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Global Sensitivity Analysis of Vehicle Restraint Systems with Screening Methods and Sobol' Indices

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ABSTRACT

Restricted by model complexity and high number of model runs for sampling-based Sensitivity Analysis (SA), qualitative SA methods are widely used for parameter study of complex engineering models (i.e. numerical model of high calculation cost with multiple variables) and quantitative SA of such models has always been a challenge. This work explores the approaches for SA of complex engineering systems and carries out quantitative SA of Vehicle Restraint System (VRS): Influential factors of VRS model were identified through screening analysis (qualitative SA with 2-level screening method and Morris method); The non-influential parameters can be fixed and reduction of inputs variables makes it possible for quantitative SA of VRS, Variance-based SA method (SOBOL' indices) was used to quantify the influence of the influential factors, and metamodel technology --- Kriging interpolation --- was used to generate surrogate model to facilitate the quantitative SA. The factors which contribute most for the robustness of VRS were identified and their influences for model uncertainties were quantified by combining screening analysis and Variance-based SA.

CCS Concepts

- Applied computing → Computer aided design
- Computing methodologies → Uncertainty quantification
- Software and its engineering → Dynamic analysis
- General and reference → General conference proceedings

Keywords

VRS crash simulation; Structural Dynamics; Sensitivity Analysis; Sobol' indices; Screening analysis; Structural Robustness

1. INTRODUCTION

Sensitivity Analysis (SA) is the study of how the uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input [1]. Uncertainty analysis and SA are essential parts of analyses for complex systems [2, 3]. SA can be useful for a range of purposes, including: robustness testing of a model in the presence of uncertainty; increased understanding of the relationships between input and output variables; identifying model inputs that cause significant uncertainty in the output. Model can be simplified through SA by fixing non influential parameters and focusing on the parameters whose uncertainties have great influence on system performances.

Vehicle Restraint Systems (VRS) are specially designed to restrain an errant vehicle by dissipating or absorbing the impact energy and redirecting the vehicles to reduce impact severity. Crashing test of VRS is commonly associated to the development of new device. But it provides a view of the performance of the device of only one set of parameters. One cannot know how robust the design is because the repetition of crash test is economically infeasible and the system uncertainties (such as uncertainty of material mechanical properties, tolerances of manufacture) can't be controlled. Dynamic simulations with FE programs such as LS-DYNA [4] allow the evaluation of the robustness of a design taking into account all these variations. SA of VRS helps to have a deep understanding of model uncertainty and identify the factors that should be taken into considerations during VRS performance studies and robust optimizations [5].

Limited by crashing simulation cost and large number of input variables, resent robust studies of VRS (such as [6]) have only analyzed qualitatively the model performances' uncertainties and it is not possible to analyze directly the quantitative influence of each variables. The approaches for SA of complex engineering systems were proposed and quantitative SA of a VRS was realized with the proposed approaches in this article.

2. Sensitivity Analysis Approaches study

SA is almost always performed by running the model a number of times, i.e. a sampling-based approach [7]. Large number of methods existed for SA, and each method run the model using specialized Design of Experiments (DOE).

2.1 Screening Methods

The central question of screening in the context of modeling and computer simulation is: which factors – among the many potentially important factors – are really important? And to do this, the choice of a well-designed experiment is essential.

2.1.1 Two-level screening Designs

Two-level screening, namely two values for each input variables are taken during DOE. Though these methods can only be used for SA of monotonous models, taking input at two levels greatly reduce the samples required. The main effect $ME_i(Y)$ of parameter X_i on Y is obtained by taking half the difference of average Y values for the two values of the parameter. i.e.

$$ME_r(Y) = \frac{1}{2} \left(\frac{1}{k} \sum_{j=1}^k y_j - \frac{1}{k} y_j \right) \quad (1)$$

The most evident way to take samples is: varies each factor independently over the two levels, holding all others at the specified baseline design. Small number of samples is used, but it does not account for interactions among factors and as it takes only one sample for each factor at each level, output uncertainties are largely influenced by single calculation result. To take into consideration of all combinations of all k factors at the 2 levels, $2k$ samples are taken for full factorial design and it can be very expensive for models of multiple factors. Fractional Factorial Designs (FFD), consisting of a carefully chosen fraction of the full factorial design, can greatly decrease the number of samples [1, 8]. Although with relatively low accuracy, DOE with Orthogonal Arrays (OA) is one of the most efficient sampling methods for FFD [9].

2.1.2 Morris Analysis

Morris Analysis (MA) [1,8,10] (i.e. Elementary Effect Method), a multi-level screening Method, is based on the concept of two successive points within a trajectory differ from each other only in one dimension, or one parameter coordinate, by a fixed amount of Δ . Consider a model with k independent inputs which varies in the k -dimensional unit cube across p selected levels. The elementary effect of the i th input factor EE_i is defined as

$$EE_i = \frac{Y(X_1, \dots, X_{i-1}, X_i + \Delta, \dots, X_k) - Y(X_1, \dots, X_k)}{\Delta} \quad (2)$$

where Y represents the model under study and k is the number of model parameters. One EE is produced per parameter from each trajectory. It estimates at different points in the input space the main effect of a factor by computing a number (say) r of trajectories with $k+1$ model evaluations for each trajectory, and then taking their average. The average for absolute value of EE_i of the r trajectories μ_i and its variance σ_i^2 were calculated with equations (3)(4).

$$\mu_i = \frac{1}{r} \sum_{j=1}^r |EE_i^j| \quad (3)$$

$$\sigma_i^2 = \frac{1}{r-1} \sum_{j=1}^r (|EE_i^j| - \mu_i)^2 \quad (4)$$

MA can determine which input factors could be considered to have effects which were negligible, linear and additive, or nonlinear or involved in interactions with other factors. This method is ideal when the number of input factors is too large to allow the application of computationally expensive quantitative analysis. It helps us to identify the few factors that are influential and reduction of input variables makes it possible for quantitative SA of complex engineering system.

2.2 Variance-Based Sensitivity Analysis

Working within a probabilistic framework, Variance-Based Sensitivity Analysis (VBSA) decomposes the variance of the output of the model or system into fractions which can be attributed to inputs or sets of inputs. They measure sensitivity across the whole input space can deal with nonlinear responses,

and measure the effect of interactions in non-additive systems. The Fourier Amplitude Sensitivity Test (FAST) [1,8], independent of model assumptions, explores the multidimensional space of the input factors by a search curve that scans the entire input space. Similar to FAST, The method of Sobol [1,8,11] assumes the total variance of the model output to be made up of terms of increasing dimensionality and is superior to FAST in that the computation of the higher interaction terms is very natural and is similar to the computation of main effects. For model of independent parameters $Y=f(X_1, X_2, \dots, X_k)$, the Sobol' indices main effect S_i and total effect S_{Ti} of X_i on Y are calculated as:

$$S_i = \frac{V_{X_i}(E_{X_i}(Y|X_i))}{V(Y)} \quad (5)$$

$$S_{Ti} = 1 - \frac{V_{X_{-i}}(E_{X_i}(Y|X_{-i}))}{V(Y)} \quad (6)$$

VBSA can quantify the influences of inputs uncertainties on model performances, but large number of samples is needed and the experiment model runs increase exponential with the number of input variables.

2.3 Approach for Sensitivity Analysis of VRS

The steps for quantitative SA of VRS summarized are as follows:

- 1) Two-level screening ---Orthogonal Array (OA)
- 2) Multi-level screening---Morris Analysis (MA)
- 3) VBSA---Sobol' indices

A complex model may have tens or hundreds of input factors, but only a few of them are influential. By carefully choosing the samples, although with low precision for SA, two-level screening methods are of lowest calculation cost for models SA to find influential factors. Considering the high simulation cost of VRS crashing test, FFD with OA will firstly be used to realize VRS crashing test model parameters screening.

Limited by calculation precision, OA screening can only preliminarily select the influential variables. Non-influential variables will then be treated as constant, which can greatly facilitate Multi-level screening. MA will then be used to classify the influential variables for Multi-level screening.

The few variables of great influence on model performance will be identified after MA. VBSA---Sobol' Analysis---will then be used to quantify the influences of the great influential variables. Even for a model of few factors, thousands of model runs might be needed for quantitative SA, metamodeling technology can be used to generate surrogate models.

3. VRS Sensitivity Analysis

3.1 VRS Crash Model

The real installation conditions of VRS and VRS crash situations are innumerable and impossible to replicate. European Norm EN1317 [12] defined the containment level of VRS and the relative standardized test conditions and criteria. A W-Beam steel VRS of containment level N2 were tested under TB32 test conditions. The tested VRS is composed with the W form beam Rail, Spacer, C form support Post and illustrated in Fig.1. Both Post-Spacer and Spacer-Rail are connected with one single bolt

and two Rails are connected with 8 bolts. A BMW 520i vehicle of 1431 kg was used in the test. The guided vehicle struck the VRS at a controlled, stabilised speed of 113.6kph, at an angle of 20°.

The FE model is illustrated in Fig.2. By simplifying small deformation components and detailed modelling of the parts which are exposed to impact loading, the crash test was simulated efficiently and accurately [13]. Fig.3 compared the experimental test and simulation results at different impact time.

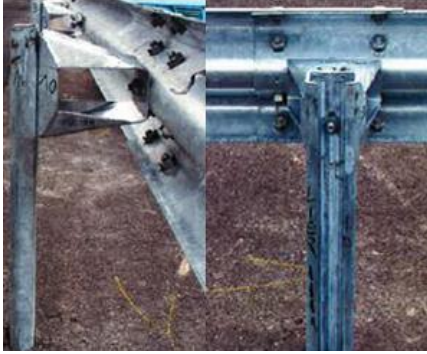


Figure 1 VRS components connections with bolts



Figure 2 Simplified FE crashing model of VRS

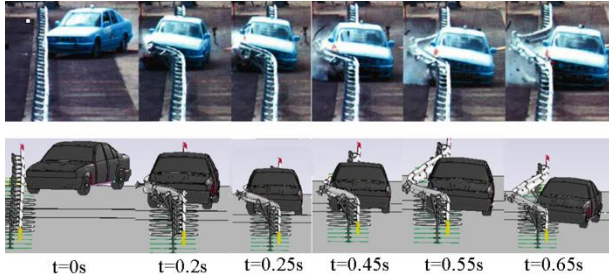


Figure 3 Crashing test and simulation at different impact time

3.2 VRS Parameters & Performance Criteria

3.2.1 Model input parameters

Due to material mechanical properties variations and tolerances in manufacturing, the uncertainties of the following parameters are considered and their probability distribution functions are defined in table 1. VRS components Rail, Spacer and Post are fabricated with structure steel S235. S235 mechanical properties have been analyzed statistically in literature studies [14]. Supposing steel tensile strength is proportional with its yield strength, uncertainties in steel mechanical properties influence Rail Yield strength (RY), Rail young Modulus (RM), Spacer Yield strength (SY), Spacer young Modulus (SM), Post Yield strength (PY) and Post young Modulus (PM); The designed Rail Thickness (RT), Spacer Thickness (ST), Post Thickness (PT) were 3mm, 3mm, 5mm separately, standard deviations of the thickness parameters caused by fabrication tolerances is defined to be 5% of their mean values; Fixing to the ground, VRS performances are affected by Soil bulk Modulus ($SoilM$); VRS components are connected by bolts and Bolt Pre-load (BP) is defined to load the tighten force.

3.2.2 Performance Criteria

Impact severity Criteria [12] – the Theoretical Head Impact Velocity ($THIV$) is used.

Deformation of VRS [12] – Dynamic deflection (D) is the maximum lateral dynamic displacement of the side facing the traffic of the restraint system.

Table 1. VRS crashing model input variables

Type	No.	Variables	Distribution
Steel S235 mechanical properties	1	RY/MPa	$N(284.5, 21.5)$
	2	RM/GPa	$N(203, 12.6)$
	3	SY/MPa	$N(284.5, 21.5)$
	4	SM/GPa	$N(203, 12.6)$
	5	PY/MPa	$N(284.5, 21.5)$
	6	PM/GPa	$N(203, 12.6)$
Tolerances of fabrication	7	RT/mm	$N(3, 0.15)$
	8	ST/mm	$N(3, 0.15)$
	9	PT/mm	$N(5, 0.25)$
Soil & Bolts pre-load	10	$SoilM/MPa$	$N(400, 100)$
	11	BP/mm	$N(0.15, 0.05)$

3.3 VRS Model Variables Screening

3.3.1 Two-level screening --- Orthogonal Array

The uncertainties of the 11 factors were studied by two-level screening with OA. The OA and the relative outputs $THIV$ and D are listed in table 2. Columns in OA represent the 11 variables listed in table 1. Every variable takes two values, $0:\mu_k-\sigma_k$ and $1:\mu_k+\sigma_k$ (μ_k : average value of factor k ; σ_k : standard deviation of factor k).

Table 2. VRS OA sampling and simulation outputs

No.	OA	$THIV[km/h]$	$D[m]$
1	1 1 1 1 1 1 1 1 1 1 1	22.4481	1.044
2	1 1 1 1 1 0 0 0 0 0 0	21.2421	1.182
3	1 1 0 0 0 1 1 1 0 0 0	20.3844	1.221
4	1 0 1 0 0 1 0 0 1 1 0	21.5142	1.160
5	1 0 0 1 0 0 1 0 1 0 1	21.4796	1.170
6	1 0 0 0 1 0 0 1 0 1 1	20.2277	1.159
7	0 1 1 0 0 0 1 0 0 1 1	21.5688	1.213
8	0 1 0 1 0 0 0 1 1 1 0	21.5030	1.186
9	0 1 0 0 1 1 0 0 1 0 1	22.9677	1.150
10	0 0 1 1 0 1 0 1 0 0 1	21.5258	1.246
11	0 0 1 0 1 0 1 1 1 0 0	22.1834	1.092
12	0 0 0 1 1 1 1 0 0 1 0	22.3825	1.167

A total number of 12 model runs were realized. Half the values in each column are 0 and that half are 1. $ME_r(Y)$ of each variable on the two outputs $THIV$ and D were calculated and their influences were ranked from the most influential 1 to the least influential 11 according to absolute value of $ME_r(Y)$ in table 3.

Limited by analysis precision, two-level OA screening can only identify qualitatively the influential parameters. The first 4 influential factors for both $THIV$ and D were selected separately and a total number of 6 variables (variables on bold in table 3) out of 11 are considered as influential after this analysis.

Table 3. Main effect for Orthogonal Array screening

Paras	$THIV$		D	
	ME [km/h]	Rank	ME [mm]	Rank
RY	-0.4029	1	-9.8	4
RM	0.1227	8	9.8	4
SY	0.1281	7	-9.7	6
SM	0.1446	6	0	11
PY	0.2896	3	-33.5	1
PM	0.2315	5	-1.2	10
RT	0.1222	9	-14.7	3
ST	-0.2402	4	-7.8	8
PT	0.3971	2	-32.2	2
$SoilM$	-0.0116	11	-9.7	6
BP	0.0840	10	-2.2	9

3.3.2 Multi-level Screening---Morris Analysis

Cumulative Distribution Function (CDF) value of a factor is unitless and can be assumed to be uniformly distributed across the internal $[0, 1]$ regardless of the factor distribution, rather than concentrated in one part of the internal. In the Morris analysis, CDF values of parameters were treated as inputs variables and 4 levels ($1/8, 3/8, 5/8, 7/8$) were taken for each variables with $\Delta=0.5$. The relative factor values were calculated through the inverse transformations. r trajectories with each trajectory corresponds to $(k+1)$ model executions basing on once-at-a-time sampling strategy were selected and a total number of 42 model runs ($r \cdot (k+1)$ model runs with $r=6, k=6$) were realized. And the analytical values were plotted in Fig.4, with the two outputs $THIV$ and D as criteria. ME represent the main effect of factors, and large value of $Inter$ implies significant interaction effects of a parameter. Considering the main effect and interaction effect with both $THIV$ and D as criteria, the three variables PT, PY and RT are of significant influence on VRS performances (see Fig.4).

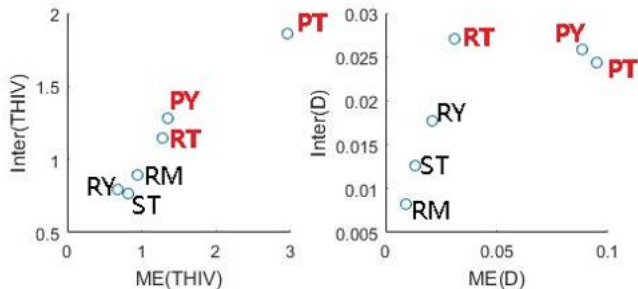


Figure 4 Main Effect (ME) and Interaction Effect (Inter) with both $THIV$ and D as criteria

3.4 VRS Model VBSA---Sobol' indices

3 variables out of 11 were identified as of great influences on VRS performances after the screening analysis. VBSA---Sobol'

indices---was used to quantify the influence of the three variables --- PT, PY and RT . 110 model runs were realized with Latin Hyper Cube sampling and the metamodel was created with Kriging interpolation method [15]. The simulation results were illustrated in Fig.5. Sobol' indices were calculated with the metamodel and plotted in Fig.6.

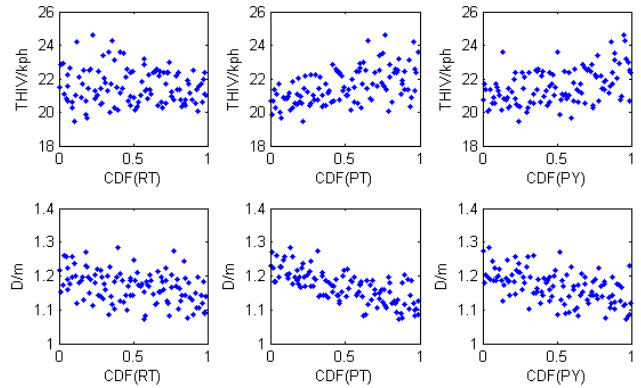


Figure 5 Scatterplots of CDF values of inputs RT, PT, PY and the outputs $THIV, D$

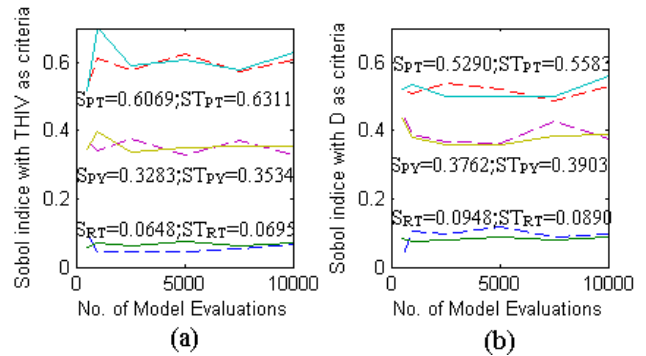


Figure 6 Evolution of the Sobol' indices against sample data size: (a) $THIV$ as criteria and (b) D as criteria (Solid line: total effects of $PT (ST_{PT}), PY (ST_{PY})$ and $RT (ST_{RT})$; Dotted line: main effects of $PT (S_{PT}), PY (S_{PY}), RT (S_{RT})$)

After the quantitative analysis, we can see that among the three influential factors, the variance of post thickness (PT) is the most influential factor for VRS performances (with $S_{PT}=0.6069, ST_{PT}=0.6311$ for $THIV$ and $S_{PT}=0.529, ST_{PT}=0.5583$ for D). Uncertainties of post yield strength (PY) also play a role important for VRS robustness (with $S_{PY}=0.3283, ST_{PY}=0.3534$ for $THIV$ and $S_{PY}=0.3762, ST_{PY}=0.3903$ for D). Relative to the other two factors, influences of rail thickness (RT) are negligible (with $S_{RT}=0.0648, ST_{RT}=0.0695$ for $THIV$ and $S_{RT}=0.0948, ST_{RT}=0.0890$ for D). For all the three variables, their main effects are approximately equal to their total effects, which indicate that there are nearly no interactions effects.

4. Conclusions

VRS must past the crashing test before being installed on the roadside. While numerical uncertainties exist in VRS model parameters and VRS performances can't be evaluated through a single experimental test. A w-beam steel VRS has been tested through a single experimental test. In this paper, the influences of model inputs' uncertainties on model robustness were analyzed with numerical simulations through sensitivity analysis approaches:

11 variables were selected initially among which 3 variables *PY*, *PT*, *RT* were identified as influential after two-level screening with Orthogonal Array design and Morris screening. Metamodel technology, Kriging interpolation, was used to generate low calculation cost model and sensitivities of the selected 3 variables were quantified with VBSA---Sobol' Analysis. 2 of the 3 influential variables were classified as of critical influence on VRS performances---tolerance of Post Thickness (*PT*) and uncertainties of Post material Yield strength (*PY*). The most efficient way to increase model robustness is to decrease the fabrication tolerance of Post Thickness. Another way to increase model robustness is to construct the Post with structure steel fabricated by the same manufacturer under the same fabrication conditions (i.e. decrease Post Yield strength uncertainty).

Sensitivity Analysis can also provide useful information for system structure design. With great influence on model performances, the uncertainties of the two variables *PT* and *PY* are need to be considered in VRS structural design.

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