# An Evaluation Tool for Backbone Extraction Techniques in Weighted Complex Networks

October 20, 2023

#### 1 Requirements

netbone is available on Pypi. But make sure you have Python version 3.10 or higher and it's a good idea to use conda, virtualeny, or pyeny.

!pip install netbone

Once installed, the *netbone* package can be imported simply

[1]: import netbone as nb

b.to\_dataframe()

## 2 Toy Example

To cover all users needs we separated the calculation process from the filtering process in *netbone*. Thus, the process of extracting the backbone follows: 1. Apply a backbone extraction method to run the computation process 2. Apply a filter to extract the backbone

To illustrate the usage of *netbone*, we consider the high salience skeleton method with the Les Misérables network. We chose this extraction technique because it can be associated with the three filtering methods provided by *netbone*. The *netbone* package can handle two types of inputs: a *networkx* graph or a *DataFrame*. In this example, we will load the Les Misérables network from *networkx* and apply the *high\_salience\_skeleton()* method.

```
[2]: import networkx as nx
g = nx.les_miserables_graph()

b = nb.high_salience_skeleton(g)
```

The resulting scores can be examined using the to\_dataframe() function as shown below:

#### [3]: in\_backbone source target weight distance salience 0 Myriel 1.000000 Napoleon 1.000000 True MlleBaptistine 1 Myriel 8 0.125000 True 0.987013 2 Myriel MmeMagloire 10 0.100000 True 0.987013 3 Myriel CountessDeLo 1.000000 True 1.000000 4 Myriel Geborand 1.000000 1.000000 True

• •	• • •	• • •		• • •	• • •	
249	Babet	Brujon	3	0.333333	False	0.025974
250	Claquesous	Montparnasse	2	0.500000	False	0.025974
251	Claquesous	Brujon	1	1.000000	False	0.000000
252	Montparnasse	Brujon	1	1.000000	False	0.000000
253	Child1	Child2	3	0.333333	False	0.025974

[254 rows x 6 columns]

The high salience skeleton method exhibits a bimodal distribution of scores centered around 0 and 1. The default approach of this method is to keep only edges with scores greater than 0.8. In *netbone*, it can be accomplished using the *boolean\_filter()*. However, in that case, two nodes are missing from the extracted backbone in this particular example. To fix this issue, users can adjust the threshold by using the *threshold\_filter()* function. One can use a threshold of 0.7 to retain all the network nodes. Additionally, users can control the size of the backbone using the *fraction\_filter()*, such as keeping 15% of the network. The following code shows how to do it in *netbone*:

```
[4]: from netbone.filters import boolean_filter, threshold_filter, fraction_filter

backbone1 = boolean_filter(b)

backbone2 = threshold_filter(b, 0.7)

backbone3 = fraction_filter(b, 0.15)
```

High Salience Skeleton Filter High Salience Skeleton Filter High Salience Skeleton Filter

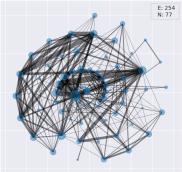
To illustrate the usage of the extracted backbones, we plot them using netowrkx.

```
nx.draw_networkx(g, ax=ax,
                 alpha=.5,
                 # width=.6,
                 node_size=sizes,
                 width = weg,
                 # node_color='k',
                 pos=pos,
                 with_labels=False,
                 font_size=50)
plt.legend([f'E: {len(g.edges())} \nN: {len(g.nodes())}'], handlelength=0,_
→handleheight=0)
titles = ['Boolean Filter', 'Threshold Filter', 'Fraction Filter']
for i, backbone in enumerate([backbone1, backbone2, backbone3]):
    sizes = [node_scale * deg[n] for n in backbone.nodes()]
    weg = [edge_scale * backbone[u][v]['weight'] for u,v in backbone.edges()]
    ax = plt.subplot(grid[1,i])
    ax.set_title(titles[i], fontsize=20)
    removed = g.nodes() - backbone.nodes()
    nx.draw_networkx_nodes(g, pos=pos, nodelist=removed, node_color='white',_
 →node_size=[node_scale * deg[n] for n in removed], alpha=0.0)
    nx.draw_networkx_nodes(g, pos=pos, nodelist=['Child1'], node_color='white',_
 →node_size=[node_scale * deg[n] for n in ['Child1']], alpha=0.0)
    nx.draw_networkx(backbone, ax=ax,
                     alpha=.5,
                     # width=.6,
                     node_size=sizes,
                     width = weg,
                     # node_color='k',
                     pos=pos,
                     with_labels=False)
    # plt.legend([r'h]$' + f': {len(backbone.nodes())} \n' + r'h[E]$' +

→f': {len(backbone.edges())}'], handlelength=0, handleheight=0)

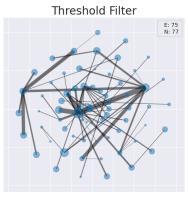
    plt.legend([f'E: {len(backbone.edges())} \nN: {len(backbone.nodes())}'],__
→handlelength=0, handleheight=0)
# plt.savefig('./images/toy.pdf', dpi=300, bbox_inches='tight')
plt.savefig('./images/toy.png', dpi=300, bbox_inches='tight', transparent=True)
```

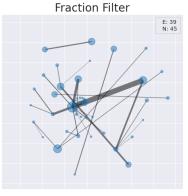
Les Misérables Original Network



Boolean Filter

E: 73
N: 75





In this experiment, we focus on assessing the connectivity of the structural backbone extraction methods in the air transportation network using *netbone*'s comparison framework. The aim is to have a connected filtered network when applying filters since connectivity is an essential property in transportation networks. To accomplish this, first we define an instance of the *Compare* class from the *compare* module.

```
[6]: from netbone.compare import Compare
framework = Compare()
```

After initialization, the first step is to add the original network to *netbone*'s comparison framework using the  $set\_network()$  function. For this purpose, we must provide a networkx graph or an edge list stored in a DataFrame object. In this experiment, we use a DataFrame object.

```
[7]: import pandas as pd
edge_list = pd.read_csv('./data/data.csv')
framework.set_network(edge_list)
```

The next step is to set up the filter in the comparison framework. It is done using the  $set\_filter()$  function. It specifies the filter used to extract the backbones before computing the properties. In this experiment, we choose to use the  $boolean\_filter()$ . Since the selected methods extract one subgraph by there definition.

```
[8]: from netbone.filters import boolean_filter framework.set_filter(boolean_filter)
```

After setting the original network and filter, the next step is to add the backbone extraction methods to the comparison framework. This is done in two stages, first we apply the backbone extraction method. Then we add them to the comparison framework using the add\_backbone() function. Here we chose to use eight structural techniques. We recall that in *netbone* the computation process is separated of the filtration process. Subsequently, the backbone extraction method in *netbone* returns an instance of the *Backbone* Class.

```
[9]: import netbone as nb
     ds = nb.doubly_stochastic(edge_list)
     hb = nb.h_backbone(edge_list)
     hss = nb.high_salience_skeleton(edge_list)
     msp = nb.maximum_spanning_tree(edge_list)
     mb = nb.metric_distance_backbone(edge_list)
     umb = nb.ultrametric_distance_backbone(edge_list)
     pmfg = nb.pmfg(edge_list)
     pla = nb.plam(edge_list)
     framework.add_backbone(ds)
     framework.add_backbone(hb)
     framework.add_backbone(hss)
     framework.add_backbone(msp)
     framework.add_backbone(mb)
     framework.add_backbone(umb)
     framework.add_backbone(pmfg)
     framework.add_backbone(pla)
```

The final step is to add the properties used to evaluate the backbones. To add a property, users can use the  $add\_property()$  function by passing it a name and a property function. Here, we use six predefined property functions from the measures module

```
from netbone.measures import node_fraction, edge_fraction, average_degree, reachability, weight_fraction, density
framework.add_property('Node Fraction', node_fraction)
framework.add_property('Edge Fraction', edge_fraction)
framework.add_property('Weight Fraction', weight_fraction)
framework.add_property('Density', density)
framework.add_property('Average Degree', average_degree)
framework.add_property('Reachability', reachability)
```

Now that everything is set up and added to the framework, we call the *properties()* function to compute the added properties. This function returns a pandas *DataFrame* that can be inspected to compare the computed properties of the backbones

```
[11]: results = framework.properties()
results
```

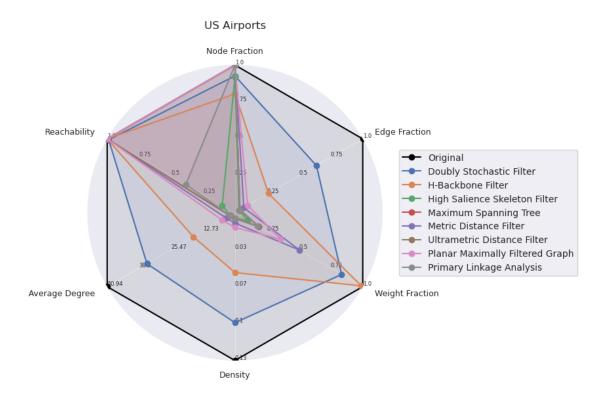
[11]:		Node Fraction 1	Edge Fractio	on \	
	Original	1.000000	1.00000		
	Doubly Stochastic Filter	0.926316	0.63804	<u>1</u> 5	
	H-Backbone Filter	0.805263	0.26224	14	
	High Salience Skeleton Filter	0.918421	0.03347	'8	
	Maximum Spanning Tree	1.000000	0.03916	31	
	Metric Distance Filter	1.000000	0.06974	16	
	Ultrametric Distance Filter	1.000000	0.03916	31	
	Planar Maximally Filtered Graph	1.000000	0.09971	.1	
	Primary Linkage Analysis	1.000000	0.03874	18	
		Weight Fraction	Density A	Average Degree	\
	Original	1.000000	0.1344	50.936842	
	Doubly Stochastic Filter	0.834884	0.1000	35.085227	
	H-Backbone Filter	0.988625	0.0544	16.588235	
	High Salience Skeleton Filter	0.096433	0.0053	1.856734	
	Maximum Spanning Tree	0.186046	0.0053	1.994737	
	Metric Distance Filter	0.503583	0.0094	3.552632	
	Ultrametric Distance Filter	0.186046	0.0053	1.994737	
	Planar Maximally Filtered Graph	0.355704	0.0134	5.078947	
	Primary Linkage Analysis	0.177826	0.0052	1.973684	
		Reachability			
	Original	1.000000			
	Doubly Stochastic Filter	0.988669			
	H-Backbone Filter	1.000000			
	High Salience Skeleton Filter	0.100007			
	Maximum Spanning Tree	1.000000			
	Metric Distance Filter	1.000000			
	Ultrametric Distance Filter	1.000000			
	Planar Maximally Filtered Graph	1.000000			

To perform the comparative analysis of backbone extraction techniques visually, we plot the properties across various dimensions using a  $radar\_plot()$  function from the visualize module. This function takes two inputs: the results DataFrame and a String representing the title of the figure and the name of the saved figure file.

0.384294

```
[12]: from netbone.visualize import plot_radar plot_radar(results, 'US Airports')
```

Primary Linkage Analysis



The Previous experiment focuses on the structural methods for backbone extraction. Some of these methods can be adjusted using a threshold on scores or selecting the top fraction of scores. In this experiment, our objective is to sparsify the network while preserving all the nodes, which is crucial in the context of a transportation network. To achieve this, we use *netbone*'s comparison framework to help us determine the appropriate fraction. We start by initiating an instance of the Compare class from the compare module. Then we add the original network to *netbone*'s comparison framework using the  $set\_network()$  function.

```
[13]: from netbone.compare import Compare
import pandas as pd

framework = Compare()

edge_list = pd.read_csv('./data/data.csv')
framework.set_network(edge_list)
```

The next step is to set up the filter in the comparison framework. In this experiment, we choose to use the *fraction\_filter()* to evaluate the backbones at the fractions from 0.01 till 0.5. Thus, we pass an array of these values while setting the filter.

```
[14]: from netbone.filters import fraction_filter

fractions = [0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5]

framework.set_filter(fraction_filter, fractions)
```

Once the original network and filter are set, the following step is to add the backbone extraction methods in the comparison framework.

```
[15]: import netbone as nb

gt =nb.global_threshold(edge_list)
hss = nb.high_salience_skeleton(edge_list)
ds = nb.doubly_stochastic(edge_list)
gspar = nb.gspar(edge_list)
bet = nb.betweenness(edge_list, weighted=True)

framework.add_backbone(gt)
framework.add_backbone(hss)
framework.add_backbone(ds)
framework.add_backbone(gspar)
framework.add_backbone(bet)
```

The last step is incorporating the properties to assess the backbones under varying fractions. In this case, we use one property function, the *node fraction()* from the *measures* module.

```
[16]: from netbone.measures import node_fraction framework.add_property('Node Fraction', node_fraction)
```

After configuring everything and adding it to the framework, the next step is to call the *properties\_progression()* function to compute the properties for the backbone at each fraction. The output of this function is a *dictionary* of *DataFrames*. One can use it to inspect the computed properties of the backbones with respect to the fractions.

\

```
[17]: results= framework.properties_progression() results['Node Fraction']
```

[17]:	Global Threshold Filter	High Salience Skeleton Filter
Fraction of Edges		
0.01	0.092105	0.3
0.05	0.226316	1.0
0.10	0.378947	1.0
0.15	0.526316	1.0
0.20	0.657895	1.0
0.25	0.773684	1.0
0.30	0.863158	1.0
0.35	0.915789	1.0
0.40	0.950000	1.0
0.45	0.971053	1.0

0.50 0.973684 1.0

	Doubly Stochastic Filter	Global Sparsification \
Fraction of Edges		
0.01	0.310526	0.155263
0.05	0.784211	0.239474
0.10	0.836842	0.294737
0.15	0.850000	0.350000
0.20	0.855263	0.381579
0.25	0.863158	0.450000
0.30	0.868421	0.500000
0.35	0.871053	0.552632
0.40	0.878947	0.626316
0.45	0.886842	0.671053
0.50	0.886842	0.705263

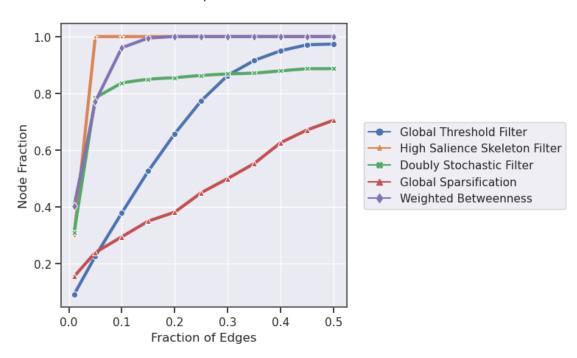
#### Weighted Betweenness

Fraction of Edges	
0.01	0.402632
0.05	0.771053
0.10	0.960526
0.15	0.994737
0.20	1.000000
0.25	1.000000
0.30	1.000000
0.35	1.000000
0.40	1.000000
0.45	1.000000
0.50	1.000000

To visualize the evolution of the properties versus the fraction values, we use the *plot\_progression()* function from the *visualize* module. This function requires two arguments: the results *dictionary* and a *String* that represents the title of the figure and the name of the saved figure file.

```
[18]: from netbone.visualize import plot_progression plot_progression(results, 'US Airports')
```

#### **US Airports**



# 5 Experiment 3

In this experiment, we use *netbone*'s comparison framework to assess the global threshold and statistical methods to capture the weight and degree distributions. We start by initiating an instance of the Compare class from the compare module. Then we add the original network to *netbone*'s comparison framework using the *set network()* function.

```
[19]: from netbone.compare import Compare
  import pandas as pd

framework = Compare()

edge_list = pd.read_csv('./data/data.csv')
  framework.set_network(edge_list)
```

Then we add the backbone extraction methods to the comparison framework. Here, the order is important because we are going to use the order of the added backbones in the next step.

```
[20]: import netbone as nb
gt = nb.global_threshold(edge_list)
df = nb.disparity(edge_list)
mlf = nb.marginal_likelihood(edge_list)
nc = nb.noise_corrected(edge_list)
```

```
ecm = nb.ecm(edge_list)
lans = nb.lans(edge_list)

framework.add_backbone(gt)
framework.add_backbone(nc)
framework.add_backbone(df)
framework.add_backbone(ecm)
framework.add_backbone(lans)
framework.add_backbone(mlf)
```

The next step is to set up the filter in the comparison framework. In this experiment, we choose to use the *threshold\_filter()* to evaluate the backbones. For the global threshold method, we set the threshold value to the average weight of 7000. For the statistical methods, we use a significance level of 0.05. Thus, we pass an array of these values while setting the filter taking into consideration the order when we added the backbones.

```
[21]: from netbone.filters import threshold_filter

values = [7000] + [0.05]*5
framework.set_filter(threshold_filter, values)
```

The last step is incorporating the property functions that will extract the values to assess the distribution of the properties in the backbones. In this case, we use two property functions, the weights() and degrees() from the measures module.

```
[22]: from netbone.measures import weights, degrees

framework.add_property('Weight', weights)
framework.add_property('Degree', degrees)
```

After configuring everything and adding it to the framework, the next step is to call the *distribution\_ks\_statistic()* function to compute the KS statistic between the original and backbone property distributions. The output of this function is a *DataFrame* and a *dictionary*. One can use the *DataFrame* to inspect the computed KS statistic for each property, and the *dictionary* is used later for visualization.

```
[23]: results, dist = framework.distribution_ks_statistic() results
```

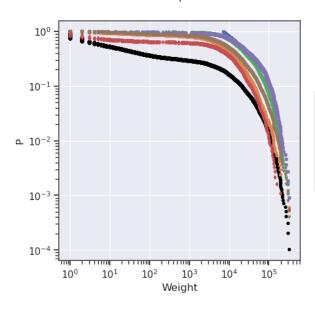
```
[23]:
                                                                  Degree
                                                        Weight
      Global Threshold Filter
                                                      0.805125
                                                                0.406337
      Noise Corrected Filter
                                                      0.517305 0.542105
     Disparity Filter
                                                      0.700747 0.494889
      Enhanced Configuration Model Filter
                                                      0.325893 0.555263
     Locally Adaptive Network Sparsification Filter
                                                     0.662704 0.665789
     Marginal Likelihood Filter
                                                      0.553501 0.415789
```

To visualize the cumulative distribution of the properties, we use the *plot\_distribution()* function from the visualize module. This function requires two arguments: the results dictionary and a

String that represents the title of the figure and the name of the saved figure file.

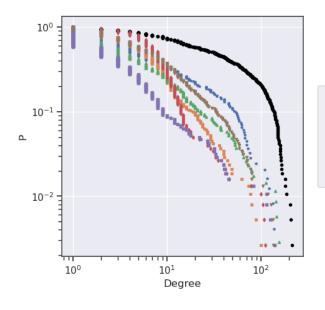
# [24]: from netbone.visualize import plot\_distribution plot\_distribution(dist, title='US Airports')

#### **US** Airports



- Original
- Global Threshold Filter
- Noise Corrected Filter
- Disparity Filter
- Enhanced Configuration Model Filter
- Locally Adaptive Network Sparsification Filter
- Marginal Likelihood Filter

#### **US Airports**



- Original
- Global Threshold Filter
- Noise Corrected Filter
- Disparity Filter
- Enhanced Configuration Model Filter
- Locally Adaptive Network Sparsification Filter
- Marginal Likelihood Filter

In this experiment, we use netbone's comparison framework to extract the consensus backbone using the statistical backbone extraction methods. We start by initiating an instance of the Compare class from the compare module. Then we add the original network to netbone's comparison framework using the  $set\_network()$  function.

```
[25]: from netbone.compare import Compare
import pandas as pd

framework = Compare()

edge_list = pd.read_csv('./data/data.csv')
framework.set_network(edge_list)
```

Then we add the backbone extraction methods to the comparison framework. Similar to the previous experiment, the order is important because we are going to use the order of the added backbones in the next step.

```
[26]: import netbone as nb

df = nb.disparity(edge_list)
mlf = nb.marginal_likelihood(edge_list)
nc = nb.noise_corrected(edge_list)
ecm = nb.ecm(edge_list)
lans = nb.lans(edge_list)

framework.add_backbone(nc)
framework.add_backbone(df)
framework.add_backbone(ecm)
framework.add_backbone(lans)
framework.add_backbone(mlf)
```

The next step is to set up the filter in the comparison framework. In this experiment, we choose to use the *threshold\_filter()* to evaluate the backbones. We set the threshold value to 0.05. Thus, we pass an array of these values while setting the filter taking into consideration the order when we added the backbones.

```
[27]: from netbone.filters import threshold_filter

values = [0.05]*5
framework.set_filter(threshold_filter, values)
```

Here we don't need to add any property function we simply use the method consent(). By taking the intersection of the extracted backbones, this method returns a netowrkx graph representing the consensus backbone.

```
[28]: consensual = framework.consent()
```

Next we extract the backbones similar to the toy example to prepare it for plotting later.

Marginal Likelihood Filter
Noise Corrected Filter
Disparity Filter
Enhanced Configuration Model Filter
Locally Adaptive Network Sparsification Filter

We extract the coordinates of the nodes and the degree of each node to plot the nodes in the right position.

```
[30]: import pyreadr
import networkx as nx
result = pyreadr.read_r('./data/data.RData')
g = nx.from_pandas_adjacency(result['airport'])
positions = {index: tuple(row) for index, row in result['latlong'].iterrows()}
deg = nx.degree(g)
```

We plot the original network with the backbones using cartopy and networkx.

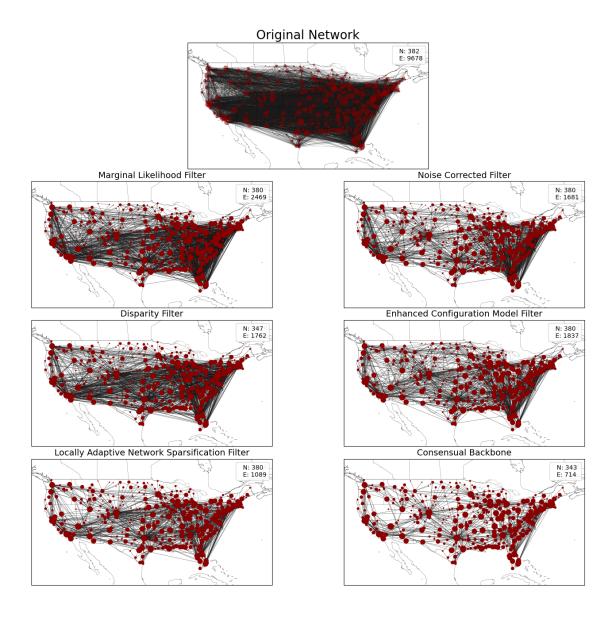
```
[31]: import cartopy.crs as ccrs
import cartopy.feature as cfeature
import matplotlib.pyplot as plt
import seaborn as sns
sns.reset_defaults()
crs = ccrs.PlateCarree()
fig = plt.figure(figsize=(18, 20))
rows = 5
columns = 2

grid = plt.GridSpec(rows, columns, wspace = .025, hspace = .1)

sizes = [.5 * deg[iata] for iata in g.nodes()]
ax = plt.subplot(grid[0,:], projection=crs)
ax.coastlines(lw=0.2)

ax.set_extent([-128, -62, 20, 50])
ax.add_feature(cfeature.BORDERS, color="k", lw=0.2)
```

```
ax.add_feature(cfeature.STATES, lw=0.1)
ax.set_title('Original Network', fontsize=20)
ax.set_aspect('equal')
nx.draw_networkx(g, ax=ax,
                 alpha=.5,
                 width=.3,
                 node_size=sizes,
                 node_color='#8b0000',
                 pos=positions,
                 cmap=plt.cm.autumn,
                 with_labels=False,
                 edge_color='k')
ax.legend([f'N: {len(g.nodes())} \nE: {len(g.edges())}'], handlelength=0,_
→handleheight=0, markerscale=0)
# b = ['Marginal Likelihood', 'Noise Corrected', 'Disparity Filter', "ECM_{\square}
→Filter", "LANS Filter", 'Consensual Backbone', 'Global Threshold']
for i, ax in enumerate(backbones):
    ax = plt.subplot(grid[int(i/2)+1,i%2], projection=crs)
    backbone = backbones[i]
    sizes = [.5 * deg[iata] for iata in backbone.nodes()]
    ax.coastlines(lw=0.2)
    ax.set_extent([-128, -62, 20, 50])
    ax.add_feature(cfeature.BORDERS, color="k", lw=0.2)
    ax.add_feature(cfeature.STATES, lw=0.1)
    ax.set_title(b[i], fontsize=14)
    ax.set_aspect('equal')
    nx.draw_networkx(backbone, ax=ax,
                     # alpha=.5,
                     width=.3,
                     node_size=sizes,
                     node_color='#8b0000',
                     pos=positions,
                     cmap=plt.cm.autumn,
                     with_labels=False,
                     edge_color='k')
    # nx.draw_networkx_nodes(backbone, pos=positions, nodelist=backbone.
→nodes()['ALB'], node_color='white', alpha=0.0)
    ax.legend([f'N: {len(backbone.nodes())} \nE: {len(backbone.edges())}'],
→handlelength=0, handleheight=0, markerscale=0)
plt.show()
plt.savefig('networks+consenual.png', dpi=300, bbox_inches='tight')
```



This experiment illustrates how users can integrate their custom backbone extraction method and custom evaluation properties into netbone's comparison framework. To illustrate this process, we define the new\_backbone\_method() function. It generates random values and keeps them in a new edge property named new\_score. The function should return a new instance of the Backbone class. To initialize an instance of the Backbone class, users should provide: 1. networkx graph containing the new edge scores 2. The name of the new method 3. The edge property name 4. The ascending parameter: It should be set to True if the edge property name represents a p-value. Otherwise, it should be False 5. An array of compatible filters. Here, the edge property is a numerical value then the appropriate filters to use in this case are the threshold\_filter() and the fraction\_filter() 6. The filter\_on parameter: should indicate whether the filter is applied to 'Edges' or 'Nodes'.

netbone allows users to implement their new custom evaluation measure. To illustrate this, we define the new\_property\_method() method. This method will imitate the node\_fraction() method; it returns the node fraction preserved in the backbone. The method should: 1. Take two inputs: (the original and backbone networks) 2. Return the computed property value.

```
[33]: def new_property(original, backbone):
    return len(backbone)/len(original)
```

Once the new backbone extraction method and evaluation measures are defined. One can easily add integrate them into the comparison framework using the  $add\_backbone()$  and  $add\_property()$  methods. The following example illustrates comparing the new defined method with the Disparity filter in terms of the new defined property.

```
[34]: from netbone.filters import threshold_filter
      from netbone.compare import Compare
      from netbone.utils.utils import edge_properties
      import pandas as pd
      import netbone as nb
      framework = Compare()
      edge_list = pd.read_csv('./data/data.csv')
      graph = nx.from_pandas_edgelist(edge_list, edge_attr=edge_properties(edge_list))
      framework.set_network(edge_list)
      thresholds = [0.05, 0.9]
      framework.set_filter(threshold_filter, thresholds)
      df = nb.disparity(graph)
      new = new_backbone_method(graph)
      framework.add_backbone(df)
      framework.add_backbone(new)
      framework.add_property('New Property', new_property)
      framework.properties()
```

[34]: New Property
Original 1.000000
Disparity Filter 0.913158
New Backbone Method 0.792105

Users also can compare different distributions. To illustrate this, we define a new method named distribution\_property(). It will imitate the weights() method; it returns all the edge weights in the backbone. The method should: 1. Take one inputs: the backbone network 2. Return and array of the computed property values

```
[35]: def distribution_property(backbone):
    return list(nx.get_edge_attributes(backbone, 'weight').values())
```

One can easily add integrate them into the comparison framework using the add\_property() method. The following example illustrates comparing the new defined method with the Disparity filter in terms of the new defined distribution property.

```
[36]: from netbone.filters import threshold_filter
      from netbone.compare import Compare
      import pandas as pd
      import netbone as nb
      from netbone.utils.utils import edge_properties
      import networkx as nx
      framework = Compare()
      edge_list = pd.read_csv('./data/data.csv')
      graph = nx.from_pandas_edgelist(edge_list, edge_attr=edge_properties(edge_list))
      framework.set_network(edge_list)
      thresholds = [0.05, 0.98]
      framework.set_filter(threshold_filter, thresholds)
      df = nb.disparity(graph)
      new = new_backbone_method(graph)
      framework.add backbone(df)
      framework.add_backbone(new)
      framework.add_property('Distribution Property', distribution_property)
      results, dist = framework.distribution_ks_statistic()
      results
```

```
[36]: Distribution Property
Disparity Filter 0.700747
New Backbone Method 0.078684
```