature of the streamlines over the crest and therefore less deviation from the upstream 27 hydrostatic pressure condition. To incorporate this effect into the head-discharge rating equation, a 28 transformation of the h term is applied by multiplying the measured h that occurs over a tilting weir 29 by a correction factor to match the effective h that would occur if the weir were aligned perpendicular 30 to the flow at the same discharge. Thus, a modified form of the classical sharp-crested weir rating 31 equation can be used as a means for determining the value of Q for tilting cases to a high degree of 32 accuracy. The degree of accuracy is dependent upon dimensionless Reynolds and Weber numbers 33 describing the flow inertia in the approach to the weir in relation to respective viscous and surface 34 tension scale effects. This approach portends marked flow measurement enhancement for flow 35 conditions above a suggested inertial threshold. 36

37 PRACTICAL APPLICATIONS

To achieve reliable and equitable water distribution it is necessary to accurately estimate the 38 volumetric discharge of water flow through rivers and channels. One such method of accomplishing 39 this is by using hydraulic structures, such as the tilting weir. This structure not only can be used 40 to measure the discharge of water on a continuous basis, but also allows for regulating the water 41 level upstream of the structure. Presently, tilting weirs are often used to control water levels, but 42 the literature on their utilization as flow measurement structures has remained sparse. This study 43 presents a thorough analysis of laboratory experiments on scaled models of tilting weirs, where the 44 flow depth upstream of the structure and the inclination angle are used to calibrate an equation to 45 estimate the discharge to a high degree of accuracy. Operational guidelines to ensure minimum 46 uncertainty in measurement are also given. 47

48 INTRODUCTION

Tilting weirs (i.e. overshot gates or pivot weirs) have been in use since the late twentieth century for the purpose of regulating upstream water levels in open channel flow, typically within

the context of irrigation systems or spillway operation (Wahlin and Replogle 1994; Stringam and 51 Gill 2012; Lee et al. 2014; Bijankhan and Ferro 2017). These simple overflow structures operate as 52 a flat rectangular plate that is hinged to the bottom of the channel and inclined from the horizontal 53 through the use of pneumatic pressure vessels, a pulley mechanism, or a piston mechanism. It is 54 generally assumed that the crest of these structures is thin, so that a springing jet (i.e. nappe) forms 55 on the downstream side of the structure. In irrigation systems, tilting weirs allow for regulation 56 of the stage upstream of the structure to provide the pressure head necessary to divert flow from 57 a main supply canal into lateral canals. In spillway operation, these structures can be installed in 58 parallel to allow for careful release of flows and regulation of the reservoir stage in response to 59 varying supply and demand levels. 60

Tilting weirs require careful consideration of aeration demands for the underside of the plunging 61 nappe to be supported by atmospheric pressure (Bos 1976). If these structures are properly 62 designed, the downstream channel bed elevation will be set low enough to allow for the energy of 63 the supercritical plunging nappe to be dissipated, while also ensuring that a sufficient supply of 64 fresh air is provided to the pocket of atmospheric pressure supporting the nappe. For these reasons, 65 tilting weirs offer the potential of being practical hydraulic structures that serve the dual functions 66 of stage regulation and discharge measurement. The initial development of these structures as stage 67 regulation devices has been well documented and widely implemented (Clemmens et al. 2001; 68 Floodlist News 2017). However, their use as flow measurement devices remains in need of further 69 investigation. 70

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Fundamentals of the Weir Rating Equation

The development of a method for predicting the volumetric discharge over tilting weirs begins with an understanding of Torricelli's principle of a jet at the base of a reservoir issuing from a small orifice. Here, the velocity (*u*) of the jet can be related to the potential energy in the static supply reservoir by the function $u = \sqrt{2gh}$, with *h* being the elevation of the water surface above the jet outlet. This principle assumes the flow is inviscid and irrotational, and that the pressure distribution over the crest is hydrostatic. In his 1717 publication "de moto aquae mixto", Giovanni Poleni applied Torricelli's principle to approximate the discharge of a fluid over a perpendicular weir as occurring in a series of horizontal elements, the velocity of each being proportional to the distance of the fluid element from the free surface (Rouse and Ince 1963). From the resultant parabolic velocity profile, the depth-integrated unit discharge (q) can be approximated using what is known as Poleni's equation.

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$$\int_0^h u \,\mathrm{d}h \approx \frac{2}{3}\sqrt{2g}h^{3/2} \approx q \tag{1}$$

⁸⁴ However, it is known that because of the simplifying assumptions implicit in Eq. 1, a dimen-⁸⁵ sionless correction factor must be applied to the idealized efflux to account for the contraction of ⁸⁶ jet at the crest of the orifice. This has typically been called the discharge coefficient (C_d) when ⁸⁷ referring to weir flow, but it is in effect a coefficient of contraction (Kindsvater and Carter 1957). ⁸⁸ Its application to Eq. (1), with the addition of the crest length (b) to account for the transversal ⁸⁹ dimension, then yields the standard volumetric discharge rating equation for a sharp-crested weir.

$$Q = C_d \frac{2}{3} \sqrt{2g} b h^{3/2}$$
 (2)

This relatively simple equation forms the theoretical basis for the discharge equation of different types of sharp-crested weirs with variable geometries (Martínez et al. 2005). However, the simplifying assumptions of this approach should not be overlooked. As previously mentioned, this derivation approximates the flow between the upstream head measurement location and the crest as being inviscid, irrotational, and hydrostatic. For the case of accelerating flow over a weir, it is often taken to be a safe assumption that over the short reach from the measurement section to the crest, the energy loss due to internal rotational shear is negligible (Kindsvater and Carter 1957).

The assumptions of inviscid flow and a hydrostatic pressure distribution can be best examined by considering the theoretical case of a sharp-crested weir of infinite height. Here the assumption of potential flow is valid because the effect of the boundary is negligible. For this case, the streamlines will approach the crest radially so that a significant vertical velocity component will be present in the flow. This causes the pressure distribution to decrease significantly from the hydrostatic ¹⁰³ condition. This pressure drop correspondingly results in an acceleration of the flow over the crest ¹⁰⁴ and a reduction in the depth of flow at the crest to satisfy the continuity principle (Rouse 1932). ¹⁰⁵ The contraction coefficient of the water surface profile over the crest of an infinitely high weir has ¹⁰⁶ been shown from potential flow theory to be equal to Kirchhoff's coefficient for a jet issuing from ¹⁰⁷ a sharp-edged orifice, where $C_d = \frac{\pi}{\pi + 2} = 0.611$ (Rouse 1946b).

If a low weir is considered, so that the effect of the boundary cannot be neglected and the 108 assumption of inviscid flow no longer holds, it can be observed that the flow separates near the 109 base of the weir and a standing eddy forms. This phenomenon was briefly verified in this study by 110 preliminary particle-image velocimetry analysis, as shown by Fig 1. A boundary layer is present 111 due to viscous effects causing a deceleration of the flow near the channel bed, as well as the 112 upstream face of the weir. The magnitude of these respective boundary layers can be described 113 by the boundary-layer displacement-thickness (δ^*) (Kindsvater and Carter 1957). The effect of 114 the flow separation on the streamlines upstream of the weir was described by Rouse (1932) as 115 being akin to tilting the weir downstream, due to the fact that the streamlines above the separation 116 zone retain more horizontal momentum. This results in less curvature of the streamlines at the 117 crest and less contraction of the overflowing jet, so that the pressure at the vena contracta more 118 closely approximate the hydrostatic condition. The result is that for a low weir, more discharge can 119 be passed given the same upstream energy condition compared to that for an infinitely high weir 120 (Rouse 1932). 121

122 **Dimensional Analysis**

If the assumption is made that the tilting weir can be treated as a modified case of the classical sharp-crested weir, the general form of Eq. (2) can be applied to establish a head-discharge rating equation for the tilting weir by adding a term that accounts for the inclination angle (θ) of the weir with respect to the horizontal. In order to understand the relevant flow parameters and fluid properties that influence this equation, a dimensional analysis of the tilting weir is now considered. Simplifying assumptions are made that the weir crest length (b) is equal to the width of the approach channel (B), and the velocity field is uniform in the transversal direction. The flow

geometry can then be described by the parameters shown in Fig. 2. Here, L is the vertical height 130 of one weir when positioned perpendicular to the channel bed; p is the elevation of the weir crest 131 above the channel bed at a given θ ; and h is the piezometric head measured with reference to the 132 crest elevation in the uniform flow section upstream from the weir. The total hydraulic head (H), is 133 the sum of the piezometric head (h) and the kinetic energy head $(\alpha \overline{U}^2/2g)$, where \overline{U} is the velocity 134 averaged over the channel cross section in the approach, α is the kinetic energy correction factor, 135 and g is the gravitational acceleration. The dimensionless term, $\sin \theta = p/L$ describes the effect 136 that the changing angle of attack of the flow on the weir has on the streamlines over the crest, 137 and relates p and L. The relevant fluid properties in determining the flow over the weir are the 138 density, ρ , the dynamic viscosity, μ , and surface tension, σ . Since this is a case of open channel 139 flow, gravitational effects are critical and thus g is included as a relevant physical parameter. A 140 functional description of the unit discharge over a fully-suppressed (b/B = 1) tilting weir can be 141 written as: 142

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$$q = f(p, h, \sin \theta, g, \rho, \mu, \sigma)$$
(3)

¹⁴⁴ Utilizing Buckingham π analysis and a yet to be determined function (ϕ), the following dimension-¹⁴⁵ less equation can be written using the scaling terms $h \equiv L$, $h/\sqrt{gh} \equiv T$, and $\rho h^3 \equiv M$:

$$C_d = \frac{3}{2\sqrt{2}} Fr = \phi(h/p, \sin\theta, Re, We)$$
(4)

wherein:

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$$Fr = \frac{q}{\sqrt{g}h^{3/2}} \tag{5}$$

$$Re = \frac{\sqrt{g}h^{3/2}}{v}$$
 where $v = \mu/\rho$ (6)

$$We = \frac{\rho g h^2}{\sigma} \tag{7}$$

The first term in Eq. (4) is the classical discharge coefficient as shown in Eq. (2), which can be considered as a dimensionless Froude number (*Fr*) for the flow over the weir crest $(q/\sqrt{g}h^{3/2})$, 155

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that has then been transformed by the coefficient $3/2\sqrt{2}$ to make the inertial term in the numerator reflect the depth-averaged velocity from Poleni's approach in Eq. (1).

Regimes of flow over a weir can be divided into four types: clinging, laminar, free, and inundated 157 flow (Johnson 1996; Sinclair et al. 2022). These regimes and their respective effect on the shape 158 of the nappe were first described by Bazin (1898), and later summarized by Rao (1975). When the 159 flow over the weir is clinging, its inertia is not sufficient to overcome scale effects due to surface 160 tension. Here, the free surface clings to the weir plate and the nappe downstream of the crest is 161 not fully formed. This causes the same discharge to pass over the weir at a lower head compared 162 to when the nappe is fully formed, thus artificially increasing the discharge coefficient (Sarginson 163 1972). An additional flow regime can occur at low heads, even when the nappe is no longer 164 clinging to the downstream face. This flow regime has also been described as a depressed nappe 165 (Rao 1975). Here, viscous effects rather than surface tension effects are dominant. The flow inertia 166 is not sufficient to overcome viscous effects in the same manner that is observed within the free 167 flow regime; hence the velocity profile deviates from that expected for turbulent open-channel flow 168 (Rouse 1946a). This reduction in the upstream bulk velocity causes h over the weir to artificially 169 increase in order to satisfy conservation of mass, thus decreasing C_d below what is expected for the 170 free flow relationship (Schoder and Turner 1929). Inundated flow occurs on the opposite end of the 171 inertial spectrum, but the effect on C_d is nominally the same as for clinging flow. Here, the pressure 172 head above the weir becomes too great for the air pocket beneath the nappe to be sustained, the 173 nappe collapses, and the head over the weir decreases. Free flow then occurs in between the limits 174 of clinging/laminar and inundated flow, when the weir is operating within the conditions assumed 175 in Eq. (2). This means that dynamic similarity is present so that the flow dynamics are independent 176 of viscous and surface tension scale effects, and that the nappe downstream of the crest is fully 177 supported by a fresh supply of air at atmospheric pressure. 178

¹⁷⁹ If viscous and surface tension effects are taken to be negligible within the normal operating ¹⁸⁰ parameters of the weir, then determining the head-discharge equation over a tilting weir converges ¹⁸¹ upon an understanding of how C_d behaves as a function of the dynamic parameters h/p and $\sin \theta$.

As Rouse (1932) speculated, the reduction in $\sin \theta$ is likely to have the same effect as increasing the h/p ratio, by which the streamlines become more horizontal over the crest and the contraction coefficient increases to allow more discharge over the weir, given the same upstream head condition. The goal of this study is to determine the behavior of the discharge capacity of a tilting weir as it relates to h/p and $\sin \theta$, as well as the range of Reynolds number (*Re*) and Weber number (*We*) values that can be considered as a normal operating regime for free flow.

188 REVIEW OF PREVIOUS WORK

189 Sharp-Crested Weirs

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¹⁹⁰ Much careful experimental work was completed on the flow measurement equation for sharp-¹⁹¹ crested weirs from the late-nineteenth until the mid-twentieth century to find an empirical equation ¹⁹² to relate C_d (as in Eq. 4) to h/p under normal operating conditions, *i.e.* when *Re* and *We* effects ¹⁹³ can be neglected. Notable works on this topic are those of Smith (1886), Bazin (1898), Rafter ¹⁹⁴ (1900), Horton (1906), Schoder and Turner (1929), Rehbock (1929), and USBR (1948).

A seminal work on the topic that aimed to integrate the work of previous researchers, while 195 offering a large amount of new experimental data, was that of Kindsvater and Carter (1957). These 196 researchers at Georgia Tech University offered the novel contribution of taking into consideration 197 viscous and surface tension effects that become relevant at low values of h. They also considered the 198 effect on the flow dynamics for weirs where b/B < 1. Kindsvater and Carter (1957) convincingly 199 showed that earlier formulas for C_d in Eq. (4) of a nonlinear form were overly complex, and that a 200 simple linear equation could be used instead. They also suggested the use of empirically calibrated 201 correction factors, k_h and k_b to account for Re and We scale effects under small values of h and 202 b, introducing equivalent head (h_e) and equivalent crest length (b_e) terms to obtain an effective 203 discharge coefficient (C_e) for sharp-crested weirs: 204

$$C_e = \frac{Q}{\frac{2}{3}\sqrt{2g}b_e h_e^{3/2}} \approx 0.602 + 0.075h/p \tag{8}$$

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with $h_e = h + k_h$ (9) $b_e = b + k_b$ (10)

The authors recommended the use of a k_h value of 0.003 ft, or approximately 1 mm. The use of 210 k_h is important only when values of h are small. For water flow through air, Kindsvater and Carter 211 (1957) estimated that prediction error due to viscous and surface tension effects would be within 2% 212 as long as h > 0.06 m. It was also emphasized that k_h is an empirically calibrated correction factor 213 that could differ significantly depending on which set of experimental data was used. The original 214 authors attributed this to differences in experimental equipment. However, we hypothesize that this 215 discrepancy may be due to the fact that at low values of h, either We or Re effects can be dominant 216 depending on whether the nappe is fully formed or not. Evidence for this bifurcation in the flow 217 behavior at low h is given in Zhang et al. (2010). By giving k_h as a positive parameter, Kindsvater 218 and Carter (1957) implicitly assumed We effects to be dominant at low values of h, as explained 219 earlier. Therefore, the specific choice of k_h may be a point in need of finer calibration on a case 220 by case basis, but we report here the recommended value $k_h = 0.001$ m of the original authors. A 221 less uncertain choice is the value of k_b . Kindsvater and Carter (1957) found little variability in the 222 value of this parameter within the experimental data they examined, likely because it is a function 223 of the static contraction ratio parameter, and not the dynamic variable h. They recommended a k_b 224 value of -0.003 ft (-0.001 m) for the case where b/B = 1. 225

After the work of Kindsvater and Carter (1957), several other publications have further elucidated the nature of sharp-crested weir flow over a wide range of h/p values, including measurements of velocity and pressure distributions near the crest. (Rajaratnam and Muralidhar 1971; Ramamurthy et al. 1987; Swamee 1988; Sinclair et al. 2022).

Tilting Weirs

With the early explorations of the sharp-crested weir as a flow measurement device, examinations were also made to investigate the effect that tilting the upstream face of the weir had on the nappe profiles, and on the head over the crest at a constant discharge (Bazin 1898; USBR 1948;
 Abou Seida and Quaraishi 1976). Hager (1994) compiled the data from several of these types of
 studies to develop a flow-rating equation for tilting weirs of the form:

$$Q = \left[\frac{2}{3}C_e + 0.05\sin\left(\frac{3}{2}(90 - \theta)\right)\right]b\sqrt{2g}h^{3/2}$$
(11)

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where C_e is Eq. 8 of Kindsvater and Carter (1957), the effective discharge coefficient for a sharpcrested weir, and θ is in degrees. The second term in the brackets on the right hand side of Eq. 11 accounts for the effect of the changing θ on the overall discharge capacity of the structure. This study, published in German, represents an interesting first investigation into the flow-rating equation of a tilting weir. However, it was limited by the fact that the early experimental studies cited by Hager (1994) and used for calibration did not cover a wide range of flow conditions (i.e. h/p values).

The seminal work to date attempting to develop a flow-rating equation for tilting weirs is that 244 of Wahlin and Replogle (1994). The authors of this technical report studied two models in a large 245 flume (B = 1.2 m). As predicted earlier by Rouse (1932), they began with the assumption that the 246 general form of the sharp-crested weir rating equation as given by Kindsvater and Carter (1957) in 247 Eq. (8), could be applied to the modified case of the tilting weir with the addition of a corrective 248 factor to account for variation in θ . They chose to call this term C_a . By studying data previously 249 collected by the US Bureau of Reclamation (USBR) in analysis of the nappe profiles for flow over 250 variously inclined weirs (USBR 1948), the authors hypothesized that the relation between C_a and 251 θ would take the form of a second order concave polynomial. C_a can also be thought of as a 252 discharge amplification factor, Q_{θ}/Q_{90} , which represents the relative effect of θ on the amount of 253 discharge that is able to pass over the structure at a given head, compared to the perpendicular 90° 254 case. The results of the study proved to be fairly true to the original hypothesis, and a second order 255 polynomial was defined as: $C_a = 1.0333 + 0.003848\theta - 0.000045\theta^2$, with θ in degrees. Although 256 by definition C_a should be unity when $\theta = 90^\circ$, the authors did not verify this experimentally, and 257

their empirical equation for C_a does not reflect this.

Furthermore, the number of observations in their study for $\theta > 45^{\circ}$ were limited. For $\theta = 54.2^{\circ}$, 259 14 data points were collected over an h/p range of 0.11-0.3 (0.04 < h < 0.11 m), and for $\theta = 63.4^{\circ}$ 260 only seven observations were made over an h/p range of 0.09-0.22 (0.03 < h < 0.09 m). With such 261 small values of h, the observations may have been subject to We and Re scale effects (Rehbock 262 1929; Kindsvater and Carter 1957). Thus, it is a concern of the current authors as to whether 263 applying the inclination-angle correction factor given by Wahlin and Replogle (1994) for $\theta > 45^{\circ}$ is 264 advisable. Wahlin and Replogle (1994) did report that when using their approach for $20^{\circ} < \theta < 45^{\circ}$, 265 an average percent error of only 0.8% was found in their predictions of the laboratory data. At 266 the very least, this means there was low prediction bias for their approach within this range of θ . 267 From the experimental data published in the appendix of the study, the current authors computed 268 the mean absolute value percent error (MAPE) to be 2.5% for this same case, which gives a more 269 complete understanding of the magnitude of residual error from this approach. When applying 270 the laboratory-derived equation to a field-scale structure, it was found that their equation under-271 predicted the discharge in the field by an average of 6.34% (Wahlin and Replogle 1994). This raises 272 the question of how directly applicable laboratory-calibrated discharge-rating equations are to field 273 conditions (Hajimirzaie and González-Castro 2021). 274

Since the first study of Wahlin and Replogle (1994) little convergence has been observed in 275 the published experimental data on flow over the tilting weir. Prakash et al. (2011) conducted a 276 laboratory experiment of flow over inclined rectangular notched weirs, with $\theta = 30^{\circ}, 45^{\circ}, 60^{\circ},$ 277 75°, and 90°. They found that the discharge capacity of the structure increases as θ decreases, 278 and proposed an equation containing two fourth-order polynomials to account for this effect that 279 was accurate to within 10%. However, the authors did not validate their findings with previous 280 experimental data for flow over sharp-crested weirs, and their data have since found little agreement 281 with results published by subsequent authors. 282

Work completed in South Korea has focused on studying the flow characteristics of free and submerged flow over tilting weirs, as well as how sedimentation patterns are affected by the

²⁸⁵ inclination angle of the weir (Lee et al. 2014; Lee 2016; Lee 2018). These works have suggested ²⁸⁶ incorporating the upstream total hydraulic head (*H*) into the tilting weir rating equation in order ²⁸⁷ to model the effect of θ on the flow. However, this suggestion does not offer the most practical ²⁸⁸ means of implementation in the field, where measuring the kinetic energy head in the approach ²⁸⁹ channel is difficult. Unfortunately, these works have also yet to be published in English; thus their ²⁹⁰ applicability to the wider engineering community remains obscure.

²⁹¹ Nikou et al. (2016) investigated tilting weirs in a laboratory setting with different side contrac-²⁹²tions for $\theta = 20^{\circ}$, 40°, 60°, 80°, and 90°. However, these authors made their observations over only ²⁹³a small number of discharge scenarios, which resulted in calibration of the empirical coefficients ²⁹⁴to only \pm 15 % in prediction accuracy. Additionally, in a discussion piece, Khalili Shayan et al. ²⁹⁵(2018) convincingly showed that the assumption of the original authors that critical flow occurs ²⁹⁶over the weir crest was inaccurate.

Bijankhan and Ferro (2018) attempted to develop a rating equation for the tilting weir that 297 was meant to be independent of the dynamic head over the structure as measured by h/p. This 298 form is simpler in the sense that it depends on fewer parameters, but also less accurate because 299 it neglects how C_d increases linearly with h/p (Kandaswamy and Rouse 1957). Furthermore, 300 Bijankhan and Ferro (2018) did not explain how a fresh air supply was provided underneath the 301 nappe to ensure the condition of atmospheric pressure. From the experimental photos published 302 by the authors, it appears the flow was not sufficiently aerated and thus subject to inundation 303 effects, especially at low θ values where the relative difference between the crest elevation and 304 tailwater depth is small. Furthermore, in a discussion piece on a follow-up study that Bijankhan 305 and Ferro (2020) completed over submerged tilting weirs, Hajimirzaie and González-Castro (2021) 306 revealed a flaw in the dimensional analysis of the authors, originally developed by Ferro (2012). 307 This error introduces a spurious correlation between the independent and dependent dimensionless 308 parameters. This leads to the perceived conclusion that C_d , as in Eq. 4, is independent of h/p. 309 Several experimental investigations have shown that this conclusion is incorrect (Kandaswamy and 310 Rouse 1957; Wahlin and Replogle 1994; Sinclair et al. 2022). 311

Other researchers have studied the flow characteristics of tilting weirs using numerical modeling. 312 Mahdavi and Shahkarami (2020) utilized smoothed particle hydrodynamics to provide helpful 313 visualizations of the flow field both upstream and downstream of the weir crest. The authors also 314 validated their simulations with good agreement to available data from physical experiments, at 315 least when $\theta = 90^{\circ}$. However, the authors did not study a sufficiently large number of flow scenarios 316 to accurately calibrate a head-discharge equation dependent on θ . Khatamipour et al. (2022) studied 317 Reynolds-averaged Navier-Stokes (RANS) simulations of flow over tilting weirs in Open FOAM 318 using a two-dimensional k- ϵ turbulence model over 12 flow cases. The authors observed similar 319 behavior to Wahlin and Replogle (1994) in the relationship between Q_{θ}/Q_{90} and θ , but at smaller 320 magnitudes of Q_{θ}/Q_{90} than the original study. It was also apparent from the published qualitative 321 flow images that the flow was submerged for $\theta < 90^\circ$, and thus not viable for calibrating a free-flow 322 rating equation. 323

Most recently, Zerihun (2022) presented a flow-rating equation for tilting weirs derived from 324 first principles using the Boussinesq-type energy equation for depth-averaged flow. This allowed 325 for the creation of a numerical code that accounted for the presence of non-hydrostatic effects in 326 the sub-critical to super-critical transition region near the crest of the tilting weir. The results 327 of the numerical simulations agreed well with the experimental nappe profiles of Bazin (1898) 328 and USBR (1948). However, the flow-rating equation presented by Zerihun (2022) lacks practical 329 applicability in that it includes terms requiring a priori knowledge of the slope of the free surface 330 and overflowing nappe. 331

To summarize, the consensus on how to properly treat the influence of θ on the flow characteristics of a tilting weir has so far been lacking in the literature. The initially proposed approach of Wahlin and Replogle (1994) has not been sufficiently validated for $\theta > 45^{\circ}$, and the large majority of subsequent work has been lacking in experimental rigor. More recent work on determining the effect of the flow attack angle over other types of weirs has further highlighted that the influence of θ should not be neglected (Schmocker et al. 2011). Higher-order approaches, such as that presented by Zerihun (2022), offer promise for further exploration of this problem numerically but remain to be clearly linked to methods for practical implementation. In pursuit of that, we present here a thorough experimental effort at two unique geometric scales that entails a large number of careful observations across all relevant θ and h/p values for channel flow.

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EXPERIMENTAL SETUP AND MEASURING DEVICES

Experiments were first conducted in a recirculating flume manufactured by Armfield and located 343 in the Environmental Fluid Mechanics Laboratory (EFML) at Colorado State University (CSU), 344 USA. The flume was 5 m in length and 0.3 m wide, with a smooth neoprene channel bed and 345 glass side walls. The flume slope was set to be horizontal. Beyond the inlet condition provided 346 by the flume manufacturers, it was found necessary to install a honeycomb-like matrix of porous 347 material at the flume inlet to straighten and normalize the incoming flow. Furthermore, a series of 348 small roughness elements were installed just downstream of the honeycomb matrix in order to trip 349 the turbulent boundary layer and reproduce as close as possible a fully-developed open-channel 350 velocity profile. The downstream channel boundary condition was a free-overfall. 351

Nine tilting weir models were constructed at $\theta = 25.7^{\circ}, 29.1^{\circ}, 36.4^{\circ}, 45^{\circ}, 50^{\circ}, 53^{\circ}, 64^{\circ}, 71.2^{\circ},$ 352 and 90° by machining two sheets of smooth acrylic plastic of 12.7 mm (0.5 in.) thickness and 353 adhering them together using a waterproof adhesive. The length of the weir plate (L), as shown in 354 Fig. 2, was consistent between all models at 150 mm. The crest of each model was precisely cut 355 to be "knife-edged", with the angle between the downstream-sloping crest and the parallel sides of 356 the acrylic plates being 45°. Due to air entrainment caused by the plunging nappe, circular tubes 357 of PVC plastic were also installed along the glass side walls of the flume to provide access to fresh 358 air supply, as is shown in Fig. 3. This ensured that the air pocket beneath the overflowing nappe 359 was fully aerated to atmospheric pressure. The diameter of these tubes was sized to 19 mm (0.75 mm)360 in.) according to specifications detailed in Bos (1976). 361

Measurements of h and steady Q were made over a full range of flow conditions for each model, ranging from clinging/laminar flow at the minimum, to fully inundated flow at the maximum. At least 20 observations were made within the free-flow regime, where dynamic similarity is achieved and the inertia of the flow is great enough to be free from scale effects caused by viscosity and

surface tension, while also avoiding the under-pressurization of the nappe caused by inundated 366 flow (see Table 1). These free-flow observations were used to calibrate C_e as a function of h/p367 for each model. Observations of h were made at a distance of 0.8 m upstream of the model crest 368 using a vernier-type point gauge to an accuracy of ± 0.3 mm (0.001 ft). Measurement of steady Q 369 through the flume was made using an electromagnetic flow meter located within the supply pipe 370 to the channel with an accuracy of ± 0.3 L/s (0.01 cfs). Measurements of water temperature also 371 were taken, at an accuracy of $\pm 0.1^{\circ}$ C in order to precisely calculate temperature-dependent values 372 for μ and σ . 373

To ensure the initial results of the EFML experiments were both reproducible and scalable, 374 follow-up experiments were conducted in a similar recirculating flume located in the Hydraulic 375 and Hydromorphology Laboratory (HHLab) of the RiverLy research unit operating in the centre of 376 the Institut National de Recherche pour l'Agriculture, l'Alimentation, et l'Environnement (INRAE) 377 located in Lyon-Villeurbanne, France. This flume was larger than the EFML flume, at 1 m wide and 378 18 m in length, with a smooth glass channel bottom and side walls (see Fig. 3). These experiments 379 were conducted in the same manner as the earlier EFML experiments, with some minor variations. 380 Here, four tilting weirs of the same design as the EFML models were examined at $\theta = 33^{\circ}, 44^{\circ}, 59^{\circ},$ 381 and 90°, and L was reduced to 126 mm. The crest location of each model structure was kept 382 consistent at 11 m downstream of the flume inlet, which was a sufficiently long entrance length to 383 ensure a fully developed velocity profile. Measurements of h were made 0.8 m upstream of the 384 crest location using a trio of ultrasonic depth sensors measuring at 50 Hz and spaced evenly in the 385 transverse direction across the 1 m wide channel. Measurements of steady Q were made using an 386 electromagnetic flow meter recording at 50 Hz. The resultant time-series of h and Q were then 387 averaged over 60 s to yield mean values, and the measurements of h were further averaged across 388 the three sensors. The standard deviation in the measurements of h and Q were approximately 1 389 mm and 0.3 L/s, respectively. 390

391 **RESULTS**

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³⁹² Verification with Previous Experimental Work

The present goal is to utilize the tilting weir as a special case of the sharp-crested weir ($\theta = 90^{\circ}$). 393 In the interest of best-practice we choose here to verify our experimental results for the vertical 394 sharp-crested weir, labeled as experiments K and O in Table 1. In Fig. 4, it can be seen that 395 there is excellent agreement between the dimensionless rating equation of Kindsvater and Carter 396 (1957), given by Eq. (8), and the experimental data for the sharp-crested weir observed within 397 the current study. For the combined 47 observations shown in Fig. 4, the MAPE between the 398 observed C_e , and that predicted by Eq. (8) is only 0.55 %. Also plotted is the C_e equation given in 399 Rehbock (1929). It corresponds to: $C_e = 0.602 + 0.082(h/p)$. Rehbock (1929) recommended the 400 use of this equation with a k_h value of 1.25 mm as a very close alternative to his equation for the 401 "non-effective" discharge coefficient (i.e. $C_d = 0.605 + 0.08h/p$). The Rehbock (1929) equation 402 has the same intercept as Eq. (8), but the slope does not match the current experimental data as well 403 as that of Kindsvater and Carter (1957). Because these experiments were completed in relatively 404 narrow channels of width b = 0.3 and 1 m, we chose to use the recommended k_b value of 0.001 405 m from Kindsvater and Carter (1957) for a full-width (b/B = 1) weir in a narrow-width channel. 406 However, we found it unnecessary to implement the introduction of k_h because for our experiments 407 K and O, we did not observe the artificial increase in C_e attributable to the decrease in h that occurs 408 when We effects are present under clinging flow conditions. Rather, Re effects appeared to be 409 dominant for these experiments. Therefore, for all of the data presented in the current study, k_h is 410 taken as 0 m. We aim to define a threshold h value below which viscous and surface tension effects 411 are significant. 412

413 Determining the Effect of the Varying Inclination Angle

Fig. 5 plots the data collected at the EFML for this study across all θ values and flow conditions examined, representing *Q* ranging from 4.1-30.5 L/s. An attempt also was made to study the flow characteristics over a model where $\theta = 20^{\circ}$, but sufficient free flow conditions could not be achieved. Furthermore, as shown in Table 1, the maximum value of h/p for most experiments was approximately unity before the flow became inundated. This was due to the bed elevation

remaining constant downstream of the weir. This h/p = 1 limit for free flow was earlier predicted 419 by Rehbock (1929) for channels where there is no drop in bed elevation downstream of the crest. 420 HHLab data are shown in Fig. 6, representing Q ranging from 10.7-84.1 L/s. In these plots, C_e 421 is calculated exactly as in Eq. (8), with $k_b = -0.001$ m and $k_h = 0$ m. It can be seen that for the 422 linear equation representing the dimensionless head-discharge rating equation, given by the form 423 $C_e = c + m(h/p)$, both the intercept (c) and slope (m) of this line tend to increase as θ decreases. 424 This can be understood by realizing that the tilting weir is an obstruction to the flow, and the amount 425 of obstruction presented to the flow by the weir depends upon θ . Generally, the streamlines over 426 the crest become more horizontal as θ decreases and as h/p increases. Due to this reduction in the 427 contraction of the overflowing jet, less upstream potential energy, represented by h, is needed for 428 the flow to pass over the obstacle presented by the weir for a given discharge. 429

To account for this effect, we propose a transformation to the h term in a manner similar to that 430 used by Kindsvater and Carter (1957) and originally examined by Rehbock (1929). These authors 431 suggested a linear transformation of the h term to account for the presence of viscous and surface 432 tension effects at low values of h for flow over a sharp-crested weir. In a similar fashion, one can 433 understand the effect of the changing θ for flow over a tilting weir as primarily affecting the h 434 term. The magnitude of this h term, representing the amount of potential energy required to pass a 435 certain flow over the weir crest, changes in response to shifting modes of approach kinetic energy 436 dissipation and momentum transfer, as brought on by the changing θ . 437

Whereas Kindsvater and Carter (1957) used a simple linear transformation to account for viscous 438 and surface tension effects for the static sharp-crested weir, an additional nonlinear transformation 439 must be utilized for the case of the variably tilting weir due to the additional degree of freedom 440 introduced by θ . Here, the flow dynamics are influenced both by the changing value of θ and the 441 relative inertial condition as defined by h/p. In keeping with the method of calculation of C_e used 442 by Kindsvater and Carter (1957), we choose to retain the notation of h_e as it is defined by Eq. (9). 443 However, it should be noted that for this study, h and h_e were equivalent since k_h was kept as 0 444 m. The purpose of our additional proposed correction factor is to transform the h_e observed over a 445

tilting weir to the h_e that would be observed under the same discharge if the weir were completely perpendicular to the bed (i.e. $\theta = 90^{\circ}$) using a term we call k_{θ} . An example transformation of the C_e values by k_{θ} is shown by Fig. 7. When applied to the h_e term, the head-discharge rating equation for a tilting weir becomes:

$$Q = C_e \frac{2}{3} \sqrt{2g} b_e (k_\theta h_e)^{3/2}$$
(12)

where $C_e = 0.602 + 0.075h/p$ from Eq. (8), and h_e and b_e are given by Eqn. (9) and Eq. (10), respectively. It should also be noted that the h/p term embedded in (12) within the C_e term has not been transformed by k_θ or k_h , but is in fact the directly measured ratio at any value of θ .

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For determining k_{θ} , the most relevant length scale in this flow scenario is in the vertical because this indicates the degree to which the pressure at the crest will deviate from hydrostatic due to the vertical momentum of the flow as it navigates the weir (Castro-Orgaz and Hager 2017). So, one can reasonably expect that k_{θ} could be understood most parsimoniously as being a function of sin θ , as was previously identified in our dimensional analysis. Using the independently measured discharge, unique values of k_{θ} can be found for each observation as in Eq. 13.

$$k_{\theta} = \left[\frac{Q}{C_e \frac{2}{3}\sqrt{2g}b_e h_e^{3/2}}\right]^{2/3}$$
(13)

For each experiment shown in Table 1, the average value of k_{θ} calculated by Eq. 13 was found and is given in Table 2. Then, a trial and error best-fit regression analysis was performed to fit an empirical function to the calibrated k_{θ} values. Gaussian, sum of sine, and power function fits were examined. The greatest agreement with the observed k_{θ} values in Table 2 was found using a two-term power function, which is given by Eq. 14.

$$k_{\theta} \simeq -\beta(\sin\theta)^{\lambda} + (1+\beta) \tag{14}$$

where for the current experimental data $\beta = 0.07$ and $\lambda = 4.5$. Eq. (14) is valid for $\theta = 25^{\circ} - 90^{\circ}$,

with RMSE = 0.0094 (see Fig. 8).

Eq. 14 is monotonic in the range of realistic θ values for tilting weirs. The implicit assumption in this approach is that the effect of the inclination on the contraction coefficient of the nappe is continuous, and that there is no intermediate value of θ between 0-90° which represents either a local maxima or minima in the discharge capacity for the tilting weir. This is an assumption supported by the hypothesis of (Rouse 1932) that the effect of increasing θ on the contraction coefficient would be akin to a decrease in h/p. As can be seen from Eq. 8 and Fig. 4, the influence of h/p on the discharge capacity of the weir also is monotonic.

The inclusion of the exponent λ on the sin θ term indicates that the effect of θ on the contraction 476 of the streamlines around the crest is nonlinear. This is likely due to the separation zone that 477 is present at higher values of θ , but eventually disappears below some threshold value. From a 478 cursory analysis of Fig. 8, a first approximation of this threshold value can be given as $\theta \approx 60^{\circ}$. 479 Zerihun (2022) also identified a similar threshold value of θ for the influence of the separation 480 zone on the flow dynamics. From the construction of Eq. 14, it is evident that when $\theta = 90^\circ$, k_{θ} 481 will be unity in Eq. (14), and Eq. (12) reduces to the classical sharp-crested weir rating equation 482 given by Kindsvater and Carter (1957). At a minimum value of $\theta = 20^{\circ}$, a maximum k_{θ} value of 483 approximately 1.07 is applied to the head term. Physically, this means that the same discharge is 484 passing over the structure inclined at $\theta = 20^{\circ}$ with a 6.5 % reduction in h compared to that discharge 485 passing over a sharp-crested weir inclined at $\theta = 90^{\circ}$. 486

487 Analyzing Uncertainty and Considering Operational Constraints

In addition to the 392 free-flow depth-discharge observations shown in Table 1, 48 additional observations across θ values were made, but were excluded from the calibration of Eq. (14) since these observations were made outside the normal operating regime of the tilting weir. This concerned flows at low values of *h* that were subject to scale effects due to viscosity and surface tension. Flows outside the normal operating regime were identified either visually by the presence of a clinging nappe, or by regression analysis. If the inclusion of a certain data point at a low value of *h* noticeably reduced the regression coefficient of the linear C_e vs. h/p equation shown in Figs. ⁴⁹⁵ 5 and 6, it was excluded. These additional observations are included in Figs. 9 and 10 to show the
 ⁴⁹⁶ influence of scale effects on the accuracy of the rating equation to predict discharge.

Fig. 9 shows a plot of C_e versus h/p computed from Eq. (12) for the 440 observed sets of Q, h, h/p and θ , in comparison to Eq. (8). In Fig. 10, \mathcal{H}_{error} is defined as the relative percent difference between the observed discharge during experimentation (Q_{obs}) and the predicted discharge (Q_{pred}) given by Eq. (12):

$$\mathscr{M}_{error} = \frac{\mathcal{Q}_{obs} - \mathcal{Q}_{pred}}{\mathcal{Q}_{obs}} \times 100 \tag{15}$$

It can be seen from Fig. 9 that there is a general relationship between h/p and \mathcal{M}_{error} , with h/p = 0.6 being an approximate divergence point below which \mathcal{M}_{error} beings to grow. This value was previously identified by a separate study as an indicator of the lower threshold for the normal operating regime of a sharp-crested weir (Sinclair et al. 2022). For all observations of the current study, when $h/p \le 0.6$, the mean of $|\mathcal{M}_{error}|$ is 1.75%. When h/p > 0.6, this value is reduced to 1.15%.

To explore thresholds related to *Re* and *We* scale effects, we also define updated *Re* and *We* numbers, where the length scale has been corrected by k_{θ} :

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$$Re_{\theta} = \frac{\sqrt{g}(k_{\theta}h)^{3/2}}{v} \tag{16}$$

$$We_{\theta} = \frac{\rho g(k_{\theta}h)^2}{\sigma} \tag{17}$$

Figure 10 and Table 3 reveal the relationship between Re_{θ} , We_{θ} , and the relative percent error ($\%_{error}$) between the predicted and observed discharge for the 440 observations within the current study. Two flow regimes were identified. The threshold values of Re_{θ} and We_{θ} were identified by determining when the MAPE for the lower regime grew beyond 2 %, and the prediction bias, represented by the mean value of $\%_{error}$, was as close to zero as possible for the upper regime. We choose $\pm 2\%$ as a somewhat arbitrary threshold, but this level of accuracy can generally be considered as excellent for flow measurements over hydraulic structures (Replogle 2002).

Regime I occurs when $Re_{\theta} \leq 3 \times 10^4$ & $We_{\theta} \leq 3 \times 10^2$ and represent flows that occur below 519 the absolute minimum required inertial state in the approach for the weir to be considered operating 520 in free flow. Below this threshold, scale effects cannot be justifiably neglected. Fig. 10 and Table 521 3 show that for the majority of points, this results in the observed discharge being smaller than 522 what is predicted for this regime, indicating that Re_{θ} effects were more significant here than those 523 due to We_{θ} . This can be most readily explained by understanding that as viscous effects become 524 more significant at low values of Re_{θ} , the piezometric head (h) must grow larger to account for the 525 decrease in the velocity head. Regime II occurs when $Re_{\theta} > 3 \times 10^4$ & $We_{\theta} > 3 \times 10^2$. Here, 526 scale effects are minimized and the accuracy of the head-discharge equation is optimized. This 527 represents the ideal operating regime for a tilting weir. Table 3 shows that for this regime, the 528 average \mathcal{M}_{error} is only -0.08 \mathcal{M} , indicating the bias in the predicted discharge has been removed 529 compared to Regime I. 530

These threshold values for Re_{θ} and We_{θ} can also be translated to more practical values of h, 531 if we assume common values for physical constants and the fluid properties of room-temperature 532 water (i.e. $g = 9.81 \ m/s^2$; $\mu = 10^{-3} \ Ns/m^2$; $\rho = 1000 \ kg/m^3$; $\sigma = 0.0728N/m$). These 533 approximated values are shown in Table 3. Unsurprisingly, we find a similar result as Kindsvater 534 and Carter (1957) for the threshold value of $h \approx 0.05$ m as the point above which scale effects 535 can be justifiably neglected. A similar finding was also reported in Hager (1994), but Ranga Raju 536 and Asawa (1977) advocate for a higher threshold for avoiding scale effects of h > 0.11 m. The 537 frequency histograms shown in Fig. 10 also can be used to estimate the probability of a certain 538 level of measurement accuracy within each regime (see Table 3). 539

540 DISCUSSION

⁵⁴¹ We now turn to some qualifications for the findings of the current study. First, it can be observed ⁵⁴² from the residuals in Fig. 8 that the greatest uncertainty in the the value of k_{θ} occurs between ⁵⁴³ $\theta = 45^{\circ}$ and 65° . We hypothesize that this is partially due to experimental error and uncertainty ⁵⁴⁴ in the aeration condition of the downstream nappe. Another possible explanation is the instability ⁵⁴⁵ of the separation zone in this region. Additionally, in Fig. 8, it can be seen that a large gap in

the observed k_{θ} values exists between the four experiments with $\theta > 60^{\circ}$ and the remainder of 546 the experiments. One possible explanation for this behavior could be the disappearance of the 547 separation zone below $\theta \approx 60^\circ$, which merits further exploration. 548

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An analysis of Fig. 7 reveals that k_{θ} may be dependent on h/p as well as sin θ . This dependence 549 is observable for experiment D in Fig. 7; but a consistent dependence was not found between all experiments upon further investigation. Additional experimentation to investigate the link between k_{θ} with both h/p and $\sin \theta$ is warranted. 552

Finally, additional observations are needed for θ between 60° and 90°. The experimental data 553 of the current study is sparse in this range, where the dynamics of the separation zone are likely 554 to change most significantly. It also remains to be seen how applicable these laboratory findings 555 can be when the flow problem is scaled up and implemented in the field. Additional complexities 556 related to the amount of friction in the approach channel, the approach geometry, the impact of 557 sedimentation over time, and changes to the surface roughness of the weir plate must be considered. 558

CONCLUSIONS 559

Observations of head and steady discharge were made for 440 flow cases in two unique ex-560 perimental facilities, over hydraulic models of tilting weirs with inclination angles ranging from 561 $\theta = 25^{\circ}$ to 90°. This allowed for the calibration of a head multiplication term as a function of the 562 inclination angle, given by k_{θ} in Eq. (14). This empirical equation accounts for the fact that at 563 a constant discharge, the head upstream of a tilting weir generally decreases with the inclination 564 angle, due to the improved navigability of the streamlines over the crest and reduction in non-565 hydrostatic effects near the crest. The factor k_{θ} then acts to adjust the observed head over a tilting 566 weir to that expected if the weir was perpendicular to the channel bed. Then, a modified form 567 of the classical sharp-crested weir rating equation, given by Eq. (12), can be applied to predict 568 steady discharge. If the inertial state of the approach flow is kept above a threshold value of at least 569 $Re_{\theta} = 3 \times 10^4$ & $We_{\theta} = 3 \times 10^2$ ($h \approx 0.05$ m) and the nappe remains fully aerated, the tilting weir 570 can be considered within an ideal operating regime where the mean absolute value of the error 571 in the predicted discharge is only 1.3 % for the current experimental data (see Table 3). We also 572

⁵⁷³ identify an h/p value of 0.6 as a more immediately applicable threshold for the normal operating ⁵⁷⁴ regime, but note that this value will likely not hold for especially small weirs (i.e. L < 0.1 m).

The results of this study work to synthesize and expand significantly upon the current level of 575 knowledge within the literature. The most extensive experimental study to date on developing a 576 head-discharge rating equation for tilting weirs is that of Wahlin and Replogle (1994). The current 577 study expands on this work by developing an equation having greater continuity with previous work, 578 in that it links back to the classical empirical data for the sharp-crested weir, being applicable for 579 $25^{\circ} \le \theta \le 90^{\circ}$. We also offer a more parsimonious monotonic solution to correct for the effect of 580 the dynamic weir inclination angle that relies less on arbitrarily maximizing goodness-of-fit metrics 581 for the observed empirical data, and instead aims to give insight to the underlying flow dynamics. 582 In terms of calibration accuracy, the current study contains a much larger number of observations 583 of the flow at θ between 45° and 90° than in previous studies. As shown by Fig. 11, this results in 584 the data from the current study agreeing well with the findings of previous researchers for certain 585 values of θ , while also being able to more fully reveal the nature of the relationship between Q_{θ}/Q_{90} 586 and θ across a wide range of operational θ values. In summary, the findings of this study promote 587 the tilting weir as a practical and versatile structure not only for stage regulation but also for flow 588 measurement, giving operators of open-channel flows the opportunity for much greater control and 589 efficiency within their system. 590

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TOWARDS PRACTICAL IMPLEMENTATION

⁵⁹² Step-by-step instructions taken from the approach laid out in this study for using the tilting weir ⁵⁹³ as a water flow measurement structure are as follows:

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1. Measure the inclination angle (θ) of the weir from the horizontal and the crest elevation (p) relative to the upstream channel bed.

- ⁵⁹⁶ 2. Measure the depth of water flowing over the weir crest (*h*) at a distance of 3-4 times the ⁵⁹⁷ anticipated *h*, upstream from the crest location. For most accuracy, ensure h > 0.05 m.
- ⁵⁹⁸ 3. Calculate h/p. For channels without a downstream drop in bed elevation below the crest,

599	h/p should remain below 1.
600	4. Using h/p , calculate C_e from Eq. 8.
601	5. Calculate h_e and b_e using the recommended values of k_h and k_b of Kindsvater and Carte
602	(1957), as shown by Eqs. 9 and 10, respectively.
603	6. Calculate k_{θ} using Eq. 14. For most accuracy, ensure $25^{\circ} \le \theta \le 90^{\circ}$.
604	7. Using Eq. 12, estimate the discharge Q .
605	APPENDIX
606	DATA AVAILABILITY STATEMENT

- ⁶⁰⁷ All 440 head (h) and discharge (Q) measurements across all experiments are available in a ⁶⁰⁸ repository online in accordance with funder data retention policies at
- https://efml.engr.colostate.edu/jhe_pugh_etal_2024/.

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614 DISCLAIMERS

- ⁶¹⁵ The authors declare that they have no known competing financial interests or personal relation-
- ships that could have appeared to influence the work reported in this paper.

617 NOTATION LIST

618 The following symbols are used in this paper:

 θ = angle between the tilting weir and the horizontal channel bed;

 \overline{U} = bulk (cross-section averaged) channel velocity [L/T];

 C_d = discharge coefficient;

 C_e = effective discharge coefficient;

 b_e = effective crest length as corrected by k_b [L];

 h_e = effective head as corrected by k_h [L];

p = elevation of the weir crest above the channel bed [L];

 Q_{θ}/Q_{90} = flow amplification factor

 ρ = fluid density [M/L³];

 μ = fluid dynamic viscosity [M/LT];

 σ = fluid surface tension [M/T²];

Fr = Froude number;

g = gravitational acceleration constant [L/T²];

 α = kinetic energy correction factor;

 $\beta = k_{\theta}$ Eq. empirical coefficient 1;

 $\lambda = k_{\theta}$ Eq. empirical coefficient 2;

b = length of weir crest [L];

 k_b = linear weir crest length correction term [L];

 k_h = linear head correction term [L];

c = linear intercept;

m = linear slope;

 k_{θ} = nonlinear head multiplication factor;

h = piezometric head above the weir crest [L];

Re = Reynolds number;

 $Re_{\theta} = \theta$ -adjusted Reynolds number;

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	H	=	total hydraulic head above the weir crest [L];					
	q	=	unit discharge per weir crest length $[L^2/T]$;					
	и	=	velocity of a jet issuing from a small orifice [L/T];					
621	L	=	vertical height of weir when perpendicular [L];					
	Q	=	volumetric discharge $[L^3/T]$;					
	We	=	Weber number;					
	$We_{ heta}$	=	θ -adjusted Weber number;					
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	Location	$\theta(^{\circ})$	<i>n</i> (mm)	n	h/n		Rea		Wea	
	Location	0()	p (mm)	п	. <i>n</i> / <i>P</i>				· · · ·	
					mın.	max.	mın.	max.	mın.	max.
Α	EFML	25.7	65.0	27 (22)	0.50	0.94	2.0E+04	5.2E+04	1.6E+02	5.7E+02
В		29.1	73.0	26 (21)	0.57	0.91	2.9E+04	5.9E+04	2.7E+02	6.7E+02
С		36.4	89.0	36 (29)	0.36	0.98	2.0E+04	8.8E+04	1.6E+02	1.1E+03
D		45.0	106	43 (39)	0.38	0.97	2.7E+04	1.1E+05	2.4E+02	1.6E+03
Е		45.0	106	23 (22)	0.43	1.05	3.3E+04	1.3E+05	3.1E+02	1.9E+03
F		50.0	115	20 (20)	0.35	0.96	2.8E+04	1.2E+05	2.5E+02	1.8E+03
G		50.0	115	27 (27)	0.34	1.02	2.6E+04	1.3E+05	2.2E+02	2.0E+03
Η		53.0	120	50 (43)	0.43	0.99	3.9E+04	1.4E+05	3.9E+02	2.1E+03
Ι		64.0	135	27 (27)	0.32	0.99	2.9E+04	1.6E+05	2.7E+02	2.6E+03
J		71.2	142	27 (24)	0.39	0.95	4.1E+04	1.6E+05	4.2E+02	2.5E+03
Κ		90.0	150	27 (26)	0.24	0.93	2.2E+04	1.6E+05	1.8E+02	2.6E+03
L	HHLab	33.0	68.6	32 (27)	0.53	1.08	2.4E+04	7.0E+04	2.1E+02	8.5E+02
Μ		44.0	87.5	25 (20)	0.53	0.99	3.4E+04	8.7E+04	3.3E+02	1.1E+03
Ν		59.0	108	28 (24)	0.41	0.88	3.0E+04	9.6E+04	2.8E+02	1.3E+03
0		90.0	126	22 (21)	0.38	0.96	3.2E+04	1.3E+05	3.0E+02	2.0E+03

TABLE 1. Description of experimental data collected. The number in parentheses after the number of observations per experiment (*n*) indicates the number of points used to calibrate the free-flow head-discharge equation. The range of h/p, Re_{θ} , and We_{θ} given for each experiment constitute the cases that were considered as free flow and used to calibrate the stage-discharge equation.

Experiment	θ (°)	$\overline{k_{ heta}}$	std. dev.
А	25.7	1.074	0.0081
В	29.1	1.064	0.0045
С	36.4	1.065	0.0083
D	45	1.050	0.0040
E	45	1.042	0.0040
F	50	1.064	0.0025
G	50	1.066	0.0036
Н	53	1.053	0.0016
Ι	64	1.009	0.0047
J	71.2	1.012	0.0034
Κ	90	1.000	0.0022
L	33	1.055	0.0055
Μ	44	1.054	0.0070
Ν	59	1.043	0.0032
0	90	1.000	0.0038

TABLE 2. Mean and standard deviation values of k_{θ} distributions for each experiment.

TABLE 3. Relationship between head-discharge equation prediction accuracy and inertial regimes as defined by Re_{θ} and We_{θ} . For use as a best practice guide for the normal operating regimes of tilting weirs.

Regime	Re_{θ}	We_{θ}	$\approx h (\mathrm{cm})$	п	prob. ±2%	Merror	$\left \overline{\mathcal{N}_{error}} \right $
Ι	$\leq 3 \times 10^4$	$\leq 3 \times 10^2$	$h \leq 5$	72	0.53	-1.45	2.08
II	$> 3 \times 10^4$	$> 3 \times 10^2$	h > 5	368	0.80	-0.08	1.25

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