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Realization of a fractional quantum Hall state with ultracold atoms

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Strongly interacting topological matter [1] exhibits fundamentally new phenomena with potential applications in quantum information technology [2, 3]. Emblematic instances are fractional quantum Hall states [4], where the interplay of magnetic fields and strong interactions gives rise to fractionally charged quasi-particles, long-ranged entanglement, and anyonic exchange statistics. Progress in engineering synthetic magnetic fields [5–21] has raised the hope to create these exotic states in controlled quantum systems. However, except for a recent Laughlin state of light [22], preparing fractional quantum Hall states in engineered systems remains elusive. Here, we realize a fractional quantum Hall (FQH) state with ultracold atoms in an optical lattice. The state is a lattice version of a bosonic $\nu = 1/2$ Laughlin state [4, 23] with two particles on sixteen sites. This minimal system already captures many hallmark features of Laughlin-type FQH states [24–28]: we observe a suppression of two-body interactions, we find a distinctive vortex structure in the density correlations, and we measure a fractional Hall conductivity of $\sigma_H/\sigma_0 = 0.6(2)$ via the bulk response to a magnetic perturbation. Furthermore, by tuning the magnetic field we map out the transition point between the normal and the FQH regime through a spectroscopic probe of the many-body gap. Our work provides a starting point for exploring highly entangled topological matter with ultracold atoms [29–33].

The FQH effect emerges in two-dimensional electron gases from the combination of a magnetic field and repulsive interactions [4]. The magnetic field quenches the kinetic energy into highly degenerate Landau levels, among which the particles arrange themselves to minimize their interaction energy. In many cases FQH states are described by Laughlin’s wave function [23], whose characteristic pairwise correlated vortex motion results in a screening of interactions and strong anti-correlations at distances below the vortex size (Fig. 1a). FQH states show a topological robustness with exotic properties that are unseen in non-interacting systems, including quasi-particles with fractional charge, non-local topological en-

tanglement, and anyonic exchange statistics [4].

The desire to study these phenomena in a controlled environment has triggered effort to realize FQH states in quantum-engineered systems. Since the constituents of those platforms are typically charge neutral, synthetic magnetic fields are introduced through the Coriolis force in rotating systems [5–8, 35], or by engineering geo-

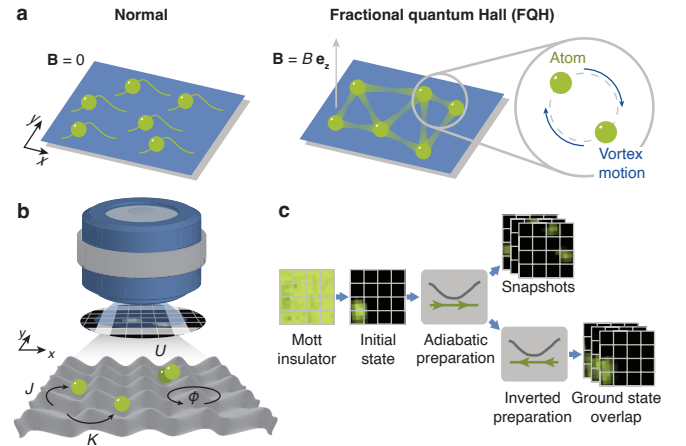


FIG. 1. **Realizing a fractional quantum Hall state in an optical lattice.** **a**, Without magnetic field, a two-dimensional gas remains in a superfluid (normal) state with weak correlations. In the presence of a strong magnetic field, the system may enter a FQH state with strong correlations, which (for Laughlin states) manifest through a simultaneous vortex motion between all pairs of atoms. The system thereby minimizes interactions while incorporating the angular momentum induced by the magnetic field. **b**, We realize such a system with two bosonic ^{87}Rb atoms in an optical lattice potential with 4×4 sites. The system is placed in the focus of a high-resolution imaging system, which allows us to take projective measurements of the quantum state with single lattice site resolution. The system is described by the Harper-Hofstadter model with tunneling rates K and J along x and y , respectively, magnetic flux ϕ per plaquette and on-site interaction U . **c**, The experimental protocol begins with a Mott insulator, from which we prepare the initial state with both atoms on neighbouring edge sites. We adiabatically change the Hamiltonian parameters until reaching the FQH state. We either take snapshots of the final state, or we invert the preparation protocol and map the final state back to the initial state in order to characterize the adiabaticity of the protocol.

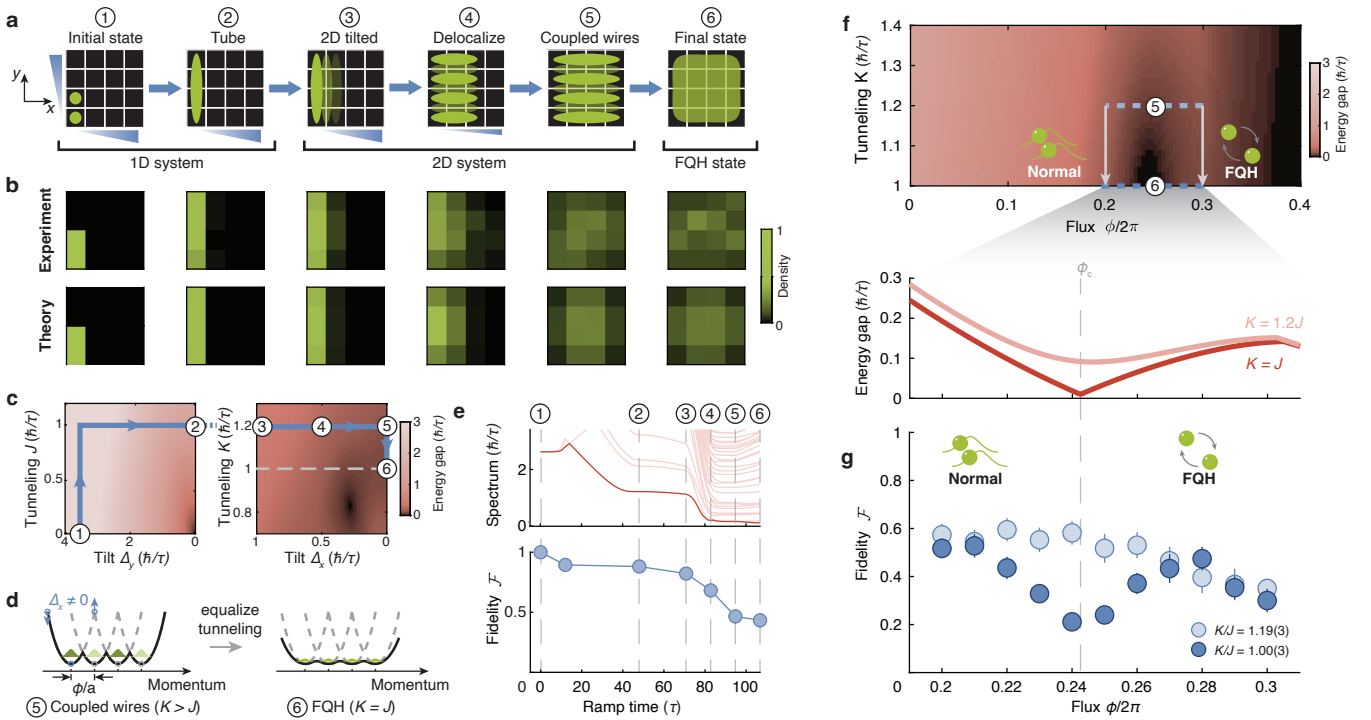


FIG. 2. FQH state preparation and gap diagram. **a**, Adiabatic preparation: (1) The ground state of the initial Hamiltonian corresponds to two repulsively interacting bosons on neighbouring sites. (2) Enabling tunneling J along y and reducing the gradient Δ_y homogeneously delocalizes the particles into one column. (3) Switch on tunneling K along x in the presence of a strong tilt and at flux $\phi/2\pi = 0.27$. (4) Particles spread over the entire 2D system as the tilt is reduced. (5) Up to this point $K > J$, such that the system resembles coupled horizontal wires. (6) Ramp to $K = J$ to reach the final state. **b**, Measurements of the site-resolved density confirm the delocalization into the 2D box potential, in agreement with exact numerical calculations [34]. **c**, The preparation scheme ensures optimal adiabaticity by avoiding closing of the energy gap between the ground state and the excited states (shown in units of the inverse tunneling time $\tau = 4.3(1)$ ms), as confirmed by numerical calculations of the many-body spectrum. **d**, The robustness of the scheme can be understood in a coupled-wire picture, where the quadratic dispersions of weakly coupled rows are offset by multiples of the momentum ϕ/a (with a the lattice constant). While excited single-particle states get shifted for $\Delta_x > 0$ (blue circles), the ground state in each wire remains robust. The two-body ground state is reminiscent of a charge density wave and adiabatically connects to a FQH state as the tunneling ratio approaches $K = J$. **e**, We quantify the preparation fidelity through the quantum state overlap $\mathcal{F} = \langle \psi_0 | \hat{\rho}_{\text{Final}} | \psi_0 \rangle$, inferred from measurements after inversion of the protocol. Despite a decreasing energy gap to the first (solid line) and higher excited states (shaded lines) in the energy spectrum during the preparation, a significant population remains in the ground state throughout the evolution. **f**, For $K = J$ the numerically calculated energy gap shows a closing, whereas it remains open for $K > J$. **g**, We spectroscopically reveal the gap closing through a loss of adiabaticity, signaled by a reduction of the ground-state overlap when preparing the system at the flux $\phi_c/2\pi \approx 0.25$ (dark). The reduction is absent when ending the preparation at step (5) with $K > J$ (light blue). Errorbars denote the s.e.m. and are smaller than the marker size if not visible.

metric phases [9–13, 15, 18, 19]. Recently, interaction-induced behaviour has been observed in several systems [14, 16, 20, 21], including a Laughlin state made of light [22]. Quantum gases in driven optical lattices [36] constitute a particularly promising platform to study FQH physics due to their exquisite control and large attainable system sizes, yet, reaching the strongly interacting regime remains a challenge.

Here, we realize a bosonic FQH state in a bottom-up approach using two particles in a driven optical lattice. The presence of few-particle FQH states in lattice models, also called fractional Chern insulators, is numerically well-established [24–28, 30, 37]. Conceptually they orig-

inate from flat Chern bands that take the role of the Landau levels. The proposed preparation schemes for those states, however, have exceeded experimental capabilities so far [38–40]. In our work, we devise and apply a novel adiabatic state preparation scheme, enabled by site-resolved control in a quantum gas microscope with bosonic ^{87}Rb (Fig. 1b) [14, 41]. We verify that the prepared state corresponds to the target FQH state by inverting the preparation scheme (Fig. 1c), and we sample density snapshots from the prepared state to confirm that it exhibits key properties of a FQH state, including a screening of two-body interactions, a vortex structure in the density correlations, and a fractional Hall conduc-

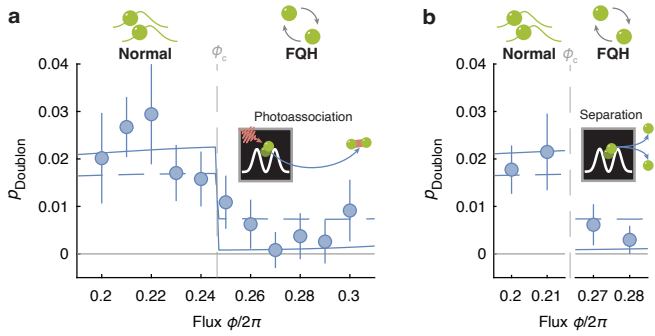


FIG. 3. **Suppression of two-body interactions.** We find a reduction of the doublon fraction in the FQH state, revealing the screening of interactions in the many-body wave function. **a**, Doublon fraction measurement by photoassociation, converting the doubly occupied lattice site into an empty one. **b**, Doublon fraction measurement by separating the particles on two different lattice sites prior to fluorescence imaging. Solid lines show exact calculations for the ground state, dashed lines take into account the finite ground-state overlap [34]. All errorbars denote the s.e.m.

tivity.

The system is governed by the interacting Harper-Hofstadter model (Fig. 1c), which describes the motion of particles on a square lattice in the presence of a magnetic field. In our setup the effective magnetic field is realized by Floquet engineering complex tunneling amplitudes with Raman-assisted tunneling processes [34, 36]. Within the Floquet-engineered Hamiltonian, we achieve independent control of the flux $\phi/2\pi$ per unit cell, the tunneling rates K and J along x and y , respectively, as well as the gradients Δ_x and Δ_y . The on-site interaction U remains constant and large compared to all other energy scales.

Our state preparation scheme is based on an adiabatic path that keeps the finite-size gap between the ground state and the excited states open throughout the protocol. It begins from an initial state of two localized atoms in the absence of any tunneling. First, we increase the tunneling J along y in the presence of a gradient Δ_y , while tunneling along x remains inhibited (Fig. 2a). Since J remains approximately constant for the remainder of the protocol [34], it sets the tunneling time $\tau = \hbar/J = 4.3(1)$ ms and the interaction strength $U = 6.7(1) J$. Subsequently Δ_y is adiabatically removed and we obtain a one-dimensional system in its ground state. A similar procedure is then performed along x : tunneling K is increased at constant flux $\phi/2\pi = 0.27$ and gradient Δ_x , then the gradient is adiabatically removed. Up to this point we keep the tunneling ratio at $K/J = 1.19(3)$. In a final step we bring the tunneling amplitudes to $K = J$ to reach the target state. At each step of the evolution we measure the system's density distribution and find agreement with an exact numerical prediction at the respective Hamiltonian parameters

(Fig. 2b).

The asymmetric tunnelling $K > J$ is key to realizing a favourable path between the initial product state and the final FQH state. This can be understood in a coupled-wire picture [34, 42], which interprets the single-particle spectrum as parabolic dispersions with tunnelling K , which are offset in momentum by ϕ/a . The tunnelling J creates a weak interwire coupling that enables particle motion in elongated cyclotron orbits. Repulsive interactions let particles avoid each, resulting in a Tao-Thouless-type charge-density wave (CDW) along y . This CDW state acts as a bridge between the initial state and the final state: on the one hand it is adiabatically connected to a 1D CDW by adding a gradient along x , and on the other hand it is adiabatically connected to a Laughlin state with vortex binding of composite particles by ramping to isotropic cyclotron orbits at $K = J$.

The success probability of the state preparation is given by the fidelity $\mathcal{F} = \langle \psi_0 | \hat{\rho}_{\text{Final}} | \psi_0 \rangle$, which measures the overlap between the density operator $\hat{\rho}_{\text{Final}}$ describing the state after the preparation protocol, and the ground state $|\psi_0\rangle$ at the final Hamiltonian parameters. Since $|\psi_0\rangle$ is a delocalized and entangled state, measuring \mathcal{F} directly with local observables is not possible. Instead, we map $|\psi_0\rangle$ back to the initial state by following the preparation with an identical, but reversed protocol. Assuming that the evolution during both ways is independent, the final ground state population is given by \mathcal{F}^2 , which can be directly measured because it equals the probability to measure the initial density distribution. We find a dominant ground state population throughout the evolution, and a fidelity of $\mathcal{F} = 43(6)\%$ to prepare the final state (Fig. 2e).

In the thermodynamic limit, N charge carriers are expected to enter the lowest bosonic Laughlin state at a filling factor $\nu = N/N_\phi = 1/2$, where N_ϕ is the number of magnetic flux quanta in the system. Including corrections for the finite size and particle number we expect the transition in our system to occur approximately within the flux range of $\phi/2\pi \approx 0.2 - 0.33$ [34]. A systematic understanding of the transition point in small systems is subject of current research [28]. In order to map out the transition between the normal state and the FQH state we use the adiabaticity of the preparation scheme as a spectroscopic signature for the energy gap (Fig. 2g). The fidelity \mathcal{F} is limited by the smallest energy gap during the preparation protocol, and therefore decreases when the energy of the ground and excited states approach each other. Repeating the protocol for different flux values shows a breakdown of the adiabaticity at $\phi/2\pi \approx 0.25$, indicating the location of the transition point. The observed transition point is in agreement with exact numerical calculations of the gap diagram (Fig. 2f,g). In contrast, when repeating the measurement at $K > J$ no breakdown of adiabaticity is visible, indicating that the many-body gap remains open until we reach homo-

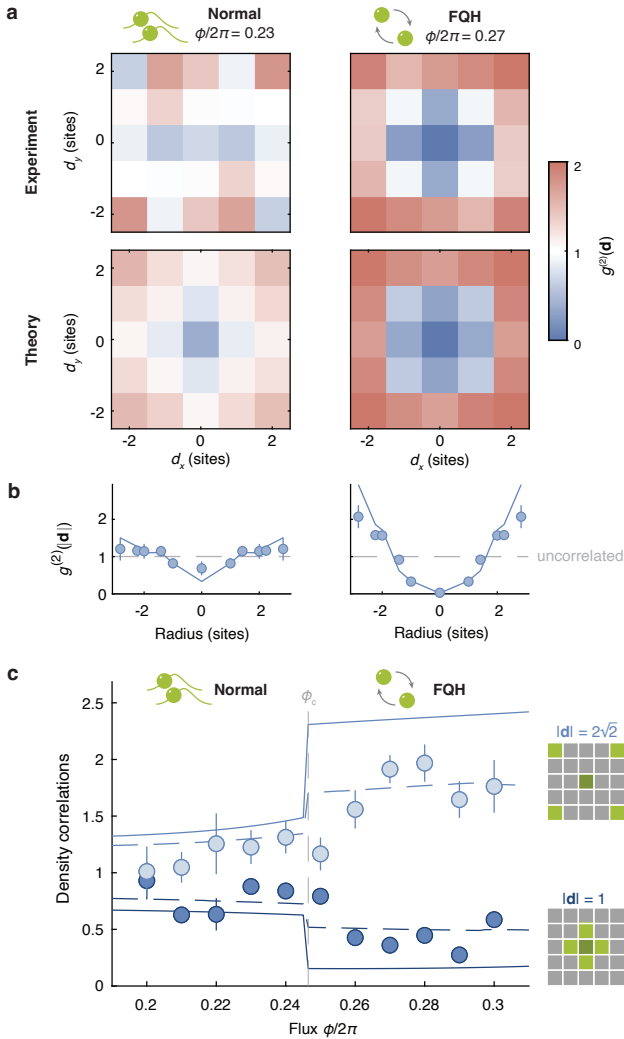


FIG. 4. **Vortex structure of correlations.** **a**, The density correlations (averaged over all bulk sites) $g^{(2)}(\mathbf{d})$ show a homogeneous behaviour in the normal regime, whereas they show a ring structure in the FQH regime. By construction the correlations are inversion-symmetric. **b**, The radial average shows how correlations in the FQH state are suppressed at short distance and enhanced at larger distance compared to the trivial state, in agreement with binding to a doubly charged vortex. **c**, The divergence between correlations at short distance ($|\mathbf{d}| = 1$) and large distance ($|\mathbf{d}| = 2\sqrt{2}$) coincides with the previously established transition point. Solid lines show exact calculations for the ground state, dashed lines take into account the finite ground-state overlap [34]. Error bars denote the s.e.m.

geneous tunneling $K = J$.

A hallmark of Laughlin-type FQH states is the screening of on-site interactions due to the pairwise vortex motion. In our two-particle system, the interaction energy simplifies to $E_{\text{int}} = \langle \psi_0 | \sum_{\mathbf{i}} U \hat{n}_{\mathbf{i}} (\hat{n}_{\mathbf{i}} - 1) / 2 | \psi_0 \rangle = U \times p_{\text{Doublon}}$, where $\hat{n}_{\mathbf{i}}$ is the number operator on site \mathbf{i} and p_{Doublon} is the global probability to observe the two particles on the same lattice site. We measure the dou-

blon probability in two different ways. In a first set of measurements we perform photo-association of the doublons into molecular states, whose excess energy ejects them from the lattice and converts the doublon probability p_{Doublon} into the probability to image neither atom (Fig. 3a). This process happens naturally at the beginning of the fluorescence imaging. In a second set of measurements, we break up the doublon prior to the imaging step and measure each of the two atoms individually (Fig. 3b) [34]. Both techniques reveal a reduction of the doublon fraction as the flux is increased beyond the transition point. Our results agree with exact numerical calculations for the ground state. When approximately accounting for the imperfect preparation fidelity by assuming only half the population to be in the ground-state, we numerically find a reduced contrast of the doublon suppression.

The mechanism from which the screening of interactions originates is the pairwise correlated vortex motion. In a simplified picture the $\nu = 1/2$ Laughlin state can be thought of as a correlated motion where each particle remains bound to the core of a doubly-charged vortex around every other particle. This results in a flat overall bulk density, however, the density correlations contain information about the underlying vortex structure. Averaging over many independent experimental realizations, we determine the reduced density correlations

$$g^{(2)}(\mathbf{d}) = \frac{N}{N-1} \frac{1}{N_{\text{Bulk}}} \sum_{\mathbf{i}, \mathbf{i}+\mathbf{d}} \frac{\langle \hat{a}_{\mathbf{i}}^\dagger \hat{a}_{\mathbf{i}+\mathbf{d}}^\dagger \hat{a}_{\mathbf{i}+\mathbf{d}} \hat{a}_{\mathbf{i}} \rangle}{\langle \hat{a}_{\mathbf{i}}^\dagger \hat{a}_{\mathbf{i}} \rangle \langle \hat{a}_{\mathbf{i}+\mathbf{d}}^\dagger \hat{a}_{\mathbf{i}+\mathbf{d}} \rangle}, \quad (1)$$

where the creation (annihilation) of a boson on site \mathbf{i} is described by the operator $\hat{a}_{\mathbf{i}}^\dagger$ ($\hat{a}_{\mathbf{i}}$). We average over all terms with \mathbf{i} and/or $\mathbf{i}+\mathbf{d}$ within the $N_{\text{Bulk}} = 4$ central bulk sites to obtain the reduced correlations at relative particle distance vector $\mathbf{d} = (d_x, d_y)$. The prefactor normalizes the correlations for finite particle number N such that $g^{(2)}(\mathbf{d})$ is larger (smaller) than unity if the densities are correlated (anti-correlated). By construction the correlator is inversion-symmetric, i.e. $g^{(2)}(\mathbf{d}) = g^{(2)}(-\mathbf{d})$. In the normal regime we observe approximately homogeneous correlations, whereas the FQH regime shows a contrast in correlations with increasing distance (Fig. 4a). To quantitatively analyze the density correlations we compute the azimuthal average as a function of distance $|\mathbf{d}|$. While the normal state remains mostly uncorrelated, the FQH state shows anti-correlations up to $r \lesssim \sqrt{2}$ sites and increased correlations for $r \gtrsim 2$ sites (Fig. 4b). Taking into account that the core size of a doubly charged vortex is about twice the magnetic length $l_B = 1/\sqrt{\phi} \approx 0.8$ sites, the observed pair correlations are consistent with the pattern of particles binding to doubly charged vortices. When measuring the correlations for different flux, we find that the onset of the vortex pattern coincides with the previously established transition point of $\phi/2\pi \approx 0.25$ (Fig. 4c).

Our observed FQH properties suggest that, despite its small size, our system may already exhibit precursors of a topological robustness. The paradigmatic signature of FQH states is the Hall conductivity $\sigma_H = \mathcal{C}\sigma_0$, which, normalized by von Klitzing's constant $\sigma_0^{-1} = R_K$, is directly related to the topological many-body Chern number \mathcal{C} . Although the Hall conductivity is a transport property, it is also encoded into the system's density distribution through Štředa's formula [28, 43–45]:

$$\frac{\partial \rho_{\text{Bulk}}}{\partial(\phi/2\pi)} = \frac{\sigma_H}{\sigma_0} \equiv \mathcal{C}. \quad (2)$$

Štředa's formula predicts an increase in the bulk density as a response to an increasing magnetic field that is directly proportional to the Hall conductivity. In an isolated system (such as ours) the response of the bulk density is enabled by particle exchange with the edge. Founded on general thermodynamics relations, it is valid for any insulating state [4], including strongly correlated states, and has been explicitly verified for few-particle FQH states on a lattice [28]. We measure the response of the density to an increasing flux and observe an enhanced bulk density within the FQH regime (Fig. 5a). Mapping out the density averaged over all bulk sites reveals a monotonous increase, from which we extract the Hall conductivity $\sigma_H/\sigma_0 = 0.6(2)$ through a linear fit to the data, comparable to the expectation $\sigma_H/\sigma_0 = 1/2$ for the $\nu = 1/2$ FQH state in the thermodynamic limit. The extracted Hall conductivity agrees with the exact numerical prediction of $\sigma_H/\sigma_0 = 0.6$ for the ground state. We find numerically that populations in the lowest excited states decrease the bulk density, however, for moderate variations in the ground-state fidelity (as measured in Fig. 2g) this does not substantially alter the prediction for the Hall conductivity.

Our work establishes ultracold atoms as a viable platform to study strongly correlated topological matter. The results provide a basis for extensions in several directions. Extracting the many-body Chern number from Hall responses [28, 46] or randomized measurements [47] is within reach for comparable system sizes. Realizing larger systems is challenging due to their reduced finite-size gap, which requires enhanced coherence times to maintain a significant ground-state overlap during the adiabatic preparation. Several approaches are currently explored to improve the coherence of Floquet systems, for instance, by destructive interference of heating channels [48]. Those larger systems host more complex FQH states such as Pfaffian states with non-abelian anyonic excitations [30, 33]. Furthermore, methods to isolate and move fractional quasi-particles with local potentials [31] and impurities [29, 32] can be implemented in our system, paving the way towards experiments on braiding statistics and fault-tolerant quantum information processing.

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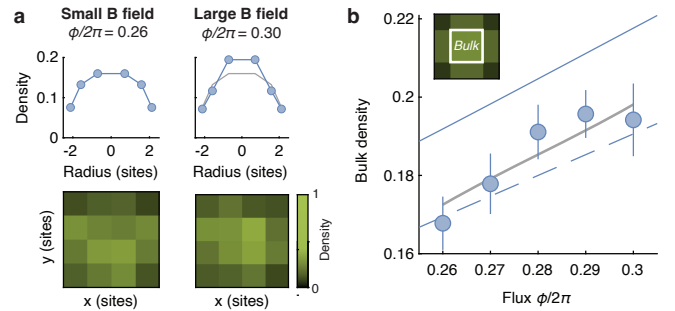


FIG. 5. **Fractional Hall conductivity.** **a**, Radial averages of the density (always for total particle number $N = 2$) show an enhanced probability to occupy the bulk sites for larger flux, in agreement with Štředa's prediction. The grey line in the right subpanel repeats the experimental data from the left panel for comparison. **b**, The bulk density is related to the Hall conductivity through the derivative $\partial \rho_{\text{Bulk}}/\partial(\phi/2\pi) = \sigma_H/\sigma_0$, where $\sigma_0^{-1} = R_K$ is von-Klitzing's constant. We extract a Hall conductivity $\sigma_H/\sigma_0 = 0.6(2)$ through a linear fit (grey line) to the data, in agreement with the fractional value $\sigma_H/\sigma_0 = 1/2$ in the thermodynamic limit. Solid blue lines show exact calculations for the ground state, the blue dashed line takes into account the finite ground-state overlap [34]. Error bars denote the s.e.m.

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METHODS

Experimental details

Mott insulator In each experimental realization we prepare a Bose-Einstein condensate of bosonic ^{87}Rb atoms in the $|F = 1, m_F = -1\rangle$ hyperfine state, which is loaded into a single plane of a one-dimensional lattice along the z direction with $a_z = 1.5\mu\text{m}$ lattice constant and $250 E_R$, where $E_R = \hbar^2/(2ma_z^2) = \hbar \times 0.25\text{ kHz}$ is the recoil energy for an ^{87}Rb atom of mass m . This lattice will remain on for the remainder of the experiment. We generate a superfluid with well-defined particle number from this quantum gas by first isolating a controlled number of atoms from the gas with an attractive dimple beam, and then loading them into a ring-shaped repulsive potential. The Mott insulating state is then reached by ramping up two optical lattices along x and y with lattice constants $a = 680\text{ nm}$ and depths $V_x = V_y = 45 E_R$ over 250 ms to create a repulsive two-dimensional square lattice, where $E_R = \hbar \times 1.24\text{ kHz}$ is the recoil energy.

Initial state preparation The initial two-atom state is prepared from the Mott insulator by holographically shaping two laser beams at 760 nm with digital micromirror devices (DMD, model: DLP5500 from Texas Instruments) to project site-resolved, repulsive potentials onto the optical lattice. The DMDs are placed in the Fourier plane with respect to the atoms, which allows us to project diffraction-limited arbitrary potentials that correct for optical wavefront aberrations in the imaging system [49]. The procedure is similar to the one described in [50]. In brief, we first optically confine a single line of lattice sites along y within the Mott insulator's unity-filling shell, and subsequently reduce down V_x . All atoms outside the confinement potential are ejected with a repulsive deconfinement beam of large gaussian shape, while the atoms within the projected confinement potential remain pinned on their lattice site. We then increase the lattice depth back to $V_x = 45 E_R$ and remove the confining DMD potential. In a second step, we select two atoms out of the single line of atoms by projecting a TEM20-like potential along y , and subsequently reducing down V_y . After removing all atoms outside the projected potential with the deconfinement beam, we end up with the initial two-atom state with a success rate of typically 95%. After post-selecting for the atom number $N = 2$, this procedure results in an initial state fidelity of 99.1(2) %.

Floquet engineering The Harper-Hofstadter Hamiltonian is generated through Floquet driving in the Bose-Hubbard model. The setup is described in detail in previous work [14]. In brief, we send two laser beams at a wavelength of 760 nm through the high-resolution objective, which overlap at the position of the atoms. The two beams have a detuning of $\omega_{\text{Raman}} = 2\pi \times 700\text{ Hz}$

with each other and therefore generate an moving lattice in the imaging plane. The angle of incidence of the beams can be controlled with piezo mirrors that are positioned in a plane that is conjugate to the atomic plane. We choose the angle such that the interference pattern has a wave vector of $k_x = \pi/a$ along x , i.e. with twice the lattice constant. The angle along y determines the Peierls phase ϕ per plaquette via $k_y = \phi/a$. The moving lattice also creates an off-resonant Floquet drive along the non-tilted direction, which leads to a renormalization of J by the factor $j_0(V_{\text{Raman}}/\hbar\omega_{\text{Raman}}) \approx 0.9$ in the final state. This is taken into account for the ratios K/J given in the manuscript. At the end of the preparation protocol the interaction strength in units of J is therefore $U = 8.1(1)J$. To simplify the description of Fig. 2 we show all parameters in units of the bare tunneling time τ . We also restrict the discussion to the effective Hamiltonian. A detailed representation of the preparation scheme in terms of the driven Bose-Hubbard model is shown in Fig. 6.

Fluorescence imaging and photoassociation The fluorescence imaging is performed after handing over the atoms to a lattice at 795 nm with the same lattice constant $a = 680\text{ nm}$ as the previous lattice.

Doublon splitting For the data in Fig. 2e,g and in Fig. 3b we use a full counting procedure to split the doublon prior to fluorescence imaging, similar to the one described in [50]. In brief, we first capture the atoms in a deep optical lattice. We then abruptly lower V_x such that the atoms can expand freely within each row, while we switch on a vertical wall potential between the first and the second column of the system. After a short evolution time we recapture the atoms on either side of the wall. Each atom distribution corresponds to a different Fock state. The populations of states involving atoms in the right side of the system are not resolved individually but only their sum. For Fig. 3b we remove atoms from the right half of the system prior to the expansion. In this case, double occupancies of left (right) half-rows signal the presence of a doublon on the left edge (left bulk) lattice site in the respective row.

Calibration The tunneling amplitudes J and K are calibrated by fitting the density after a single-particle quantum walk with $n(i,t) = j_i(Kt)$, where j_i are the Bessel functions of the first kind and i is the distance from the initial site. The interaction strength is calibrated through amplitude modulation of the lattice in the presence of a tilt. We also use amplitude modulation to calibrate the potential gradients Δ_x and Δ_y . The flux is calibrated with a precision of $\Delta\phi/2\pi = \pm 0.013$ by imaging the interference pattern of the Raman beams on a camera and extracting its wave vector through a fit.

Coherence time We perform Bloch oscillations of a single atom starting on one lattice site in order to measure the coherence time in the presence of Raman driving (Fig. 7). We find a $1/e$ -lifetime of $\tau_{\text{Raman}} = 1.25(7)\text{ s}$.

Box potential shaping We use the first DMD to project two wall potentials along y to confine the system along the x direction. Each wall has a gaussian shape along x with a $1/e^2$ -width of $w_0 = 0.7$ sites and is positioned at a distance of 1.5 lattice sites away from the edge site. Along y the wall has a smoothed flat-top profile with an extension of ≈ 10 lattice sites. We choose a wall height of 3.3 kHz, such that the energy offset on the first site outside the system is larger than the Raman frequency to suppress possible resonances. The gaussian distribution of the wall potential also causes a small energy offset on the edge site, which we confirm through density measurements in the static Bose-Hubbard system to be $< 6(1)$ Hz, small compared to the tunneling. With the second DMD we generate two wall potentials of the same kind along x to confine the system along the y direction.

Data analysis

Density We obtain the mean density on each site by averaging the site occupations from the density snapshots. This procedure neglects contributions from doublons, however, this effect is small compared to our statistical error.

Fidelity measurements We compute the ground state fidelity as the fraction of snapshots showing the initial density distribution after inverting the preparation scheme. This protocol also maps all excited states back to different initial density distributions, which allows us to determine their populations. We confirm that all excited state populations remain small compared to the ground state population. See section *Excited state populations* for more details.

Doublon measurement For the photoassociation measurement, the doublon fraction is given by $p_{\text{Doublon}} = p_0 - p_{\text{Offset}}$, where p_0 is the probability to detect zero atoms. The offset probability p_{Offset} contains errors in state preparation (zero atoms in the initial state), during the evolution (loss of both particles during the preparation), and the detection fidelity (false negative to detect zero particles instead of two). We calibrate the latter through repeated measurements of a Mott insulating state. The first two contributions are calibrated by fluorescence imaging of the final state after a short expansion in the lattice, which separates the two atoms. For the separation measurement we detect the doublons with a full counting procedure described in earlier work [14]. Since the procedure only works for two columns, we first remove the particles in one half of the system and then detect the doublons in the remaining half. The total doublon fraction shown in Fig. 3b is twice the detected doublon fraction.

Density correlations To extract the correlation function $g^{(2)}(\mathbf{i}, \mathbf{j})$ we first compute the correlation for each

pair of sites (\mathbf{i}, \mathbf{j}) within each snapshot, and then average them over all snapshots. We then move to the relative coordinate $\mathbf{d} = \mathbf{i} - \mathbf{j}$ by keeping the position of particle \mathbf{i} fixed and averaging over all available sites \mathbf{j} . We only take into account correlations with at least one particle on one of the four bulk sites. To obtain the radial average $g^{(2)}(|\mathbf{d}|)$ we compute the mean of the correlations over all pairs of sites with the same Euclidean distance $|\mathbf{d}|$. Our density snapshots do not account for on-site correlations, because these are determined by the doublon probability. The on-site correlations are given by $g^{(2)}(\mathbf{d} = 0) = (\sum_{i \in \text{Bulk}} p_{\text{Doublon}, i} / \langle \hat{n}_i \rangle^2) / N_{\text{Bulk}} \times N / (N - 1)$, where $p_{\text{Doublon}, i}$ is the probability for a doublon on site i . We use the doublon measurements from Fig. 3b at flux $\phi/2\pi = 0.21$ ($\phi/2\pi = 0.27$) to extract $p(\hat{n}_i = 2)$ in the normal (FQH) regime. The resulting on-site correlations are the ones shown in Fig. 4a,b.

Fractional Hall conductivity We obtain the mean density on each site as described above. We then compute the bulk density by taking the mean over the densities on the four central sites. For the linear fit we only include data for $\phi/2\pi \geq 0.26$ in order to remove contributions from adiabaticity breakdown at the transition point.

Uncertainties All given uncertainties are s.e.m., and are computed through bootstrapping.

Excited state populations

In the same way as the ground state overlap $\mathcal{F} = p_0 = \langle \psi_0 | \hat{\rho}_{\text{Final}} | \psi_0 \rangle$ is determined by the probability to recover the initial density distribution after reversing the preparation protocol, the overlaps $p_i = \langle \psi_i | \hat{\rho}_{\text{Final}} | \psi_i \rangle$ with excited states $i \geq 2$ appear as density distributions different from the initial state. The histogram of density patterns therefore provides information about the population of the individual excited states. Assuming that $p_i \ll p_0$ and neglecting transfer between excited states during the reversed preparation, the adiabatic transfer during the preparation and the reversion is the same and the final population of state i is given by $2p_i$.

With our full counting protocol we are able to detect the atom number on each site in the left column, as well as the sum of the atom numbers in each row of the other three two columns (Fig. 8a). Sorted by energy, eigenstates 1 – 10 after inversion involve both atoms in the left column, eigenstates 11 – 36 involve both atoms in the left two columns, and all higher eigenstates up to the Hilbert space dimension 136 involve at least one atom in the right two columns of the system. As a consequence, the protocol allows us to determine all excited state populations individually if only eigenstates 1 – 36 are occupied; populations in higher eigenstates are measured together with the respective eigenstate with both atoms in the left two columns.

The inferred populations are shown in Fig. 8b for flux

$\phi/2\pi = 0.27$. We find all excited state populations to be small compared to the population of the ground state.

Numerical simulation

All theoretical predictions were obtained via exact diagonalization of the interacting Harper-Hofstadter Hamiltonian for $U = 8J$. The results in Fig. 2 were obtained by computing the energy difference between the lowest two eigenvalues. The ground-state predictions for Figs. 3–5 were computed from the lowest eigenstate. We also compute a numerical prediction for a mixed state in order to have a closer comparison with the prepared state. The dashed lines in Figs. 3–5 show the results for a statistical average of 50% ground state population, where the remaining population was randomly distributed over the lowest 10 excited states.

Adiabatic preparation scheme

The adiabatic preparation scheme introduced in the main text realizes a favorable path connecting the initial product state to the two-particle lattice-Laughlin state. As we now discuss, interactions, gradients and kinetic terms complement each other along the chosen route and lead to minimal frustration, which ultimately leads to the large finite-size gap observed along the entire path.

In the first step, the initial two-site Mott state is melted into a one-dimensional liquid. The large on-site interactions effectively fermionize the bosons in this step, allowing to balance kinetic and interaction energies by introducing Pauli-type correlations. In the small four-site system with hard walls, this fermionized state has a strong charge-density wave (CDW) character. In the second step, this CDW-like state is adiabatically stretched along x-direction by lowering the gradient in this direction. Since the tunneling along x exceeds the tunneling along y, and the synthetic magnetic field is switched on, the particles remain localized in single-particle states resembling elongated cyclotron orbits. This allows them to minimize their kinetic (single-particle) energy while maintaining a strong CDW character that simultaneously minimizes interaction energies. The state reached at the end of this step resembles a Tao-Thouless CDW along y, which is known to be adiabatically connected to the isotropic Laughlin-like state. The latter is reached in the last step, by equalizing hopping amplitudes along x and y, which renders the underlying cyclotron orbits spatially isotropic. This still minimizes the kinetic and interaction energies without causing frustration: now vortex-binding fermionizes the underlying bosons and a liquid of composite fermions is realized, i.e. the two-particle Laughlin state.

Shape of the adiabatic ramp

All lattice and DMD laser beam powers are ramped exponentially in time. The tilt lowering along y is performed with a local adiabatic ramp, which optimizes the total adiabaticity for a given ramp duration by adapting the speed to the instantaneous many-body gap. We define the adiabaticity parameter γ through the differential equation:

$$1/\gamma = \frac{\partial \Delta_y(t)/\partial t}{\delta^2(\Delta_y)}. \quad (3)$$

Here $\delta(\Delta_y) = E_1(\Delta_y) - E_0(\Delta_y)$ is the instantaneous gap between the ground state and the first excited state, and $\Delta_y(t)$ is the profile of the tilt along y during the ramp. The local adiabatic ramp follows the specific profile $\Delta_y(t)$ that solves the above differential equation for a fixed γ . The ramp is adiabatic in the limit $\gamma \gg 1$. Here we use a generalized form of the above differential equation, which takes into account the lowest ten excited states, and additionally weighs them by the overlap matrix element with the ground state.

We use a local adiabatic ramp computed in the same way when decreasing Δ_x . During the last step of the preparation scheme (decreasing K from $K > J$ to $K = J$) the many-body gap remains almost constant, hence the local adiabatic ramp becomes approximately linear. We therefore use a linear ramp during this step.

On the existence of few-boson FQH states

In the following we discuss several aspects concerning the existence of FQH states in interacting Harper-Hofstadter systems with few bosons. First, we verify that the experimentally observed signatures are genuine FQH properties and not related to the discreteness of the lattice potential. We then use additional, numerically accessible observables to identify the FQH state: the orbital occupations, and (via comparison with an $N = 4$ system) topological quasi-hole excitations in the particle entanglement spectrum. Finally, we close with a qualitative discussion on the circumstances at which further FQH states than the $\nu = 1/2$ state may exist in few-boson systems.

Lattice-continuum crossover

We numerically verify that the observed signatures are genuine properties of the $\nu = 1/2$ FQH state and we hereby discuss their robustness to finite size effects. This is done by exploring the lattice-continuum crossover with a system size scaling at constant particle number $N = 2$. Consequently, the flux per plaquette at which the FQH

state appears decreases $\phi_c/2\pi \rightarrow 0$ as the system size is increased. In Fig. 9 we show exact diagonalization calculations for the experimental signatures for system sizes ranging from 3×3 sites to 14×14 sites, while the particle number remains constant at $N = 2$. We find that already the 3×3 system qualitatively agrees with the FQH state for all signatures:

Gap diagram (Fig. 2): In the left panel of Fig. 9a we show the energy gap between the ground state and the first excited state as a function of the flux $\phi/2\pi$. We find a single gap closing around $\phi_c/2\pi \approx 0.32$, marking the transition point between the normal and the FQH state. For imbalanced tunneling $K > J$ the gap closing disappears. The gap diagram looks qualitatively similar for all considered system sizes $L \times L$.

As L increases, we find that the flux $\phi_c/2\pi$ where the gap closing occurs becomes reduced. We extract the filling via $\nu_c = \rho_{\text{Bulk}}/(\phi_c/2\pi)$, where ρ_{Bulk} is the density in the bulk region, averaged over all sites within a radius of $3l_B = 3/\sqrt{\phi}$ sites from the system's center. Our choice of the bulk extension ensures a sufficiently large number of lattice sites to obtain a stable average, yet small enough to avoid contributions from edge sites. We find that the extracted transition point remains approximately constant at $\nu_c \approx 0.7$ for all considered system sizes (right panel), showing a consistent behavior of the normal to FQH state phase transition across the lattice-continuum crossover. The value $\nu_c > 0.5$ may be explained by edge effects, which are present at the phase transition even for large L because the transition occurs when there are just enough fluxes through the lattice to host a two-boson FQH state. While it is challenging to evaluate this effect quantitatively, our calculations of orbital occupations (see section *Additional numerical signatures of the FQH state*) show that the two bosons occupy three orbitals in the FQH regime, as expected from the few-particle expression $N_\phi = 2N - 1$.

Doublon suppression (Fig. 3): In the 3×3 system we compute the doublon probability p_{Doublon} and find a sharp reduction at the transition point $\phi_c/2\pi$ (Fig. 9b). We quantify the reduction by the suppression ratio $p_{\text{Doublon}}^{\text{FQH}}/p_{\text{Doublon}}^{\text{Normal}}$. The probability $p_{\text{Doublon}}^{\text{Normal}}$ ($p_{\text{Doublon}}^{\text{FQH}}$) is extracted from an interval of $\Delta\phi/2\pi = 0.02$ right before (after) the transition point. The resulting suppression ratio of $\approx 3\%$ is already remarkably close to the large-system limit of $\approx 3\%$ (right panel), suggesting an efficient screening of on-site interactions already in the 3×3 system.

Density correlations (Fig. 4): In Fig. 9c we show the reduced density correlations $g^{(2)}(|\mathbf{d}|)$. We find a depletion of correlations at $|\mathbf{d}|$ followed by an increase with the radial distance, which shows that the vortex structure is already present in the 3×3 system. As the system size is increased, the correlations at $|\mathbf{d}|$ approach zero, whereas at a distance of $|\mathbf{d}| = 3l_B$ they stabilize at a

value between one and two (right panel).

Fractional Hall conductivity (Fig. 5): We obtain the bulk density from an average over all sites within a radial distance $|\mathbf{d}| = 3l_B$ from the central site. For the 3×3 system this corresponds to all sites except for the corner sites; as L increases this condition includes sites within a circle with a diameter of about half the length of the system. We extract the Hall conductivity from a fit over an interval of $\Delta\phi_c/2\pi = 0.1\phi_c/2\pi$ with Středa's formula. The left panel of Fig. 9d shows the density at flux $\phi/2\pi \approx 0.34$, approximately at the center of the fitting window. When increasing the system size, the extracted Hall conductivity rapidly converges towards the fractional value of $\sigma_{\text{H}}/\sigma_0 = 1/2$, in agreement with the expectation for the thermodynamic limit (right panel).

Additional numerical signatures of the FQH state

We additionally verify the presence of a FQH state by computing its orbital occupations. The $\nu = 1/2$ Laughlin state with N bosons occupies $2N - 1$ orbitals in the lowest Landau level in the disk geometry. The distribution of the occupations approach a uniform distribution in the thermodynamic limit, in finite systems it shows moderate variations thereof.

We first study a system with $N = 4$ bosons on 7×7 lattice sites and repulsive interactions $U = 8J$ (Fig. 10a). The many-body spectrum shows three local minima of the energy difference between the group state and the first excited state, which we interpret as finite-size signatures of phase transitions. The occupation of the single-particle orbitals shows a qualitatively different distribution each time a minimum is crossed (histograms). Within the range of $0.2 < \phi/2\pi < 0.3$ it agrees with the expectation of a four-particle $\nu = 1/2$ Laughlin state.

An unambiguous identification of the FQH state can be obtained through the particle entanglement spectrum (PES) [51]. We choose the bipartition $N_A = N_B = 2$ of the particle sector. The PES is the spectrum of $-\log \hat{\rho}_A$, where $\hat{\rho}_A = \text{Tr}_B \hat{\rho}$ is the reduced density matrix obtained by tracing $\hat{\rho} = |\Psi_0\rangle\langle\Psi_0|$ over subsystem B . Laughlin quasiholes show a degeneracy due to a generalized exclusion principle, which is 15-fold in the present case of two bosons in seven orbitals. Indeed, we find a gap above the lowest fifteen states within the flux range $0.2 < \phi/2\pi < 0.3$, confirming the FQH nature of the ground-state in this parameter range.

We now turn to the experimentally studied system with $N = 2$ bosons on 4×4 lattice sites and repulsive interactions $U = 8J$ (Fig. 10b). We find a single minimum in the energy spectrum between the ground state and the first excited state, indicating a finite-size precursor of a phase transition. The transition is witnessed by a change in the orbital occupation (histograms): the occupations change abruptly at the transition point, in-

dicating a change of the ground state from absent orbital momentum to one with an orbital population that is in agreement with the prediction for a $\nu = 1/2$ Laughlin wave-function. For the $N = 2$ system the PES cannot provide a topological signature of the FCI, because a bipartition results in the spectrum of the single-particle density matrix which cannot probe topological order.

For further numerical signatures on FQH states with $N \geq 3$ we refer the reader to Ref. [28].

Absence of other topological states

Few-boson FQH systems generally may exhibit other topological states in addition to the $\nu = 1/2$ Laughlin state. Two factors that determine the presence and the properties of these states are the particle number and the interaction range. In two-boson systems, several states cannot exist, such as fractions with a numerator larger than 2, the $\nu = 1$ bosonic Pfaffian state, or the $\nu = 2/3$ Jain state. Indeed, for two bosons in a geometry with edges, the Pfaffian and $2/3$ -Jain states would be supported by a single lowest Landau level orbital, due to their respective relationship between number of particles and number of flux quanta on the plane geometry: $N_\phi = N - 1$ for the Pfaffian state, $N_\phi = N \times 3/2 - 2$ for the $\nu = 2/3$ Jain state [52]. Both two-body wavefunctions would thus be identical, and topologically trivial condensates living in a one-dimensional Hilbert space. Other FQH states with a smaller fraction, such as the $\nu = 1/4$ Laughlin state, could in principle be stabilized in a two-boson system, but would be gapless due to the on-site range of interactions. Indeed, as the flux is increased beyond the regime where a gapped $\nu = 1/2$ Laughlin state is observed, the creation of quasihole states becomes possible at zero interaction energy cost. For on-site interactions, these quasi-hole states are degenerate (quasidegenerate if lattice effects are taken into account) with the $\nu = 1/2$ Laughlin state, and remain degenerate at larger flux. The $\nu = 1/4$ Laughlin state (as well as other higher order Laughlin states), can become the ground state at higher flux, but it will be degenerate with the $\nu = 1/2$ Laughlin state and its quasihole states [53]. So, we would not expect a gapped $\nu = 1/4$ FQH state as the ground state in this system, and the $\nu = 1/2$ Laughlin state remains the most dilute FQH state for which we would expect a many-body gap for on-site interactions. These considerations hold for $\phi/2\pi < 0.3$; at flux $\phi/2\pi > 0.3$ numerical studies have shown that lattice effects lead to a more complex picture [24, 26, 54].

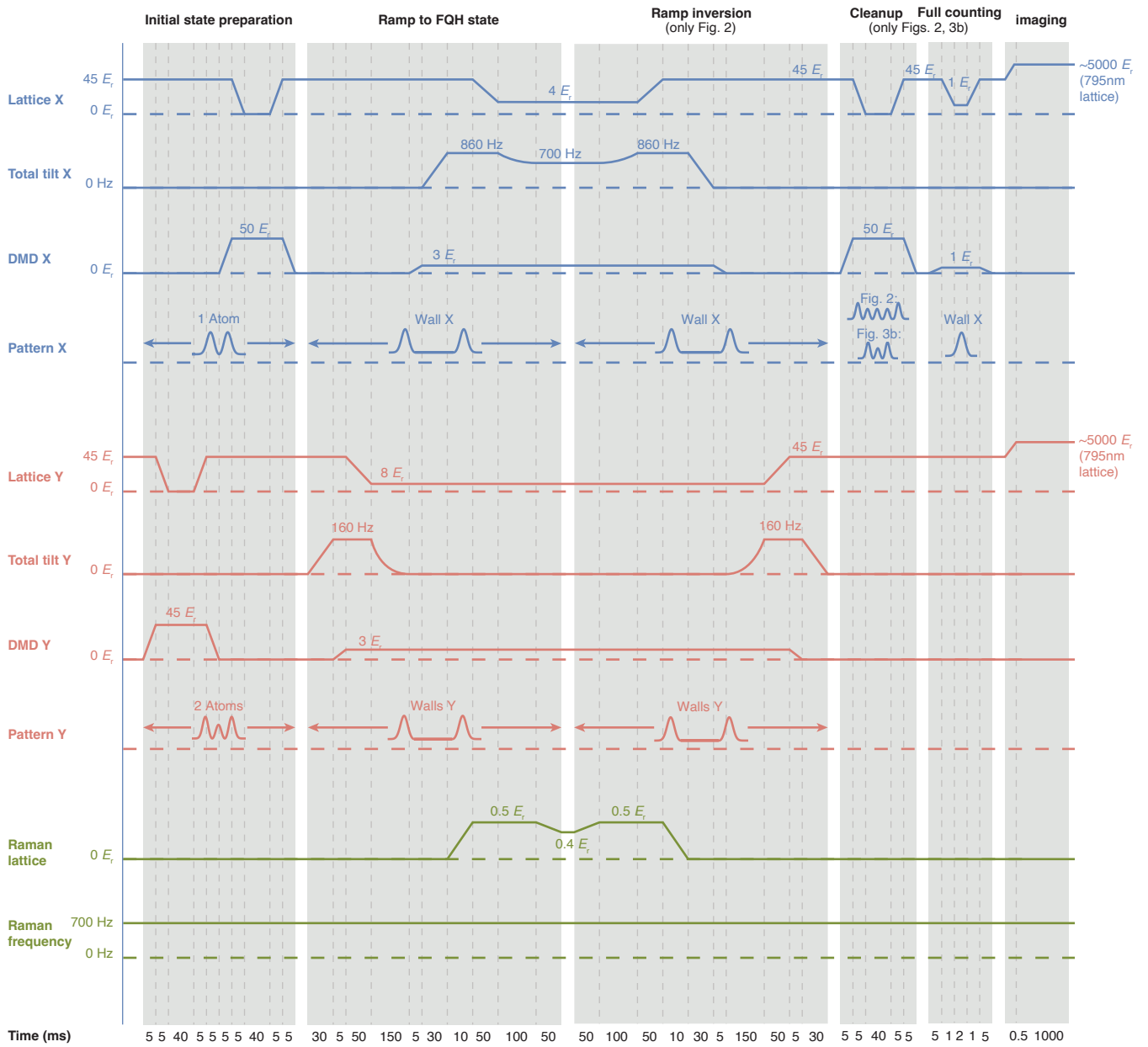


FIG. 6. **Sequence.** Full sequence for the performed experiments. All parameters are given for the driven Bose-Hubbard Hamiltonian (not the effective Hamiltonian).

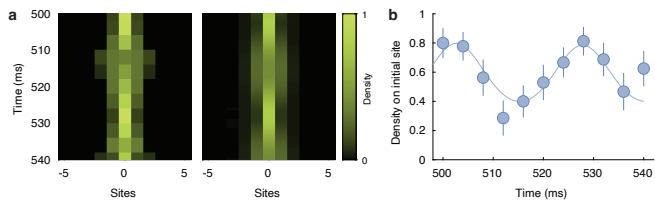


FIG. 7. **Coherence time.** **a**, Long-term evolution of an atom under 1D Raman tunneling. The atom initially occupies site 0. The tunneling along x is set to $K/2\pi = 13$ Hz (smaller than for the main measurements), the tilt along x is $\Delta_x/2\pi = 40$ Hz per site. The tilt leads to a rephasing of the quantum walk and converts it to Bloch oscillations, which show coherent revivals after the evolution times of 500 ms. The evolution is in agreement with a fit to the data, which incorporates decoherence through the Monte-Carlo wave function technique. **b**, Cut through the data in **a** of the initial site 0. The fit result (solid line) yields a decay rate of $\tau_{\text{Raman}} = 1.25(7)$ s. Error bars denote the s.e.m.

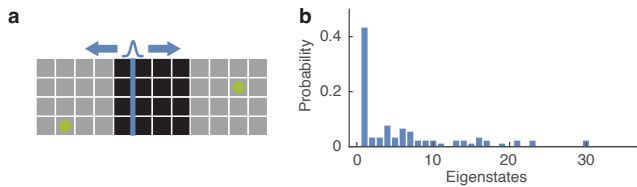


FIG. 8. **Excited state populations.** **a**, Protocol for measuring the full density distribution in the left two columns. We project a repulsive wall potential between the left two columns and enable tunneling along the horizontal direction, thereby ejecting the atoms into half-rows and breaking up potential doublons. This allows us to identify the populations of the lowest 36 Fock states in energy. Fock states that are higher in energy involve at least one atom in the right two columns; their populations are allotted to their low-energy counterpart with both atoms in the left two columns. **b**, Excited state populations. Inferred excited state overlap $\langle \psi_i | \hat{\rho}_{\text{Final}} | \psi_i \rangle$ of the prepared state $\hat{\rho}_{\text{Final}}$ with the eigenstates $|\psi_i\rangle$ of the final Hamiltonian. We find a dominant population in the ground state and most of the excited state population in the lowest few eigenstates.

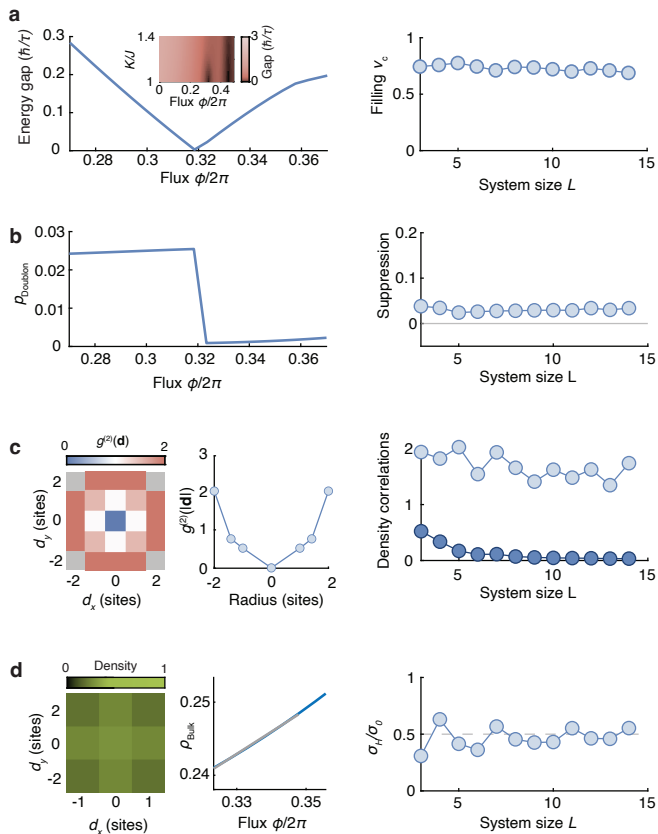


FIG. 9. **System size scaling.** Numerical system size scaling of the observed FQH signatures for $N = 2$ particles in quadratic box potentials. Left panels show data for a 3×3 system, right panels the behaviour when increasing the length L of the system. **a**, Energy gap diagram with a gap closing at the flux $\phi_c/2\pi$ for tunneling $K/J = 1$. For each system size we compute the corresponding filling factor via $\nu_c = \rho_{\text{Bulk}}/(\phi_c/2\pi)$ (right panel). **b**, Doublon fraction with suppression at $\phi_c/2\pi$. We extract the ratio $p_{\text{Doublon}}^{\text{FQH}}/p_{\text{Doublon}}^{\text{Normal}}$ from the doublon fraction $p_{\text{Doublon}}^{\text{FQH}}$ in the FQH state and $p_{\text{Doublon}}^{\text{Normal}}$ in the normal state, each extracted over an interval of $\Delta\phi = 0.1 \times \phi_c$. **c**, The reduced density correlations show already the vortex pattern for the 3×3 system (left panel). As the system size is increased, the correlations for neighbouring sites ($|\mathbf{d}| = 1$), dark blue) approach zero, and the correlations at a distance of $3l_B$ (light blue, similar to Fig. 4c) stabilize at a value between one and two. **d**, Increase of the bulk density and extracted Hall conductivity σ_H/σ_0 from a linear fit via Štředa's formula. When increasing the system size, the obtained Hall conductivity converges to $\sigma_H/\sigma_0 = 1/2$.

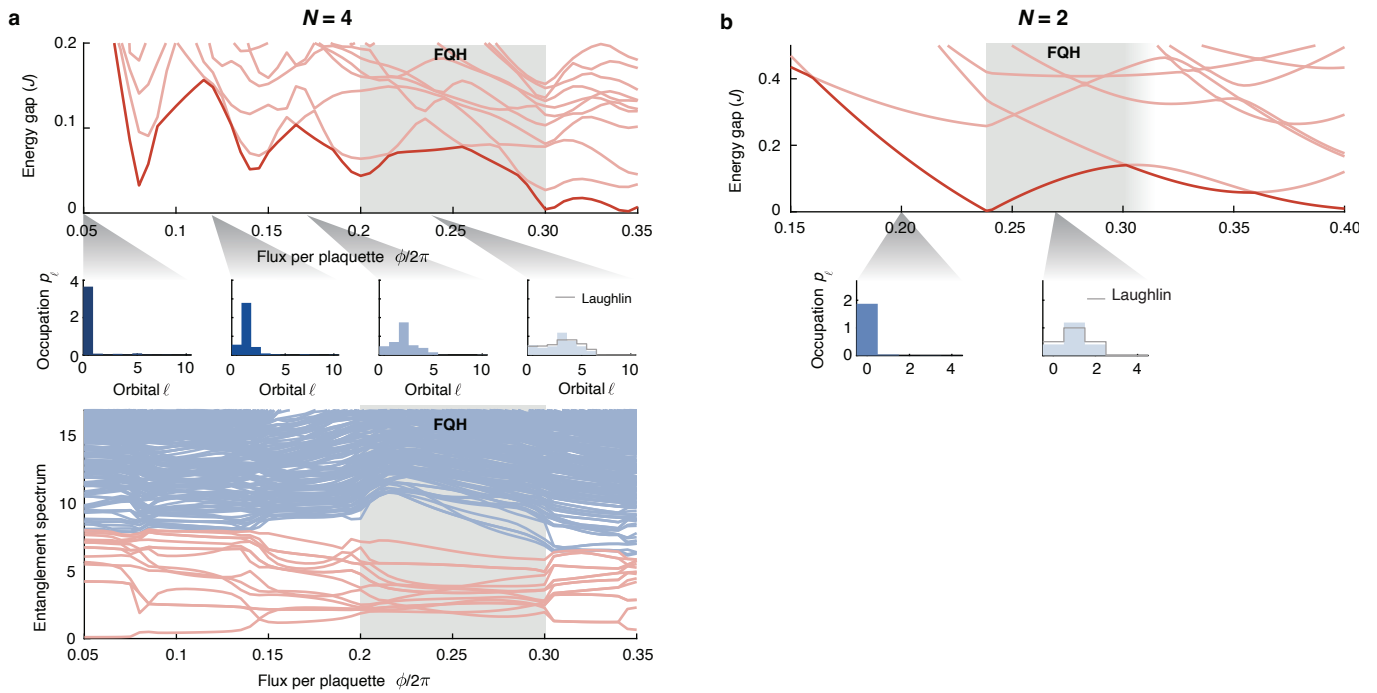


FIG. 10. **Orbital occupations and topological properties for systems with $N = 2$ and $N = 4$ bosons.** **a**, We first consider a system with $N = 4$, $U = 8J$, and 7×7 lattice sites. The many-body spectrum shows several local minima between the ground-state and the first excited state (dark red), which we interpret as finite-size signatures of phase transitions. In the range $0.20 < \phi/2\pi < 0.3$, between two such minima, the occupations of the single-particle orbitals (histograms) approximately match the expectation for a Laughlin state. This interpretation is solidified by the particle entanglement spectrum (PES), which shows a gap between the lowest 15 eigenstates (red) and all higher lying states (blue), indicating (quasi-)degenerate quasihole states that identify the Laughlin state. **b**, For a system with $N = 2$, $U = 8J$ and 4×4 lattice sites, we find only a single local minimum between the ground-state and the first excited state ($\phi/2\pi \approx 0.25$), which we interpret as the transition from the normal to the FQH state. This is confirmed by the large overlap of the ground-state occupations of the single-particle orbitals with the Laughlin state. For the $N = 2$ system an identification of the topological signatures in FQH states with the PES is not possible.