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# The Role of Index Traders in the Financialization of Commodity Markets: A Behavioral Finance Approach

Camille Aït-Youcef\*      Marc Joëts†

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## Abstract

This paper investigates the impact of financialization on commodity prices across various markets, particularly over recent decades. We introduce a groundbreaking theoretical model that incorporates both chartist-fundamentalist traders and institutional investors, targeting trading signals in two distinct commodity markets. In alignment with empirical data, our model enables institutional investors to participate in multiple markets simultaneously through index investing. Our findings indicate that the interactions between traditional traders and index investors create price dynamics that closely mirror observed patterns in commodity markets. Specifically, index investors not only cause prices to diverge from their fundamental values but also substantially influence the trading positions of other market actors. Moreover, we elucidate the crucial role of index investors in amplifying market correlations—both among different commodities and between commodities and equities, especially during periods of intense price fluctuations. Our innovative theoretical model goes beyond conventional chartist-fundamentalist frameworks, offering a robust alternative for understanding the complex pricing dynamics of commodities at large.

JEL Classification: Q4, G41, G15, D81

Keywords: Commodity prices; speculation; agents-based model; nonlinear dynamics.

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# 1 Introduction

The role of speculators in financial markets, particularly in commodity markets, has been a subject of intense debate among policymakers, media, and academics in recent years. There are traditionally two conflicting views on the intrinsic nature of speculation. The classical view, as exemplified by Keynes (1923) and Friedman et al. (1953), posits that irrational speculators cannot survive long-term in the marketplace, as asset prices invariably reflect their fundamentals.<sup>1</sup> In contrast, the behavioral finance perspective contends that bounded rationality—manifested through herding behaviors, noise trading, speculative bubbles, and the like—can destabilize markets and cause prices to deviate from their fundamentals (see, Hirshleifer (1989), Shleifer & Summers (1990), De Long et al. (1990), and Shiller (2003) to name a few).

This debate has gained renewed urgency with the surge in commodity prices since the early 2000s and the increasing participation of financial institutions in commodity markets. Notable concerns have been directed toward institutional investors, such as hedge funds or index traders, who trade a wide array of financial products and generally operate with limited public disclosure. According to the U.S. Commodity Futures Trading Commission (CFTC), the scale of financial institutions' involvement in commodity index-related instruments expanded dramatically over the past decade. Such developments have stoked fears that the financialization of commodity markets is disrupting price mechanisms and cross-market linkages (Tang & Xiong (2012) and Büyüksahin & Robe (2014)).

This paper aims to address these concerns by focusing on areas where the impact of financialization has significant social and economic repercussions, as outlined by Myers (2006), Baumeister & Kilian (2014), and Joëts (2015). For example, fluctuations in food prices present considerable challenges for low-income countries and prompt questions regarding the role of private financial interests in essential public utilities. The same holds true for the energy supply in low-income nations. Moreover, these issues are frequently discussed in political debates concerning the regulation of commodity derivatives.

A substantial body of research has sought to empirically investigate the role of financial institutions in shaping commodity prices. Some studies have primarily focused on the comovements between commodity and financial assets (see, e.g., Tang & Xiong (2012), Silvennoinen & Thorp (2013), Creti et al. (2013)), Büyüksahin & Robe (2014), Brunetti et al. (2016), Bruno et al. (2017), and Le Pen & Sévi (2018)), while others have examined markets influenced by various structural shocks (see, e.g., Kilian (2009), Kilian & Murphy (2012), and Juvenal & Petrella (2015)).<sup>2</sup> The results are mixed and often hinge on how speculation is quantified. Some studies indirectly measure speculation through correlations across asset classes, while others capture it through the accumulation of commodity inventories. Remark-

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<sup>1</sup>Arbitrageurs buying when the price is low and selling when the price is high quickly counter mispricings.

<sup>2</sup>See Fattouh et al. (2013) and Joëts (2015) for a review.

ably, none of these studies explicitly model the trading behaviors that operate within these markets. A common thread in earlier research is the limiting assumption of market efficiency, rational investors, and representative agents, often neglecting the heterogeneity among market participants. Although contributions in behavioral finance suggest that heterogeneous, boundedly rational speculators can endogenously generate complex price dynamics<sup>3</sup> few theoretical attempts have been made to model commodity price movements. Notable exceptions include the chartist-fundamentalist models developed by He & Westerhoff (2005), Ter Ellen & Zwinkels (2010), and more recently Joëts (2015). Despite empirical evidence suggesting the significance of institutional investors, these models do not account for the role of index traders in commodity price dynamics.

This paper aims to fill an existing research gap by modeling the role of index traders in commodity markets. We construct an agent-based model featuring two types of traders: boundedly rational chartist-fundamentalist traders and index traders. These actors operate in two distinct markets. Index traders allocate resources in both markets simultaneously, guided by index-based profit considerations. In contrast, chartist-fundamentalist traders engage in one market at a time, utilizing either technical analyses based on market trends or fundamental analyses grounded in supply and demand factors. We establish analytical conditions for the local asymptotic stability of the model's steady-state equilibrium and find that the incorporation of index traders leads to significant deviations of commodity prices from their fundamental values. Our model also produces price dynamics that more accurately reflect empirical statistical features of commodity markets, such as kurtosis, skewness, and clustered volatility, compared to traditional models that only include chartist and fundamentalist traders. Overall, our results underscore the importance of incorporating index traders in agent-based models to capture the financialization dynamics in commodity markets more accurately. We further propose several extensions that assess the role of speculation in influencing asset co-movements. These extensions emphasize the impact of index traders on evolving market correlations. Our model is versatile enough to be applied to virtually any commodity market where institutional investors are prevalent, serving as a robust tool for forecasting commodity price trends, especially in volatile environments.

The remainder of the paper is organized as follows: The next section briefly reviews the traditional chartist-fundamentalist approach, presents our agent-based model featuring index traders, and discusses stability and bifurcation properties. Section 3 outlines the model's calibration and analyzes the role of index traders in commodity price dynamics. Section 4 delves into the impact of index traders on commodity price co-movements, and Section 5 concludes the paper.

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<sup>3</sup>See, e.g., Kirman (1991), Lux (1995), and Brock & Hommes (1998) among others.

## 2 The theoretical model

Our objective is to provide an accurate representation of speculative behaviors in commodity markets. The architecture of our model relies on the chartist-fundamentalist framework, in which traders exhibit bounded rationality and operate based on predefined rules. Depending on market conditions, traders may generate trading signals from historical price trends (i.e., trend followers) or opt to capitalize on discrepancies between current prices and market fundamentals. Price dynamics are shaped by the interplay between these two rule-based behaviors, as described by Brock & Hommes (1998). This section is divided into two parts for clarity: First, we offer a brief overview of the benchmark chartist-fundamentalist model as developed by Westerhoff & Dieci (2006); second, we introduce an extension tailored to the specific realities of commodity markets, which includes the role of index traders.

### 2.1 The benchmark model

In the literature on Heterogeneous Agent Models (HAMs), markets are commonly characterized by two types of beliefs: chartists and fundamentalists (Boswijk et al. (2007)). In our study, we build upon the work of Westerhoff & Dieci (2006) and posit that trading activities transpire in two separate speculative markets, labeled as Market 1 and Market 2. Traders employing a chartist (or technical) strategy base their decisions on historical price movements in these assets, while fundamentalists choose to trade based on the asset's price convergence toward an equilibrium value, which is determined by the market fundamentals. The choice to invest in either Market 1 or Market 2 is influenced by the historical and current profitability of each respective strategy.

The aggregate price adjustment for Markets 1 and 2 at time  $t + 1$  is described by the following log-linear equations:

$$P_{t+1}^1 = P_t^1 + a(W_t^{C^1} D_t^{C^1} + W_t^{F^1} D_t^{F^1}) + S_t^{P^1} \quad (1)$$

$$P_{t+1}^2 = P_t^2 + a(W_t^{C^2} D_t^{C^2} + W_t^{F^2} D_t^{F^2}) + S_t^{P^2} \quad (2)$$

$P_{t+1}^1$ ,  $P_{t+2}^2$ ,  $P_t^1$ , and  $P_t^2$  represent the log-linear prices for Markets 1 and 2 at times  $t + 1$  and  $t$ . The variable  $a$  indicates a positive price adjustment.  $D_t^{C^1}$ ,  $D_t^{C^2}$ ,  $D_t^{F^1}$ , and  $D_t^{F^2}$  denote the trading orders placed by chartists ( $C$ ) and fundamentalists ( $F$ ) in Markets 1 and 2 at time  $t$ . Similarly,  $W_t^{C^1}$ ,  $W_t^{C^2}$ ,  $W_t^{F^1}$ , and  $W_t^{F^2}$  signify the proportions of traders adhering to each strategy at time  $t$ . Both  $S_t^{P^1}$  and  $S_t^{P^2}$  are random variables<sup>4</sup>, where  $S_t^{P^1} \sim N(0, \sigma^{P^1})$  and  $S_t^{P^2} \sim N(0, \sigma^{P^2})$  for each respective market.

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<sup>4</sup>These shock variables represent the macroeconomic and financial factors that influence commodity prices.

At any given moment, market price dynamics adjust in response to the buying and selling behaviors exhibited by each category of traders. Excessive buying or selling will influence the aggregate price of the corresponding market in an upward or downward direction. The demands originating from chartists and fundamentalists in Markets 1 and 2 are defined as

$$D_t^{C^i} = c(P_t^i - P_{t-1}^i) + S_t^{C^i} \quad (3)$$

$$D_t^{F^i} = f(F_t^i - P_t^i) + S_t^{F^i} \quad (4)$$

where  $i = 1, 2$  for markets  $i$ ;  $S_t^{C^i} \sim N(0, \sigma^{C^i})$  and  $S_t^{F^i} \sim N(0, \sigma^{F^i})$  capture deviations from the first deterministic component.  $P$  signifies the log-linear price, while  $c > 0$  quantifies the potency of the price signal in the chartist strategy. For any given asset, a  $P_t > P_{t-1}$  condition in Equation (3) implies a positive signal, suggesting a buy action. In Equation (4),  $F_t^i$  represents the fundamental value at time  $t$  for each market  $i$ . The parameter  $f > 0$  measures the strength of the price deviation relative to its fundamental value, thereby signaling either market undervaluation ( $P < F$ ) or overvaluation ( $P > F$ ), which in turn suggests either a buying or selling rule, respectively.

Trading behaviors are not static and are permitted to transition between rules within each trading cycle. This switching mechanism is a crucial component of Heterogeneous Agent Models (HAMs) as it accounts for non-linear adjustments in pricing. Specifically, the switching mechanism—also referred to as ‘fitness’—in market  $i$  is predicated on the performance of each trading rule and can be described as follows:

$$A_t^{C^i} = (\exp[P_t^i] - \exp[P_{t-1}^i])D_{t-2}^{C^i} + bA_{t-1}^{C^i} \quad (5)$$

$$A_t^{F^i} = (\exp[P_t^i] - \exp[P_{t-1}^i])D_{t-2}^{F^i} + bA_{t-1}^{F^i} \quad (6)$$

The investment timing stipulates that orders submitted during period  $t - 2$  are executed at prices from period  $t - 1$ , and the profits are contingent on prices in period  $t$  (as captured in the first part of the equations). In the second part of the equations, the memory parameter  $b$  gauges the extent to which current fitness relies on past gains for strategic selection.<sup>5</sup> Therefore, the fitness or switching rule for strategies hinges on both current and past performance metrics.

In market  $i$ , the market shares for each type of trader—specifically  $W^{F^i}$  for fundamentalists and  $W^{C^i}$  for chartists—are influenced by their relative appeal. To ascertain these proportions, we adopt the discrete choice approach as laid out by Brock & Hommes (1998).

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<sup>5</sup>For a more in-depth discussion on the timing and the influence of past performances, refer to Brock & Hommes (2001) and Westerhoff & Dieci (2006).

$$W_t^{ij} = \frac{\exp[rA_t^{ij}]}{V} \quad (7)$$

$$W_t^0 = 1 - W_t^{C1} - W_t^{F1} - W_t^{C2} - W_t^{F2} \quad (8)$$

where  $V = \exp[rA_t^{C1}] + \exp[rA_t^{C2}] + \exp[rA_t^{F1}] + \exp[rA_t^{F2}] + \exp[0]$ ,  $i = 1, 2$  and  $j = C, F$ . The parameter  $r > 0$  gauges the sensitivity of traders when selecting the most viable strategy; specifically, higher values of  $r$  lead to more traders opting for the strategy with the highest fitness.

In the benchmark model, chartists and fundamentalists are limited to investing in a single market at any given time. Consequently, at each decision-making step, traders must select both the trading rule and the target market based on performance metrics. However, in commodity markets, a prevalent strategy for institutional investors involves diversifying their portfolio by investing in an index of multiple commodities. Known as index traders, these investors emphasize strategic asset allocation between classes of commodities and other financial assets like equities and bonds. They typically enter and exit positions in all commodities within a given index simultaneously. Our extended model incorporates this nuanced trading behavior, shedding light on the impact it has on both the price dynamics and co-movements within commodity markets.<sup>6</sup>

## 2.2 Heterogenous agent model with index traders: the HAM-IT

Commodity indices are generally constructed based on the values of nearby futures contracts with a delivery period extending beyond one month. This approach avoids the costs associated with holding physical commodities. At the conclusion of each maturity period, the commodity index outlines a so-called "roll," which involves replacing the current contract (known as the front month) with the subsequent contract. This mechanism facilitates passive long positions in the listed commodities. The two most prevalent commodity indices are the SP GSCI and the DJ-UBSCI. Index traders typically mimic the index by diversifying their portfolios across a basket of commodity markets. Upon reaching maturity, they often roll their positions forward, moving from one futures contract to another. This process creates a "weight-of-money" effect on prices, a phenomenon where large position changes by individual market participants can significantly influence market prices. Index traders are generally indifferent to price movements in individual commodities since they allocate their funds across all markets within the index based on strategic portfolio considerations.

The aim of our extension is to model the trading strategies of index traders and to analyze

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<sup>6</sup>See, for example, Barberis & Shleifer (2003), Tang & Xiong (2012), Büyüksahin & Robe (2014), and Brunetti et al. (2016).



their impact on the dynamics of commodity prices. Given that these traders participate in commodities futures and options markets for reasons other than hedging against specific commodity risks, our model provides a direct account of speculative activity (see citations Brunetti et al. (2016) and Bruno et al. (2017) for reference).

To align with market realities, our model assumes that Markets 1 and 2 are part of a commodity index. Consequently, when index traders take positions in the index, they are effectively investing in both markets simultaneously.

The commodity index at time  $t$  is defined as

$$I_t = \frac{1}{K} \sum_{k=1}^K P_{kt}$$

In this formulation,  $K = 2$  represents the total number of markets included in the index. The index is calculated as an arithmetic equally weighted average of the two commodity futures prices. It should be noted that the simulations presented in the paper are robust to variations in the weighting scheme. Additional results supporting this claim are available upon request from the authors. Index traders make investment decisions based on the historical performance of the commodity index, as elaborated further in the following sections.

$$D_t^{I^i} = \frac{1}{2} \times d(I_t - I_{t-1}) + S_t^{I^i} \quad (9)$$

where  $d > 0$  is the strength of the signal from  $t - 1$  to  $t$ , and  $S_t^{I^i} \sim N(0, \sigma^{I^i})$ .  $i$  is for market 1 and 2 respectively.

At each step, the performance of the strategy is evaluated by the following fitness equation

$$A_{t+1}^I = (\exp[P_{t+1}^1] - \exp[P_t^1])D_{t-1}^{I^1} + (\exp[P_{t+1}^2] - \exp[P_t^2])D_{t-1}^{I^2} + bA_t^I \quad (10)$$

In accordance with the timing framework set forth by Westerhoff & Dieci (2006), the fitness measure incorporates both the historical and most recent market performances for each market  $i$ . Orders submitted at time  $t - 1$  are executed at  $t$ . The parameter  $b$  serves as a memory factor, quantifying the influence of past performances on the current trading strategy. Specifically, as  $b \rightarrow 1$  the fitness is calculated as the cumulative sum of all past profits.

To streamline the discussion, the trading rule for index traders is succinctly outlined as follows:

$$\begin{aligned}
z_{t+1} &= I_t \\
v_{t+1} &= z_t \\
A_{t+1}^I &= (\exp[P_{t+1}^1] - \exp[P_t^1])\left(\frac{1}{2} \times d(z_t - v_t) + S_{t-1}^{I1}\right) \\
&\quad + (\exp[P_{t+1}^2] - \exp[P_t^2])\left(\frac{1}{2} \times d(z_t - v_t) + S_{t-1}^{I2}\right) + bA_t^I
\end{aligned}$$

The market shares of index traders in market  $i$  are exogenous and range from approximately 15% to 50% of the commodity market positions.<sup>7</sup> This share varies depending on the composition of the commodity index and shifts in relation to futures markets. According to their trading rules, investors either opt to invest (i.e., take a long position) or abstain from trading in the commodity markets.<sup>8</sup> The market shares of index traders ( $W_t^I + W_t^{-I}$ ) are calibrated based on the works of Mayer (2012) and Gilbert (2019). Utilizing the average percentage of total open interest held by index investors in commodity futures, we set the market shares of index investors at 30%. We define the market share of index investors as follows:

$$W_t^I = 0.3 \times \frac{\exp[rA_t^I]}{V^I} \quad (11)$$

with

$$V^I = \exp[rA_t^I] + \text{Exp}[0]$$

then,

$$W_t^{-I} = 0.3 - W_t^I$$

Investment from institutional investors has no direct linkage with the investment from other investors. Thus, we change the weights of chartists and fundamentalists discussed in the baseline model corresponding to 70% and no more 100% of the total market shares.

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<sup>7</sup>This share is based on the average percentage of total open interest held by long commodity index traders (see Capelle-Blancard & Coulibaly (2011) and Irwin & Sanders (2011)).

<sup>8</sup>The proportion of index investors within the overall pool of investors which includes both index and traditional investors is exogenously determined. However, the choice of whether or not to invest in each period is endogenous to index investors, contingent upon their confidence in the success of their trading strategy.

$$\begin{aligned}
W_t^{C^1} &= 0.7 \times \frac{\text{Exp}[rA_t^{C^1}]}{VG}, & W_t^{C^2} &= 0.7 \times \frac{\text{Exp}[rA_t^{C^2}]}{VG}, \\
W_t^{F^1} &= 0.7 \times \frac{\text{Exp}[rA_t^{F^1}]}{VG}, & W_t^{F^2} &= 0.7 \times \frac{\text{Exp}[rA_t^{F^2}]}{VG}, \\
VG &= \text{Exp}[rA_t^{C^1}] + \text{Exp}[rA_t^{C^2}] + \text{Exp}[rA_t^{F^1}] + \text{Exp}[rA_t^{F^2}] + \text{Exp}[0] \\
W_t^0 &= 0.7 - W_t^{C^1} - W_t^{C^2} - W_t^{F^1} - W_t^{F^2}
\end{aligned}$$

### 2.2.1 Dynamic and steady state of the HAM-IT

Both the dynamic and steady-state outcomes are shaped by incorporating the behavior of index traders into the benchmark model. The dynamical system of HAM-IT is defined by the following set of eleven equations, where the exponent "I" signifies the behavior of index traders. Appendix A offers further details on the analytical solutions and elaborates on the characteristics of the steady state.

$$\begin{aligned}
P_{t+1}^1 &= P_t^1 + a(W_t^{C^1}(c(P_t^1 - x_t^1) + S_t^{C^1}) + W_t^{F^1}(f(F^1 - P_t^1) + S_t^{F^1}) \\
&\quad + W_t^I \frac{1}{2} d(\frac{1}{2}(P_t^1 + P_t^2) + S_t^{I^1} - \frac{1}{2}(x_t^1 + x_t^2) + S_{t-1}^{I^1}) + S_t^{I^1}) + S_t^{P^1} \\
x_{t+1}^1 &= P_t^1 \\
y_{t+1}^1 &= x_t^1 \\
P_{t+1}^2 &= P_t^2 + a(W_t^{C^2}(c(P_t^2 - x_t^2) + S_t^{C^2}) + W_t^{F^2}(f(F^2 - P_t^2) + S_t^{F^2}) \\
&\quad + W_t^I \frac{1}{2} d(\frac{1}{2}(P_t^1 + P_t^2) + S_t^I - \frac{1}{2}(x_t^1 + x_t^2) + S_{t-1}^{I^2}) + S_t^{P^2}) \\
x_{t+1}^2 &= P_t^2 \\
y_{t+1}^2 &= x_t^2 \\
A_{t+1}^{C^1} &= (\text{exp}[P_{t+1}^1] - \text{exp}[P_t^1])(c(x_t^1 - y_t^1) + S_{t-1}^{C^1}) + bA_t^{C^1} \\
A_{t+1}^{F^1} &= (\text{exp}[P_{t+1}^1] - \text{exp}[P_t^1])(f(F^1 - x_t^1) + S_{t-1}^{F^1}) + bA_t^{F^1} \\
A_{t+1}^{C^2} &= (\text{exp}[P_{t+1}^2] - \text{exp}[P_t^2])(c(x_t^2 - y_t^2) + S_{t-1}^{C^2}) + bA_t^{C^2} \\
A_{t+1}^{F^2} &= (\text{exp}[P_{t+1}^2] - \text{exp}[P_t^2])(f(F^2 - x_t^2) + S_{t-1}^{F^2}) + bA_t^{F^2} \\
A_{t+1}^I &= (\text{exp}[P_{t+1}^1] - \text{exp}[P_t^1])(\frac{1}{2}d(\frac{1}{2}(x_t^1 + x_t^2) + S_{t-1}^I - (\frac{1}{2}(y_t^1 + y_t^2) - S_{t-1}^I) + S_t^{I^1})) \\
&\quad + (\text{exp}[P_{t+1}^2] - \text{exp}[P_t^2])(\frac{1}{2}d(\frac{1}{2}(x_t^1 + x_t^2) + S_{t-1}^I - (\frac{1}{2}(y_t^1 + y_t^2) - S_{t-1}^I) + S_t^{I^2})) + bA_t^I
\end{aligned}$$

Assuming that the reaction parameters  $a$ ,  $c$ ,  $d$ , and  $f$  are strictly positive, and the memory parameter  $b$  lies in the range  $0 \leq b < 1$ , the dynamical variables at the unique steady state

can be obtained from the system as follows:

$$\begin{aligned}\bar{P}^1 &= \bar{x}^1 = \bar{y}^1 = \bar{F}^1 \\ \bar{P}^2 &= \bar{x}^2 = \bar{y}^2 = \bar{F}^2 \\ \bar{A}^{C^1} &= \bar{A}^{F^1} = \bar{A}^{C^2} = \bar{A}^{F^2} = \bar{A}^I = 0\end{aligned}$$

In the long run, the last expression indicates that both prices and the index converge to their fundamental levels. Agents do not realize any profits, leading to a null average realized profit for all market participants. Consequently, at the steady state, the market share is as follows:

$$\bar{W}^{C^1} = \bar{W}^{F^1} = \bar{W}^{C^2} = \bar{W}^{F^2} = \bar{W}^I = \bar{W}^{I^0} = \bar{W}^0 = \frac{1}{7}$$

The local stability of the steady state in the HAM-IT model is determined through the derivation of the Jacobian matrix. Further details on the derivation and the eigenvalues of this matrix, as well as the necessary conditions for local stability, are provided in Appendix A. We demonstrate that the model's stability is contingent on the behavior of index traders, specifically the parameter  $d$ . When  $d \rightarrow 0$ , the steady state is locally stable in the long run—meaning all eigenvalues have a modulus less than one under the following conditions:

$$\begin{aligned}\frac{7}{4a} &> c \\ \sqrt{7}\sqrt{7-4ac} &> 0\end{aligned}$$

When  $d \rightarrow \infty$ , the eigenvalue  $\lambda_6$  exceeds one in modulus, leading to explosive price behavior in the long run. This finding underscores the destabilizing role of index traders in the dynamics of commodity prices. Our simulations indicate that the Hopf bifurcation occurs at the threshold level of  $d = 13.31$  assuming that  $a = 1$  and  $c = f = 0.05$ .

### 3 Dynamics of commodity prices with index traders

After establishing the asymptotic properties of our model, we turn our focus to exploring how Heterogeneous Agent Models (HAMs) enhance our comprehension of the impact of speculation on commodity prices. To achieve this, we carry out extensive numerical simulations across various scenarios and benchmark our results against the statistical attributes of selected 1-month futures contracts for commodities including crude oil, natural gas, copper, gold, corn, and cocoa.<sup>9</sup> These simulations yield 5,000 daily price observations, covering a timespan of approximately 20 years from 2000 to 2020.

We begin by outlining the calibration framework for each model (i.e., the HAM-IT and the benchmark) and present some preliminary results. Following this, we engage in a thorough

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<sup>9</sup>For additional details on the markets under consideration, see Appendix B.

Monte Carlo analysis to assess which models are most effective in aligning with the statistical properties of actual commodity prices. This analysis enables us to delve into how index traders exacerbate price fluctuations and influence trading strategies, thereby illuminating the financialization of commodity markets.

### 3.1 Models' calibration and preliminary results

Our dual simulation experiments rely on the parameter settings outlined in Table 1 for the benchmark model, and in Table 2 for the HAM-IT model. Although both tables are largely similar, they differ in parameters associated with the behavior of index traders—specifically, the strength of the index signal ( $d$ ), the deviation from the deterministic component ( $\sigma^I$ ), and traders' sensitivity to performance ( $r$ ). Key parameters in the model are those that quantify the impact of each investor type on price—namely,  $a^c$ ,  $a^f$ , and  $d$  for chartists, fundamentalists, and index traders, respectively. While calibration can be a critical step, numerous empirical studies suggest that the value for these parameters typically falls within the range of 0 to 0.1 (see, for example, Westerhoff & Reitz (2003), He & Westerhoff (2005), and Dieci & He (2018)). To generate plausible dynamics and maintain consistency with the benchmark model, we follow Westerhoff & Dieci (2006) and Reitz & Westerhoff (2007), selecting a value of 0.05 for each investor category.<sup>10</sup> Another crucial parameter is traders' sensitivity in choosing the optimal strategy, denoted as  $r$ . In our study,  $r$  is set at 250 for the benchmark model and 60 for our extension.<sup>11</sup>

Table 1: Parameters setting for the benchmark HAM

$a = 1$	$a^c = 0.05$	$a^f = 0.05$	$b = 0.975$	$\sigma^{P1} = \sigma^{P2} = 0.01$
$r = 250$	$F^1 = 0$	$F^2 = 0$	$\sigma^{C1} = \sigma^{C2} = 0.05$	$\sigma^{F1} = \sigma^{F2} = 0.05$

<sup>10</sup>Simulation results are robust to other values within this range. Additional results are available upon request from the authors.

<sup>11</sup>The value of  $r$  was chosen to yield reasonable dynamics, as supported by Westerhoff & Dieci (2006).

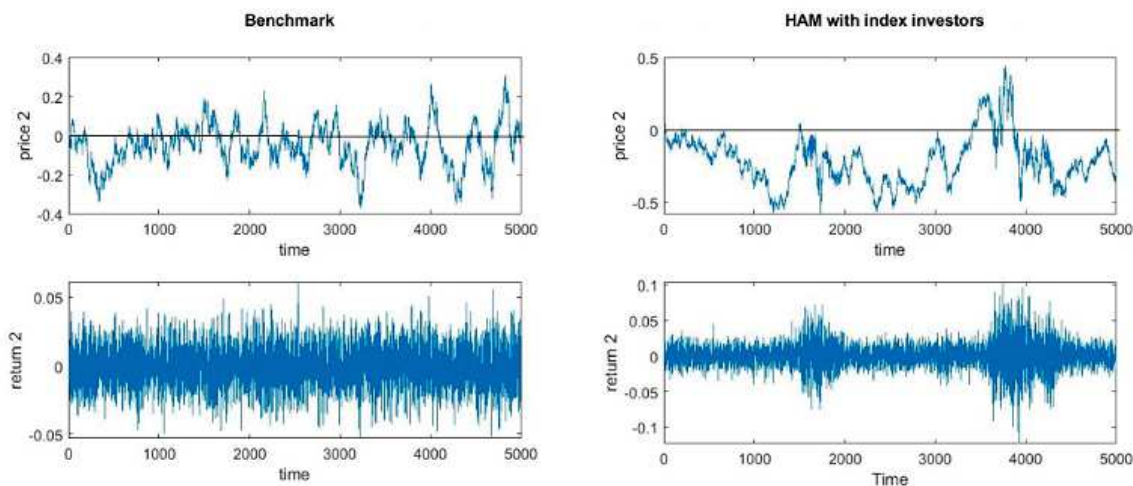
Table 2: Parameters setting for HAM-IT

$a = 1$	$a^c = 0.05$	$a^f = 0.05$	$\sigma^{P^1} = \sigma^{P^2} = 0.01$
$r = 60$	$F^1 = 0$	$F^2 = 0$	$\sigma^{C^1} = \sigma^{C^2} = 0.05$
$b = 0.975$	$\sigma^{F^1} = \sigma^{F^2} = 0.05$	$d = 0.05$	$\sigma^{I^1} = \sigma^{I^2} = 0.05$

Figure 1 presents our simulated log price and return series for market 2, comparing the benchmark model (left panel) with the HAM-IT model (right panel).<sup>12</sup> In both models, log prices appear to oscillate around their fundamental levels of zero, exhibiting intermittent phases of boom and bust. For example, prices in the HAM-IT model are predominantly undervalued relative to fundamentals from period 0 to 3000, and overvalued from period 3500 to 4000. The last two panels show log price returns for market 2, derived from each model. These returns serve as proxies for market volatility. As evidenced, fluctuations in price returns are significant throughout the observed period. Intriguingly, while the benchmark model does not display a clear pattern, the HAM-IT model manifests clustering in return volatility. In line with the behavior often observed in commodity markets, this suggests that periods of low volatility alternate with periods of high volatility (see, Creti et al. (2013)).

<sup>12</sup>Simulations for market 1 yield materially similar results. These are available upon request from the authors.

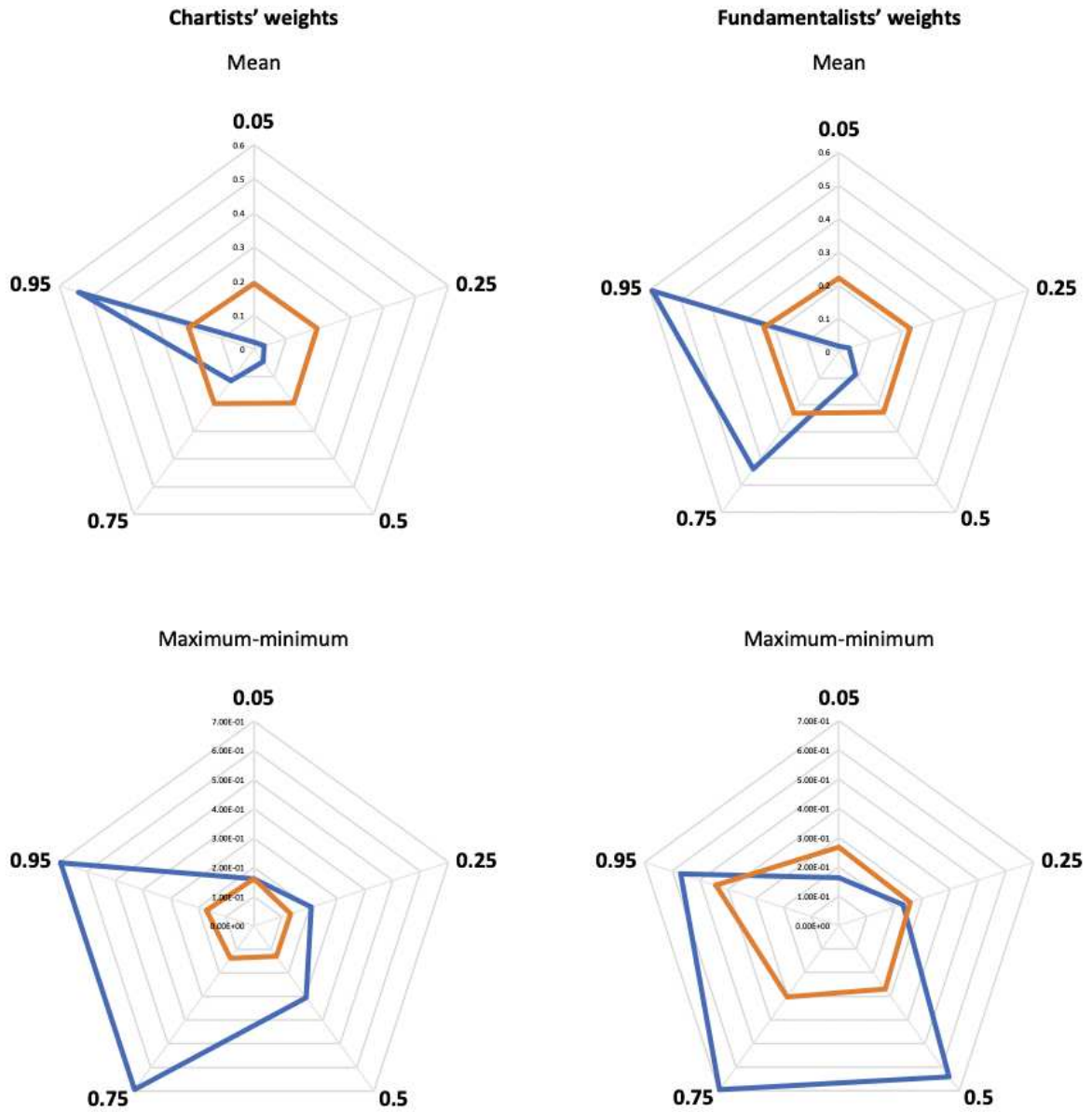
Figure 1: Simulated log prices and returns for market 2



Note: These figures report simulated log prices and returns for market 2 using parameters setting for the benchmark HAM (see Table 1) and HAM with index investors (see Table 2). Black line is the fundamental value.

The dynamics of trading strategies fundamentally drive price behavior over time. We hypothesize that the positions taken by index traders not only influence price dynamics but also shift the trading positions of traditional investors, namely chartists and fundamentalists. To test this hypothesis, Figure 2 displays radar plots of the trading weights for chartists and fundamentalists, comparing results from the benchmark model (shown in orange) with those from the HAM-IT model (in blue). The top panel represents the average weights across our 5,000 simulations, while the bottom panel illustrates the range between the maximum and minimum weights. Additionally, we present these metrics at various quantile levels (e.g., 0.05, 0.25, 0.5, 0.75, and 0.95) to capture shifts in trading strategies more comprehensively. On average, the trading weights in the benchmark model remain relatively consistent across quantiles. In contrast, the HAM-IT model reveals a pronounced shift toward extreme positions for both chartists and fundamentalists, as seen in the top panel. This indicates that the inclusion of index traders generally pushes traditional investors into taking more extreme positions. Moreover, the gap between the maximum and minimum trading positions also diverges substantially into extreme regions under the influence of index traders. Overall, the presence of index traders in the market prompts traditional investors to significantly amplify their extreme long positions on price, potentially leading to heightened price fluctuations.

Figure 2: Weights of chartist and fundamentalist strategies



Note: These figures report the weights associated with chartists and fundamentalists' strategies, comparing the benchmark model (shown in orange) to the HAM-IT model (in blue) at different quantiles (0.05, 0.25, 0.5, 0.75, 0.95). The top panel portrays the average weights based on the 5,000 simulations, while the bottom panel illustrates the range between the maximum and minimum weights.

### 3.2 Stylized facts on commodity price dynamics

To further assess the accuracy of our model in capturing the statistical properties of real commodity prices, we conduct an exhaustive Monte Carlo analysis using our simulated price data. Building on the work of Westerhoff & Dieci (2006), speculative markets can be characterized



by four key features<sup>13</sup>: (i) the presence of bubbles and crashes, leading to price distortions; (ii) elevated levels of volatility and clustering; (iii) pronounced kurtosis, making the distribution leptokurtic; and (iv) temporal dependence manifested by strong autocorrelation in returns. To quantify these characteristics, we employ the following statistical measures<sup>14</sup>: minimal and maximal daily returns; return volatility  $V$  calculated as the average of absolute returns; distortion  $D$  relative to fundamental values; kurtosis  $K$ ; and the coefficients of autocorrelation for both returns  $ac_r^i$  and absolute returns  $ac_{|r|}^i$ .

Table 3 presents the estimated statistics for artificial agricultural futures returns in market 2, generated by the HAM-IT model across the 5, 25, 50, 75, and 95 percent quantiles.<sup>15</sup> The statistics from the benchmark model are shown in parentheses for comparison. Each model comprises 1,000 simulations, each with 5,000 observations. Broadly speaking, key statistics suggest that HAM-IT yields higher estimates compared to the benchmark model. The range of extreme negative returns varies between -14.89 and -7.37 percent, and extreme positive returns fluctuate between 7.42 and 14.86 percent. While the models don't differ significantly in terms of volatility, both mispricing and kurtosis estimates from HAM-IT substantially exceed those from the benchmark. For example, mispricing at the 95th percentile is more than ten-fold higher in HAM-IT (160.63 percent vs. 10.98 percent). Furthermore, at least 95 percent of our simulations display excess kurtosis of 12.81. Autocorrelation coefficients for returns are not significant in either model. However, the autocorrelation coefficients for absolute returns are both significant and higher in HAM-IT, especially more than 90 percent of the time (0.29 vs. 0.03 at the 95th percentile quantile). The persistence of these autocorrelation functions across the five lags considered reveals long memory in volatility dynamics. In summary, incorporating index traders into the traditional Heterogeneous Agent Model (HAM) appears to significantly amplify price fluctuations and results in an unstable, non-fundamental steady state commonly referred to as Hopf bifurcation (see Brock & Hommes (2001)). Consequently, price movements during certain periods can deviate substantially from their fundamental values, i.e., supply-demand conditions, resembling the behavior of speculative assets.

We now assess the model's efficacy by juxtaposing our estimated statistics with those of real commodity futures prices from 2000 to 2020. We consider a diverse set of commodity future prices at 1-month intervals, including crude oil, natural gas, copper, aluminium, corn, cocoa, gold, and silver.<sup>16</sup> Table 4 summarizes the statistics of these commodity prices. Evidently, extreme returns vary significantly across all markets. For instance, corn price returns fluctuate between -26.36 and 12.72. Such extreme returns, coupled with high volatility—ranging from 1.21 to 2.20—point to substantial price distortions. All markets exhibit either high or extremely high kurtosis, ranging from 7.30 to 20.30. Except for gold, the autocorrelation co-

<sup>13</sup>For a theoretical discussion, see Cont (2001), Sornette & Andersen (2002), and Lux & Ausloos (2002). In our specific context, existing literature has shown that the properties of commodity markets closely resemble those of traditional financial assets (see Creti et al. (2013), Joëts (2014), and Joëts (2015)).

<sup>14</sup>For additional details, refer to Westerhoff & Dieci (2006).

<sup>15</sup>Results for market 1 are closely aligned and are available upon request.

<sup>16</sup>For further details, see Appendix B.

efficients of absolute returns display strong temporal dependence in volatility. A comparison between Table 3 and Table 4 demonstrates that our HAM-IT model more accurately replicates the statistical properties of commodity futures prices, capturing crucial stylized facts of speculative markets. Overall, our findings suggest that incorporating index traders enhances the model's ability to faithfully represent commodity price behaviors when compared to existing models. Index traders' strategies impact not only the behaviors of other traders in the market but also the dynamics of asset prices, making markets more erratic. In the subsequent section, we utilize HAM-IT as a computational laboratory to conduct artificial experiments investigating the role of index traders in commodity price co-movements.

Table 3: Stylized facts on artificial commodity markets

Quantile	$r_{min}$	$r_{max}$	$V$	$D$	$K$
0.05	-14.89 (-6.77)	7.42 (4.98)	0.97 (1.20)	18.77 (7.67)	3.52 (2.52)
0.25	-13.15 (-6.00)	9.57 (5.32)	1.23 (1.21)	26.03 (8.50)	4.92 (2.87)
0.50	-11.88 (-5.65)	11.76 (5.62)	1.72 (1.23)	38.34 (9.16)	5.34 (3.22)
0.75	-9.88 (-5.35)	13.06 (5.98)	2.32 (1.24)	72.44 (9.94)	7.71 (3.71)
0.95	-7.37 (-4.99)	14.86 (6.68)	2.85 (1.25)	160.63 (10.98)	14.81 (5.40)
Quantile	$ac_r^1$	$ac_r^2$	$ac_r^3$	$ac_r^4$	$ac_r^5$
0.05	0.00 (-0.01)	-0.03 (-0.03)	-0.03 (-0.03)	-0.03 (-0.03)	-0.03 (-0.03)
0.25	0.01 (0.01)	-0.01 (-0.01)	-0.01 (-0.01)	-0.01 (-0.01)	-0.01 (-0.01)
0.50	0.03 (0.02)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
0.75	0.04 (0.03)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.00)
0.95	0.07 (0.04)	0.03 (0.02)	0.03 (0.02)	0.03 (0.02)	0.03 (0.02)
Quantile	$ac_{ r }^1$	$ac_{ r }^2$	$ac_{ r }^3$	$ac_{ r }^4$	$ac_{ r }^5$
0.05	0.01 (-0.02)	0.00 (-0.02)	0.00 (-0.02)	0.00 (-0.02)	0.00 (-0.02)
0.25	0.05 (0.00)	0.05 (0.00)	0.05 (0.00)	0.05 (0.00)	0.04 (0.00)
0.50	0.18 (0.01)	0.17 (0.01)	0.18 (0.01)	0.18 (0.01)	0.18 (0.01)
0.75	0.24 (0.02)	0.23 (0.02)	0.24 (0.02)	0.23 (0.02)	0.24 (0.02)
0.95	0.29 (0.03)	0.29 (0.03)	0.28 (0.03)	0.28 (0.03)	0.28 (0.03)

Note: Statistics estimated from the benchmark model are shown in parentheses.  $r_{min}$  and  $r_{max}$  respectively represent the minimum and maximum daily returns, measured as log price changes.  $V$  denotes volatility, defined as the average absolute return.  $D$  is the distortion coefficient, representing the deviation of observed log prices from their fundamental value.  $K$  stands for kurtosis.  $ac_r^i$  and  $ac_{|r|}^i$  where  $i = 1, \dots, 5$ , are the autocorrelation coefficients for daily returns and absolute daily returns, ranging from lag 1 to lag 5.

Table 4: Stylized facts of real agricultural returns (covering the period 2000-2020)

Commodity returns	$r_{min}$	$r_{max}$	$V$	$D$	$K$
Crude oil	-10.15	15.85	1.30	-	20.30
Natural gas	-7.69	8.75	1.45	-	10.60
Copper	-8.36	10.58	1.50	-	10.40
Aluminium	-6.19	7.75	1.82	-	7.30
Corn	-26.86	12.75	2.20	-	12.41
Cocoa	-17.09	7.81	1.72	-	7.40
Gold	-11.68	11.42	1.21	-	10.82
Silver	-13.41	19.32	1.84	-	12.27
Commodity returns	$ac_r^1$	$ac_r^2$	$ac_r^3$	$ac_r^4$	$ac_r^5$
Crude oil	0.02	0.00	0.01	0.01	0.00
Natural gas	-0.01	0.01	0.00	0.01	0.03
Copper	-0.02	-0.01	0.01	0.00	0.00
Aluminium	0.00	0.01	0.01	0.02	0.00
Corn	0.02	-0.01	0.01	0.01	-0.01
Cocoa	0.06	0.00	-0.03	-0.03	0.00
Gold	-0.04	0.00	0.01	0.00	0.01
Silver	0.01	-0.01	0.00	-0.02	-0.05
Commodity returns	$ac_{ r }^1$	$ac_{ r }^2$	$ac_{ r }^3$	$ac_{ r }^4$	$ac_{ r }^5$
Crude oil	0.25	0.18	0.16	0.16	0.08
Natural gas	0.21	0.15	0.24	0.11	0.15
Copper	0.18	0.11	0.19	0.09	0.10
Aluminium	0.19	0.19	0.12	0.09	0.03
Corn	0.11	0.08	0.06	0.09	0.08
Cocoa	0.11	0.06	0.03	0.07	0.05
Gold	0.09	0.09	0.03	0.02	0.05
Silver	0.14	0.14	0.11	0.11	0.09

Note:  $r_{min}$  and  $r_{max}$  represent the minimum and maximum daily returns, respectively, calculated using log price changes.  $V$  denotes volatility, defined as the average absolute return. Since the fundamental values of these markets are unknown, distortion cannot be computed.  $K$  stands for kurtosis.  $ac_r^i$  and  $ac_{|r|}^i$  where  $i = 1, \dots, 5$ , are the autocorrelation coefficients for daily returns and absolute daily returns from lag 1 to lag 5.

## 4 Index traders and commodity prices co-movements

A salient feature of the financialization of commodities is the heightened co-movements among various market prices. Financial institutions, such as index traders or hedge funds, often trade across a diverse range of asset classes, facing fewer restrictions compared to tradi-

tional commodity traders. Their trading activities thus frequently induce significant risk transfer between markets, amplifying price co-movements (Basak & Pavlova (2016), Rahi & Zigrand (2009)). The literature has both theoretically and empirically examined this phenomenon, considering co-movements both within commodities (Joëts (2014), Büyüksahin & Robe (2014), Brunetti et al. (2016), Bruno et al. (2017)) and between equity and commodity markets (Creti et al. (2013), Tang & Xiong (2012), Silvennoinen & Thorp (2013), Cheng et al. (2015), Le Pen & Sévi (2018)). Index traders, particularly when motivated by portfolio rebalancing needs and leverage constraints, appear to markedly enhance the propagation of shocks across markets, especially during periods of extreme volatility (Vayanos et al. (2010), Aït-Youcef (2019)).

In the upcoming simulated experiments, our objective is to ascertain the extent to which index traders exacerbate market co-movements. To this end, we extend our HAM-IT model to incorporate a third market.<sup>17</sup> We focus on three distinct trading scenarios:

1. Scenario 1 - No Restrictions on All Trading Strategies (Baseline Model): In this case, both traditional traders (fundamentalists and chartists) and index traders have the liberty to invest in all three markets.
2. Scenario 2 - Restrictions on Fundamentalists and Chartists Only: Here, traditional traders are constrained to investments in only the first two markets, whereas index traders remain unrestricted.
3. Scenario 3 - Restrictions on Index Traders Only: In this scenario, fundamentalists and chartists can invest across all markets, but index traders are limited to participating in the first two markets.

Each trading scenario serves as a lens through which we can evaluate the influence of index investors on the co-movements in various contexts: (i) commodity futures prices within the same asset class; (ii) a basket of similar commodity prices in relation to other commodities or asset classes; and (iii) indexed commodities as compared to those that are off-index. The dynamic systems governing these extensions are comprehensively outlined in Appendix C.

#### 4.1 No restrictions on trading strategies

In the first extension, each asset is considered as an individual commodity futures price of the same class, and we construct the index as an equally weighted amalgamation of these prices.<sup>18</sup> Aligning with industry standards, the commodity index is formed based on the values of nearby futures contracts that have delivery times exceeding one month. To streamline the model, we abstain from incorporating the 'roll-over' process, which entails substituting an expiring contract with the next available one upon maturity. In this extension, both traditional traders—comprising fundamentalists and chartists—and index traders have the freedom

<sup>17</sup>The model can, in theory, be extended to include an infinite number of markets.

<sup>18</sup>The specific weightings assigned to individual futures prices within the index do not have a material effect on this extension, thereby allowing our framework to accommodate various weighting schemes.

to invest in all available assets. The primary distinction lies in their investment scope: while fundamentalists and chartists concentrate their investments on single assets, index traders are free to diversify their positions across all commodities represented in the index. Consequently, index investors predominantly focus on broader asset allocation and display minimal sensitivity to the fluctuations in individual commodity prices. This model serves as an expansion of the framework delineated in Section 2 and includes three markets. For further elaboration, readers are directed to Appendix C.1.

Table 5 presents estimated correlations of commodity prices across the three scrutinized markets, calculated for different quantiles through the unrestricted HAM-IT model. For comparative purposes, we also furnish—in parentheses—correlations derived from the benchmark model, which includes only chartists and fundamentalists. The objective is to determine whether integrating index traders into the HAM-IT model significantly boosts market correlations, thereby more accurately reflecting the complex dynamics among commodity assets. Our findings affirm that HAM-IT surpasses the benchmark model in capturing elevated correlations. Importantly, consistent with existing empirical studies (Gorton & Rouwenhorst (2006), Bhardwaj & Dunsby (2013), Creti et al. (2013), Joëts (2014), and Ait-Youcef (2019)), we identify robust correlations for each pair of assets in over 95 percent of our simulations—namely, 0.632, 0.610, and 0.664—compared to notably weaker correlations in the benchmark model, with values of 0.268, 0.263, and 0.258, respectively.

Table 5: Commodity returns co-movements: no restrictions

Quantile	P1/P2	P1/P3	P2/P3
0.05	-0.598 (-0.268)	-0.587 (-0.281)	-0.571 (-0.255)
0.25	-0.191 (-0.112)	-0.191 (-0.121)	-0.181 (-0.116)
0.50	-0.105 (0.007)	-0.056 (0.003)	0.016 (0.0002)
0.75	0.376 (0.115)	0.380 (0.113)	0.298 (0.108)
0.95	0.632 (0.268)	0.610 (0.263)	0.664 (0.258)

Note: This table presents the estimated inter-market correlations of commodity returns across various quantiles, as generated by the HAM-IT model. For comparative purposes, the correlations calculated using the benchmark model are included in parentheses.

## 4.2 Restrictions on fundamentalists and chartists

In the second extension, we narrow our focus to two markets that represent commodity futures prices within the same class, such as agriculture, and introduce a third market representing a different asset class—for instance, crude oil. Crude oil is a noteworthy asset in this context due to its substantial weight in popular commodity indices. However, the same framework could be extended to cover other commodities or asset classes, including industrial metals, precious metals, equities, or bonds. In this configuration, traditional traders (fundamentalists and chartists) are confined to investing solely in the two agricultural markets, representing their

area of specialization. Conversely, index traders are not subject to these limitations and can freely invest across all available markets via index allocation. The portfolio strategy for index traders could thus incorporate combinations such as agricultural-energy or agricultural-equity blends, offering greater diversification.<sup>19</sup> The primary goal of this extension is to investigate whether the inclusion of index traders amplifies correlations across disparate types of commodities or between commodities and other financial assets. Additional elaborations on this theme can be found in Appendix C.2.

The findings are summarized in Table 6, which highlights the significant increase in correlations across all asset classes in the presence of index traders, as modeled in HAM-IT. Our results are consistent with previous findings for the correlation between P1 and P2, the two agricultural futures prices, as showcased in Table 5. Notably, there is also a substantial rise in correlations between different asset classes, namely P1/P3 and P2/P3. These augmented correlations appear in more than 90 percent of our simulations, with values reaching 0.630 and 0.624, respectively. These findings reinforce the conclusions reached in earlier studies, such as that by Brunetti et al. (2016), affirming that index traders play a pivotal role in amplifying co-movements across diverse asset classes.

Table 6: Commodity returns co-movements: restrictions on chartists-fundamentalists

Quantile	P1/P2	P1/P3	P2/P3
0.05	-0.514 (-0.268)	-0.569 (-0.281)	-0.538 (-0.255)
0.25	-0.138 (-0.112)	-0.200 (-0.121)	-0.200 (-0.116)
0.50	-0.008 (0.007)	-0.006 (0.003)	0.009 (0.0002)
0.75	0.305 (0.115)	0.399 (0.113)	0.371 (0.108)
0.95	0.632 (0.268)	0.630 (0.263)	0.624 (0.258)

Note: This table presents the estimated inter-market correlations of commodity returns across various quantiles, as generated by the HAM-IT model. For comparative purposes, the correlations calculated using the benchmark model are included in parentheses.

### 4.3 Restrictions on index traders

The third extension of the study focuses on understanding how index traders influence the co-movement of prices between commodities that are part of an index (in-index) and those that are not (off-index). The chosen scenario models three commodity futures prices: P1 and P2 are part of an index (in-index), while P3 is not (off-index). The investment behavior of index traders is restricted to P1 and P2, while traditional traders (chartists and fundamentalists) can invest in any of the three markets. Additional details are provided in Appendix C.3.

<sup>19</sup>See Tang & Xiong (2012) for further discussion on this matter.

Table 7 presents the correlations between these assets as determined by the HAM-IT model, with the benchmark model’s correlations included in parentheses for comparison. Our findings reveal a pronounced difference in the correlation patterns between in-index and off-index commodities. Specifically, the correlation between P1 and P2 is significantly higher than that between P1 and P3 or P2 and P3. These results are consistent in over 90 percent of our simulations, showing correlations of 0.630 for P1-P2 compared to 0.456 and 0.461 for P1-P3 and P2-P3, respectively. This confirms that the presence of index traders in commodity markets increases the correlation of in-index commodities. Most notably, our extended HAM-IT model more accurately captures the nuanced co-movement patterns of indexed commodity prices compared to the benchmark model. This implies that our approach offers a more appropriate framework for understanding the dynamics and correlations among commodity prices, particularly in the context of financialization and the role of index traders.

Table 7: Commodity returns correlations: restrictions on index traders

Quantile	P1/P2	P1/P3	P2/P3
0.05	-0.397 (-0.268)	-0.553 (-0.281)	-0.564 (-0.255)
0.25	-0.080 (-0.112)	-0.182 (-0.121)	-0.173 (-0.116)
0.50	0.098 (0.007)	0.011 (0.003)	0.016 (0.0002)
0.75	0.290 (0.115)	0.173 (0.113)	0.197 (0.108)
0.95	0.630 (0.268)	0.456 (0.263)	0.461 (0.258)

Note: This table presents the estimated inter-market correlations of commodity returns across various quantiles, as generated by the HAM-IT model. For comparative purposes, the correlations calculated using the benchmark model are included in parentheses.

## 5 Conclusion

The dynamics of commodity prices and market co-movements—both within the commodity-equity space and across different commodities—have seen a sharp increase over recent decades. Given the global implications of these fluctuations, understanding their causes has become a topic of critical importance. This is especially true in the current climate of global uncertainty and inflation, driven by factors such as the Russia-Ukraine conflict and the COVID-19 pandemic. The surge in commodity prices has fueled debates among scholars and policy-makers alike, questioning the role of financial participants in magnifying market responses to fundamental shifts in supply and demand.

In this paper, we explore the underlying causes of commodity price movements through an agent-based framework, inspired by the works of Reitz & Westerhoff (2007) and Joëts (2015). We introduce a novel behavioral agent-based model that features three distinct categories of traders: chartists, fundamentalists, and a new class called index traders. Designed to

encapsulate the financialization of commodity markets, our theoretical model incorporates index traders alongside traditional market participants. Index traders, in keeping with observed market behavior, engage in multi-commodity investments via index funds and act as catalysts for price changes. The interplay between these various trading strategies and the influence of index traders lead to complex market dynamics. Our model aims to provide a more nuanced understanding of these intricacies, enriching the current academic discourse.

Simulations indicate that our model adeptly replicates the statistical characteristics of commodity markets, capturing features such as volatility clustering, leptokurtic returns, and episodes of bubbles and crashes. When compared to traditional models that only incorporate chartists and fundamentalists, our approach—enhanced by the inclusion of index traders—offers a more accurate representation of the financialization of commodity markets. The model reveals that the actions of index traders not only divert prices from their fundamentals but also influence the trading positions of other traditional market participants. Moreover, our extensions demonstrate that the presence of index traders substantially heightens asset co-movements, both within the realm of cross-commodity and between commodity and equity markets. This underscores the limitations of existing literature in adequately capturing the complexities of commodity pricing. The HAM-IT model can be reasonably extended to other commodities and asset classes influenced by institutional investors. As such, it offers a robust, micro-founded alternative to conventional models, providing both industry professionals and scholars with a more nuanced tool for predicting and explaining the future trajectories of commodity prices. Future research could explore this model’s implications for finance and operations research, among other potential applications.



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# Appendix

## A Local stability of the steady state in HAM-IT

The Appendix provides both the derivation of the Jacobian Matrix associated with our HAM-IT model and an analysis of the eigenvalues at the steady state.

Simplifying the dynamic system outlined in Section 2.2.1 and omitting the random terms, we obtain the following:

$$\begin{aligned}
P_{t+1}^1 &= P_t^1 + a(W_t^{C^1}(c(P_t^1 - x_t^1)) + W_t^{F^1}(f(F^1 - P_t^1)) + W_t^I \frac{1}{2} d(\frac{1}{2}(P_t^1 + P_t^2) - \frac{1}{2}(x_t^1 + x_t^2))) \\
x_{t+1}^1 &= P_t^1 \\
y_{t+1}^1 &= x_t^1 \\
P_{t+1}^2 &= P_t^2 + a(W_t^{C^2}(c(P_t^2 - x_t^2)) + W_t^{F^2}(f(F^2 - P_t^2)) + W_t^I \frac{1}{2} d(\frac{1}{2}(P_t^1 + P_t^2) - \frac{1}{2}(x_t^1 + x_t^2))) \\
x_{t+1}^2 &= P_t^2 \\
y_{t+1}^2 &= x_t^2 \\
z_{t+1} &= I_t = \frac{1}{2}(P_t^1 + P_t^2) \\
v_{t+1} &= z_t = \frac{1}{2}(x_t^1 + x_t^2) \\
A_{t+1}^{C^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(c(x_t^1 - y_t^1)) + bA_t^{C^1} \\
A_{t+1}^{F^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(f(F^1 - x_t^1)) + bA_t^{F^1} \\
A_{t+1}^{C^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(c(x_t^2 - y_t^2)) + bA_t^{C^2} \\
A_{t+1}^{F^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(f(F^2 - x_t^2)) + bA_t^{F^2} \\
A_{t+1}^I &= (\exp[P_{t+1}^1] - \exp[P_t^1])(\frac{1}{2}d(z_t - v_t)) + (\exp[P_{t+1}^2] - \exp[P_t^2])(\frac{1}{2}d(z_t - v_t)) + bA_t^I
\end{aligned}$$

For brevity, we use a prime symbol to denote variables at time  $t+1$ . Specifically  $P'^i$  represents the price at time  $t+1$  for market  $i$  where  $i = 1, 2$ . We proceed to examine the partial derivatives of  $P'^i$  with respect to  $P^i$ ,  $x'^i$ , and  $y^i$ .

$$\begin{aligned}
\frac{\partial P'^1}{\partial P^1} &= 1 + a(W^{C^1}c - W^{F^1}f + \frac{1}{4}W^I d) \\
\frac{\partial P'^1}{\partial x^1} &= -a(W^{C^1}c + \frac{1}{4}W^I d)
\end{aligned}$$

which gives at the steady state

$$\left. \frac{\partial P^1}{\partial P^1} \right|_{s.s.} = 1 + \frac{1}{7}a(c - f + \frac{d}{4}); \quad \left. \frac{\partial P^1}{\partial x^1} \right|_{s.s.} = -\frac{1}{7}a(c + \frac{d}{4})$$

while

$$\begin{aligned} \frac{\partial P^1}{\partial P^2} &= -\frac{\partial P^1}{\partial x^2} = \frac{1}{28}ad \\ \frac{\partial P^1}{\partial y^1} &= \frac{\partial P^1}{\partial y^2} = 0 \end{aligned}$$

Similarly, at the steady state,

$$\left. \frac{\partial P^2}{\partial P^2} \right|_{s.s.} = 1 + \frac{1}{7}a(c - f + \frac{d}{4}), \quad \left. \frac{\partial P^2}{\partial x^2} \right|_{s.s.} = -\frac{1}{7}a(c + \frac{d}{4})$$

and

$$\begin{aligned} \frac{\partial P^2}{\partial P^1} &= -\frac{\partial P^2}{\partial x^1} = \frac{1}{28}ad \\ \frac{\partial P^2}{\partial y^1} &= \frac{\partial P^2}{\partial y^2} = 0 \end{aligned}$$

The partial derivatives of  $P^i$  with respect to  $A^{K^i}$  and, with  $K = C, F, I$  and  $i = 1, 2$  are

$$\frac{\partial P^i}{\partial A^{K^i}} = a \left[ c(P^i - x^i) \frac{\partial W^{C^i}}{\partial A^{C^i}} + f(F^i - P^i) \frac{\partial W^{F^i}}{\partial A^{K^i}} + \frac{1}{2}d \left( \frac{1}{2}(P_t^1 + P_t^2) - \frac{1}{2}(x_t^1 + x_t^2) \right) \frac{\partial W^I}{\partial A^I} \right]$$

At the steady state  $P^i = x^i = F^i$ . Therefore, all these partial derivatives vanish (i.e.  $\frac{\partial P^i}{\partial A^{K^i}} = 0$ ).

Let's define  $U^i \equiv (Exp [P^i] - Exp [P^i])$  for markets  $i = 1, 2$ . This expression vanishes at the steady state because  $P^i = P^i$ . Therefore, the partial derivatives of  $A'^{K^i}$  with respect  $P^i$ ,  $x^i$  and  $y^i$  at the steady state are as follows:

$$\begin{aligned} \frac{\partial A'^{C^i}}{\partial P^i} &= c(x^i - y^i) \frac{\partial U^i}{\partial P^i} = 0; \quad \frac{\partial A'^{F^i}}{\partial P^i} = f(F^i - x^i) \frac{\partial U^i}{\partial P^i} = 0, \\ \frac{\partial A'^{C^i}}{\partial x^i} &= c \left( (x^i - y^i) \frac{\partial U^i}{\partial x^i} + U^i \right) = 0; \quad \frac{\partial A'^{F^i}}{\partial x^i} = f \left( (P^i - x^i) \frac{\partial U^i}{\partial x^i} - U^i \right) = 0, \\ \frac{\partial A'^{C^i}}{\partial y^i} &= -cU^i = 0; \quad \frac{\partial A'^{F^i}}{\partial y^i} = 0 \end{aligned}$$

For  $j = 1, 2$ ,  $i = 1, 2$  and  $i \neq j$

$$\frac{\partial A^{C^i}}{\partial P^j} = \frac{\partial A^{F^i}}{\partial P^j} = \frac{\partial A^{C^i}}{\partial x^j} = \frac{\partial A^{F^i}}{\partial x^j} = \frac{\partial A^{C^i}}{\partial y^j} = \frac{\partial A^{F^i}}{\partial y^j} = 0$$

It turns out that due to  $(x^i - y^i)$  and  $(F^i - x^i)$  in the system, all partial derivatives are null at the steady state with exception of

$$\frac{\partial A^{C^1}}{\partial A^{C^1}} = \frac{\partial A^{F^1}}{\partial A^{F^1}} = \frac{\partial A^{C^2}}{\partial A^{C^2}} = \frac{\partial A^{F^2}}{\partial A^{F^2}} = b$$

Considering the partial derivatives of  $A^I$ , with respect to  $P^i$ ,  $x^i$  and  $y^i$ , with  $i = 1, 2$  and the variable  $U^i \equiv (Exp[P^{i'}] - Exp[P^i]) = 0$  at the steady state (with  $P^{i'} = P^i$ ), we find that:

$$\frac{\partial A^I}{\partial P^i} = \frac{\partial A^I}{\partial x^i} = \frac{\partial A^I}{\partial y^i} = \frac{\partial A^I}{\partial A^{K^i}} = 0; \quad \frac{\partial A^I}{\partial A^I} = b$$

The Jacobian matrix of partial derivatives considering the following variable ordering  $P^1$ ,  $x^1$ ,  $y^1$ ,  $P^2$ ,  $x^2$ ,  $y^2$ ,  $A^{C^1}$ ,  $A^{F^1}$ ,  $A^{C^2}$ ,  $A^{F^2}$  and  $A^I$  is

$$J = \begin{pmatrix} 1 + \frac{1}{7}a(c - f + \frac{d}{4}) & -\frac{1}{7}a(c + \frac{d}{4}) & 0 & \frac{1}{28}ad & -\frac{1}{28}ad & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{28}ad & -\frac{1}{28}ad & 0 & 1 + \frac{1}{7}a(c - f + \frac{d}{4}) & -\frac{1}{7}a(c + \frac{d}{4}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b \end{pmatrix}$$

which can be decompose as two sub-matrices A and B:

$$A = \begin{pmatrix} 1 + \frac{1}{7}a(c - f + \frac{d}{4}) & -\frac{1}{7}a(c + \frac{d}{4}) & 0 & \frac{1}{28}ad & -\frac{1}{28}ad & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{28}ad & -\frac{1}{28}ad & 0 & 1 + \frac{1}{7}a(c - f + \frac{d}{4}) & -\frac{1}{7}a(c + \frac{d}{4}) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{and } B = \begin{pmatrix} b & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 \\ 0 & 0 & b & 0 & 0 \\ 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & b \end{pmatrix}$$

The local stability of the steady state is determined if all the eigenvalues of matrices  $A$  and  $B$  have a modulus less than 1. Matrix  $B$  is diagonal, so its four eigenvalues are all equal to  $b$ , which by definition satisfies  $0 \leq b < 1$ . Therefore, the local stability is primarily dependent on matrix  $A$ . To simplify the expressions for the eigenvalues of  $A$ , we assume that  $c = f$ . This assumption does not impact the stability of the steady state, as discussed in Westerhoff & Reitz (2003).

$$\begin{aligned} \lambda_1 &= 0 \\ \lambda_2 &= 0 \\ \lambda_3 &= \frac{1}{2} \left( 1 - \frac{1}{7} \sqrt{7} \sqrt{7 - 4ac} \right) \\ \lambda_4 &= \frac{1}{2} \left( 1 + \frac{1}{7} \sqrt{7} \sqrt{7 - 4ac} \right) \\ \lambda_5 &= \frac{1}{28} ad - \frac{1}{28} \sqrt{a^2 d^2 - 28ad - 112ac + 196} + \frac{1}{2} \\ \lambda_6 &= \frac{1}{28} ad + \frac{1}{28} \sqrt{a^2 d^2 - 28ad - 112ac + 196} + \frac{1}{2} \end{aligned}$$

Both eigenvalues  $\lambda_1$  and  $\lambda_2$  are equal and inherently stable. For  $\lambda_3$  and  $\lambda_4$  the necessary and sufficient conditions for their moduli to be less than 1 are:

$$\frac{7}{4a} > c \tag{12}$$

$$\sqrt{7} \sqrt{7 - 4ac} > 0 \tag{13}$$

The stability properties of  $\lambda_5$  and  $\lambda_6$  are contingent upon the parameter  $d$  that characterizes index traders. We can distinguish two cases based on the value of  $d$ . On the one hand, as  $d \rightarrow 0$  both  $\lambda_5$  and  $\lambda_6$  have moduli less than 1 if conditions (12) and (13) are met. This suggests that when index investors place little trust in their trading rule, the model's local stability aligns with that of the baseline model under similar conditions. On the other hand, as  $d \rightarrow \infty$ ,  $\lambda_6$  has a modulus greater than 1. This finding corroborates the destabilizing influence of index traders, pushing the long-term price trajectory from a state of equilibrium to one of explosiveness. Additional simulations—available upon request from the authors—indicate that a Hopf bifurcation occurs at a threshold value of  $d = 13.31$ , assuming  $a = 1$  and  $c = f = 0.05$ .

## B Commodity futures price

Table 8: Commodity futures prices

Commodity prices	Markets
<b>Energy</b>	
Crude oil (Brent)	NYMEX
Natural Gas (Henry Hub)	NYMEX
<b>Industrial metals</b>	
Copper	LME
Aluminium	LME
<b>Agricultural</b>	
Corn	CME
Cocoa	CME
<b>Precious metals</b>	
Gold	LME
Silver	LME

This table presents the commodity futures prices selected for our study. NYMEX refers to the New York Mercantile Exchange, LME denotes the London Metal Exchange, and CME represents the Chicago Mercantile Exchange.

## C HAM-IT with three markets

This Appendix delves into the technical nuances of the Heterogeneous Agent Model with Index Traders (HAM-IT), specifically its extensions with three markets. We examine three different scenarios: (i) a setting with no trading restrictions on any category of traders; (ii) a framework where traditional traders face certain market restrictions; and (iii) a scenario where trading limitations are imposed solely on index investors.

### C.1 No restrictions on trading strategies

In this initial extension, we examine commodity futures prices within a single asset class. The index is conceived as an equally weighted amalgamation of these individual futures prices. Importantly, there are no constraints placed on traders regarding which assets they can invest in. Traditional traders, such as fundamentalists and chartists, are designed to invest in only one asset at a time. In contrast, index traders have the latitude to take positions in all assets simultaneously via the commodity index. This unrestricted setting serves as a foundational model to analyze the investment strategies and market dynamics associated with different types of traders.



$$\begin{aligned}
P_{t+1}^1 &= P_t^1 + a(W_t^{C^1}(c(P_t^1 - x_t^1) + S_t^{C^1}) + W_t^{F^1}(f(F^1 - P_t^1) + S_t^{F^1}) \\
&\quad + W_t^I \frac{1}{3} d(\frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I - \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I) + S_t^{I^1}) + S_t^{P^1} \\
x_{t+1}^1 &= P_t^1 \\
y_{t+1}^1 &= x_t^1 \\
P_{t+1}^2 &= P_t^2 + a(W_t^{C^2}(c(P_t^2 - x_t^2) + S_t^{C^2}) + W_t^{F^2}(f(F^2 - P_t^2) + S_t^{F^2}) \\
&\quad + W_t^I \frac{1}{3} d(\frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I - \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I + S_t^{I^2}) + S_t^{P^2} \\
x_{t+1}^2 &= P_t^2 \\
y_{t+1}^2 &= x_t^2 \\
P_{t+1}^3 &= P_t^3 + a(W_t^{C^3}(c(P_t^3 - x_t^3) + S_t^{C^3}) + W_t^{F^3}(f(F^3 - P_t^3) + S_t^{F^3}) \\
&\quad + W_t^I \frac{1}{3} d(\frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I - \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I) + S_t^{I^3}) + S_t^{P^3} \\
x_{t+1}^3 &= P_t^3 \\
y_{t+1}^3 &= x_t^3 \\
z_{t+1} &= I_t = \frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I \\
v_{t+1} &= z_t = \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I \\
A_{t+1}^{C^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(c(x_t^1 - y_t^1) + S_{t-1}^{C^1}) + bA_t^{C^1} \\
A_{t+1}^{F^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(f(F^1 - x_t^1) + S_{t-1}^{F^1}) + bA_t^{F^1} \\
A_{t+1}^{C^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(c(x_t^2 - y_t^2) + S_{t-1}^{C^2}) + bA_t^{C^2} \\
A_{t+1}^{F^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(f(F^2 - x_t^2) + S_{t-1}^{F^2}) + bA_t^{F^2} \\
A_{t+1}^{C^3} &= (\exp[P_{t+1}^3] - \exp[P_t^3])(c(x_t^3 - y_t^3) + S_{t-1}^{C^3}) + bA_t^{C^3} \\
A_{t+1}^{F^3} &= (\exp[P_{t+1}^3] - \exp[P_t^3])(F^3 - x_t^3) + S_{t-1}^{F^3} + bA_t^{F^3} \\
A_{t+1}^I &= (\sum_{i=1}^3 (\exp[P_{t+1}^i] - \exp[P_t^i])) (\frac{1}{3} \cdot d(z_t - v_t) + S_{t-1}^{I^1} + S_{t-1}^{I^2} + S_{t-1}^{I^3}) + bA_t^I
\end{aligned}$$

With

$$\begin{aligned}
D_{t-1}^I &= \frac{1}{3}(z_t - v_t) + S_{t-1}^{I^1} + S_{t-1}^{I^2} + S_{t-1}^{I^3} \\
D_t^{C^i} &= c(P_t^i - P_{t-1}^i) + S_t^{C^i} \\
D_t^{F^i} &= c(F^i - P_t^i) + S_t^{F^i}
\end{aligned}$$

## C.2 Restrictions on traditional investors only

In the second extension, we expand our focus to include three markets: two of which feature commodity futures prices within the same asset class, and the third represents a commod-

ity from a different class, such as crude oil. Traditional traders, namely fundamentalists and chartists, are restricted to investing solely in the first two markets, reflecting their specialization in a given class of commodities. On the other hand, index traders face no such restrictions and are free to invest across all available markets through index allocation strategies. This setup allows us to delve deeper into how different trading restrictions influence market dynamics and asset correlations, particularly when commodities from diverse classes are involved.

$$\begin{aligned}
P_{t+1}^1 &= P_t^1 + a(W_t^{C^1}(c(P_t^1 - x_t^1) + S_t^{C^1}) + W_t^{F^1}(f(F^1 - P_t^1) + S_t^{F^1}) \\
&\quad + W_t^I \frac{1}{3} d(\frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I - \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I) + S_t^{P^1}) \\
x_{t+1}^1 &= P_t^1 \\
y_{t+1}^1 &= x_t^1 \\
P_{t+1}^2 &= P_t^2 + a(W_t^{C^2}(c(P_t^2 - x_t^2) + S_t^{C^2}) + W_t^{F^2}(f(F^2 - P_t^2) + S_t^{F^2}) \\
&\quad + W_t^I \frac{1}{3} d(\frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I - \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I) + S_t^{P^2}) \\
x_{t+1}^2 &= P_t^2 \\
y_{t+1}^2 &= x_t^2 \\
P_{t+1}^3 &= P_t^3 + W_t^I \frac{1}{3} d(\frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I - \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I) + S_t^{P^3} \\
x_{t+1}^3 &= P_t^3 \\
y_{t+1}^3 &= x_t^3 \\
z_{t+1} &= I_t = \frac{1}{3}(P_t^1 + P_t^2 + P_t^3) + S_t^I \\
v_{t+1} &= z_t = \frac{1}{3}(x_t^1 + x_t^2 + x_t^3) + S_{t-1}^I \\
A_{t+1}^{C^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(c(x_t^1 - y_t^1) + S_{t-1}^{C^1}) + bA_t^{C^1} \\
A_{t+1}^{F^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(f(F^1 - x_t^1) + S_{t-1}^{F^1}) + bA_t^{F^1} \\
A_{t+1}^{C^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(c(x_t^2 - y_t^2) + S_{t-1}^{C^2}) + bA_t^{C^2} \\
A_{t+1}^{F^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(f(\bar{P}^2 - x_t^2) + S_{t-1}^{F^2}) + bA_t^{F^2} \\
A_{t+1}^I &= (\sum_{i=1}^3 (\exp[P_{t+1}^i] - \exp[P_t^i])) (\frac{1}{3} d(z_t - v_t) + S_{t-1}^{I^1} + S_{t-1}^{I^2} + S_{t-1}^{I^3}) + bA_t^I
\end{aligned}$$

With

$$\begin{aligned}
D_{t-1}^{I^i} &= \frac{1}{2}(z_t - v_t) + S_{t-1}^{I^1} + S_{t-1}^{I^2} \\
D_t^{C^i} &= c(P_t^i - P_{t-1}^i) + S_t^{C^i} \\
D_t^{F^i} &= c(F_t^i - P_t^i) + S_t^{F^i}
\end{aligned}$$

### C.3 Restrictions on index investors only

In the third extension, the model focuses on three distinct commodity futures markets. Among these, the first two markets (P1 and P2) are part of a predefined index, while the third one (P3) remains off-index. Traditional traders, such as chartists and fundamentalists, are not subject to any asset-specific restrictions; they are free to invest in any of the three markets, though only one at a time. In contrast, index traders are limited to positions in only the two in-index markets, via the index investment strategy. This extension enables us to examine the role of index traders more critically, especially in terms of how their restrictions impact price co-movements and market dynamics between in-index and off-index commodities.

$$\begin{aligned}
P_{t+1}^1 &= P_t^1 + a(W_t^{C^1}(c(P_t^1 - x_t^1) + S_t^{C^1}) + W_t^{F^1}(f(F^1 - P_t^1) + S_t^{F^1}) \\
&\quad + W_t^I \frac{1}{2} d(\frac{1}{2}(P_t^1 + P_t^2) + S_t^I - \frac{1}{2}(x_t^1 + x_t^2) + S_{t-1}^I) + S_t^{P^1}) \\
x_{t+1}^1 &= P_t^1 \\
y_{t+1}^1 &= x_t^1 \\
P_{t+1}^2 &= P_t^2 + a(W_t^{C^2}(c(P_t^2 - x_t^2) + S_t^{C^2}) + W_t^{F^2}(f(F^2 - P_t^2) + S_t^{F^2}) \\
&\quad + W_t^I \frac{1}{2} d(\frac{1}{2}(P_t^1 + P_t^2) + S_t^I - \frac{1}{2}(x_t^1 + x_t^2) + S_{t-1}^I) + S_t^{P^2}) \\
x_{t+1}^2 &= P_t^2 \\
y_{t+1}^2 &= x_t^2 \\
P_{t+1}^3 &= P_t^3 + a(W_t^{C^3}(c(P_t^3 - x_t^3) + S_t^{C^3}) + W_t^{F^3}(f(F^3 - P_t^3) + S_t^{F^3}) + S_t^{P^3}) \\
x_{t+1}^3 &= P_t^3 \\
y_{t+1}^3 &= x_t^3 \\
z_{t+1} &= I_t = \frac{1}{2}(P_t^1 + P_t^2) + S_t^I \\
v_{t+1} &= z_t = \frac{1}{2}(x_t^1 + x_t^2) + S_{t-1}^I \\
A_{t+1}^{C^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(c(x_t^1 - y_t^1) + S_{t-1}^{C^1}) + bA_t^{C^1} \\
A_{t+1}^{F^1} &= (\exp[P_{t+1}^1] - \exp[P_t^1])(f(F^1 - x_t^1) + S_{t-1}^{F^1}) + bA_t^{F^1} \\
A_{t+1}^{C^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(c(x_t^2 - y_t^2) + S_{t-1}^{C^2}) + bA_t^{C^2} \\
A_{t+1}^{F^2} &= (\exp[P_{t+1}^2] - \exp[P_t^2])(f(\bar{P}^2 - x_t^2) + S_{t-1}^{F^2}) + bA_t^{F^2} \\
A_{t+1}^I &= \left(\sum_{i=1}^2 (\exp[P_{t+1}^i] - \exp[P_t^i])\right) \left(\frac{1}{2} \cdot d(z_t - v_t) + S_{t-1}^{I^1} + S_{t-1}^{I^2}\right) + bA_t^I
\end{aligned}$$

With

$$\begin{aligned}D_{t-1}^{I^i} &= \frac{1}{3}(z_t - v_t) + S_{t-1}^{I^1} + S_{t-1}^{I^2} + S_{t-1}^{I^3} \\D_t^{C^i} &= c(P_t^i - P_{t-1}^i) + S_t^{C^i} \\D_t^{F^i} &= c(\bar{P}_t^i - P_t^i) + S_t^{F^i}\end{aligned}$$