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► **To cite this version:**

Sasha Gasquet, Laurent Desbat, Pierre-Yves Solane. Motion Estimation in Parallel-Beam Linogram Geometry Using Data Consistency Conditions. 17th International Meeting on Fully 3D Image Reconstruction in Radiology and Nuclear Medicine, Jul 2023, Stony Brook (NY), United States. hal-04266404

HAL Id: hal-04266404

<https://hal.science/hal-04266404>

Submitted on 31 Oct 2023

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Motion Estimation in Parallel-Beam Linogram Geometry Using Data Consistency Conditions

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Abstract Data consistency conditions (DCCs) express the redundancy in the projections. In X-ray computed tomography, the most common conditions are expressed pairwise on the projections or as equality between projection-based moments and polynomials. The latter is better known in the parallel-beam geometry as the Helgason-Ludwig consistency conditions (HLCCs). The DCCs are often used to self-calibrate radiography systems. In this paper, we adjust data consistency conditions to a time-dependent model of the data in the parallel linogram geometry. We show that it is not possible to estimate the parameters of a uniform motion of a translating object using the DCCs. However, we show that we can estimate the average speed with prior information on the object's center of mass. Then, we model and estimate the parameters of a periodical variation of the motion. Finally, we run simulations to assess the performances of our method.

1 Introduction

In X-ray computed tomography, the data consistency conditions (DCCs) give information on the behavior of the radiography system based on the redundancy of the projections. If some changes occur in the system, the conditions are no longer satisfied. The changes can be detected or even estimated with proper modeling.

In the literature, some conditions are derived from the Helgason-Ludwig consistency conditions (HLCCs) [1][2]. These conditions have been used to estimate the motion of a moving object in the fan-beam geometry with a circular trajectory of the source and in the parallel geometry [3]. A more suitable representation of the data in the geometry with the source on a line is the linogram. The HLCCs have been expressed in the linogram geometry [4]. Results on the estimation of the source position and motion have been published for the fan-beam linogram geometry [5][6][7].

In this work, we consider a radiography system composed of a X-ray source, a horizontal linear detector and an object translating on a conveyor belt. The source and detector are supposed stationary. The object position is defined by its center of mass $c(t)$. The system is represented in the Fig. 1. Equivalently, this system can be considered as a system with a stationary object and a translating source and detector. The translation is the same as in the original radiography system but in the opposite direction. The equivalent system is represented in Fig. 2. At a fixed viewing angle ϕ , all the X-rays are parallel. The source position and the projection on the detector are supposed point-like. The X-rays are defined in the coordinate system (x_1, x_2) as segment from the source $S(t) = (x(t), 0)$ to the detector point with the direction

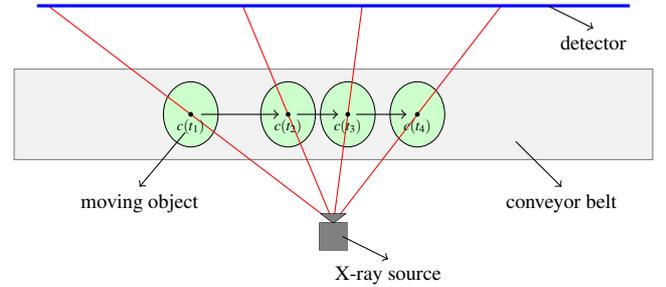


Figure 1: The radiography system is constituted of a X-ray source, a horizontal linear detector and a conveyor belt. Everything is stationary, except for the object translating on the conveyor belt.

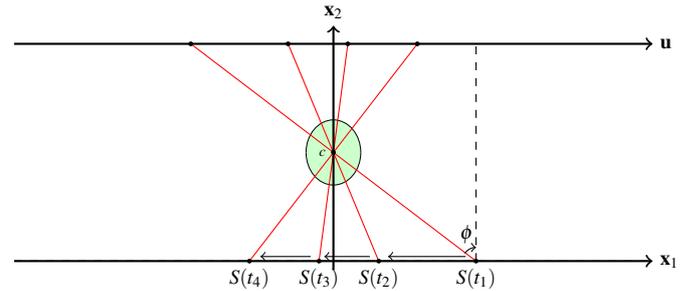


Figure 2: The equivalent radiography system. The source and the detector are moving at the same speed in the opposite direction to the translation of the conveyor belt in the Fig. 1. The object is stationary.

$\gamma_\phi = (\sin \phi, \cos \phi) \in S^1$, where S^1 is the unit sphere.

We suppose that the measured object $\mu : \mathbb{R}^2 \rightarrow \mathbb{R}$ has a compact support. The projections are modeled by the Beer-Lambert absorption law which makes the link between the object μ , the initial intensity I_0 of the X-rays and the intensity I acquired by the detector:

$$I = I_0 \exp \left(- \int_{\mathbb{R}} \mu \left(S(t) + r\gamma_\phi \right) dr \right) \quad (1)$$

A logarithm transform leads to the classical projection form:

$$p(\phi, x(t)) = \int_{\mathbb{R}} \mu \left((x(t), 0) + r\gamma_\phi \right) dr \quad (2)$$

In the following, we first define the parallel linogram geometry and recall the parallel linogram consistency conditions derived from the HLCCs to our geometry. We show that we cannot estimate the parameters of a uniform motion. Then, we model and estimate the parameters of a non uniform mo-

tion with the DCCs. Finally, we run simulations to evaluate the accuracy of the method.

2 Theory

2.1 Parallel linogram geometry

In an equivalent radiography system, the object is considered stationary. The source and the detector are moving at the same speed on two parallel lines separated by a distance $D > 0$. The parallel beam X-rays in the linogram geometry are parallel segments from a source at $S(t) = (x(t), 0)$ on the $x_2 = 0$ axis to a detector at $(x(t) + u, D)$ on the parallel axis $x_2 = D$ where $u \in \mathbb{R}$ and the distance D is fixed. The object position is defined by its center of mass $c = (c_1, c_2)$. We assume $x'(t) < 0, \forall t \in \mathbb{R}$. The offset u on the x_1 axis between the source at $(x(t), 0)$ and the detector point at $(x(t) + u, D)$ is bijectively linked to the projection angle ϕ with $u = D \tan(\phi)$, $\phi \in]-\pi/2, \pi/2[$, $u \in \mathbb{R}$, or equivalently $\phi_u = \arctan(u/D)$. This system is represented in Fig. 3.

The parallel-beam linogram l is defined by:

$$l(u, x) = \int_{\mathbb{R}} \mu((x, 0) + r(u, D)) dr \quad (3)$$

In our geometry, the X-rays are indexed by the time t at which they are measured. Thus, we define the parallel linogram \bar{l} and the parallel linogram operator $\bar{\mathcal{L}}$ as:

$$\bar{\mathcal{L}} \mu(u, t) = \bar{l}(u, t) \quad (4)$$

$$= l(u, x(t)) \quad (5)$$

$$= \int_{\mathbb{R}} \mu((x(t), 0) + r(u, D)) dr \quad (6)$$

The linogram \bar{l} is a weighted linogram. Using the change of variable $r' = r\sqrt{u^2 + D^2}$, we have:

$$\bar{l}(u, t) = \frac{1}{\sqrt{u^2 + D^2}} \int_{\mathbb{R}} \mu((x(t), 0) + r' \gamma_{\phi_u}) dr' \quad (7)$$

$$= \frac{1}{\sqrt{u^2 + D^2}} p(\phi_u, x(t)) \quad (8)$$

where $\gamma_{\phi_u} = (\sin(\phi_u), \cos(\phi_u)) = \frac{1}{\sqrt{u^2 + D^2}}(u, D)$.

2.2 Helgason-Ludwig Consistency Conditions

In the parallel-beam linogram geometry, the order $n \in \mathbb{N}$ moment of the projections is defined by:

$$J_n(u) = \int_{\mathbb{R}} l(u, x) x^n dx \quad (9)$$

For the Radon transform, the Helgason-Ludwig theorem states that the order n moment is an homogeneous polynomial of order n in $\cos \phi$ and $\sin \phi$ [8]. Such DCCs can be derived for the parallel linogram geometry [4]. We adjust these

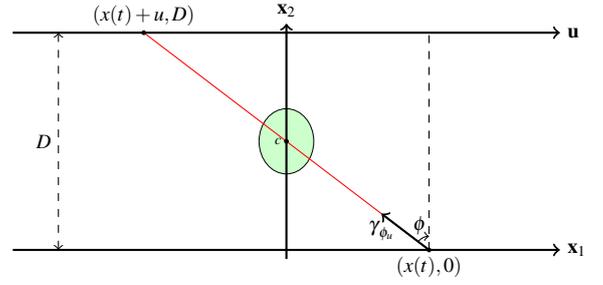


Figure 3: The parallel linogram. The object is supposed stationary and its center of mass is denoted by c . For a viewing angle ϕ , the offset $u = D \tan \phi$ between the source and the projection point on the detector is constant. The position of the source and the projection point associated to ϕ are respectively $(x(t), 0)$ and $(x(t) + u, D)$.

conditions to our geometry. Using a change of variable, we define the order n moment of the projections as:

$$\bar{J}_n(u) = \int_{\mathbb{R}} \bar{l}(u, t) x^n(t) |x'(t)| dt \quad (10)$$

Proposition 1 (from [4]) The data \bar{l} are consistent, i.e. \bar{l} is in the range of $\bar{\mathcal{L}}$, if and only if

$$\bar{J}_n(u) = \sum_{k=0}^n c_{n,k} u^k \quad (11)$$

From the data \bar{l} , we can only compute the time related moment $\tilde{J}_n(u)$ defined as follow:

$$\tilde{J}_n(u) = \int_{\mathbb{R}} \bar{l}(u, t) t^n dt \quad (12)$$

In the following sub-sections, we use the proposition 1 to estimate parameters related to the motion $x(t)$.

2.3 Uniform motion

We first model the source position using 2 real parameters x_0 and v_0 .

$$x(t) = x_0 + v_0 t \quad (13)$$

The parameter x_0 cannot be estimated using DCCs [9]. We arbitrarily set $x_0 = 0$. Thus, $\bar{J}_n(u)$ can be rewritten as:

$$\bar{J}_n(u) = |v_0| v_0^n \int_{\mathbb{R}} \bar{l}(u, t) t^n dt \quad (14)$$

$$= \text{sgn}(v_0) v_0^{n+1} \tilde{J}_n(u) \quad (15)$$

We want to estimate v_0 using the Eq. (15). Since $\bar{J}_n(u)$ is defined relatively to v_0 , we use the proposition 1. Therefore, in addition to v_0 , we need to estimate the parameters $c_{n,k}$ for $k = 0, \dots, n$. Let's now consider u_1, \dots, u_{n_a} where n_a is

the number of projections. We get a non-linear system of equations from the Eq. (15).

$$\begin{cases} \operatorname{sgn}(v_0) \sum_{k=0}^n c_{n,k} u_1^k - v_0^{n+1} \tilde{J}_n(u_1) = 0 \\ \vdots \\ \operatorname{sgn}(v_0) \sum_{k=0}^n c_{n,k} u_{n_a}^k - v_0^{n+1} \tilde{J}_n(u_{n_a}) = 0 \end{cases} \quad \forall n \in \mathbb{N} \quad (16)$$

The system of Eqs. (16) has an infinity of solutions: if $\{(c_{n,k}, v_0), \forall n \in \mathbb{N}, k = 0, \dots, n\}$ is a solution then $\{(\lambda^{n+1} c_{n,k}, \lambda v_0), \forall n \in \mathbb{N}, k = 0, \dots, n\}$ is a solution for any $\lambda \in \mathbb{R}$. Thus, we cannot determine v_0 from the DCCs.

2.4 Estimating v_0 from a center of mass property

The parameters of the uniform motion cannot be estimated using the DCCs *only*. However, we can use DCCs with a calibration object to estimate v_0 . The DCCs of order 0 and 1 are related to the center of mass of an object (This property can be used for misalignment correction of the projections) [9]. We show in this sub-section that we can use the center of mass coordinates to estimate the average velocity v_0 of the source from two different projections. Using the definition of $\bar{J}_n(u)$ and $x(t)$ in the Eqs. (10) and (13), we get the following:

$$\frac{\bar{J}_1(u)}{\bar{J}_0(u)} = \frac{\int_{\mathbb{R}} \bar{l}(u, t) (x_0 + v_0 t) v_0 dt}{\int_{\mathbb{R}} \bar{l}(u, t) v_0 dt} \quad (17)$$

$$= x_0 + v_0 t_c(u) \quad (18)$$

where $t_c(u) = \tilde{J}_1(u)/\tilde{J}_0(u)$ is the temporal center of mass of the projection u . Now, using the Eqs. (4) and (10), we get:

$$\frac{\bar{J}_1(u)}{\bar{J}_0(u)} = \frac{\int_{\mathbb{R}} \int_{\mathbb{R}} \mu(x(t) + ru, rD) x(t) x'(t) dr dt}{\int_{\mathbb{R}} \int_{\mathbb{R}} \mu(x(t) + ru, rD) x'(t) dr dt} \quad (19)$$

We make the following change of variables:

$$\begin{cases} x_1 = x(t) + ru \\ x_2 = rD \end{cases} \quad (20)$$

Additionally, we have $dx_1 dx_2 = |-Dx'(t)| dr dt$. We recall that $x'(t) < 0, \forall t \in \mathbb{R}$. Then, we have $dx_1 dx_2 = -Dx'(t) dr dt$. Hence, applying the change of variables, we get:

$$\frac{\bar{J}_1(u)}{\bar{J}_0(u)} = \frac{\int_{\mathbb{R}} \int_{\mathbb{R}} \mu(x_1, x_2) \left(x_1 - \frac{x_2}{D} u\right) dx_1 dx_2}{\int_{\mathbb{R}} \int_{\mathbb{R}} \mu(x_1, x_2) dx_1 dx_2} \quad (21)$$

$$= c_1 - \frac{u}{D} c_2 \quad (22)$$

where $c = (c_1, c_2)$ is the center of mass of the calibration object μ . From the Eqs. (18) and (22), we have:

$$c_1 - \frac{u}{D} c_2 = x_0 + v_0 t_c(u) \quad (23)$$

For two different projections u_1 and u_2 , we can write the following formula using a linear combination of the Eq. (23).

$$v_0 = -\frac{u_1 - u_2}{D(t_c(u_1) - t_c(u_2))} c_2 \quad (24)$$

We do not need to know c_1 nor x_0 here but only c_2 .

2.5 Non uniform motion estimation

We now assume the conveyor belt has a non uniform motion due to mechanical instabilities. We model the variations with the time dependent function $\delta(t)$. The position of the source is then defined by:

$$x(t) = x_0 + v_0 t + \delta(t) \quad (25)$$

The motion δ is assumed to be periodic.

$$\delta(t) = A \sin(\omega t + \psi) \quad (26)$$

We assume $x'(t) < 0, \forall t \in \mathbb{R}$. The DCCs can be rewritten using Eqs. (25) and (26) as:

$$\bar{J}_n(u) = -\int_{\mathbb{R}} \bar{l}(u, t) (x_0 + v_0 t + A \sin(\omega t + \psi))^n \times (v_0 + A \omega \cos(\omega t + \psi)) dt \quad (27)$$

Eq. (27) is non-linear in A, ω, ψ for all $n \in \mathbb{N}$. We apply the proposition 1 as in the subsection 2.3. We use the 0-order condition to estimate the parameters $A, \omega, \psi, c_{0,0}$ by solving a non-linear system of equations using the Gauss-Newton algorithm based on:

$$-v_0 \tilde{J}_0(u) = c_{0,0} + A \omega \int_{\mathbb{R}} \bar{l}(u, t) \cos(\omega t + \psi) dt \quad (28)$$

3 Simulations

The mean velocity v_0 is assumed to be known and $\delta(t)$ is estimated by solving the Eq. (28). The simulation are done using the library RTK [10]. Our phantom is composed of two cylinders respectively with a radius of 45mm and 50mm, and of density -0.2 and 0.2 . It is placed midway between the source and the detector. The source to detector distance is $D = 480$ mm. The linear detector is composed of 500 pixels with a pitch of 0.4mm. The leftmost pixel is the pixel 0. Thus, we set its origin at $u = -200$ mm. The moving source position is defined by $x(t) = x_0 + v_0 t + A \sin(\omega t + \psi)$ where we fix $x_0 = 0$ mm, $v_0 = -1000$ mm/s, $A = 2$, $\omega = 40$, $\psi = \pi/4$. We acquire the projection at a rate of 2000Hz within the interval $[-T/2, T/2]$ where we set $T = 0.6$ s. Gaussian noise is added to the projections. The standard deviation of the noise is defined for each pixel as a percentage of its value.

Noise	0%	1%	3%
A	1.988	1.976 ± 0.030	1.959 ± 0.104
ω	39.966	39.886 ± 0.181	39.731 ± 0.688
ψ	0.785	0.783 ± 0.007	0.775 ± 0.028

Table 1: Results of 50 simulations with Gaussian noise added to the projections. The parameter values are $A = 2$, $\omega = 40$, $\psi = \pi/4 \approx 0.785$.

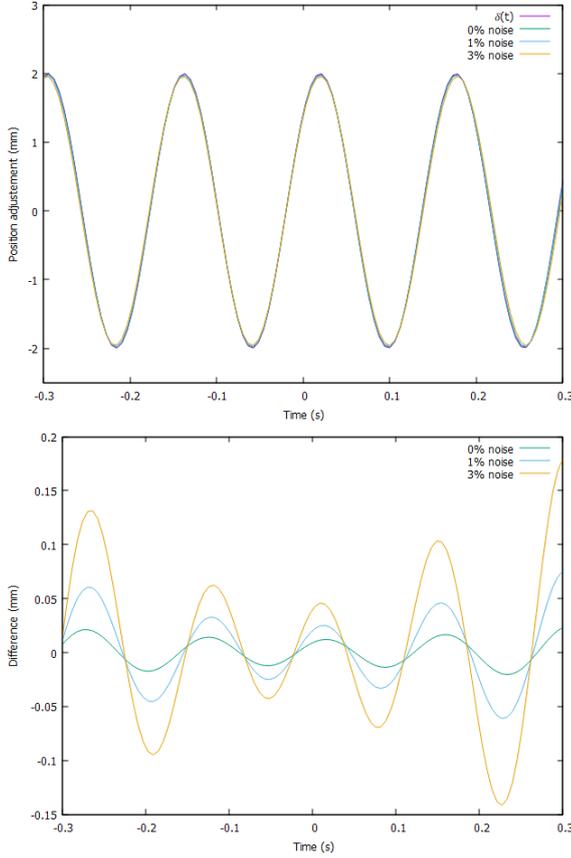


Figure 4: Estimation of the motion variation $\delta(t) = A \sin(\omega t + \psi)$ with 0%, 1% and 3% Gaussian noise. *Top:* the function $\delta(t)$ and its estimates. *Bottom:* difference between the theoretical value of $\delta(t)$ and its estimates.

The initialization of the Gauss-Newton algorithm is set close to the real solution. Often, the convergence of the Gauss-Newton algorithm is local. The parameters are therefore initially set to $A = 3$, $\omega = 42$, $\psi = \pi/8$, $c_{0,0} = 0$. The results are given in the table 1. Noticing that $\pi/4 \approx 0.785$, we see that the estimates are quite good. In the Fig. 4, we show the estimation of the function $\delta(t)$ and the difference between the theoretical value of $\delta(t)$ and its estimates. The differences are respectively up to $25\mu\text{m}$ and $80\mu\text{m}$ for the simulations with 0% and 1% noise. Except for $c_{0,0}$, all the parameters can be roughly estimated a priori using external tools. With different set of initial solution, we could see that the most sensitive parameter is ω . It's worthwhile noticing that the solution is not unique. Indeed, it depends on the definition interval of ψ . We have $A \sin(\omega t + \psi + k\pi) = (-1)^k A \sin(\omega t + \psi)$ with $k \in \mathbb{Z}$.

4 Conclusion

We have adjusted the Helgason-Ludwig consistency conditions expressed in the parallel linogram geometry to a time-dependent self-calibration problem. We have proven that we cannot estimate the mean velocity v_0 using the DCCs. However, we have shown that we can estimate v_0 using a priori information on center of mass of a calibration object. We have modeled a non uniform motion with a periodical function and proposed a method to estimate the motion based on 0-order DCC. As in [5], we experimented that higher order moments ($n \geq 1$, cf Eq. (27)) do not provide significant improvements. Moreover, the results can easily be extended to the 3D using multiple 2D plane as done by Nguyen *et al.* [7]. The redundancy in the data will most likely help to get more robust estimates. Nonetheless, the results obtained in the simulations are already good enough for our needs.

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