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# Exploring Statistical Backbone Filtering Techniques in the Air Transportation Network

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**Abstract**—The dense nature of transportation networks expands the challenge of their visualization and processing. Several statistical backbone extraction techniques are proposed to reduce their size while keeping essential information. Here, we perform a comparative evaluation of seven prominent statistical backbone extraction techniques in the USA weighted air transportation network. One can classify the airports into hubs, spokes, and focus airports based on the business models used by the airlines. We compare the extracted backbones using various performance measures. We consider the number of components, sizes, the fraction of airport type, edge type, and weights preserved by each method. Results show that the Enhanced Configuration Model (ECM) Filter tends to preserve edges between spoke airports uncovering the infrastructure connecting the regional spoke airports. In contrast, the alternative filters (Disparity, Poly Urn, Marginal Likelihood, Noise Corrected, Global Statistical Significance (GLOSS), Locally Adaptive Network Sparsification (LANS)) highlight edges between the hub and spoke, focus and spoke, and spoke and spoke airports revealing more of the hub and spoke foundation used by airlines. Moreover, the Disparity Filter, Marginal Likelihood Filter, and Noise Corrected Filter preserve the highest proportion of weights while Poly Urn Filter and ECM Filter keep the lowest. The GLOSS and LANS Filters maintain a moderate fraction of weights between the two extremes.

**Index Terms**—Complex Networks, Backbone Filtering Techniques, Network Compression, Graph Summarization, Sparsification, Transportation Networks

## I. INTRODUCTION

Networks are widely adopted for analyzing complex systems in many fields. In particular, there is a significant interest in analyzing transportation networks [1]–[4]. In these networks, nodes represent spatial units such as cities, airports, ports, and stations, while edges describe transport-related interactions between the nodes. Edges are typically weighted by frequency, distance, capacity, or the time it takes to travel between nodes. Since transport networks are spatial, nodes

are preferably visualized in their location, producing dense networks with regionally clustered edges. This type of locally dense representation makes processing and visualization challenging tasks. Filtering the edges with weights below a global threshold value is a naive approach to reducing the network size. Unfortunately, this approach destroys the heterogeneity of the edge weight distribution. Consequently, various backbone extraction techniques have been proposed to reduce the network size while preserving the highest amount of “information”. One can classify current backbone extraction techniques into structural and statistical methods. Structural techniques remove edges and nodes according to a criterion allowing the latent structure of the network to emerge [5]–[11]. They are generally designed to preserve critical topological properties of the network, such as the average distance, the community structure, and so on. In contrast, statistical techniques consider that noisy edges that need to be filtered hide the network structure [12]–[18]. They assess the significance of an edge through a statistical test, eliminating the least significant edges.

A previous study [19] compared six backbone extraction techniques in the Southeast Asian intercity air transport network [20]. It includes one statistical method (disparity filter) and five structural techniques (global weight thresholding, k-core decomposition, minimum spanning tree, primary linkage analysis, and multiple linkage analysis) [12], [21]–[24]. They evaluate the extracted backbones based on their geographical and topological structures, pointing out the potential of each technique in the light of different transport research applications. For instance, they recommend k-core decomposition to analyze the best-connected core for multiplex networks. They suggest that Primary linkage analysis is well-suited for functional/nodal regions analysis and highlighting hub-and-spoke structures. Finally, they recommend the Disparity Filter for analyzing the overall topological and spatial information for large-scale networks and possible hidden structures.

After the Airline Deregulation Act in 1978, airport hubs have formed the backbone of the USA's present-day flying network and marketplace. Many regular airline companies such as American Airlines and United Airlines embrace the hub-and-spoke business model. However, not all airlines rely on this system. Many low-cost airlines provide point-to-point networks to route passengers on strategically chosen flight paths between focus airports, such as Southwest Airlines and JetBlue. In this study, we first consider a toy example (a part of the US Air Transportation Network) to understand better the behavior of various statistical backbone extraction techniques under test. We compare their ability to preserve different types of airports (hub, focus, and spoke) and edges. Finally, we extend our investigation to the North American air transportation Network to consolidate our findings

The rest of the paper is organized as follows. Section II introduces the backbone extraction techniques under test. Section III compares the backbone extraction techniques in a toy example. Section IV compares the backbone extraction techniques in the North American air transportation network. Finally, we make some concluding remarks in Section V.

## II. BACKBONE EXTRACTION TECHNIQUES

This section briefly presents the statistical backbone extraction techniques under evaluation. Assume that  $G(N, E, W)$  is a simple, undirected, and weighted graph where  $N$  is the set of nodes of size  $n = |N|$  nodes and  $E \subseteq N \times N$  is the set of edges of size  $m = |E|$  edges.  $W$  represents the weight matrix, whose element  $w_{ij}$  indicates the weight of the edge joining node  $i$  and  $j$ . We recall the degree  $k_i$  of node  $i$  is the number of its neighbors, and its strength  $s_i = \sum_j w_{ij}$  is the sum of the weights of the edges incident to it.  $S = \sum w_{ij}$  is the strength of the network

All these statistical techniques share a common framework. Given an edge connecting node  $i$  to node  $j$ , they perform a statistical test based on a defined null model or the empirical data to compute the p-value  $\alpha_{ij}$ . Edges with p-values larger than a predefined statistical significance threshold  $\alpha$  are considered non-significant and thus removed.

### A. Global Threshold

The most straightforward technique is filtering edges with weights lower than a predefined threshold  $\alpha$ .

### B. Disparity Filter [12]

This popular technique assesses the significance of an edge locally from the viewpoint of the incident nodes considered independently. It assumes that a node  $i$  distributes its total strength  $s_i$  uniformly among its edges. Based on the defined model, it normalizes the weights following:  $x_{ij} = \frac{w_{ij}}{s_i}$ . Then it performs a one-sided right-tail test computing  $\alpha_{ij}$  the probability of observing a normalized weight  $X$  at least as "extreme" as  $x_{ij}$ . It is given by:

$$\begin{aligned} \alpha_{ij} &= \mathbb{P}(X > x_{ij} | k_i) \\ &= 1 - (k_i - 1) \int_0^{x_{ij}} (1 - x)^{k_i - 2} dx \end{aligned} \quad (1)$$

### C. Polya Urn Filter [13]

In the Pólya Urn [25], observing a particular event increases the probability of further observing it. Similarly, one can assume that edge weights are due to an aggregation process of nodes interacting with each other over time. Based on this, it assumes that a node  $i$  distributes its strength  $s_i$  randomly through a Pólya process. For each edge, it performs a one-sided right-tail test computing  $\alpha_{ij}$  the probability that node  $i$  will assign randomly through a Pólya process a weight  $X$  larger than or equal to  $w_{ij}$ . It is given by:

$$\begin{aligned} \alpha_{ij} &= \mathbb{P}(w \geq w_{ij} | k_i, s_i, a) \\ &= 1 - \sum_{w=0}^{w_{ij}-1} \binom{s_i}{w} \frac{B(\frac{1}{a} + w, \frac{k_i-1}{a} + s_i - w)}{B(\frac{1}{a}, \frac{k_i-1}{a})} \end{aligned} \quad (2)$$

A reinforcement parameter  $a$  governs this mechanism. The higher the value of the reinforcement mechanism  $a$ , the larger the weight must be for the edge to be considered significant. Note that the disparity filter is a special case of the Polya Urn Filter.

### D. Marginal Likelihood Filter [14]

While the Disparity Filter and Polya Urn Filter assess the significance of an edge in the light of each node it connects independently, the Marginal Likelihood Filter considers the two nodes the edge connects. An integer-weighted edge is treated as multiple unit edges. The null model assumes that each unit edge randomly chooses two nodes, which results in a binomial distribution. For each edge, it performs a one-sided right-tail test computing  $\alpha_{ij}$  the probability of drawing at least  $w_{ij}$  unit edges from the strength  $S$  of the network with probability  $p = \frac{s_i s_j}{2S^2}$  proportional to both nodes' strengths. It is given by:

$$\begin{aligned} \alpha_{ij} &= \mathbb{P}(w \geq w_{ij} | s_i, s_j, S) \\ &= 1 - \sum_{w=0}^{w_{ij}-1} \binom{S}{w} p^w (1-p)^{S-w} \end{aligned} \quad (3)$$

### E. Noise Corrected Filter [15]

Similar to the Marginal Likelihood Filter, it assumes edge weights are drawn from a binomial distribution. However, it estimates the probability of observing a weight connecting two nodes using a Bayesian framework. This framework enables us to generate posterior variances for all edges. This posterior variance allows us to create a confidence interval for each edge weight. Finally, we remove an edge if its weight is less than  $\delta$  standard deviations stronger than the expectation, where  $\delta$  is the only parameter of the algorithm. It also provides a direct approximation through Binomial distribution similar to the Marginal Likelihood Filter. however, the probability of a unit edge connecting node  $i$  with node  $j$  is defined as follows:  $p = \frac{s_i s_j}{S^2}$ .

### F. GLOSS Filter [17]

The GLOSS filter assumes that one cannot assess edges independently of the overall network topology. Therefore, it defines a global null model for evaluating the significance of an edge. However, it makes no assumptions about the distribution of weights. Instead, it employs the empirical distribution of weights. The model is a network with the same topology as the original network and edge weights randomly drawn from the empirical weight distribution. For each edge, it performs a one-sided right-tail test computing  $\alpha_{ij}$  the probability of observing at least  $w_{ij}$  edge weight between node  $i$  and node  $j$  considering the nodes' observed degrees  $k_i$ ,  $k_j$  and strengths  $s_i$ ,  $s_j$  as constraints. It is given by:

$$\begin{aligned} \alpha_{ij} &= \mathbb{P}(w > w_{ij} | s_i, k_i, s_j, k_j) \\ &= \frac{\int_{w_{ij}}^{\infty} P_{obs}(w) P(s_i, s_j | w, k_i, k_j) dw}{\int_0^{\infty} P_{obs}(w) P(s_i, s_j | w, k_i, k_j) dw} \end{aligned} \quad (4)$$

where,  $P(s_i, s_j | w, k_i, k_j) = F(s_i - w_{ij}, k_i - 1) F(s_j - w_{ij}, k_j - 1)$ . The function  $F(s, k)$  is the probability of randomly extracting from the weight distribution  $P_{obs}(w)$ ,  $k$  elements whose sum is equal to  $s$ , which means that:

$$\begin{aligned} F(s, k) &= \int P_{obs}(x_1) dx_1 \int P_{obs}(x_2) dx_2 \dots \\ &\times \int P_{obs}(x_k) dx_k \delta(x_1 + x_2 + \dots + x_k - s) \end{aligned} \quad (5)$$

### G. LANS Filter [18]

Like the GLOSS Filter, it makes no assumptions about the underlying weight distribution. Instead, it employs the empirical cumulative density function to judge statistical significance. First, it normalizes the weights following:  $x_{ij} = \frac{w_{ij}}{s_i}$ . Then from the viewpoint of node  $i$  it calculates the probability of choosing an edge at random with a normalized weight at least equal to the observed normalized weight  $x_{ij}$ . It is given by:

$$\begin{aligned} \alpha_{ij} &= \mathbb{P}(X > x_{ij}) \\ &= 1 - \frac{1}{k_i} \sum_{k=1}^{k_i} 1\{x_{ik} \leq x_{ij}\} \end{aligned} \quad (6)$$

where  $1\{\}$  is an indicator function.

### H. ECM Filter [16]

Like the GLOSS Filter, it makes no assumptions about the underlying weight distribution. Using the Enhanced Configuration Model of network reconstruction, it employs a null model based on the canonical maximum-entropy ensemble of weighted networks having the same degree and strength distribution as the real network. We refer the readers to the original paper for a detailed mathematical explanation.

## III. COMPARING BACKBONE EXTRACTION TECHNIQUES ON A TOY EXAMPLE

We begin our review of the backbone extraction techniques using a "toy network," a subgraph from the original worldwide air transportation network [26], including 42 US Airports

and 378 edges. Edges represent routes between airports, and weights are the number of flights offered by different companies flying between 17 May and 22 May 2018. After the Airline Deregulation Act in 1978, airport hubs have formed the backbone of the country's present-day flying network and marketplace. Many regular airline companies, such as American Airlines and United Airlines, embraced the hub-and-spoke business model. A traveler from a spoke airport would fly using a small carrier into a hub, then to another hub with a large carrier, and then reach the final destination by flying from the hub to another spoke airport. However, some airlines do not rely on the hub-and-spoke system. Indeed, Low-cost companies such as Southwest Airlines and JetBlue provide point-to-point networks to route passengers on strategically chosen flight paths between focus airports. Therefore, we can consider three types of airports (Hubs, Spokes, and Focus). The toy example network contains 20 Hubs (Stars), 20 Spokes (Triangles), and 3 Focus (Square) airports. We extract the top most significant 30 edges for each backbone filtering technique to build the backbones. We use the Global Threshold Filter as a reference in the comparative evaluation. Figure 1 represents the backbone extracted by the Global Threshold Filter in the toy example. It shows the top 30 edges with the highest weights connecting the Hubs, Focus, and Spokes airports.

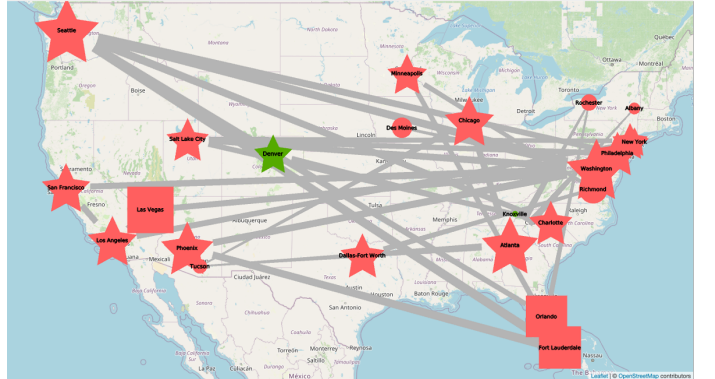


Fig. 1. The backbone preserves the 30 top most significant edges extracted by the Global Threshold Filter. Stars, squares, and circles represent hub, focus, and spoke airports. The node's size and the edge's width are proportional to the weighted degree and weight, respectively.

Table I reports the number of components identified by each filtering technique and their relative size. Except for LANS and ECM, the filters demonstrate their ability to extract a large connected component with a few small ones. In contrast, the backbones of LANS and ECM spread across multiple components of more homogeneous sizes.

Table II reports the percentage of nodes, hub, focus, and spoke airports preserved by each filtering technique. LANS and ECM Filters keep a significant proportion of nodes (above 80%). The other filters are far behind, with a score similar to the Global Threshold Filter. Indeed, they keep around 60% of the nodes of the toy network. All the filters are pretty effective at keeping the hub airports. The LANS Filter backbone includes them all. The GLOSS and ECM Filters

TABLE I

RELATIVE SIZE OF THE COMPONENTS EXTRACTED BY THE FILTERING TECHNIQUE IN THE TOY EXAMPLE IN DECREASING ORDER. NOTE THAT VALUES BETWEEN () REPRESENT THE NUMBER OF COMPONENT WITH THE SAME SIZE.

Filter	% C1	% C2	% C3	% C4	% C5
<b>Global Threshold</b>	<b>92.3</b>	<b>7.7</b>	-	-	-
Disparity	93.1	6.9	-	-	-
Polya Urn	92	8	-	-	-
Marginal Likelihood	93.1	6.9	-	-	-
Noise Corrected	70	23.33	6.67	-	-
GLOSS	56.67	26.67	10	6.66	-
LANS	27.79	25	19.45	11.1(2)	5.56
ECM	28.95	15.79	10.52(2)	7.9(3)	5.26(2)

follow. They preserve a significant fraction of hub airports (95%). Polya Urn Filter and the global threshold filter are the less effective, with a score of 85%. The third column of the table shows that the filters preserve all the focus airports. It is not the case for GLOSS and the Polya Urn Filter. Indeed, they isolated one focus airport. The last column reports the percentage of spoke airports preserved. The Global Threshold Filter and Polya Urn Filter keep the lowest fraction of spoke airports (30%). The Disparity Filter, Marginal Likelihood Filter, Noise Corrected Filter, and GLOSS Filter preserve a medium proportion between 40% and 45%. LANS Filter and the ECM Filter are the more effective. Indeed, they keep a significant balance of spoke airports (65% and 80%).

TABLE II

FRACTION OF NODES, HUB, FOCUS, AND SPOKE AIRPORTS PRESERVED BY THE FILTERING TECHNIQUES IN THE TOY EXAMPLE.

Filter	% Nodes	% Hubs	% Focus	% Spokes
<b>Global Threshold</b>	<b>60</b>	<b>85</b>	<b>100</b>	<b>30</b>
Disparity	67	90	100	40
Polya Urn	58	85	67	30
Marginal Likelihood	67	90	100	40
Noise Corrected	69	90	100	45
GLOSS	69	95	67	45
LANS	83	100	100	65
ECM	88	95	100	80

Table III reports the fraction of various edge types preserved in the extracted backbones (hub-hub, hub-focus, hub-spoke, and focus-spoke).

Let's start with the hub-hub edges. One can observe that the GLOSS Filter and Polya Urn Filter score similarly to the Global Threshold Filter at preserving the hub-hub edges ( $\approx 8\%$ ). In contrast, the other filters keep a lower fraction. In particular, LANS Filter and ECM Filter preserve a lower portion (5.95% and 4.32%), nearly half the amount of the reference filter.

Let's now turn to the hub-focus edges. The Global threshold Filter keeps 10% of hub-focus edges. The other filters preserve a lower proportion ( $\leq 8.6\%$ ). The GLOSS Filter is the worst, with only 1.72% of the hub-focus edges kept.

Let's now consider the hub-spoke edges. The Global threshold and Polya Urn Filter preserve the same fraction of the hub-spoke edges (6.72%). The other filters preserve a slightly higher fraction. In particular, the LANS and ECM Filters retain the highest fraction of the hub-spoke edges ( $\approx 12.6\%$ ).

Finally, results are more mixed for focus-hub edges. Indeed, the Disparity Filter, Polya Urn Filter, and GLOSS Filter follow the reference filter. They preserve the same amount of the focus-hub edges (7.69%). The Marginal Likelihood Filter, Noise Corrected Filter, and ECM Filter are more effective. Indeed, they preserve more than twice the fraction of these edges compared to the reference. The LANS Filter does not retain any of the focus-spoke edges.

TABLE III

THE PROPORTION OF HUB-HUB (H-H), HUB-FOCUS (H-F), HUB-SPOKE (H-S), AND FOCUS-SPOKE (F-S) EDGES PRESERVED BY THE FILTERING TECHNIQUES IN THE TOY EXAMPLE.

Filter	% H-H	% H-F	% H-S	% F-S
<b>Global Threshold</b>	<b>8.10</b>	<b>10.34</b>	<b>6.72</b>	<b>7.69</b>
Disparity	7	8.62	9.24	7.69
Polya Urn	8.65	8.62	6.72	7.69
Marginal Likelihood	7	6.9	9.24	15.38
Noise Corrected	6.48	6.9	10	15.38
GLOSS	8.10	1.72	10.92	7.69
LANS	5.95	6.9	12.61	0
ECM	4.32	6.9	12.61	23

Table IV reports the fraction of weights associated with the different types of edges kept in the backbone. The first column gives the total fraction of weight independently of their class. One can see that the Global Threshold Filter preserves the highest possible fraction of weights (36.10%). Indeed, it retains the links with the top weights exclusively. The Disparity Filter, Marginal Likelihood Filter, and Noise Corrected Filter are not far behind. They retain  $\approx 34\%$ . In contrast, the Polya Urn Filter preserves the smallest portion of weights (7.6%), showing its ability to keep edges with low weights in the backbone. The other filters are distributed between these two extremes. When we look at the fraction of hub-hub edges retained in the various backbones, we observe the same ranking with all types of nodes. Indeed hub-hub links have generally heavy weights. Concerning the hub-focus links, the Polya Urn Filter and GLOSS Filter preserve a tiny portion of weights ( $\approx 4.8\%$ ). The global Threshold filter, as expected, keeps the highest fraction ( $\approx 50\%$ ). The other filters preserve  $\approx 10\%$  fewer edge weights than the Global Threshold Filter. Now, look at the hub-spoke links results reported in the fourth column. Once again, the Polya Urn Filter preserves a tiny portion of this type of edge ( $\approx 6.2\%$ ). The ECM Filter follows. Indeed, it keeps a smaller fraction of edges than the Global Threshold Filter. All the other filters outperform the Global Threshold Filter at preserving a high fraction of hub-spoke links. Indeed, this type of link usually does not have the top weights. The last column reports the fraction of focus-spoke links kept in the various backbones. The Global Threshold, Disparity, and GLOSS Filters preserve

the same fraction ( $\approx 23\%$ ). The Marginal Likelihood, Noise Corrected, and ECM Filters retain a significant proportion ranging between 38% and 45%. In contrast, the Poly Urn Filter preserves a tiny portion of weights associated with the Focus-Spoke edges (1.1%). The LANS backbone do not preserve any of this type of edges.

Table V summarizes the strengths and weaknesses of the backbone filtering techniques based on these results.

TABLE IV

FRACTION OF WEIGHTS PRESERVED BY THE FILTERING TECHNIQUES IN THE TOY EXAMPLE FOR THE DIFFERENT TYPE OF EGDES: ALL TYPE OF EDGE TYPE (W), HUB-HUB (H-H), HUB-FOCUS (H-F), HUB-SPOKE (H-S), AND FOCUS-SPOKE (F-S)

Filter	% W	% H-H	% H-F	% H-S	% F-S
<b>Global Threshold</b>	<b>36.10</b>	<b>36.32</b>	<b>49.55</b>	<b>28.35</b>	<b>23.94</b>
Disparity	34.74	32.58	44.9	34.37	23.94
Polya Urn	7.61	9.62	4.92	6.29	1.10
Marginal Likelihood	34.78	32.58	39.71	35.82	38.30
Noise Corrected	34	30.3	39.7	37.9	38.29
GLOSS	20.38	18.35	4.69	35.72	23.94
LANS	30.22	26.31	39.71	37.38	0
ECM	26.10	22.65	39.71	20.91	45.31

#### IV. COMPARING BACKBONE EXTRACTION TECHNIQUES IN THE NORTH-AMERICAN AIR TRANSPORTATION NETWORK

We reproduce the same experiment with a bigger network to validate our findings. We extract the nodes and edges of North America from the original world transportation network. Indeed, it is easier to classify the airports in this region than in the rest of the world. The network contains 839 nodes and 4,046 edges. Finally, we extracted the network’s backbone preserving the most significant 10% edges using all the filtering techniques under test.

Table VI reports the size and the number of components of the various backbones. Results are similar to the toy example. Indeed, the LANS and ECM filters split the network into several parts while the other filtering techniques retain a large component.

Table VII reports the fraction of nodes kept by the backbones when filtering 90% of edges. One can consider three categories when comparing the total number of nodes preserved in the various backbone. The first class contains ECM, LANS, and GLOSS filters. They keep the higher fraction of nodes (mean value around 45%). Poly Urn and Disparity filters form the second category. In that case, the mean value of the fraction of kept nodes drops to 30%. Finally, Marginal Likelihood, Noise Corrected, and Global threshold filters are similar. Indeed, they keep around 15% of the nodes. Moreover, except for the ECM Filter, all filters effectively keep Hub and Focus airports. Indeed, they retain between 80 % and 100 % of these types of nodes characterized by high traffic. This observation is in line with the toy example. In contrast, The ECM Filter preserves only 32% of the hubs and 17% of the focus airports. The last column reports the fraction of spokes contained in the backbones. Globally, the results are

comparable with the toy example network. The hierarchy is unchanged except for the Poly Urn filter and the Marginal Likelihood Filter. Indeed, compared to the global threshold filter, this time, the Poly Urn Filter preserves more spoke airports while the Marginal preserves less.

Table VIII reports the fraction of edges preserved by the filtering technique in the North American network. Concerning the hub-hub edges, results correlate well with the behavior observed in the toy example. Indeed, this time all the filters contain a lower amount of this type of node than the global threshold filter. We notice a different behavior for the ECM, LANS, and GLOSS Filters for the hub-focus edges. The ECM and LANS Filters preserve a low fraction of these edges, while the GLOSS Filter preserves a higher amount. Except for the ECM Filter, all the filters behave similarly as in the toy example for the hub-spoke edges. Indeed, the ECM Filter preserves a very tiny portion of these edges. The ECM filter and LANS filter behavior deviate from the toy example for the focus-spoke edges. Indeed, the LANS Filter starts to keep this type of edge, while the ECM Filter barely retains any edges of this type. This network contains two new classes of edges that do not exist in the toy example: the focus-focus and spoke-spoke edges. Disparity, Marginal Likelihood, and Noise Corrected Filters keep nearly the same amount of these focus-focus nodes as the Global Threshold Filter ( $\approx 10\%$ ). The other techniques hold a lower share. In particular, the ECM Filter removes all of them. One can notice that all the backbone filtering methods preserve a higher fraction of the spoke-spoke edges than the Global Threshold Filter. It is particularly true for the ECM Filter that keeps almost 40%.

Table IX reports the fraction of weights kept by the filtering techniques in the North-American Network. The results of the total fraction of weights and the hub-hub edge weights are similar to the toy example network. Only the ECM Filter deviates. It preserves a tiny fraction of the weights. LANS and ECM Filters keep a low fraction contrasting with the toy example for the hub-focus weights. Looking at the hub-spoke edge weights, unlike in the toy example, GLOSS and LANS Filter preserve a lower fraction than the Global Threshold this time. For the focus-spoke edge weights, the LANS Filter does not remove all edges of this type, thus increasing the weight. On the other hand, the ECM Filter almost removes all these edges, thus decreasing the preserved weight. The other filters exhibit the same behavior as in the toy example. We observe two categories for the focus-focus edge weights. The first category contains the Disparity, Marginal Likelihood, Noise Corrected, and the Global Threshold Filters. They preserve more than 50% of these links. The techniques in the second category keep a lower amount. In particular, the Poly Urn Filter preserves a tiny portion (7%), and the ECM Filter does not preserve any. Finally, all techniques preserve a fraction similar to the reference filter for the spoke-spoke edge weights, except the Poly Urn Filter. Indeed, it keeps a low amount of spoke-spoke edge weights (7%).

All filtering techniques generally maintain a similar behavior except for the ECM Filter. They preserve a lower fraction

TABLE V  
SUMMARY ON STRENGTHS AND WEAKNESSES OF THE STATISTICAL BACKBONE FILTERING TECHNIQUES.

Filter	Strengths	Weaknesses
Global Threshold	<ul style="list-style-type: none"> <li>Preserves the highest possible fraction of weights</li> <li>Retains a large connected component</li> <li>Prioritizes hub-hub and hub-focus edges with high weights</li> </ul>	<ul style="list-style-type: none"> <li>Preserves a medium portion of nodes (60%)</li> <li>Ignores edges with low weights</li> </ul>
Disparity	<ul style="list-style-type: none"> <li>Preserves a very high fraction of weights</li> <li>Retains a large connected component</li> <li>Prioritizes hubs-spoke edges with high weights</li> </ul>	<ul style="list-style-type: none"> <li>Preserves a medium portion of the nodes (67%)</li> <li>Biased to edges with high weights</li> </ul>
Polya Urn	<ul style="list-style-type: none"> <li>Retains a large connected component</li> <li>Preserves similar portions for each edge type</li> </ul>	<ul style="list-style-type: none"> <li>Preserves a medium portion of the nodes (58%)</li> <li>Preserves very low fraction of weights (7.6%)</li> </ul>
Marginal Likelihood	<ul style="list-style-type: none"> <li>Preserves a high portion of weights (34.7%)</li> <li>Retains a large connected component</li> <li>Prioritizes edges between hub-spoke and focus-spoke edges, preserving more hubs and spoke airports</li> </ul>	<ul style="list-style-type: none"> <li>Preserves a medium portion of nodes (67%)</li> </ul>
Noise Corrected	<ul style="list-style-type: none"> <li>Preserves a high portion of weights (34%)</li> <li>Retains a large connected component</li> <li>Prioritizes edges between hub-spoke and focus-spoke edges, preserving more hubs and spoke airports</li> </ul>	<ul style="list-style-type: none"> <li>Preserves a medium portion of nodes (67%)</li> </ul>
GLOSS	<ul style="list-style-type: none"> <li>Preserve a medium portion of the nodes (70%)</li> <li>Prioritizes hub-spoke edges</li> <li>Preserves almost all the hub and half of the spoke airports</li> </ul>	<ul style="list-style-type: none"> <li>Preserves a medium portion of weights (20%)</li> <li>Do not retain a large connected component</li> </ul>
LANS	<ul style="list-style-type: none"> <li>Preserves a high portion of nodes (83%)</li> <li>Preserves a high portion of weights (30%)</li> <li>Prioritizes hub-spoke edges</li> </ul>	<ul style="list-style-type: none"> <li>Do not retain a large connected component</li> <li>Ignores focus-spoke edges.</li> </ul>
ECM	<ul style="list-style-type: none"> <li>Preserves a high portion of nodes (88%)</li> <li>Prioritizes hub-spoke and focus-spoke edges</li> <li>Preserves a medium portion of weights (26%)</li> </ul>	<ul style="list-style-type: none"> <li>Do not retain a large connected component</li> </ul>

TABLE VI

RELATIVE SIZE OF THE COMPONENTS EXTRACTED BY THE FILTERING TECHNIQUE IN THE THE NORTH AMERICA NETWORK IN DECREASING ORDER. THE FIRST THREE COLUMNS SHOWS THE SIZE OF COMPONENTS WITH A SIZE LARGER THAN 10%, THE LAST COLUMN SHOWS THE NUMBER OF COMPONENTS WITH SIZE LOWER THAN 10%.

Filter	% C1	% C2	% C3	# C $\leq$ 10%
<b>Global Threshold</b>	<b>100</b>	-	-	-
Disparity	96	-	-	4
Polya Urn	81	-	-	20
Marginal Likelihood	100	-	-	-
Noise Corrected	100	-	-	-
GLOSS	85	-	-	16
LANS	26	17	13	10
ECM	22	20	-	54

of nodes than in the toy example. Still, the LANS and ECM Filters maintain the highest percentage of nodes compared to the other techniques. Moreover, due to the spoke-spoke edges in the network, the Disparity Filter, Polya Urn Filter, GLOSS

TABLE VII

FRACTION OF NODES, HUB, FOCUS, AND SPOKE AIRPORTS PRESERVED BY THE FILTERING TECHNIQUES IN THE NORTH AMERICA NETWORK.

Filter	% Nodes	% Hubs	% Focus	% Spokes
<b>Global Threshold</b>	<b>15.6</b>	<b>100</b>	<b>82.6</b>	<b>10.6</b>
Disparity	26.8	100	95.6	22.2
Polya Urn	33.3	100	95.6	29.1
Marginal Likelihood	17.4	100	91.3	12.3
Noise Corrected	16.3	100	82.6	11.4
GLOSS	35.3	100	86.9	31.5
LANS	49.9	100	91.3	46.95
ECM	48.9	32.1	17.3	50.5

Filter, and LANS Filter shift their priority in preserving hub-spoke edges to the spoke-spoke edges. Additionally, the ECM Filter almost ignores all types of edges and keeps only the spoke-spoke edges. These edges are associated with small weights usually, which explains the low percentage of weight

TABLE VIII

THE PROPORTION OF HUB-HUB (H-H), HUB-FOCUS (H-F), HUB-SPOKE (H-S), FOCUS-FOCUS (F-F), FOCUS-SPOKE (F-S), AND SPOKE-SPOKE (S-S) EDGES PRESERVED BY THE FILTERING TECHNIQUES IN THE NORTH-AMERICAN NETWORK.

Filter	% H-H	% H-F	% H-S	% F-F	% F-S	% S-S
<b>Global Threshold</b>	<b>23.4</b>	<b>15.8</b>	<b>10.9</b>	<b>10.3</b>	<b>8.5</b>	<b>1.9</b>
Disparity	16.5	14.2	10.8	12.3	8.9	4.7
Polya Urn	13	13.1	10.7	6.1	7.7	7.7
Marginal Likelihood	10.7	13.1	12.1	11.3	12.8	2.4
Noise Corrected	11.4	14.7	12.3	10.3	10.9	2.3
GLOSS	10.4	9.8	9.8	6.1	8.7	11.3
LANS	5.3	3.2	14.8	5.1	7.1	6.5
ECM	0.3	0.2	1.6	0	1.3	39.6

TABLE IX

FRACTION OF WEIGHTS PRESERVED BY THE FILTERING TECHNIQUES IN THE NORTH-AMERICAN NETWORK FOR THE DIFFERENT TYPE OF EDGES: ALL TYPE OF EDGES (W), HUB-HUB (H-H), HUB-FOCUS (H-F), HUB-SPOKE (H-S), FOCUS-FOCUS (F-F), FOCUS-SPOKE (F-S), AND SPOKE-SPOKE (S-S)

Filter	% W	% H-H	% H-F	% H-S	% F-F	% F-S	% S-S
<b>Global Threshold</b>	<b>47.6</b>	<b>61.8</b>	<b>56.3</b>	<b>44.5</b>	<b>54.9</b>	<b>39.2</b>	<b>23.5</b>
Disparity	41.1	51.7	52.3	37.2	56.8	33.7	21.1
Polya Urn	11.6	12.7	12	13.2	7.1	6.3	6.9
Marginal Likelihood	44.8	38.5	50	45.6	56.7	50.1	28.1
Noise Corrected	45.3	38.9	52.8	46.9	54.9	45.8	27.5
GLOSS	25.4	27.9	28.7	24.4	21.1	23.8	24.1
LANS	24.1	23.3	16.8	28.8	30.6	15.4	20.2
ECM	2.4	1.6	0.8	1.5	0	0.5	21.1

preserved by the ECM Filter.

## V. CONCLUSION

This paper evaluates statistical backbone extraction techniques in filtering the hub and spoke infrastructure of the air transportation network. Results show that the reference (Global Threshold Filter) extracts a backbone mainly composed of the hub and focus airports, revealing the rich-club phenomena in the air transportation network. In contrast, other statistical filters tend to preserve various categories of links (edges connecting the hub to spoke, focus to spoke, and spoke to spoke airports), revealing more information on the hub and spoke foundation used by airline companies. The ECM filter has a unique behavior. It keeps mainly edges between spoke airports uncovering the infrastructure connecting the regional spoke airports. Moreover, LANS and ECM split the backbone into multiple components. It is an undesired property for a transportation network. Indeed, both filters preserve a low portion of hub-hub and focus-hub edges which usually hold the network together. Furthermore, results show that the Disparity Filter, Marginal Likelihood Filter, and Noise Corrected Filter preserve the highest fraction of weights. In contrast, Polya Urn Filter and ECM Filter perform poorly. The other techniques retain a moderate weight fraction between these extremes. Future work will consider an extensive investigation of various transportation networks to consolidate these findings.

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