T -Square Dependence of the Electronic Thermal Resistivity of Metallic Strontium Titanate
Shan Jiang, Benoît Fauqué, Kamran Behnia

To cite this version:

HAL Id: hal-04247339
https://hal.science/hal-04247339
Submitted on 18 Oct 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
The temperature dependence of the phase space for electron-electron (e-e) collisions leads to a T-square contribution to electrical resistivity of metals. Umklapp scattering are identified as the origin of momentum loss due to e-e scattering in dense metals. However, in dilute metals like lightly doped strontium titanate, the origin of T-square electrical resistivity in absence of Umklapp events is yet to be pinned down. Here, by separating electron and phonon contributions to heat transport, we extract the electronic thermal resistivity in niobium-doped strontium titanate and show that it also displays a T-square temperature dependence. Its amplitude correlates with the T-square electrical resistivity. The Wiedemann-Franz law strictly holds in the zero-temperature limit, but not at finite-temperature, because the two T-square prefactors are different by a factor of ≈ 3, in other Fermi liquids. Recalling the case of $^3$He, we argue that T-square thermal resistivity does not require Umklapp events. The approximate recovery of the Wiedemann-Franz law in presence of disorder would account for a T-square electrical resistivity without Umklapp.
scattering [24, 25]. Experiments on various metals, including Ni [26], Al [27], W [28], Sb [29], CeRhIn$_5$ [30], WP$_2$ [31], UPt$_3$ [32] have confirmed both these expectations.

A quantitative connection between this physics and heat transport in the normal liquid $^3$He was recently highlighted [33]. In normal liquid $^3$He, thermal conductivity becomes proportional to the inverse of temperature $\kappa$ at very low temperatures, which means that thermal resistivity $WT = T/\kappa$, is proportional to $T^2$. The evolution of this T-square resistivity with pressure follows the scaling seen for $A$ and $B$ with $E_F$ in metals [33]. There is no Umklapp in normal liquid $^3$He and the Fermi surface is a single sphere. Thus, T-square thermal resistivity can occur in a Fermi liquid without Umklapp and the amplitude of $B$ is directly linked to its Landau parameters, which also set the Fermi temperature.

Here, we present a study of electric and thermal conductivity in SrTi$_{1-x}$Nb$_2$O$_3$ at two different carrier concentrations ($n = 3.1 \times 10^{20} \text{cm}^{-3}$ and $n = 1.8 \times 10^{20} \text{cm}^{-3}$). Despite the dominance of the lattice contribution to the heat transport in strontium titanate [35, 37], we succeeded in extracting the electronic contribution to heat transport by exploiting the differentiating effect of the magnetic field on phonons and electrons. Such a method was employed previously in the case of bismuth and antimony [29, 38, 40]. We found that $WT$ follows $\sigma$ and $L_0 B > A$. Thus T-square resistivity in SrTi$_{1-x}$Nb$_2$O$_3$ cannot be distinguished from other metals in which the e-e origin of the T-square resistivity is uncontested. This leads us to conclude that T-square (electric and thermal) resistivity can be caused without Umklapp as a consequence of the T-square decrease in the amplitude of the (momentum and energy) diffusivity in a Fermi liquid caused by fermion-fermion scattering. A comprehensive theory of this phenomenon is yet to be elaborated.

Fig. 1 shows the temperature dependence of the total thermal conductivity in two samples of SrTi$_{1-x}$Nb$_2$O$_3$. As discussed in the supplementary material [41], the temperature dependence of electrical resistivity in our samples is comparable to what has been reported previously for this doping level and their residual resistivity is close to the lower end of the spectrum. In order to see the relative share of the electronic and the phononic contributions to the total heat transport, $\kappa_{xx}/T$ is compared with $L_0 \sigma_{xx}$, which represents the upper boundary of electronic thermal conductivity according to the WF law. With decreasing temperature, $\kappa_{xx}/T$ approaches $L_0 \sigma_{xx}$. As the temperature tends to zero (see inset), they tend to join each other. $\kappa_{xx}/T$ and $L_0 \sigma_{xx}$ are both modified by the presence of a perpendicular 12 T magnetic field. To quantify longitudinal conductivity in presence of magnetic field, we measured both the electrical and the thermal Hall resistivities and inverted the resistivity tensor.

The field-induced decrease in electrical conductivity $\sigma_{xx}$ (i.e. the magnetoresistance) was the subject matter of a previous study [42], which found that both longitudinal and transverse conductivity follow the behavior expected in the semi-classical picture:

$$\sigma_{xx} = \frac{n e \mu}{1 + \mu^2 B^2}$$

$$\sigma_{xy} = \frac{n e \mu}{1 + \mu^2 B^2} \mu B$$

Here, $n$ is the carrier density, and $e$ is the electron charge. Mobility, $\mu(B, T)$, is the only adjustable parameter depending on temperature and magnetic field. It monotonically decreases with increasing magnetic field and/or temperature. A remarkable (and poorly understood) fact about metallic STO is that the field dependence of mobility shows little dependence on the orientation of the magnetic field [42].

The thermal conductivity tensor $\kappa$, on the other hand has an electronic $\kappa^e$ and a lattice $\kappa^ph$ component in longitudinal: $\kappa_{xx} = \kappa^e_{xx}(B) + \kappa^ph_{xx}$. As in previous studies on semi-metals [29, 38, 40], one can separate the two components by assuming that the field dependence of the lattice thermal conductivity is negligible compared to the field dependence of electronic thermal conductivity. In insulating strontium titanate, thermal conductivity is purely phononic. There, at 24 K, a magnetic field of 12 T reduces $\kappa_{xx}$ at most by $7 \times 10^{-3}$ and generates a finite thermal Hall conductivity of $\kappa^ph_{xy} \approx 0.09 \text{W/K.m}$. In our metallic samples, the effect of magnetic field on $\kappa_{xx}$ and the amplitude of $\kappa_{xy}$ (see below) are orders of magnitude larger.

Fig. 2 shows the temperature dependence of the transverse thermal conductivity divided by temperature $(-\kappa_{xy}/T)$. In the whole temperature range, it remains close (but smaller than $L_0 \sigma_{xy}$), which is what is expected for the electronic part. The measured signal is much larger than $\kappa^ph_{xy}/T$ measured in insulating STO [43, 44].

Thus, we can safely identify the field-induced change in thermal conductivity $\Delta \kappa_{xx}$ with the thermal magnetoresistance of electrons:

$$\Delta \kappa_{xx} = \kappa^e_{xx}(B = 0) - \kappa^e_{xx}(B)$$

Fig. 2 (a) and (c) compare $\Delta \kappa_{xx}/T$ with $L_0 \Delta \sigma_{xx}$. In both samples, these two quantities converge at low temperature and their difference grows with increasing temperature. This implies the validation of the WF law at zero temperature, a departure from it at finite temperature. The finite-temperature departure from the WF law is more significant in the sample with lower carrier density.

In order to extract the longitudinal electronic thermal conductivity at zero magnetic field ($\kappa^e_{xx}(B = 0)$) from thermal magneto-conductivity ($\Delta \kappa_{xx}$), we need an additional assumption: At any given temperature, the field dependence of $\kappa^e_{xx}$, is similar to the field dependence of the electrical conductivity (expressed by Eq. 3). Since the field-induced reduction in conductivity, in both thermal and electrical channels, is due to the same Lorentz
FIG. 1. Thermal conductivity in Nb-doped SrTiO$_3$: Thermal conductivity divided by temperature ($\kappa_{xx}/T$) at B= 0 and B= 12 T compared with the electrical conductivity multiplied by the Sommerfeld value ($L_0\sigma_{xx}$) in sample #1 (a) and in sample #2 (b). $\kappa_{xx}/T$ increases with warming, because of the phonon contribution, which rises faster than $T$. $L_0\sigma_{xx}$, which is a rough estimate of electronic contribution to $\kappa_{xx}/T$ decreases with warming due to the reduction of electrical conductivity by inelastic scattering. Note the reduction induced by magnetic field in both. The inset is a zoom on the low-temperature data, showing that they tend to join in the zero-temperature limit.

FIG. 2. Longitudinal and Hall conductivity: (a) The difference in longitudinal thermal conductivity divided by temperature between zero field and 12 T. ($\Delta\kappa_{xx} = \kappa_{xx}(0T) - \kappa_{xx}(12T)$) in sample #1. Also shown is the difference in the longitudinal electric conductivity multiplied by the Sommerfeld value ($\Delta\sigma_{xx} = \sigma_{xx}(0T) - \sigma_{xx}(12T)$). (c) Same for sample #2. (b) The transverse thermal conductivity $\kappa_{xy}$ divided by temperature, compared with the transverse electric conductivity $\sigma_{xy}$ multiplied by $L_0$ (d) same for sample #2. Also shown in (b) and (d) is the $\kappa_{xy}/T$ caused by phonons in undoped pure STO [37, 43].
force, this is a reasonable assumption. It implies that the Lorenz ratio ($L = \frac{\kappa_e}{\sigma_{xx}}$) has a negligible field dependence. This assumption is consistent with our field-dependent data, which shows at a given temperature $\frac{L}{T}$ is less than unity, but its amplitude does not depend on magnetic field (See Fig. S2 in the supplement [11]). Thus, the magnetoresistance, which is not quadratic in magnetic field is set by the field dependence of residual resistivity and there is no detectable field-induced change in inelastic scattering. This approach leads us to [11]:

$$\kappa_{xx}^e(B = 0) = \Delta \kappa_{xx} - \frac{\sigma_{xx}(B = 0)}{\Delta \sigma_{xx}}$$

Having quantified $\kappa_{xx}^e$ from our data, we can deduce $\kappa_{xx}^{ph}$ by subtracting the electronic component from the total conductivity. Fig. 3 shows the results. One can see in panels (a) and (b) that, above 20 K, $\kappa_{xx}^{ph}$ becomes an order of magnitude larger than $\kappa_e$. However, since $\kappa_{xx}^{ph}$ decreases faster than $\kappa_{xx}^e$, with cooling, the electron contribution becomes prominent below 10 K. It is almost equal to $L_0\sigma_0T$ at low temperature, but becomes significantly lower at higher temperatures.

The extracted $\kappa_{xx}^{ph}$, shown in Fig. 3(c), is significantly lower than $\kappa_{xx}^{ph}$ in undoped STO [10]. As one can see in the figure, $\kappa_{xx}^{ph}$ of our metallic samples, with about 1% of Ti atoms replaced by Nb, is similar to the total $\kappa$ of insulating samples of Sr$_{1-x}$Ca$_x$TiO$_3$, with about 1% of Sr atoms are replaced by Ca. In both cases, $\kappa_{xx}^{ph}$ is reduced in comparison to pristine STO, because the substituting atoms are randomly distributed and their average distance is comparable with the order of magnitude of the wavelength of thermally excited phonons. The rough similarity between Nb doping (which brings mobile electrons) and Ca substitution (which does not), indicates that scattering by mobile electrons plays a minor role.

Let us now turn our attention to the electronic thermal resistivity, $WT$, obtained by inverting $\kappa_{xx}^e/T$. Fig. 4 shows $\rho$ and $L_0WT$ as a function of $T^2$ for the two samples. One can see that in both samples, Equations 1 and 2 hold. $\rho_0$ and $L_0WT_0$ are identical at low temperature confirming the validity of the WF Law in the zero-temperature limit. In both samples, the slope of $L_0WT(T^2)$ (B in equation 2) is larger than the slope of $\rho(T^2)$ (A in equation 2). This behavior is similar to what has been observed in semimetals (like W, WP, and Sb) and heavy-fermions (like UPt$_3$, and CeRhIn$_5$) (See...
in analogy with the case of normal liquid

\[ v_F = \frac{\kappa}{m_B \tau_{\text{N}}} \]  

for, assuming a rough recovery of the Wiedemann-Franz law, one can quantify the Fermi velocity and find \( \tau_{\text{N}} T^2 \). The results are listed in Tab. II. Unsurprisingly, \( \tau_{\text{N}} T^2 \) is orders of magnitude larger in STO than in \( ^3\text{He} \), which has a lower Fermi temperature and higher fermion-fermion collision cross section.

A more instructive basis for comparison is a dimensionless collision cross-section defined as

\[ \zeta = \frac{h E_F}{\tau_{\text{N}} T^2 k_B} \]  

The amplitude of this quantity in a Fermi liquid is set by a combination of its Landau parameters \( \kappa, m_B, \) and phonon scattering. As for T-square electrical resistivity (at low temperatures, that is below the degeneracy temperature of electrons and the minimum energy of the soft TO phonons), it could be accounted for, assuming a rough recovery of the Wiedemann-Franz law in presence of disorder. However, the theory for such a scenario is yet to be elaborated. It may require including the gradient of momentum flow caused by disorder and phonon scattering.

We thank L. Hechler, M. Feigel’man, X. Li, A. Marguerite, D. Maslov, D. Vollhardt P. Wölfle, and Z. Zhu for discussions. This work was supported by the Agence Nationale de la Recherche (ANR-19-CE30-0014-04), by Jeunes Equipes de l’Institut de Physique du Collège de France and by a grant attributed by the Ile de France regional council. S.J. acknowledges a grant from China Scholarship Council.

---

TABLE I: T-square resistivity in metals—Residual resistivity $\rho_0$, and electrical (A) and thermal (B) T-square prefactors and their ratio in several Fermi liquids. In the case of CeCoIn$_5$ [61], the Fermi liquid behavior appears only in presence of a magnetic field larger than the upper critical field of the superconductor. $B/A$ is always found to be larger than unity, varying between 1.5 and 6.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho_0$ (n$\Omega \cdot cm$)</th>
<th>$A$ (n$\Omega \cdot cm \cdot K^{-2}$)</th>
<th>$B$ (n$\Omega \cdot cm \cdot K^{-2}$)</th>
<th>$B/A$</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>(5.66 ± 0.04) $\times 10^{-2}$</td>
<td>(8.7 ± 0.3) $\times 10^{-4}$</td>
<td>(5.3 ± 0.15) $\times 10^{-3}$</td>
<td>6.1 ± 0.4</td>
<td>[28]</td>
</tr>
<tr>
<td>Sb</td>
<td>30</td>
<td>0.3</td>
<td>0.63</td>
<td>2.1</td>
<td>[29]</td>
</tr>
<tr>
<td>WP$_2$</td>
<td>4.7</td>
<td>$1.66 \times 10^{-2}$</td>
<td>7.56 $\times 10^{-2}$</td>
<td>4.55</td>
<td>[31]</td>
</tr>
<tr>
<td>CeRhIn$_5$</td>
<td>37</td>
<td>21</td>
<td>57</td>
<td>2.7</td>
<td>[30]</td>
</tr>
<tr>
<td>CeCoIn$_5$ (B=7 T)</td>
<td>370</td>
<td>2600</td>
<td>5500</td>
<td>2.1</td>
<td>[61]</td>
</tr>
<tr>
<td>CeCoIn$_5$ (B=10 T)</td>
<td>530</td>
<td>900</td>
<td>1900</td>
<td>2.1</td>
<td>[61]</td>
</tr>
<tr>
<td>UPT$_3$ (a-axis)</td>
<td>230</td>
<td>590</td>
<td>905</td>
<td>1.5</td>
<td>[32]</td>
</tr>
<tr>
<td>UPT$_3$ (c-axis)</td>
<td>610</td>
<td>1600</td>
<td>2445</td>
<td>1.5</td>
<td>[32]</td>
</tr>
<tr>
<td>Nb:STO (S#1)</td>
<td>6 $\times 10^4$</td>
<td>33</td>
<td>90 ± 20</td>
<td>2.7 ± 0.6</td>
<td>This work</td>
</tr>
<tr>
<td>Nb:STO (S#2)</td>
<td>4.5 $\times 10^4$</td>
<td>40</td>
<td>185 ± 20</td>
<td>4.6 ± 0.5</td>
<td>This work</td>
</tr>
</tbody>
</table>

TABLE II: Thermal and electronic properties of $^3$He and SrTi$_{3-x}$Nb$_x$O$_5$: Thermal conductivity, fermion-fermion scattering and Fermi liquid properties in a strongly correlated and a weakly correlated fermionic system.

<table>
<thead>
<tr>
<th>System</th>
<th>$\kappa$ \unit{(W.m.K^{-1})}</th>
<th>$v_F$ \unit{(m.s^{-1})}</th>
<th>$k_F$ \unit{(nm^{-1})}</th>
<th>$E_F$ \unit{(K)}</th>
<th>$\tau_T$ \unit{(ns.K^{-1})}</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$He (saturating vapor pressure)</td>
<td>2.9 $\times 10^{-4}$</td>
<td>60</td>
<td>7.9</td>
<td>1.8</td>
<td>3.9 $\times 10^{-4}$</td>
<td>35</td>
</tr>
<tr>
<td>$^3$He (melting pressure)</td>
<td>7.3 $\times 10^{-5}$</td>
<td>32.4</td>
<td>8.9</td>
<td>1.1</td>
<td>1.4 $\times 10^{-4}$</td>
<td>58</td>
</tr>
<tr>
<td>SrTi$_{3-x}$Nb$_x$O$_5$ (n=3.1$\times 10^{20}$cm$^{-3}$)</td>
<td>27 ± 6</td>
<td>$6 \times 10^4$</td>
<td>2.1</td>
<td>480</td>
<td>1.4</td>
<td>2.6 ± 0.6</td>
</tr>
<tr>
<td>SrTi$_{3-x}$Nb$_x$O$_5$ (n=1.8$\times 10^{20}$cm$^{-3}$)</td>
<td>13 ± 1.5</td>
<td>$5 \times 10^4$</td>
<td>1.7</td>
<td>330</td>
<td>0.8</td>
<td>3.2 ± 0.3</td>
</tr>
</tbody>
</table>

In STO samples with a carrier density two orders of magnitude lower ($n \approx 10^{18} \text{cm}^{-3}$), $\kappa_{xy}/T$ is larger than $L \sigma_{xy}$, in contrast with the samples studied here, where the carrier density is two orders of magnitude larger ($n \approx 10^{20} \text{cm}^{-3}$) and the Hall angle is much smaller than unity. In this case, one does not expect to see a detectable phonon drag contribution to $\kappa_{xy}$ (See the supplement [41] for a more detailed discussion).
Supplemental Material for “T-square electron thermal resistivity in metallic strontium titanate”

S1. Materials and methods

SrTi$_{1-x}$Nb$_x$O$_3$ crystals were commercially provided by SurfaceNet GmbH. The nominal Nb content for sample #1 (#2) is 1 wt% (0.5 wt%). The expected carrier concentration for #1 and #2 is $3.3 \times 10^{20} \text{cm}^{-3}$ and $1.7 \times 10^{20} \text{cm}^{-3}$, respectively, in good agreement with the carrier concentration obtained by measuring the Hall coefficient (#1 $n_H = 3.1 \times 10^{20} \text{cm}^{-3}$ and #2 $n_H = 1.8 \times 10^{20} \text{cm}^{-3}$). Fig S1 compares the temperature-dependence of their resistivity with previous data on SrTi$_{1-x}$Nb$_x$O$_3$ [13, 20, 62] and Sr$_{1-x}$La$_x$TiO$_3$ [13] at similar carrier doping levels. Above 100 K, the resistivity of samples with the same carrier concentration is very similar. Below 80 K, they show different residual resistivities. Our samples tend to display a comparatively lower residual resistivity.

FIG. S1. Electrical resistivity ($\rho$): $\rho$ vs. temperature in log scale from 2 K to 300 K. Our samples (#1 and #2) are compared with those with similar carrier concentrations in previous studies of SrTi$_{1-x}$Nb$_x$O$_3$ [20, 62] and Sr$_{1-x}$La$_x$TiO$_3$ [13].

Fig S2 shows a sketch of the setup used to measure the electrical and thermal resistance reported in the main manuscript. Longitudinal ($\rho_{xx} = \frac{E_x}{J_y}$) and transverse resistivity ($\rho_{xy} = \frac{E_y}{J_x}$) are measured in absence of heat current ($J_Q = 0$ and $J^e \neq 0$). Longitudinal ($W_{xx} = \frac{\nabla T_x}{J_y}$) and transverse ($W_{xy} = \frac{\nabla T_y}{J_x}$) thermal resistivity are measured.
in absence of electrical current \((J^e=0\) and \(J^Q \neq 0\)). In the measurement of \(W_{xx}\) the temperature is the average temperature between \(T_1\) and \(T_2\). In the measurement of \(W_{xy}\), the average temperature is taken between \(T_1\) and \(T_3\). The (electric or thermal) conductivity tensor is the inverse of the (electric or thermal) resistivity tensor. Hence the longitudinal and transverse electric and thermal conductivity are respectively equal to \(\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}\), \(\sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}\) and \(\kappa_{xx} = \frac{W_{xx}}{W_{xx}^2 + W_{xy}^2}\), \(\kappa_{xy} = \frac{-W_{xy}}{W_{xx}^2 + W_{xy}^2}\).

### S2. Field independence of the Lorenz ratio

Fig S3 shows the field-dependent electric and thermal conductivity in sample \#1. The variation of thermal and electric conductivity caused by the magnetic field are similar. This is true for both the longitudinal and transverse response. The Lorentz force is thus the main source for the variation of thermal conductivity. The field-dependent Lorenz ratio in longitudinal and transverse are shown in Fig S3 (c) and (f). The finite temperature departure of the Wiedemann Franz law does not change with the magnetic field. This confirms our assumption of a field-independent Lorenz number.

### S3. Absence of phonon drag thermal Hall effect

Fig S4 shows the Seebeck effect of sample \#2 compared with a SrTiO\(_{3-x}\) sample with a carrier density two orders of magnitude lower \(n = 1.6 \times 10^{18} \text{cm}^{-3}\). While a phonon drag peak is found in the low-doped material it is absent in sample \#2. Moreover samples \#1 and \#2 have a much smaller Hall angle compared with low doped SrTiO\(_{3-\sigma}\) \(n = 1 \times 10^{18} \text{cm}^{-3}\). Therefore, no phonon drag contribution to \(\kappa_{xy}\) is expected to arise in sample \#2 and also \#1 [37].

### S4. Extracting electronic thermal conductivity from thermal and electrical magnetoresistance

Two assumptions have been done to extract the temperature dependence of the electronic thermal conductivity. First we assume that the phonon contribution is independent of the magnetic field. The change in the total conductivity \((\kappa_{xx})\) is thus equal to the change in field of the electronic thermal conductivity :

\[
\kappa_{xx}(B) = \kappa_{xx}^e(B) + \kappa_{xx}^{ph}
\]  

(S1)

Second we assume that that the Lorenz number \((L = \frac{\kappa_{xx}^e}{\sigma_{xx} T})\) is independent of the magnetic field. This assumption is backed by our data (See Fig. S3c) in section S2). This implies that :

\[
L = \frac{\Delta \kappa_{xx} T}{\Delta \sigma_{xx}}
\]  

(S2)

where \(\Delta \kappa_{xx} = \kappa_{xx}(0) - \kappa_{xx}(B)\) and \(\Delta \sigma_{xx} = \sigma_{xx}(0) - \sigma_{xx}(B)\). Using Eq S1 and S2 we can extract the electron component of the thermal conductivity at zero magnetic field :

\[
\kappa_{xx}^e(0) = \frac{\Delta \kappa_{xx}}{\Delta \sigma_{xx}} \cdot \sigma_{xx}(0)
\]  

(S3)
FIG. S3. Field dependence of the electric and thermal conductivity: (a) Longitudinal thermal conductivity ($\Delta \kappa_{xx} = \kappa_{xx}(0) - \kappa_{xx}(B)$) divided by temperature, (b) Longitudinal electric conductivity ($\Delta \sigma_{xx} = \sigma_{xx}(0) - \sigma_{xx}(B)$) and (c) Longitudinal Lorenz ratio ($L_{xx}/L_0 = \Delta \kappa_{xx}/T L_0 \Delta \sigma_{xx}$) as a function of the magnetic field for three temperatures. (d), (e) and (f) are the same as (a), (b), and (c) in transverse configuration. The Lorenz ratios $L_{xx}$ and $L_{xy}$ are constant in field but smaller than $L_0$.

FIG. S4. Seebeck effect and Hall angle: (a) Seebeck coefficient ($S$), (b) Hall angle as a function of temperature for sample #2 ($n=1.8e20$ cm$^{-3}$) and in a lightly doped SrTiO$_{3-x}$ ($n=1.6e18$ cm$^{-3}$). No phonon drag peak is observed in sample #2. The Hall angle in sample #1 is much lower than in lightly doped SrTiO$_{3-x}$. 