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Pseudo-3d Modeling Of The Lamb Wave Field Generated By A Surface-Bonded, Finite-sized Piezoelectric Transducer

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Abstract— The aim of this work is to predict the acoustic field radiated in an isotropic plate by an integrated piezoelectric transducer. A "pseudo-3D" formulation has been developed in order to take into account the 3D aspects of the problem in a simple way. The basic principles of this model have been published recently. An adaptation of the model to an actual piezoelectric source bonded to the plate surface is presented here. First, a 2D finite element - normal mode expansion hybrid model is applied to account for the behavior of the piezoelectric transducer. Then the 3D effects are predicted by applying a corrective term to the 2D result. Theoretically speaking, this corrective term is rigorously applicable in particular situations only (low directivity angle, small transducer compared to the wavelength). However, looser validity conditions are precisely discussed, for which a very satisfying approximation is still obtained. Finally, the predicted results are compared to 3D numerical data. Though approximate, the model thus appears to provide a very satisfying estimate of the acoustic field generated in the whole plate.

Keywords: *Lamb waves; pseudo-3D modeling; SHM system; acoustic field prediction; PZT transducer; Lamb waves excitation; finite element method.*

I. INTRODUCTION

In the last decade, Lamb waves have been used extensively in non-destructive testing (NDT) [1,2]. In particular, they are used to inspect the integrity of aeronautical structures [3]. This use can be explained by their attractive characteristics, such as propagation over a long distance, good interaction with different kinds of defects such as corrosion, delamination and fibers breakage. In this context, many works are done to study the Integrated Health Monitoring System (IHMS) dedicated to evaluate the integrity of metallic or composite materials [4,5,6]. The idea of such a system consists in integrating sensors in the specimen. IHMS gave already good results by improving the quality of the control and reducing the time of evaluation. Nevertheless, optimization of such systems requires knowing the behaviors of the emitters which are stuck on the surface of

the specimen. It is then important to develop the appropriate modeling in order to take into account the influence of the specimen material nature and conditions of integration of the emitter inside the structure to evaluate.

In this context, theoretical models are developed in the literature. For instance, normal mode expansion model was initially employed to study the response of a plate subjected to a well defined excitation [7]. And in order to account for the actual behavior of the piezoelectric transducer a hybrid model was employed [8]. The common point of these models is that they treat the problem of excitation of these ultrasonic waves in 2D only. This means that the problem is treated in the plane defined by the thickness and width directions of the transducer. Accordingly, the length of the actuator is taken infinite. Only 3D analysis can take into account the actual transducer dimensions.

The goal of this paper is not to present a complete 3D model which would be difficult to exploit. The idea is to have an approached model allowing predicting the 3D behavior from a 2D computation.

The basis of this pseudo-3D modeling was already presented in [9,10]. Its basic idea is to adapt the hybrid model using conjointly 2D finite element method (2D FEM) and normal mode expansion, in such a way that it is possible to take into account the 3D aspect of the problem. The results provided by this approached model was already compared with two methods, firstly, a 3D finite element method (3D FEM) by using a well known excitation (uniform pressure over the excitation area) as it was done in [9], secondly, an exact complicated model developed by Ditre [11] and synthesized by Wilcox [12]. Under some conditions this approximate model gives excellent results.

The idea now is to compare this pseudo-3D modeling with 3D finite element method but here the influence of a real piezoelectric transducer will be taken into account.

II. PSEUDO-3D MODELING PRINCIPLE

The principle of this model is already presented in [9]. Only the final results will be reminded here. Indeed, in the case of an isotropic plate with thickness d excited by a given stress applied in its upper surface. The excitation is supposed to be rectangular-shaped and it is localized in an area S , with width W and length L , as indicated in the Fig.1.

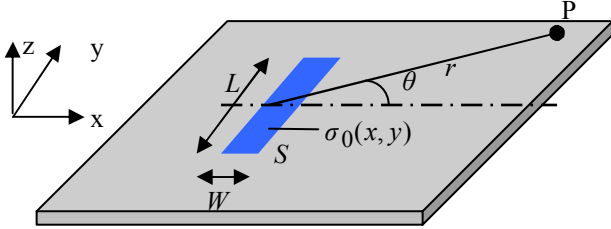


Figure 1. Description of the 3D problem geometry

The applied stress due to the surface source S is supposed to be described by [9]:

$$\sigma_0(x, y) = \sigma_{0x}(x)\sigma_{0y}(y) = \begin{bmatrix} \sigma_{0zz} \\ \sigma_{0xz} \end{bmatrix}, \quad (1)$$

with

$$\sigma_{0x} = \begin{bmatrix} \sigma_{zz}^{2D} \\ \sigma_{xz}^{2D} \end{bmatrix}, \quad (2)$$

where σ^{2D} are the stress computed with the help of the 2D-FEM. The term $\sigma_{0y}(y)$ will be named stress profile. Then, it has been shown [9] that the pseudo-3D displacement field generated in the plate can be approximated by:

$$u(r, \theta) = \alpha(r, \theta)u_{2D}, \quad (3)$$

where u_{2D} is the 2D acoustic field computed with the help of a hybrid finite element- normal mode expansion method. Then characteristic of the transducer bar (material, dimensions, working frequency) is taken into account [8,13]. The factor $\alpha(r, \theta)$ takes into account the effect of the length of the transducer bar. If the displacement field is that of a single Lamb wave mode of wavelength λ_n ($k_n = 2\pi / \lambda_n$ is being the wave number). The 2D displacement field, at a point $P(r, \theta)$ situated in the far field zone, and the factor $\alpha(r, \theta)$ are then given by:

$$u_{2D} = A_n u_n \exp(jk_n r), \quad (4)$$

and

$$\alpha(r, \theta) = \frac{1-j}{2\sqrt{\pi}} \sqrt{\frac{k_n}{r}} \int_{-L/2}^{L/2} \sigma_{0y} \exp(-jk_n y \sin \theta) dy, \quad (5)$$

respectively. Where A_n and u_n are modal amplitude and modal displacement field of the n^{th} Lamb mode, respectively. The stress σ_{0y} is taken uniform and equal to unity (see [9]). The amplitude A_n is computed with the help of the normal mode expansion method and is given by:

$$A_n = \frac{j\omega e^{-jk_n x}}{4P_n} \left[u_{nx}^*(d)I_{xz}(k_n) + u_{nz}^*(d)I_{zz}(k_n) \right] \quad (6)$$

where P_n is the average power flow. The star * denotes the complex conjugate. The influence of the piezoelectric transducer is contained in the terms I_{xz} and I_{zz} . Their expressions are given by these equations:

$$I_{xz} = \int_{-W/2}^{W/2} \sigma_{xz}^{2D}(x, d) e^{jk_n x} dx, \quad (7)$$

and

$$I_{zz} = \int_{-W/2}^{W/2} \sigma_{zz}^{2D}(x, d) e^{jk_n x} dx. \quad (8)$$

In the present case, when a transducer bar is used, it will be shown a non-uniform stress has to be taken into account. The case of a uniform applied pressure (σ_{zz}^{2D}) has already been treated in a previous paper [9].

It is interesting to note that, as indicated by (3), the approximated 3D displacement field is the product of the 2D displacement field and the factor $\alpha(r, \theta)$. This factor is then the link parameter between 2D and 3D case. Nevertheless, as detailed in [10], (3) is applicable, provided the following condition is respected:

$$g(W, \theta) = 4\pi \frac{W}{\lambda_n} \sin^2\left(\frac{\theta}{2}\right) \ll 1. \quad (9)$$

The function $g(W, \theta)$ is represented in contour line (Fig.2), as a function of angle θ and W/λ_n . According to this figure, having $g(W, \theta) \ll 1$ requires a value of the product $(\theta)(W/\lambda_n)$ smaller than unity.

In the following, two cases will be considered as shown by the Fig.2. These two cases correspond to A_0 Lamb mode with wavelength equal to 3.8 mm ($\lambda_n = 3.8 \text{ mm}$).

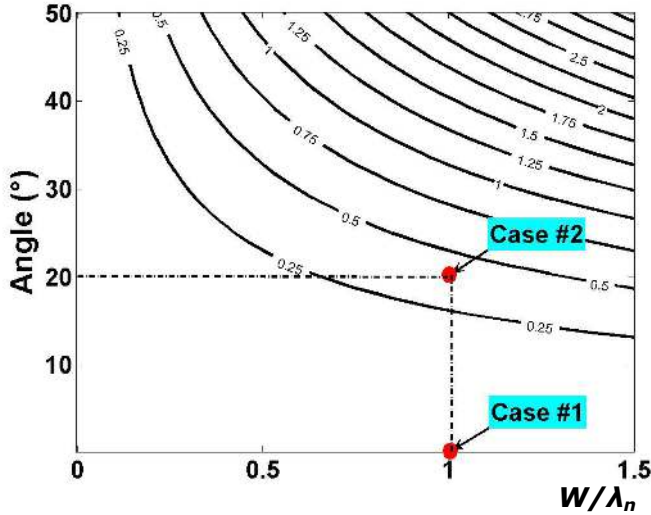


Figure 2. Function g evolution and different situation studied

III. APPLICATION TO A REAL TRANSDUCER

The goal here is to apply this pseudo-3D modeling to a real case where a PZT transducer is acting. So in order to predict the acoustic field radiated by a surface-bonded finite-sized piezoelectric transducer, a 1-mm thick aluminum plate is considered and is excited by an actual transducer having $W = 4 \text{ mm}$ and $L = 20 \text{ mm}$. The emitter is excited by 5-cycle Hanning windowed sinusoid at 500 kHz. In fact, to reduce side lobe level in the power spectrum energy of the excited signal the Hanning window is used.

The pseudo-3D modeling consists, first, to treat the problem in 2D (infinite length of transducer) as indicated in the figure below:



Figure 3. Description of 2D problem geometry

This problem can be solved by employing a hybrid model using conjointly 2D FEM and normal mode expansion method. More information can be found in [7,8,13]. In fact the 2D FEM enables to have the evolution of the stress field. As said in section II, Fig. 4 shows clearly that the stress field of the surface of the transducer, in contact with the plate, is not uniform.

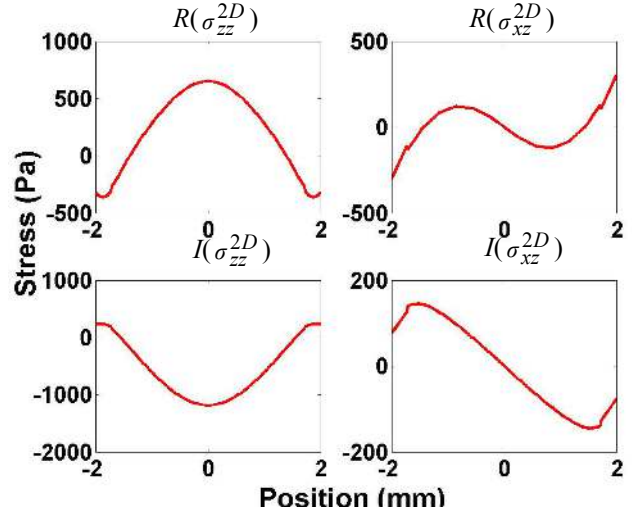


Figure 4. Stress profiles for a 1 mm thick aluminium plate and a 1 mm thick piezoelectric transducer

IV. RESULTS AND COMPARISON

In order to validate the approach, a 3D finite element method (FEM) is used and two cases, as said before, will be considered:

- Case #1: $\theta = 0^\circ$, $W/\lambda_n = 1.08$
- Case #2: $\theta = 19^\circ$, $W/\lambda_n = 1.08$

The problem in this 3D FEM is that the elements should be smaller than a quarter of the shorter wavelength and the dimension of plate has to be large enough to avoid reflection from the plate edges. This causes a huge memory and time consumption. Then, only two test cases are treated here. Moreover, In order to simplify the FE meshes, the value of r is taken equal to 20 mm and 21 mm for the first and second case, respectively. A square aluminium plate of 100 mm side is meshed. The transducer used is a PZ27 having the same dimensions as that described in the previous paragraph. The excitation frequency is always 500 kHz. The normal displacements computed by the FEM are shown in Figs. 5-6.

As shown on Figs. 5-a and 6-a, the normal 2D displacement field computed with the help of (4) is compared to those obtained by the FEM. This 2D displacement field has been multiplied by the factor $\alpha(r, \theta)$ and shown on Figs. 5-b and 6-b. A good agreement is obtained between the pseudo 3D

displacement and those obtained by the FEM which validate our approach.

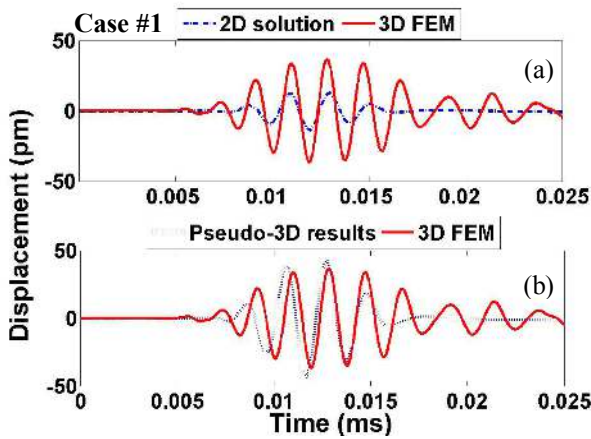


Figure 5. Comparison of (a) 2D solution to 3D FEM results (b) pseudo-3D results to 3D FEM results

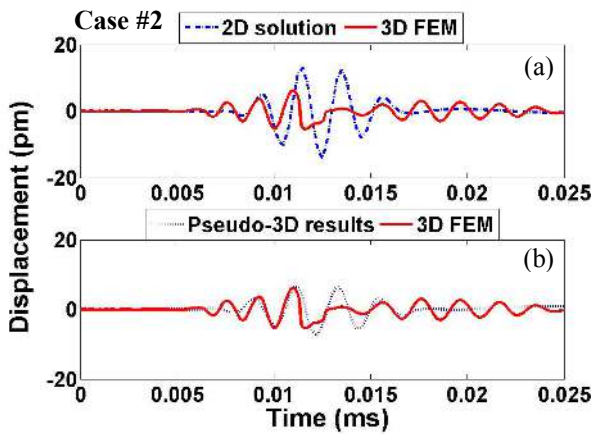


Figure 6. Comparison of (a) 2D solution to 3D FEM results (b) pseudo-3D results to 3D FEM results

In the first case, where the evaluation point is situated in the emitter axis we have $g = 0$ and we are well in the far field. Therefore all hypotheses are satisfied. In the second case, the evaluation point is situated in out of axis of the emitter, the pseudo-3D model gives satisfying results. However, the light shift noted between the two methods (Fig.6-b) can be explained by two reasons: (i) g is not lower enough than 1 (see Fig.2) (ii) the stress profile σ_{0y} is taken equal to unity. It should be noted that to compute the exact σ_{0y} , a 3D FEM is necessary. Other profiles of σ_{0y} can be tried, like parabolic profile. This work is under progress. Finally, in the two cases, a difference is observed (Fig. 5 and 6) in the end of 3D FEM signals. This difference is in fact due to the reflection of Lamb wave modes from the plate edges.

V. CONCLUSION

The model presented in this paper makes it possible to solve the problem of Lamb waves excitation by a surface mounted transducer. Indeed, pseudo-3D modeling is developed to compute the amplitude of 3D acoustic field of Lamb waves radiated in an isotropic plate. This computed field takes into account the finite size and the material characteristic of the emitter transducer. This pseudo-3D modeling scheme contributes to improve the structural health monitoring system because it permits to have a correct and light modeling of this system based on the integration of PZT transducers on the surface of the specimens.

This model has many advantages. One can firstly quote its ease of use, secondly the results can be easily interpreted and it enables to save calculation time. Moreover, its flexibility should make it possible to relatively easily take into account important additional physical effects in practice. One can in particular quote plate anisotropy, attenuation and interaction of these Lamb waves with defect. This work is currently under progress.

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