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# A Level-Set Based Topology Optimization for Maximizing the Torque of Synchronous Reluctance Machines

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The paper presents a Level-Set based topology optimization method for improving the magnetic torque of electrical motors such as synchronous reluctance machines using a 2D magnetostatic finite element context modelling. The novelty is the use of a fully analytical expression for the torque sensitivity analysis based on the virtual work principle which is more accurate and general than other methods, and its application to a Level-Set implementation. The numerical study of a synchronous reluctance machine is presented. Then, the Level-Set method is detailed in a general context. The corresponding analytical continuous sensitivity analysis using the adjoint method to maximize the torque is expressed and validated. Finally, the application to the optimization of the rotor of a synchronous reluctance machine is shown to illustrate the efficiency of the method.

*Index Terms*—Topology Optimization, Torque Sensitivity Analysis, Level-Set method, Synchronous Reluctance Machines.

## I. INTRODUCTION

**S**YNCHRONOUS RELUCTANCE MACHINES (SynRM) have many benefits. They offer interesting performances and do not need permanent magnets which are rare and expensive. However, their torque is mainly determined by the topology of their rotor. This topology can be hard to guess and requires an expensive design time for engineers [1], [2]. Topology optimization is a method allowing to find an optimal material distribution on a target region. Thus, it seems to be a really interesting approach to discover new designs in a finite element modeling context since it completely avoids the need of any prior intuition on the optimal design of the machine [1], [2].

There are two main gradient-based methods of topology optimization [1]. The first is the so-called "Solid Isotropic Material with Penalization" (SIMP) method [2], which is a density based approach to parametrize a material distribution on the mesh elements of the target region. This density can then be used as an optimization parameter to find an optimal distribution. The SIMP method is known to have numerical issues such as checkerboard patterns and "gray scales" results [1]. An alternative is the Level-Set (LS) method [3]–[5]. The principle is to track a design by its borders using the zero isovalue of a tracking function  $\phi$ . Then,  $\phi$  is updated iteratively in a descent direction for the given optimization problem. Both methods are gradient-based, and thus need the sensitivity analysis (SA) of the optimization objective and constraints with respect to the optimization variables as a descent direction provider.

The LS method used in the paper is presented in [4], [5] and uses a density-based approach to fit in the finite element modeling of the SynRM. In such context, the SA is usually done using the adjoint method [1], [2], [6], [7]. The most

widely used method for the computation and the SA of the torque is the Maxwell stress tensor [2], [3]. The virtual work method (VWM) [8] is an alternative to this method. It is known to have a better numerical behavior and is not dependent on the choice of a surrounding computation path [8], [9]. In the previous work [6], a discrete SA of the torque based on the VWM has been presented and detailed in the case of free shape optimization. The contribution of this paper is the presentation of a continuous version of this SA in a topology optimization context and its application on a LS optimization algorithm. In Section II, the 2D nonlinear magnetostatic numerical analysis context of a generic SynRM is presented, along with the integration of a material density in this resolution process. Then, the complete LS method and the corresponding SA are presented in Sections III and IV, respectively. Finally, Section V presents the application of this method to the optimization of a SynRM.

## II. MAGNETOSTATIC CONTEXT

### A. Finite element model

A SynRM is made of distinct parts. The rotor  $\Omega_R$  is the most important part for the design of the machine since its topology mainly determines the performances of the machine. The stator  $\Omega_S$  is made of iron and coils. The last parts are the air gap  $\Omega_A$  and the shaft  $\Omega_T$ . The air gap is numerically important since the torque is computed in this area with the VWM [8]. The complete domain of study is denoted  $\Omega = \Omega_A \cup \Omega_T \cup \Omega_R \cup \Omega_S$ .

The motor is studied under 2D nonlinear magnetostatic conditions. The Newton-Raphson (NR) algorithm is used to solve the problem. The matrix iterative NR assembly at iteration  $k$  reads  $S(\mathbf{A}^{(k)})\Delta\mathbf{A}^{(k+1)} = \mathbf{b}^{(k)}$  with  $\mathbf{A}$  the unknown nodal 2D magnetic vector potential and  $\Delta\mathbf{A}^{(k+1)} = \mathbf{A}^{(k+1)} - \mathbf{A}^{(k)}$ .

This system is built from the following classical weak B-conforming formulation of the 2D magnetostatic problem:

$$\forall A' \in \mathbf{H}_0^1(\Omega), \int_{\Omega} (\nabla \times \mathbf{H} - \mathbf{J}) \cdot A' d\Omega = 0 \quad (1)$$

$$\mathbf{H} = \nu(|\mathbf{B}|)\mathbf{B} \quad (2)$$

with  $\nabla \times$  the 2D curl operator,  $\mathbf{H}$  the magnetic field,  $\mathbf{B} = \nabla \times A$  the magnetic induction ( $T$ ),  $A$  the unknown continuous 2D potential vector,  $\nu$  the reluctivity following the nonlinear  $\mathbf{B}(|\mathbf{H}|)$  curve in the ferromagnetic parts,  $\nu = \nu_0$  the reluctivity of vacuum in air parts, and  $\mathbf{J}$  the supply current density ( $A.m^{-2}$ ) in the coils. The classical expressions of  $S = (S_{ij})_{1 \leq i, j \leq n}$  and  $\mathbf{b} = (b_i)_{1 \leq i \leq j}$  read:

$$S_{ij} = \int_{\Omega} (\nabla \times \tau_i)^T \frac{\partial \mathbf{H}}{\partial \mathbf{B}} \nabla \times \tau_j d\Omega, \quad (3)$$

$$\mathbf{b}_i = \int_{\Omega} \mathbf{J} \cdot \tau_i - \mathbf{H}^T \nabla \times \tau_i d\Omega, \quad (4)$$

$$\frac{\partial \mathbf{H}}{\partial \mathbf{B}} = \nu \mathbf{I} + \frac{1}{|\mathbf{B}|^2} \left( \frac{\partial |\mathbf{H}|}{\partial |\mathbf{B}|} - \frac{|\mathbf{H}|}{|\mathbf{B}|} \right) \mathbf{B} \mathbf{B}^T, \quad (5)$$

where  $n$  is the number of mesh nodes,  $\tau_i$  is the 2D nodal shape function at mesh node  $i$ ,  $\frac{\partial \mathbf{H}}{\partial \mathbf{B}}$  the incremental reluctivity tensor and  $\mathbf{I}$  the identity tensor.

### B. Nonlinear resolution in a density-based optimization context

Topology optimization methods are usually based on the definition of a continuous density  $\rho$  such that  $0 \leq \rho \leq 1$  which parametrizes the pointwise behavior of the ferromagnetic material in the optimization area [1], [2].  $\rho = 0$  represents the behavior of air,  $\rho = 1$  the one of the material and the non-physical intermediate density values ( $0 < \rho < 1$ ) should be avoided in the final result. Using this definition, the optimization problem can be rewritten as finding an optimal density on a given region, and  $\rho$  can be used as the design variable of the problem [1], [2], [5], [7].

The density is mapped to the material behavior by editing the reluctivity definition. Thus, the constitutive law (2) becomes:

$$\hat{\mathbf{H}} = [(1 - \rho)\nu_0 + \rho\nu] \mathbf{B} = \hat{\nu} \mathbf{B}, \quad (6)$$

where  $\nu$  is the initial reluctivity of the ferromagnetic material, and  $\hat{\nu}$  the one taking into account the density. This new material law must be taken into account in the NR resolution [7]. Equations (3)-(5) are transformed into (7)-(9):

$$S_{ij} = \int_{\Omega} (\nabla \times \tau_i)^T \frac{\partial \hat{\mathbf{H}}}{\partial \mathbf{B}} \nabla \times \tau_j d\Omega, \quad (7)$$

$$\mathbf{b}_i = \int_{\Omega} \mathbf{J} \cdot \tau_i - \hat{\mathbf{H}}^T \nabla \times \tau_i d\Omega, \quad (8)$$

$$\frac{\partial \hat{\mathbf{H}}}{\partial \mathbf{B}} = \hat{\nu} \mathbf{I} + \frac{\rho}{|\mathbf{B}|^2} \left( \frac{\partial |\mathbf{H}|}{\partial |\mathbf{B}|} - \frac{|\mathbf{H}|}{|\mathbf{B}|} \right) \mathbf{B} \mathbf{B}^T. \quad (9)$$

This adapted nonlinear resolution is used to implement the optimization method presented in Section III.

## III. THE LEVEL-SET METHOD

The principle of the LS method is to track the design of the rotor using an implicit definition of the design boundaries. A Level-Set function  $\phi : \mathbb{R} \times \Omega_R \mapsto \mathbb{R}$  is defined such that:

$$\phi(t, x) < 0 \text{ if } x \text{ is in air,}$$

$$\phi(t, x) > 0 \text{ if } x \text{ is in a ferromagnetic part,}$$

$$\phi(t, x) = 0 \text{ if } x \text{ is at boundary,}$$

$$\phi(0, x) \text{ matches with the input design,}$$

where  $t$  is a pseudo-temporal variable used to advect the design.

### A. A Level-Set method based on radial basis functions

The radial basis function method [4] is used. The principle is to discretize  $\phi$  in a basis of radial functions  $(\phi_i(x))_{1 \leq i \leq n_R}$  defined on the  $n_R$  nodes of the mesh of  $\Omega_R$ . Thus,  $\phi$  can be written as:

$$\phi = \sum_{i=1}^{n_R} \alpha_i \phi_i. \quad (10)$$

Note that  $\phi$  is determined by the coefficient  $\alpha = (\alpha_i)_{1 \leq i \leq n_R}$ . As mentioned in introduction, the principle of the optimization is to iteratively update  $\alpha$  in a descent direction for the optimization problem until convergence. Thus,  $\alpha$  is the optimization design variable, and as any gradient-based optimization, the sensitivity analysis of the objective with respect to the design variable is needed and is presented in Section IV.

### B. Length scale control and numerical stability

In order to enforce numerical stability and achieve length scale control on the method, a differentiable radius based filter  $\mathcal{F}$  is applied to  $\alpha$  as suggested in [5]:

$$\mathbf{v} = \mathcal{F}(\alpha)_j = \frac{\sum_{k=1}^{N_j} w_{j,k} \alpha_k}{\sum_{k=1}^{N_j} w_{j,k}}, \quad (11)$$

where the sums are on the  $N_j$  mesh nodes in the neighborhood within  $R$  radius around mesh element  $j$  and  $w_{j,k}(x_k)$  weights depending on  $R$  and on the distance of node  $k$  to element  $j$ . This filter enforces numerical stability of the method and prevent the presence of small holes or hinges in the process [5].

### C. Level-Set mapping to a density-based process

After the filtering step, values are mapped to a density-like quantity  $\rho$  which parametrizes the reluctivity as in Section II. This mapping is done using a smooth Heaviside projection approximation  $\rho = \mathcal{H}(\mathbf{v})$  to ensure differentiability of the problem. The final density-like quantity used reads:

$$\rho = \mathcal{H}(\mathcal{F}(\alpha)) = \mathcal{H}(\mathbf{v}), \quad (12)$$

$$\hat{\nu} = (1 - \rho)\nu_0 + \rho(\nu - \nu_0). \quad (13)$$

This formulation has been integrated into the nonlinear magnetostatic resolution (7)-(9).

#### IV. CONTINUOUS SENSITIVITY ANALYSIS OF THE MAGNETIC TORQUE

The advection process used to update  $\phi$  is achieved by solving an ordinary differential equation system for  $\alpha$  [4]. This step requires the sensitivity of the objective with respect to the  $\alpha_i$ . In the case of a SynRM, the objectives are usually functions of the torque  $T$  of the machine. Thus, the fundamental analysis that has to be done is to express the sensitivity  $\frac{dT}{d\alpha}$ . Using the derivation chain rule, one has:

$$\frac{dT}{d\alpha} = \frac{dT}{d\rho^T} \frac{d\rho}{d\mathbf{v}^T} \frac{d\mathbf{v}}{d\alpha}. \quad (14)$$

Since operators  $\mathcal{H}$ ,  $\mathcal{F}$  and their derivatives are analytically known, the only unknown term is  $\frac{dT}{d\rho^T}$ . The analytical sensitivity analysis of such quantity which depends on a partial differential equation (1) has become a well-known problem using the adjoint method [1], [2], [6], [7], [10]. The continuous sensitivity in a direction  $k \in \mathcal{L}^\infty(\Omega_R)$  reads [7], [10]:

$$\left\langle \frac{dT}{d\rho}, k \right\rangle = - \int_{\Omega} (\nu - \nu_0) \left[ (\nabla \times A)^T \nabla \times \lambda \right] \cdot k \, d\Omega \quad (15)$$

where  $A$  is the physical solution to (1) solved using (6)-(9) and  $\lambda$  is the adjoint state, solution to the following weak form for  $u \in \mathbf{H}_0^1(\Omega)$ :

$$\int_{\Omega} \left[ \nabla \times \left( \frac{\partial \hat{\mathbf{H}}}{\partial \mathbf{B}} \nabla \times \lambda \right) - \frac{\partial T}{\partial A}(A, \rho) \right] \cdot u \, d\Omega = 0, \quad (16)$$

where  $\frac{\partial \hat{\mathbf{H}}}{\partial \mathbf{B}}$  is the incremental reluctivity tensor (9) assembled with the physical solution  $A$ . Note that (16) can be easily solved using an additional iteration to the Newton-Raphson resolution process of (1), making it easy to solve. The novelty of this paper is the use of the virtual work method [8] to compute and optimize the torque of the SynRM in the LS process. Starting from the discrete approach of  $\frac{\partial T}{\partial A}$  in [6], a continuous version can be expressed as follows:

$$\left\langle \frac{\partial T}{\partial A}(A, \rho), u \right\rangle = \sum_e \int_{\Omega_e} \hat{\nu} \mathbf{B}^T (M_e + M_e^T) \nabla \times u \, d\Omega, \quad (17)$$

$$M_e = -G_e^{-1} \frac{\partial G_e}{\partial \omega} + \frac{1}{2|G_e|} \frac{\partial |G_e|}{\partial \omega} I, \quad (18)$$

where  $u$  is the computation direction,  $G_e$  is the Jacobian matrix of the transformation from the surface  $\Omega_e$  of mesh element  $e$  to the corresponding reference element  $\Delta_e$  [8],  $|G_e|$  its determinant,  $I$  the identity matrix and  $\omega$  the virtual rotation of the rotor around its rotation axis.

#### V. APPLICATION

##### A. Reference Machine

In Figure 1a, a reference SynRM model is presented. The numerical model is accessible in Altair Flux<sup>TM</sup> motor database. The rotor is made of a nonlinear ferromagnetic material BM\_330\_35A (in orange) and air regions (in light blue). The other components are the statoric part supplied by three alternating phases. (phase 1<sup>-</sup> is in dark red, phase 2<sup>+</sup> in green,

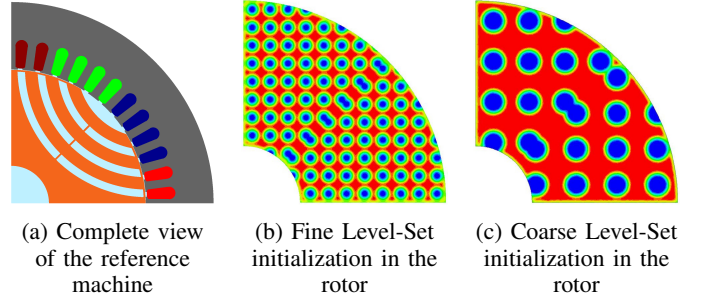


Fig. 1: View of the optimization setup

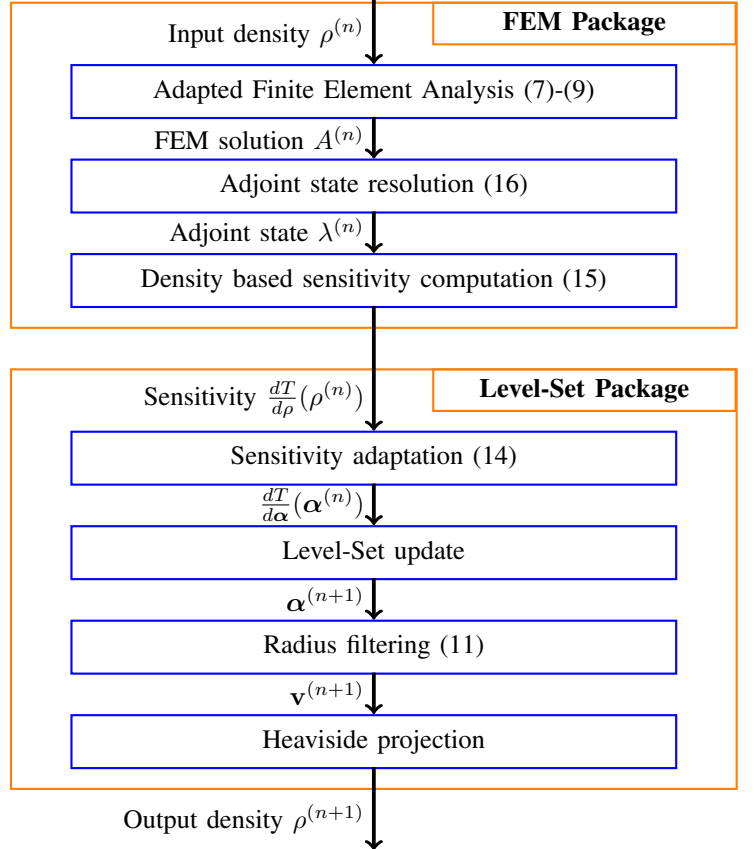


Fig. 2: Flowchart of the optimization algorithm at iteration  $n$

phase 3<sup>-</sup> in dark blue and phase 1<sup>+</sup> in red). The considered current supply  $I_i$  in phase  $1 \leq i \leq 3$  is the following:

$$I_i(\theta) = I_{max} \times \sin \left( \theta + \frac{2(i-1)}{3} \pi + \gamma \right) \quad (19)$$

with  $I_{max} = 48.17A$  and  $\gamma = -22^\circ$ . The performances of the machine are measured using the average torque  $T_{avg}$  and torque ripple  $T_r$  [2], [3] measured over  $N = 30$  rotor angular positions:

$$T_r = 100 \times \frac{T_{max} - T_{min}}{T_{avg}}, \quad (20)$$

where  $T_{min}$  and  $T_{max}$  are the minimum and maximum torque values, respectively. The torque ripple is a measurement of the relative amplitude of the torque, which must remain relatively low to control the noise of the machine [2].

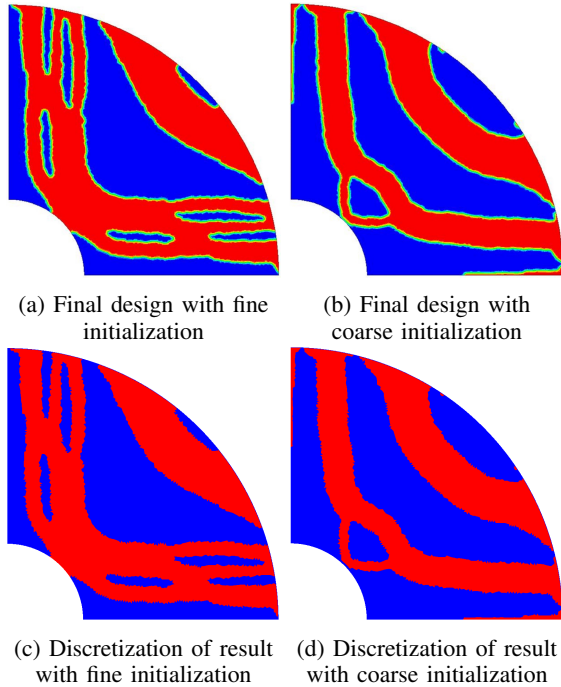


Fig. 3: Overview of the optimized designs

### B. Optimization

The Level-Set method (Section III) and the adjoint method (Section IV) have been combined in an optimization algorithm presented in Figure 2. The finite element package is implemented in the software Flux<sup>TM</sup>, and the Level-Set package in Altair Optistruct<sup>TM</sup>. We propose to test the method on the reference machine presented above. The proposed formulation is to minimize the volume of the rotor part. Constraints on average torque and torque ripple are set such that the optimized results must have at least the same torque performances as the reference machine. The Level-set function is initialized in the rotor with a perforated design. This can be used to tackle the incapacity of the LS method to create holes during the optimization process. Two refinements are tested for the initialization. They are represented in figures 1b and 1c (red:  $\phi > 0$  (material), blue:  $\phi < 0$  (air), green:  $\phi \approx 0$  (boundaries)). The two optimizations are launched on a computer with an Intel Xeon 2.90 GHz and 64 Go of RAM. A global overview of the optimization runs are presented Figure 3. Figures 3a and 3b presents the final designs. Since some intermediate density values remain in those results, discrete projections are proposed in Figures 3c and 3d. Finally, Table I highlights the magnetic performances of all designs.

Case	$T_{avg}$ (N.m)	$T_r$ (%)	Volume (%)	Iteration number
Reference machine	245.5	32.4	100	-
Result 3a	245.4	32.2	75	61 (2h03)
Projection 3c	246.26	30.4	74	-
Result 3b	243.5	32.1	61	69 (1h20)
Projection 3d	247.4	34.6	59	-

TABLE I: Overview of performances for all cases.

As shown in Table I, optimized designs of Figure 3 are interesting because they propose the same torque performances as the reference machine 1a but with a significant rotor mass reduction. The two optimization results (Figures 3a and 3b) are different, because gradient-based optimization methods are sensitive to the starting point. Note that the projection step used to remove intermediate values in those results do not degrade torque and volume performances and provide similar performances. The main issue of the actual optimization formulation is the incapacity of the method to converge toward manufacturable results. For instance results presented here are not connected and have no mechanical strength. A hint to fix this problem is to add mechanical topology optimization in the actual formulation.

## VI. CONCLUSION

The magnetostatic resolution of a SynRM in a density-based topology optimization context is detailed in depth. Then, a Level-set method based on [4], [5] is proposed. In order to optimize the SynRM using a gradient-based optimization, the fully analytical sensitivity analysis of the magnetic torque using the virtual work method [6], [8] is expressed. The Level-Set method and sensitivity analysis are integrated in an optimization algorithm (Figure 2). Finally, two optimization examples are provided to illustrate the capacity of the method to find designs with equivalent torque performances but reduced rotor mass. However, the results are not connected yet and have poor mechanical strength. Thus, the next step of this work is to take into account mechanical responses such as the compliance or the stress in the optimization problem to tackle this issue and be able to find more realistic designs.

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