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A state-space approach for remaining useful life control

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Abstract: This paper presents a state-space approach for controlling the Remaining Useful Life (RUL) of deteriorating systems. The proposed approach supposes the availability of a deterioration model that links the deterioration rate with some manipulable control inputs. Such a link can be a non-linear monotonic function between the deterioration-rate and the manipulable control inputs. We propose a method for designing both state observer and state feedback controller for RUL prediction and control. We consider that the manipulable inputs are affected by additive random disturbances and possible multiplicative unknown parameters. In addition, we assume that the control decisions can not be applied instantaneously on the system, which is modeled as a time-delay between control decisions and manipulable inputs of the deterioration dynamics. The proposed methodology is illustrated on a simulated study case which considers an exponential degradation model.

Keywords: Health-aware control, deterioration-aware control, RUL control, deterioration estimation, prognosis, reliability adaptive systems, state-space approach.

1. INTRODUCTION

Mechanical systems deteriorate during use due to wear and tear, stress, strain, friction, etc. leading to damage, fatigue and eventual equipment failure, affecting the system's reliability. For example, a friction drive system deteriorates as friction stresses and damages the actuator over time. Within this context, adapted control strategies can contribute to ensure the reliability of such degrading systems and guarantee that the controlled system remains functional and keeps operating in the same way throughout its life despite possible failures or changes in its dynamics due to degradation.

According to Rausand et al. (2021), the Remaining Useful Life (RUL) of a system is modeled as a random variable that expresses the time until the system is no longer functional, and its distribution can be used as a reliability metric. If a degradation model is available and if the degradation dynamics can be changed through the control inputs, adaptive control can be used to control the RUL of the system and influence the system reliability, as proposed in Rodriguez Obando et al. (2021). This control strategy, referred to as Health-Aware Control (HAC) in Do and Söffker (2021), proposes to incorporate health state information, such as prognostic and degradation states, into the control loop using available dynamic knowledge about the degradation. In the case where the degradation is influenced by operational decisions and control inputs, the system can be reconfigured to control the potential stress and wear factors that degrade the system. According to Do and Söffker (2020), this type of control can be used in Prognostic and Health Management (PHM) approaches to integrate maintenance requirements into global control objectives and to trade-off load mitigation and control performance.

Many works address this problem, referred here to as RUL Control, as a multi-objective control problem. With an available degradation model, it is possible to model an optimal control solution to track the desired RUL. For example, Meyer et al. (2013) has proposed modeling RUL control to incorporate RUL into the control objectives of a driven system. Gokdere et al. (2005) proposed a model to extend the lifetime of electromechanical actuators. Also, in a more recent work, Sanchez et al. (2015) proposed a Model Predictive Control (MPC) method to control the RUL of a wind turbine system to meet maintenance requirements while satisfying other control performance requirements. In addition, Rodriguez Obando et al. (2018) has proposed an architecture to incorporate the RUL requirement into the model to extend the lifetime of a friction drive system.

However, most works propose RUL control methods that model RUL as a deterministic variable. Only some works, such as Langeron et al. (2015), Mo et al. (2018) and Pedrosa Alias et al. (2022), proposed methods that establish a relationship between control and degradation by considering RUL as a random variable and using knowledge of the stochastic dynamics of degradation.

In this paper, we propose to solve the RUL control tracking problem following a state-space control approach. We also consider the stochastic dynamics of the degradation and possible delay in the decision within the relationship between control actions on the system and the degradation rate.

This paper is organized as follows: Section 2 discusses the definition of the problem, then Section 3 describes the deterioration model with the corresponding dynamic states. Next, in Section 4, the stated problem is solved

by proposing a state observer to estimate the states of the chosen deterioration process and a state-feedback controller. Finally, a study case which illustrates the proposed approach is presented in Section 5.

2. PROBLEM STATEMENT

The RUL of a system is defined as the time until it can no longer be used to meet the desired performance, i.e., the remaining time before the system reaches a maximum allowable deterioration, denoted here as D_{max} . Therefore, for a given current time t , the RUL verifies the following equality:

$$D(t + RUL) = D_{max} \quad (1)$$

Let us assume that the deterioration dynamics obeys the following differential equation:

$$\frac{dD}{dt} = \beta_1 D + \beta_0 \quad (2)$$

where the parameters β_1 and β_0 dictate the deterioration-rate. If β_1 and β_0 are random (e.g., white noise for β_0), the deterioration D is then a stochastic process determined by a Stochastic Differential Equation (SDE) resulting from (2), and particular choices of β_0 and β_1 allow this equation to represent well-known deterioration process dynamics such as Wiener or Gamma processes, often used in reliability and maintenance studies, see for instance Rausand et al. (2021).

2.1 Link function definition

Moreover, we assume that the median values of parameters β_1 and β_0 can be governed by a certain number of inputs w that we can control, and the vector of parameters $\beta = [\beta_0 \ \beta_1]^T$ can be written as a function of the input w . In particular, we can model the behavior of β as a function of w by using a monotonic nonlinear function $f(w)$ with possible additive perturbations η and uncertain multiplicative parameters γ as follows:

$$\beta = \gamma f(w) + \eta \quad (3)$$

Equation (3) models the so-called link function. In particular, the function $f(w)$ is based on the link between controllable process variables w and the degradation rate β . Of course, this relationship depends on the complexity of system dynamics, which may be nonlinear or not straightforward. However, instead of using the full dynamics of the process states to have an explicit link function, we assume that a metamodel is available. The form presented in (3) can therefore be seen as an approximation of the degradation rate dynamics with stochastic behavior. Nevertheless, the unmodeled parts of this dynamic are accounted for by the uncertainty parameters γ and η , which contain the random influence of different operating points and exogenous inputs on the degradation rate.

In this paper, we make the simplifying assumption that w can be directly controlled, and we propose to act on β by making decisions on the value of w , denoted by w^d . Of course, in practice, the considered deterioration would take place in a controlled system, and it has to be noted that w may be different from the control inputs to this deteriorating system, usually expressed by u , whose purpose is to make the system work according to a reference or set point. For example, w could be a state variable of

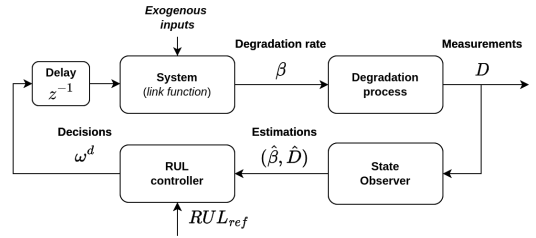


Fig. 1. Proposed RUL control architecture.

the deteriorating system, that could be controlled by u . We also consider that the decisions w^d are not perfectly executed at a given time t and may contain delays t_d . Thus, the behavior of β is described by the following time-delay relationship:

$$\beta(t + t_d) = \gamma f(w^d(t)) + \eta \quad (4)$$

2.2 RUL control definition

Now, the RUL control problem can be stated as follows: at every time instant t , for a given desired median value of the RUL, denoted $RUL^{ref}(t)$, decide the values of w^d to modify the predicted RUL, denoted $RUL(t)$, in such a way that the deterioration D , starting at $D(t)$, achieves its maximal and admissible value D_{max} at time $t + RUL^{ref}$, respecting both the deterioration dynamics (2) and the deterioration-rate parameter dynamics (4).

Remark that the desired $RUL^{ref}(t)$ can be chosen according to maintenance constraints or operational requirements for the given deteriorating system.

In order to control the RUL, we also assume that β and D can be estimated by using an available dynamical model of deterioration and the available data coming from noisy deterioration measurements or imperfect estimates. Thus, we propose a state observer that can estimate both $\hat{\beta}$ and \hat{D} , which allows us to continuously compute RUL control decisions w^d . The final proposed control architecture is depicted in Fig. 1.

In the following sections, we propose to solve the control and observer design problems using a state space approach. We will show that, based on a given link function model and a chosen degradation model, we can control degradation by making decisions on w_d to impose properties on the lifetime distribution, such as, e.g., its median value.

3. DETERIORATION MODELS

According to the nature of the deterioration behavior and to model the item-to-item variability and heterogeneity of deterioration paths, the parameters β can be considered as random variables that follow a particular *a priori* probability distribution, see Gebraeel et al. (2005). Here we will focus on deterioration models for which the parameter β follows a normal distribution, i.e., $\beta \sim \mathcal{N}(\mu, \sigma^2)$, as described in Rausand et al. (2021).

In doing so, we consider the evolution of deterioration as a stochastic process with random effects. For the control design, it is more interesting to model it as an SDE-

like equation (2) and to consider a “median” solution of Equation (2) described by:

$$\bar{D}(t) = D(0)e^{\mu_1 t} + \mu_0 t \quad (5)$$

where μ_1 and μ_0 are the median values of β_0 and β_1 , as previously defined as β , and with an initial deterioration state $D(0)$.

Equation (5) can also be expressed in terms of RUL:

$$\bar{D}(t + RUL(t)) = D(t)e^{\beta_1 RUL(t)} + \beta_0 RUL(t) \quad (6)$$

where $RUL(t)$ is the interval time between the time instant t and the time when the degradation first reaches a maximum degradation $\bar{D}(t + RUL(t)) = D_{max}$.

Finally, based on (6), for a given D_{max} , the value of $RUL(t)$ can be determined as a function of β and D , variables that can be estimated. However, the solution would require a numerical procedure. Therefore, we will proceed with the linear and exponential deterioration models, each separately, to continue the investigation with an analytical solution.

3.1 Linear model

Used to model mechanisms of wear from surface contact Archard (1953), the wear rate \dot{D} is a constant β_0 proportional to dynamical variables such as the load N and the speed of sliding of contact \tilde{v} :

$$\dot{D} = \beta_0(N, \tilde{v}) \quad (7)$$

This model represents a linear behavior of the deterioration process that obeys approximately:

$$D(t) = D(0) + \beta_0 \cdot t \quad (8)$$

with an initial deterioration state $D(0)$ and a random parameter β_0 .

Based on (8), we then consider a constant value for β_0 to predict when the deterioration at a given time t will reach its maximum D_{max} in $D(t + RUL(t))$, as follows:

$$D_{max} = D(t) + \beta_0 \cdot RUL(t) \quad (9)$$

And therefore, $RUL(t)$ at the given time t can be written as a function of the chosen value of β_0 for the prediction and the current state of deterioration $D(t)$ as follows:

$$RUL(t) = \frac{1}{\beta_0}(D_{max} - D(t)) \quad (10)$$

3.2 Exponential model

Let us now consider degradation with exponential behavior, which is common in processes where degradation occurs in a cumulative manner (e.g., wear, fatigue), especially in bearing applications:

$$\dot{D} = \beta_1 D \quad (11)$$

It will approximately follow an exponential trend:

$$D(t) = D(0)e^{\beta_1 t} \quad (12)$$

with a random degradation-rate β_1 . If we consider now a constant value of degradation-rate for β_1 to predict when the degradation reaches D_{max} , then we can write $RUL(t)$ at time t as:

$$RUL(t) = \frac{1}{\beta_1} \ln \left(\frac{D_{max}}{D(t)} \right) \quad (13)$$

Note that if we consider that β obeys a given probability distribution function $f(\cdot)$, then the predicted RUL(t) will obey its inverse one.

Now we are ready to introduce the state-space approach to observe the states of the deterioration dynamics (β and $D(t)$) as the system deteriorates and then control the RUL. The following section presents the design of a state-observer and a state-feedback controller to be implemented within the proposed RUL control architecture. For the sake of simplicity, we will focus on the exponential deterioration model and a given link function.

4. RUL CONTROL DESIGN

4.1 State-observer problem

The contribution of this paper is to present a method for synthesizing a RUL controller to impose some desired characteristics to the RUL distribution law (e.g. its median value) by modifying w_d . In the proposed framework (see Figure (1)), the RUL controller finds the optimal corrections w_d based on the current estimated degradation and observations of the degradation rate.

Let us consider a deteriorating system that follows an exponential behavior (11). The problem of the state-observer is then to estimate the parameters β and $D(t)$.

Note that β is considered as a random variable influenced by w with a relation described in (3). Nevertheless, the model (3) cannot be used to design the observer here, since this model is treated as uncertain and may have a complex form inherent to each application. Therefore, we propose a more general equation:

$$\dot{\beta} = -c\beta + \epsilon \quad (14)$$

based on the Langevin equation to estimate the deterioration rate parameter β , with a model parameter c and a noise term ϵ . Although the Equation (14) dynamics is not determined by w , it allows us to represent the fluctuating behavior of β while suffering from state-observer corrections.

Finally, D can be measured or calculated from health condition indicators. In this case, the model can consider (11) and (14), leading to the following observer:

$$\dot{\hat{x}} = \begin{bmatrix} 0 & D \\ 0 & -c \end{bmatrix} \hat{x} + K(y - \hat{y}) \quad (15)$$

where $x := [D \ \beta]^T$ and the outputs are simply $y = D + \epsilon_m$. Here, ϵ_m denotes a possible measurement error or noise. Then, the observer gain K can be obtained as a solution of an Extended Kalman Filter problem, for instance.

4.2 Control problem

For the control problem, we assume that the parameter β follows a monotonic uncertain behavior wrt w , which allows us to write β as the following link function:

$$\beta = \gamma w + \eta \quad (16)$$

with an uncertain parameter $\gamma > 0$, a normal noise η , and one-step delay on decisions, defined as w_d .

Thus, by writing the model dynamics in a discrete-time representation, we have:

$$w_{k+1} = w_k^d \quad (17)$$

$$\beta_k = \gamma \cdot w_k + \eta_k \quad (18)$$

Hence, the problem of tracking a desired median value for RUL, RUL^{ref} , can be rewritten into the problem of guaranteeing that β tracks a desired reference value β^{ref} that can be obtained from the following reference generator:

$$\beta_k^{ref} = \frac{1}{RUL_k^{ref}} \ln \left(\frac{D_{max}}{\hat{D}_k} \right) \quad (19)$$

which is based on (13), where β is written in terms of RUL and D is substituted by its estimate \hat{D} .

Remark that RUL_k^{ref} can be calculated, at each iteration k , given a desired End of Life (EoL), EoL^{ref} , since

$$RUL_k^{ref} = EoL^{ref} - k. \quad (20)$$

In order to minimize the tracking errors on β , the control design should include an integral action, that can be implemented as follows:

$$z_{k+1} = z_k + (\hat{\beta}_k - \beta_k^{ref}) \quad (21)$$

where $\hat{\beta}_k$ is the estimated value of β_k , which is used instead of the real one, because we assume that this value can not be measured directly from the process.

Then, the control design considers the models in (17) and (21) written in terms of a state vector defined as $x := [w \ z]^T$. That is, find a state feedback control w_k^d which stabilises the following state-space model:

$$\begin{bmatrix} w_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \gamma & 1 \end{bmatrix} \begin{bmatrix} w_k \\ z_k \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_k^d + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \beta_k^{ref} \quad (22)$$

for $0 < \gamma_{min} \leq \gamma \leq \gamma_{max}$.

Therefore, the state feedback control law will be given by:

$$w_k^d = -K_p \cdot w_k - K_i \cdot z_k \quad (23)$$

where the control gain parameters can be obtained by using robust control design techniques, for instance. Remark that in practice the value of w_k can be obtained by using (17), and then, the control law becomes:

$$w_k^d = -K_p \cdot w_{k-1}^d - K_i \cdot z_k \quad (24)$$

Thus, the RUL controller can be implemented by using (19), (21) and (24), as it is depicted in Fig. 2. Remark that the same approach can be applied for controlling a linear degradation dynamics. In that case, we only need to generate the references on β_0 from (10). In all cases, estimations of D and parameters β are necessary for implementing the proposed RUL control law in (24). These estimations come from an observer stated previously.

Finally, the values of K_p and K_i in (24) can be found with any optimal controller, such as H_∞ , Robust Control, Linear-Quadratic Regulator (LQR), etc. In this paper, we solve the study case controller design using a LQR.

5. STUDY CASE

In this section, the proposed approach is illustrated using an exponential deterioration model.

5.1 Description of the system

For this study case, we consider an exponential deterioration model, as described in (11), and we assume that this deterioration is measured. That is,

$$y_k = D_k + \eta_{noise} \quad (25)$$

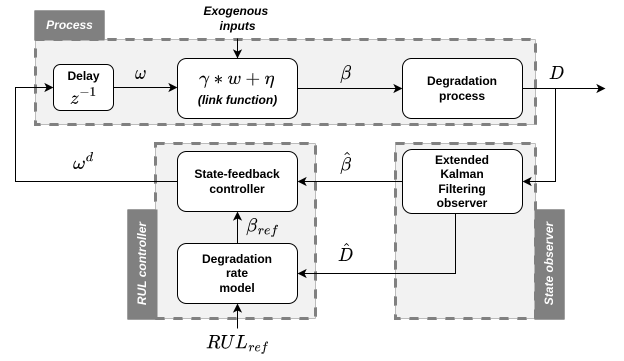


Fig. 2. Complete proposed RUL controller algorithm.

where η_{noise} is a white noise of 10^{-1} amplitude.

Thus, the state-space representation of the deterioration dynamics can be written as follows:

$$\dot{x} = H(x) \quad (26)$$

$$y = Cx + \eta_{noise} \quad (27)$$

where

$$H(x) = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}, \quad (28)$$

$C = [1 \ 0]$ and $x := [D \ \beta]^T$.

Now, by considering a sampling time t_s to be exactly equal to the actuator time-delay, the exponential model of deterioration (11) has a discrete-time representation:

$$D_{k+1} = D_k + t_s \beta_k D_k \quad (29)$$

where we will consider $D_0 = 1$ because D starts with a minimum deterioration which has to be a strictly positive measure.

Discrete-time observer — Here the chosen observer will estimate the states $\hat{x} = [\hat{D} \ \hat{\beta}]$ using an Extended Kalman Filter, as proposed in Rodriguez Obando et al. (2021). We consider a initialization of the estimates as follows:

$$\hat{x}_0 = [\hat{D}_0 \ 0.1]^T \quad (30)$$

with the initial estimate of deterioration $\hat{D}_0 = 1$.

The EKF also requires a choice of the covariance matrices. Then, we consider:

$$R = 0.01 \quad (31)$$

which assumes that the sensor noise variances are known and equal to 0.01. The matrix Q has been selected to consider innovations of the system with the following variance:

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 2.5 \cdot 10^{-5} \end{bmatrix} \quad (32)$$

Finally, an initial matrix $P_{0|0}$ was chosen as follows:

$$P_{0|0} = \begin{bmatrix} 100 & 0 \\ 0 & 2.5 \cdot 10^{-3} \end{bmatrix} \quad (33)$$

Study case degradation parameters — The chosen link function for this example is described in (16). Without control intervention, β should vary randomly depending on probability distribution of random parameters η and γ .

In order to achieve maximum deterioration at around a EoL equal to $k = 100$ (in units of time), the distribution

Table 1. Control effect on the distribution of EoL for different cases of γ .

System in	EoL for $\gamma = 1.2$		EoL for $\gamma \sim \mathcal{U}(0.7, 1.3)$	
	median	std	median	std
Open-loop	86	1.66	103	18.60
Closed-loop	100	0.63	100	0.79

of random parameters follows $\eta \sim \mathcal{N}(0, 0.01)$, and $\gamma \sim \mathcal{U}(0.7, 1.3)$.

RUL controller parameters — For this example, we establish an admissible maximum deterioration $D_{max} = 100$ where it should be reached at $EoL^{ref} = 100$ (units of time). Thus, by considering (20), a value for β_{ref} is calculated from (19). Finally, the control gain parameters are calculated using the LQR-based control synthesis, which leads to

$$K_p = 0.088, K_i = 0.955. \quad (34)$$

According to (24), we calculated the initial value for the integral action as:

$$z_0 = \frac{w_0^d + K_p w_0^d}{-K_i}. \quad (35)$$

where we will choose the initial decision parameter to be $w_0^d = 1$. This value concerns the nominal one, and then we suppose that ideally w_0 has to remain around this value for achieving the desired EoL^{ref} .

5.2 Stochastic results for 10^4 realisations

This case study aims to show that the proposed state-space feedback controller can control deterioration so that it reaches its maximum at the desired lifetime. Two applications are proposed: the first one, concerning one possible realisation of the link function, which depends on an additive noise η and a multiplicative unknown but fixed parameter γ , and the second one, intended for illustrating the case where γ may vary randomly from one realization to another.

The results for 10^4 realisations are shown in Fig. 3. In the first application, it is chosen a fixed $\gamma = 1.2$, which physically represents 20% more load than the nominal one. The results shows that the control actions dislocated the distribution of EoL, which it was around 86th cycle, towards the desired one, 100th cycle. In the second case, the system has a random γ , which physically represents random loads uniformly distributed around the the nominal one. In this case, the realizations without RUL control exhibit an EoL distribution with higher variance and a non-normal distribution. In contrast, when adding the RUL control actions, the EoL of these realisations remains closer around the desired one. Table 1 resumes the obtained distributions of EoL resulted from both cases: constant $\gamma = 1.2$ and random $\gamma \sim \mathcal{U}(0.7, 1.3)$.

5.3 RUL-control evaluation for 1 realisation

Now we are interesting in the evaluation of the response of the control system for one realisation. For this purpose, Figs. 4 and 5 show the results for the realisation $i = 42$.

The results in Fig. 4a show the assessment of RUL, represented by \hat{RUL} and estimated using (13), comparing

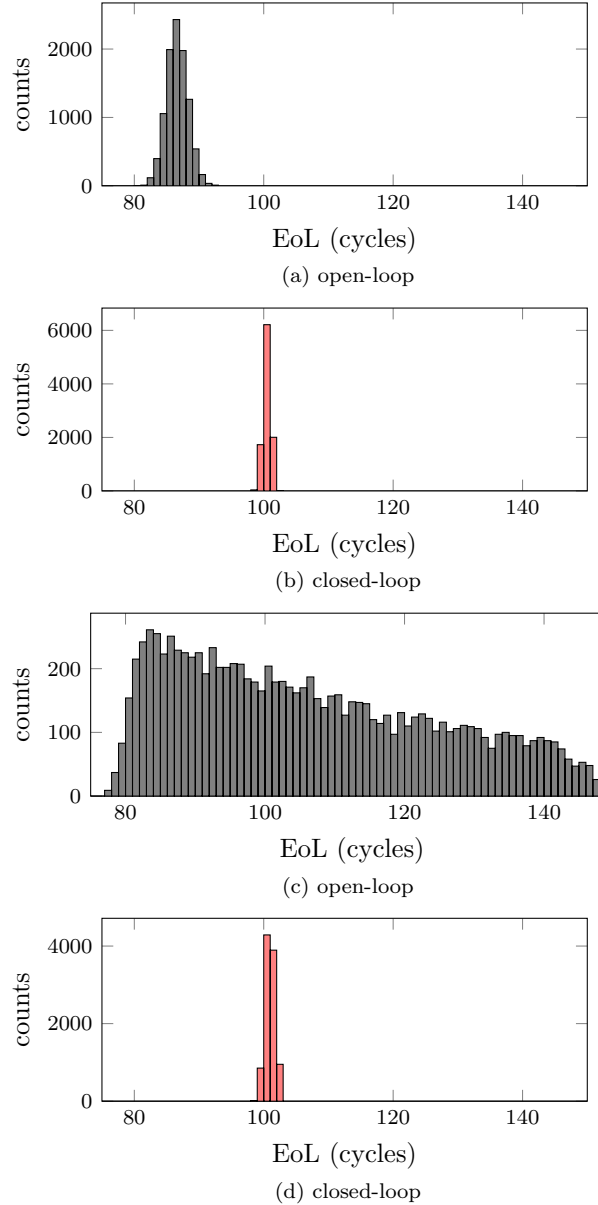


Fig. 3. Distribution of obtained EoL (when $D(t) = D_{max}$) for a (a) open-loop and (b) closed-loop with a constant $\gamma = 1.2$, and for a (c) open-loop and (d) closed-loop with a unknown random $\gamma \sim \mathcal{U}(0.7, 1.3)$

to the desired one RUL^{ref} . RUL resulted tracks the reference one thanks to the regulation of β , shown in Fig. 5a. The decisions in Fig. 4b are computed to control $\beta(k)$ based on proposed control and estimated states (\hat{D} and $\hat{\beta}$) shown in Fig. 5. In the first 40 iterations, the decision resulted is lower than the nominal value (black dashed line), which leads to a RUL greater than the desired one. After, we remark an important change on decisions to compensate and minimizing the tracking error $e(k) = RUL(k) - RUL^{ref}$, which results in a greater degradation-rate in Fig (5a).

We remark also, from Fig. 5a, that β follows the trend β_{ref} calculated as in (19). Finally, the deterioration, shown in 5b, obeys an exponential model. With the control proposed, we ensures that it reaches D_{max} at the desired EoL.

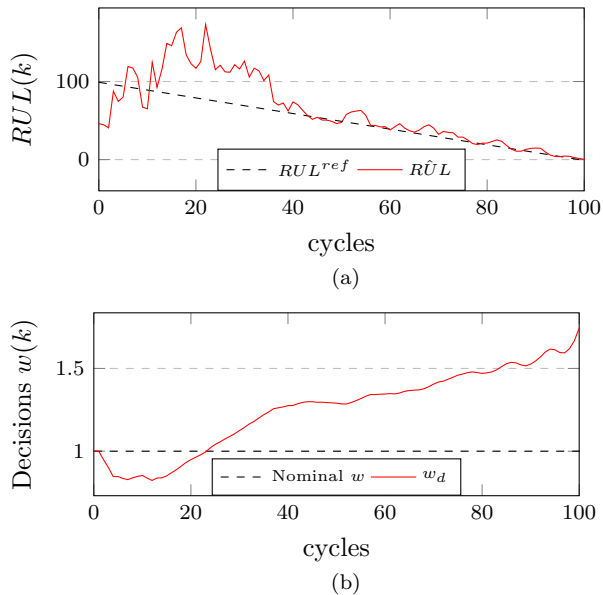


Fig. 4. Response of estimated RUL (a) in function of calculated decisions w^d (b).

6. CONCLUSION

In this paper, we have presented a state-space approach for controlling the Remaining Useful Life (RUL) of systems subject to degradation. The proposed method is based on the fact that the degradation-rate can be modeled in a state-space representation. Even if the systems, for observation and control, are linear-time varying, LPV or uncertain linear systems, this paper shows that the control design can be performed using well-known robust control techniques. This paper illustrates the approach on exponential degradation model but other degradation-rate model can be considered if they can be written under a state-space form. In cases where the degradation trends are linear or exponential, RUL predictions can be easily obtained by using explicit equations that are functions of both the current deterioration-rate and the current deterioration level. In addition, such relationships allow us to explicitly write the desired RUL in terms of the desired deterioration-rate. As it is illustrated in the presented study case, the RUL control problem can be efficiently solved by keeping state-space models expressed on the deterioration-rate space. This aspect allows us to solve the problem as a deterioration-rate tracking control problem. The solution was validated using a given link function expressing the system dynamics. Future work includes testing the presented framework for practical cases with a given physical system that can be expressed as a special case of the link function.

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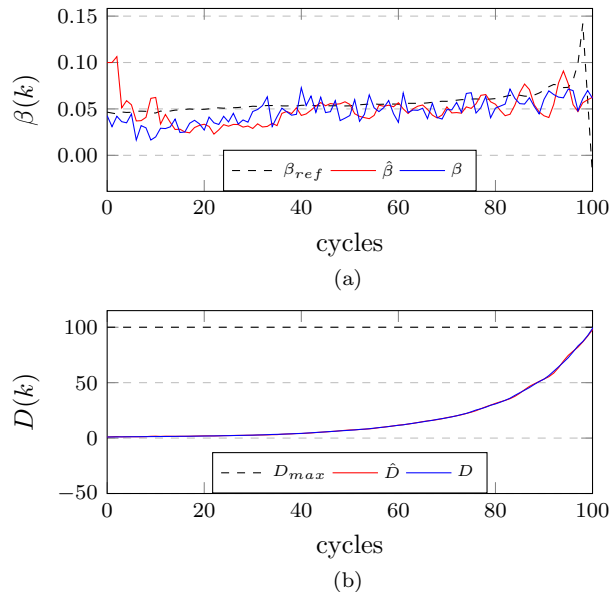


Fig. 5. Estimated and measured system states deterioration-rate β (a) resulting a deterioration D (b).

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