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# High Precision Hubble Constant Determinations Based Upon a New Theoretical Relationship Between CMB Temperature and $H_0$

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## ABSTRACT

Based on considerable progress made in understanding the Cosmic Microwave Background (CMB) temperature from a deep theoretical perspective, this paper demonstrates a useful and simple relationship between the CMB temperature and the Hubble constant. This allows us to predict the Hubble constant with much higher precision than before by using the CMB temperature. This is of great importance as it will lead to much higher precision in various global parameters of the cosmos, such as the Hubble radius and the age of the universe. We have improved uncertainty in the Hubble constant all the way down to  $66.871 \pm 0.00043$  km/s/Mpc based on data from one of the most recent CMB studies. Previous studies based on other methods have rarely reported an uncertainty much less than approximately  $\pm 1$  km/s/Mpc for the Hubble constant. Our deeper understanding of the CMB and its relation to  $H_0$  seems to be opening a new era of high-precision cosmology. Naturally, our results should also be scrutinized by other researchers over time, but we believe that, even at this stage, this deeper understanding of the CMB deserves attention from the research community.

**Key words:** Hubble constant – CMB – Planck temperature

## 1 HUBBLE CONSTANT FROM CMB TEMPERATURE

Tatum et al. (2015) provided their formula for the cosmic temperature in the following form:

$$T_H = \frac{\hbar c^3}{k_b 8\pi G \sqrt{M_c m_p}} \quad (1)$$

where  $T_H$  is the Hubble temperature,  $k_b$  is the Boltzmann constant,  $\hbar$  is the reduced Planck constant,  $c$  is the speed of light,  $G$  is the gravitational constant,  $m_p$  is the Planck mass and  $M_c$  is the critical mass in the Friedmann (1922) equation  $M_c = \frac{c^3}{2GH_0}$  that also is part of Einstein (1916b) general relativity and the  $\Lambda$ -CDM cosmological model as well as other lesser known cosmological models. Equation 1 has recently been shown to be derivable from the Stefan (1879); Boltzmann (1879) law and appears to have a solid theoretical foundation in the standard laws of physics, see Haug and Wojnow (2023). The formula above can also be expressed as:

$$T_{CMB} = \frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_{H_0}}} = \frac{T_p}{8\pi} \sqrt{\frac{2l_p H_0}{c}} \quad (2)$$

where  $T_{CMB}$  is the CMB temperature,  $R_{H_0}$  is the Hubble radius,  $H_0$  the Hubble constant,  $c$  the speed of light,  $T_p$  is the Planck (1899, 1906) temperature and  $l_p$  is the Planck length. Equations 1 and 2

are just two ways to write the same formula, as proven in Haug and Wojnow (2023), so we can start with either of these and solve for  $H_0$ . Solving for  $H_0$  gives

$$H_0 = \frac{T_{CMB}^2}{T_p^2} \frac{64\pi^2 c}{2l_p} \quad (3)$$

And since the Planck length  $l_p = \sqrt{\frac{G\hbar}{c^3}}$  and  $T_p = \frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b}$ , if we insert that into equation 3 we get

$$\begin{aligned} H_0 &= \frac{T_{CMB}^2}{\left(\frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}}\right)^2} \frac{64\pi^2 c}{2\sqrt{\frac{G\hbar}{c^3}}} \\ H_0 &= \frac{T_{CMB}^2}{\left(\frac{1}{k_b} c \hbar \sqrt{\frac{c^3}{\hbar G}}\right)^2} \frac{32\pi^2 c}{\sqrt{\frac{G\hbar}{c^3}}} \\ H_0 &= \frac{T_{CMB}^2 k_b^2 32\pi^2}{c \hbar \sqrt{\frac{c^3 \hbar}{G}}} \quad (4) \end{aligned}$$

In the equation above we can even separate out the part only containing constants:

\*

$$\frac{k_b^2 32\pi^2}{c\hbar\sqrt{\frac{c^3\hbar}{G}}} = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}$$

$$= 2.91845601539730127466404708016 \times 10^{-19} \pm 3.28 \times 10^{-29} \quad (5)$$

And we could call this composite constant<sup>1</sup> Upsilon ( $\Upsilon$ ). The relation between the Hubble constant and the CMB temperature is, therefore, just a composite constant times the CMB temperature squared:

$$H_0 = \Upsilon T_{CMB}^2 \quad (6)$$

Still, naturally, part of this Upsilon composite constant contains  $G$ , and we would still naturally need to take into account uncertainty in  $G$  as well as the uncertainty in the CMB temperature when finding the uncertainty in the Hubble constant from this method, so the uncertainty will be the same as we will get from equation 4 as we will look closely at in the next section.

To summarize this section, all of the above formulae are effectively produced by different substitutions and rearrangements of equation 1. The results are the same with respect to calculating the value and precision of the Hubble constant for a given CMB temperature value. In the next section, we will demonstrate that this formula is not only of theoretical interest to describe the relationship between the Hubble constant and the CMB, but that it surprisingly leads to much higher precision in Hubble constant predictions after properly accounting for the full uncertainty in all input parameters.

## 2 HIGH PRECISION HUBBLE CONSTANT

Since the discoveries by [Lemaître \(1927\)](#) and [Hubble \(1926\)](#) extensive observational studies have been ongoing for many decades in order to increase the precision in the Hubble constant, something that is of great importance for a more precise understanding of the cosmos. See, for example, [The Virgo Collaboration et al. \(2017\)](#); [Hotokezaka \(2019\)](#); [Freedman and et al. \(2019\)](#); [Mukherjee and et al. \(2021\)](#); [Sedgwick et al. \(2020\)](#); [Riess and et. al \(2021\)](#); [Valentino and et. al \(2021\)](#); [Kelly and et. al \(2023\)](#); [Balkenhol and et. al \(2023\)](#). Even the more precise of these studies have not much less than 1 standard deviation uncertainty in their measured or estimated Hubble constant values in units of 1 km/s/Mpc.

In our formulae, we are using the NIST CODATA (2018) value for  $G$ , which is  $6.67430 \times 10^{-11} \pm 0.00015 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$ . Therefore, we are fully accounting for the uncertainty in  $G$ . Additionally, we consider the uncertainty in CMB temperature as provided in the respective studies we represent in Table 1. The speed of light  $c = 299792458 m \cdot s^{-1}$ , the reduced Planck constant (also known as the Dirac constant)  $\hbar = \frac{h}{2\pi} = 1.054571817 J \cdot s$  and the Boltzmann constant  $k_b = 1.380649 J \cdot K^{-1}$  that we need as inputs they have no uncertainty, as they are exactly defined according to NIST 2018 CODATA. This approach allows us to incorporate the complete input uncertainty into predicting  $H_0$ .

To convert our value into units km/s/Mpc we use the resolution

<sup>1</sup> One should be aware that to achieve such high precision for Upsilon requires high precision software, such as Mathematica. The uncertainty only comes from  $G$ , as all other constants in the composite constants are defined exactly in NIST CODATA 2018.

**Table 1.** This table shows Hubble constant estimates using our new calculation method from several different CMB studies

CMB Study	Temperature Measurement	High-Precision Method for $H_0$
Dhal et. al <a href="#">Dhal et al. (2023)</a>	$2.725007 \pm 0.00024k$	$H_0 = 66.87117 \pm 0.00043$
Noterdaeme et. al <a href="#">Noterdaeme et al. (2011)</a>	$2.725 \pm 0.002k$	$H_0 = 66.87083 \pm 0.097$
Fixsen et. al <a href="#">Fixsen (2009)</a>	$2.72548 \pm 0.00057k$	$H_0 = 66.89439 \pm 0.03$
Fixsen et. al <a href="#">Fixsen and et. al (2004)</a>	$2.721 \pm 0.010k$	$H_0 = 66.67466 \pm 0.49$

B2 adopted at the 2015 General Assembly of the International Astronomical Union, where the parsec is defined as exactly  $648000/\pi$  astronomical units, and for  $AU$  we use  $149597870700 m$  (IAU 2012 Resolution B1). So the conversion factor we need to multiply the results from our formula with  $1000 \times 648000/\pi \times 149597870700$  km/Mpc. There is no uncertainty in these conversion numbers, as they are merely conversion factors that are exactly defined.

For example, from the recent [Dhal et al. \(2023\)](#) CMB study, we obtain a value of  $H_0 = 66.87117 \pm 0.00043$  km/s/Mpc. This uncertainty of  $\pm 0.00043$  km/s/Mpc represents one standard deviation. Compared to other published methods and studies, our formulae provide for dramatically improved precision. We do not know of a previous study with much less than about 1 standard deviation below 1 km/s/Mpc. The breakthrough lies in a much deeper understanding of the relationship between the CMB temperature and the Hubble constant. Table 1 displays Hubble constant values ( $H_0$ ) estimated from a series of different CMB studies, but using our new high-precision method to determine  $H_0$  while accounting for the full uncertainty in the input parameters.

An outstanding issue in relation to the Hubble constant is the Hubble tension, as discussed in for example [Valentino and et. al \(2021\)](#); [Krishnan et al. \(2021\)](#). However, we do not aim to solve it here, but we mention it as we should be humbly aware that there may still be considerably more to understand about the Hubble constant and, therefore, possibly the CMB temperature. Only future research conducted by numerous researchers over time is likely to be able to resolve or fully comprehend the Hubble tension.

## 3 HIGH PRECISION HUBBLE COSMOLOGY

Due to a significantly higher precision in the determination of the Hubble constant, we can now predict various cosmological parameters which can employ the Hubble constant, such as the Hubble time and the Hubble radius, with much greater accuracy than before. The Hubble radius, denoted as  $R_{H_0}$ , is typically calculated using the formula  $R_{H_0} = \frac{c}{H_0}$ . Since there is no uncertainty in the speed of light  $c$ , the uncertainty in  $R_H$  is essentially the same as that in  $H_0$ . The Hubble time, defined as  $t_h = \frac{1}{H_0}$ , similarly benefits from the reduced uncertainty in  $H_0$ .

As another example, in the context of the  $\Lambda$ -CDM model, the critical mass, denoted as  $M_c$ , is calculated as  $M_c = \frac{c^3}{2GH_0}$ . Here, the uncertainty is slightly higher due to the additional factor of the gravitational constant  $G$ . Nonetheless, this method still provides significantly higher precision than any other approach, thanks to the considerably reduced uncertainty in the Hubble constant value.

## 4 CONCLUSION

Any of our quantum cosmology formulae displayed in Section 1 can predict  $H_0$  with much higher precision than before due to a breakthrough in understanding the CMB temperature in relation to  $H_0$ . Based on recent high-precision CMB temperature observations in

combination with our new and deeper understanding of the relationship between CMB temperature and  $H_0$ , we obtain a 1 standard deviation uncertainty of no greater than  $\pm 0.49$  km/s/Mpc, when using the 2004 data by Fixsen and et. al (2004), to as low as one standard deviation of 0.00043 km/s/Mpc from the 2023 data provided by Dhal et al. (2023). We claim that our formulaic method to find  $H_0$  from precise CMB temperature observations is quite revolutionary and deserves attention by the research community. Over time, the research community can either confirm our findings or point out possible weaknesses in our reasoning. So far, we have not identified any such weaknesses, despite searching for them. It indeed appears that the recent breakthrough in understanding the theoretical relationship between CMB temperature and  $H_0$  offers significantly improved precision regarding the large-scale global parameters of the universe. However, a theory must undergo scrutiny by multiple researchers over time to demonstrate its robustness. Therefore, the first step must be to make our discoveries accessible. We sincerely hope that this publication will encourage more researchers to look into this relationship between CMB temperature and  $H_0$ .

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## DECLARATIONS

### Conflict of interest

The authors declares no conflict of interest.

### Data Availability Statements

All data are incorporated into the article and its online supplementary material.

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