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Investigating gas furnace control practices with reinforcement learning

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Abstract

Gas furnaces are the most widely used means of heating in industry, and with the growing concern for environmental issues, and a global energy crisis at our doorstep, the optimization of the processes related to them becomes a key challenge. This paper aims at introducing a new way of practicing gas furnace control involving simulations, virtual sensors and deep reinforcement learning (DRL) techniques. In order to do so we designed a set of simulations of conjugate heat transfer systems governed by the coupled Navier–Stokes and heat equations for single-step control. The DRL algorithm used in this paper is the policy-based optimization (PBO) algorithm specialized in singlestep (or open-loop) control. We explore its ability to find global maxima in different situations and under various constraints. Therefore, various 2D and 3D cases are tackled, in which the position of the work piece, the flow rate, and other parameters are controlled. The obtained results highlight the potential of the DRL framework combined with computational fluid dynamics (CFD) conjugate heat transfer systems for optimizing searches in large parameter spaces. For the 2D case, PPO achieved an increase of 89% in temperature homogeneity, and for the 3D case an increase of 7% in final temperature with the same total input.

Keywords: Deep Reinforcement Learning; Artificial Neural Networks; Conjugate heat transfer; Computational fluid dynamics; Thermal control; Serpentine.

1. Introduction

Just like cooling, properly heating a part is a challenge that manufacturers take up every day to obtain the desired properties in the part, whether they are mechanical, electrical, optical or aesthetic. To achieve that, gas furnaces have been studied for a long time and their control has evolved to arrive today at the cohabitation of many techniques, including manual control, fuzzy control [1], proportional-integral (PI)/proportional-integral-derivative (PID) single-loop control or cascade control [2], 3] among others. The reason for this flourishing of techniques is the difficulty to accurately perform temperature control, whether it is because of the difficulty to measure the temperature accurately inside the chamber, or because of the different characteristics of temperature variation through flow control such as non linearity, inertia and time delay.

Temperature control has been addressed by a lot of articles in the literature, and with the 11 emergence of Artificial Neural Networks (ANNs) taking advantage of the most recent advances in 12 computational power and data analysis, new control techniques that are more robust and can deal 13 with non linearity, inertia and time delay have been developed : image processing and clustering 14 to control gas burners 4.5, recurrent neural networks (RNNs) with electrical furnaces 6.7, 15 neural networks for metal quality control 8 and for flame stability control 9-11, and radial 16 basis function neural networks (RBFNNs) to control coke furnaces 12. In the aerospace industry, 17 burners are also very thoroughly studied to prevent any incident inside aircraft engines. They use 18 neural networks in the active control of the burning chambers and find ways to prevent oscillating 19 patterns to occur during combustion 13. Such patterns are generally responsible for a premature 20

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fatigue of the materials composing the chamber, as well as disturbances to the good functioning of the combustion. This is also true for industrial gas furnaces; even though these issues are less explored in the field of furnace control, they become more and more important with the use of mixed fuels, and the alternating use of natural gas and dihydrogen.

In the realm of optimal control with constraints on the state of the system, the combination of 25 deep neural networks (DNNs) and reinforcement learning (RL) algorithms (a formal framework in 26 which an agent learns by interacting with an environment and learns by gathering experience) has 27 brought new cards to the table in such a way that the best performing algorithms in a wide variety 28 of tasks (e.g., games 14, 15, cooling control 16, autonomous cars 17, medicine 18, energy 29 19, 20). In fluid dynamics, this so-called deep reinforcement learning (deep RL, or DRL) has also 30 been used with success for flow control and shape optimization with success (Refs. 21, 22 and the 31 references therein) by taking advantage of its robustness to non linearity and to high dimensional 32 spaces. This field of application is still at an early stage, as evidenced by the scarce literature 33 dedicated to DRL-based thermal control 23, 24, but it shows great promise for the future of fluid 34 related topics. 35

This work aims at introducing DRL into the field of gas furnace control. More specifically, it 36 assesses the performances of proximal policy optimization (PPO [25]) for the one-step control of a 37 heating chamber. We use an algorithm introduced in 26 and whose relevance for open-loop flow 38 control problems is assessed in **27** that is a degenerate version of the classical PPO algorithm. The 39 choice for PPO is driven by its data efficiency (a decisive criteria for computationally expensive 40 simulations), ease of implementation and already widely assessed performance. Several problems 41 of conjugate heat transfer in two and three dimensions are used as testbed to push forward the 42 development of this novel approach. To the best of the authors knowledge, this constitutes the 43 first attempt to achieve DRL-based control of conjugate forced convection heating processes. 44

45 2. Governing equations for fluid mechanics

The focus of this research is on conjugate heat transfer and laminar, incompressible fluid flow problems in two and three-dimensions, for which the conservation of mass, momentum and energy is described by the nonlinear, coupled Navier–Stokes and heat equations

$$\nabla \cdot \boldsymbol{u} = 0, \qquad (1)$$

$$\rho(\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}) = \nabla \cdot (-p\mathbf{I} + 2\mu\boldsymbol{\varepsilon}(\boldsymbol{u})), \qquad (2)$$

$$\rho c_p(\partial_t T + \boldsymbol{u} \cdot \nabla T) = \nabla \cdot (\lambda \nabla T), \qquad (3)$$

⁴⁹ where \boldsymbol{u} is the velocity field, p is the pressure, T is the temperature, $\boldsymbol{\varepsilon}(\boldsymbol{u}) = (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)/2$ is the ⁵⁰ rate of deformation tensor, we assume here constant fluid density ρ , dynamic viscosity μ , thermal ⁵¹ conductivity λ , and specific heat c_p , and we have neglected buoyancy and radiative heat transfer, ⁵² on behalf of the focus being on conjugate forced convection heat transfer.

This is solved here with an in-house stabilized finite elements environment cast in the Variational 53 Multiscale (VMS) framework. This allows using equal order linear approximations for all variables 54 (very desirable due to its simplicity of implementation and affordable computing cost) by enhancing 55 the stability of the Galerkin method via a series of additional derived residual based terms evaluated 56 over element interior. The solid is treated as an immersed body, using the Immerse Volume Method 57 to compute the amount of heat exchanged between the solid and the fluid only from the individual 58 material properties on either side of it (which in turn removes the need for a heat transfer coefficient, 59 a limiting issue for the present numerical experiments where we vary the position of the solid). For 60 details about the numerical framework, including the interface capturing method used to generated 61 strongly anisotropic meshes adapted at the fluid-solid boundary (to ensure that the fluid properties 62 are distributed as accurately and smoothly as possible over the smallest possible thickness around 63 the interface), the reader can refer to Refs. 24, 28. The relevance of this numerical method has 64 been validated on multiple benchmarks here 29 and in particular for heat transfer here 30. For 65

⁶⁶ the sake of simplicity we will not discuss it here.



Figure 1: Fully connected neural network with two hidden layers, modeling a mapping from data living in \mathbb{R}^3 to data living in \mathbb{R}^2 .

⁶⁷ 3. Deep reinforcement learning and proximal policy optimization

68 3.1. Neural networks

A neural network (NN) is a connected collection of non-linear functions. It is extremely useful 69 when trying to mimic the relationship between multiple highly non-linear phenomena. A fully 70 connected network is sketched in figure 1, it has layers (represented as columns in the sketch) and 71 within each of them the neurons are connected to all of those contained in the next layer. The input 72 layer is the one that receives information from the outside, the output layer is the one that gives the 73 result and between them are hidden layers. Building an efficient neural network requires a relevant 74 architecture (e.g., type of network, depth, width of each layer), finely tuned hyper-parameters (i.e., 75 parameters that cannot be learned directly by the neural network, e.g., optimizer, learning rate, 76 batch size) and an adequate amount of data to learn from. More details here Ref. 31 and the 77 references therein. 78

79 3.2. Deep reinforcement learning

Reinforcement learning (RL) is a type of machine learning in which an agent learns the actions to do in an environment in order to get the best reward possible. Such method is often formulated as a Markov Decision Process for which a full loop looks like this :

- Assume the environment is in state $s_t \in S$ at iteration t, where S is a set of states;
- The agent uses w_t , an observation of the current environment state (and possibly a partial subset of s_t) to take action $a_t \in A$, where A is a set of actions;
- The environment reacts to the action by transitioning from s_t to state $s_{t+1} \in S$;
- The agent is fed with a reward $r_t \in R$, where R is a set of rewards, and a new observation w_{t+1} .

⁸⁹ This repeats until a steady state is reached. The succession of states and actions defines a trajectory ⁹⁰ $\tau = (s_0, a_0, s_1, a_0, ...)$ for which the agent will try and maximize the cumulative reward at each ⁹¹ step, by choosing an action. That is why the most common quantity of interest in RL is the ⁹² discounted cumulative reward :

$$R(\tau) = \sum_{t=0}^{T} \gamma^t r_t, \tag{4}$$

where T is the final time, and $\gamma \in [0,1]$ is the discount factor that weights the importance of rewards according to their distance to the present (the agent being short-sighted in the limit where $\gamma \to 0$, and far-sighted in the limit where $\gamma \to 1$).

There exist two main types of RL algorithms : model-based methods either have access to the environment and therefore know the probability distribution of the states they end up in, or try to build an approximation of it ; model-free methods don't try to understand the environment, but only communicate with it and try to find the best actions to take, these methods are prominent in the DRL community.



Figure 2: RL agent and its interactions with its environment.

Inside the model-free family of algorithms are also two main techniques : value-based methods 101 that learn to predict the future rewards of a set of actions in order to pick the best one, and policy-102 based methods that keep in memory a policy that maps states to actions and learn to obtain the 103 best rewards by modifying this policy. Many DRL algorithms in the community, including PPO, 104 the one used in this paper, use gradient ascent to optimize a parameterized policy with respect to 105 the expected return and therefore belong to the family of policy gradient methods. For a more 106 thorough introduction to the taxonomy of RL methods (together with their respective pros and 107 \cos cons) please refer to Ref. <u>32</u>. 108

¹⁰⁹ 3.3. From policy methods to Proximal policy optimization

This section is intended for non-expert readers and provides an overview of the basic principles and prerequisites of the Policy Gradient Method and the various steps taken to improve it.

¹¹³ - Policy methods. A policy method maximizes the expected discounted cumulative reward of a ¹¹⁴ decision policy π mapping states to actions. It doesn't use a value function as explained before, ¹¹⁵ but a probability distribution to determine which actions are best at any given state. Policies being ¹¹⁶ often stochastic, the following notations are introduced:

- $\pi(s,a)$ is the probability of taking action a in state s under policy π ,
- $Q^{\pi}(s,a)$ is the expected value of the discounted cumulative reward after taking action a in state s (also termed state-action value function or Q-function)

$$Q^{\pi}(s,a) = \mathbb{E}_{\pi} \left[R(\tau) | s, a \right], \tag{5}$$

where \mathbb{E}_{π} is the expected value \mathbb{E} under policy π .

• $V^{\pi}(s)$ is the expected value of the discounted cumulative reward in state s (also termed value function or V-function)

$$V^{\pi}(s) = \mathbb{E}_{\pi} \left[R(\tau) | s \right].$$
(6)

¹²³ The V and Q functions are thus related such that

$$V^{\pi}(s) = \sum_{a} \pi(s, a) Q^{\pi}(s, a) , \qquad (7)$$

¹²⁴ so $V^{\pi}(s)$ can also be understood as the probability-weighted average of discounted cumulative ¹²⁵ rewards over all possible actions in state *s*. ¹²⁶ - Policy gradient methods. A policy gradient method aims at optimizing a parameterized policy ¹²⁷ π_{θ} , where θ denotes the free parameters whose values can be learnt from data (as opposed to the ¹²⁸ hyper parameters). In practice, one defines an objective function based on the expected discounted ¹²⁹ cumulative reward

$$J(\theta) = \mathbb{E}_{\pi_{\theta}} \left[R(\tau) \right], \tag{8}$$

and looks for θ^* maximizing $J(\theta)$:

$$\theta^* = \arg \max_{\theta} \mathbb{E}_{\pi_{\theta}} \left[R(\tau) \right].$$
(9)

One can try to do this by estimating the policy gradient $\nabla_{\theta} J(\theta)$ and using a gradient ascent algorithm. This is certainly a difficult task as one is seeking a gradient that depends on the policy parameters, but also on the whole space of state-action pairs, in a context where the effects of policy changes on the state probability distribution are unknown (since modifying the policy will most likely modify the probability distribution over the set of visited states). One commonly used estimator, derived in 32 using the log-probability trick, reads

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} \left[\sum_{t=0}^{T} \nabla_{\theta} \log \left(\pi_{\theta}(s_t, a_t) \right) R(\tau) \right] \sim \mathbb{E}_{\pi_{\theta}} \left[\sum_{t=0}^{T} \nabla_{\theta} \log \left(\pi_{\theta}(s_t, a_t) \right) \widehat{A}^{\pi}(s_t, a_t) \right], \quad (10)$$

where \widehat{A}^{π} is some biased estimate (here its normalization to zero mean and unit variance) of the advantage function

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s), \qquad (11)$$

that measures the improvement (if $A^{\pi} > 0$, otherwise the lack thereof) associated with taking action *a* in state *s* (Q-function) compared to taking the average over all possible actions (V-function). This is possible because the V-function doesn't depend on the action, and therefore doesn't change the expected value, but it shows experimentally that it also reduces the variance and speeds up the learning. When the policy π_{θ} is represented by a neural network (in which case θ simply represents the network parameters), we tend to estimate the policy loss

$$L(\theta) = \mathbb{E}_{\pi_{\theta}} \left[\sum_{t=0}^{T} \log \left(\pi_{\theta}(a_t | s_t) \right) \widehat{A}^{\pi}(s_t, a_t) \right], \qquad (12)$$

whose gradient is equal to the (approximated) policy gradient (10) and is computed with respect to each parameter of the neural network by using the chain rule at each layer with the backpropagation algorithm [33].

148

Trust regions. The learning rate of the policy gradient methods, i.e. the size of the steps taken at 149 each learning iteration, has a large impact on their performances. Too small, and the learning will 150 never end, too large, and it will be difficult to get out of degenerate regions where the gradient is 151 already high or noisy. Fine-tuning the learning rate could be a solution, but it asks sometimes for 152 too much work finding the right balance. One way to stay in the range of improvement is to define 153 a maximum distance between the new policy and the old one, this way, even when the gradient 154 becomes too high, the trust region clips the distance and avoid the aforementioned issues. We 155 will not dwell on the intricate details of the many algorithms developed to solve such trust region 156 optimization problems, e.g., natural policy gradient (NPG 34), or trust region policy optimiza-157 tion (TRPO 35). Suffice it to say that they use the MinMax algorithm to maximize iteratively 158 a surrogate policy loss (i.e. a lower bound approximating locally the actual loss at the current 159 policy), but are difficult to implement and can be computationally expensive, as they rely on an 160 estimate of the second-order gradient of the policy log probability. 161

162

- Proximal policy optimization. Proximal policy optimization (PPO) is a similar approach to TRPO (on which it is based) but with a simpler heuristic that uses a probability ratio between the two policies to maximize improvement without the risk of performance collapse [25]. The focus here is on the PPO-clip algorithm¹, that optimizes the surrogate loss

$$L(\theta) = \mathbb{E}_{\pi_{\theta}} \left[\min \left(\frac{\pi_{\theta}(a|s)}{\pi_{\theta_{old}}(a|s)}, g(\epsilon, \widehat{A}^{\pi}(s, a)) \right) \widehat{A}^{\pi}(s, a) \right],$$
(13)

167 where

$$g(\epsilon, A) = \begin{cases} 1+\epsilon & A \ge 0, \\ 1-\epsilon & A < 0, \end{cases}$$
(14)

and $\epsilon \in [0.1, 0.3]$ is the clipping range, a small hyper parameter defining how far away the new 168 policy is allowed to go from the old. Its range is adapted from the paper on Proximal Policy 169 Optimization <u>36</u> and was confirmed in our own implementation. The general picture is that 170 a positive (resp. negative) advantage increases (resp. decreases) the probability of taking action 171 a in state s, but always by a proportion smaller than ϵ , otherwise the min kicks in (13) and its 172 argument hits a ceiling of $1 + \epsilon$ (resp. a floor of $1 - \epsilon$). This prevents stepping too far away from 173 the current policy, and ensures that the new policy will behave similarly but hopefully in a better 174 way. 175

176 3.4. Single-step PPO

We now come to single-step PPO, a "degenerate" version of PPO introduced in 26 and intended 177 for situations where the optimal policy to be learnt by the neural network is state-independent, 178 i.e. $\pi_{\theta}(a,s) = \pi_{\theta}(a)$, as is notably the case in optimization and open-loop control problems 179 (closed-loop control problems conversely require state-dependent policies for which standard PPO 180 is best suited). The main difference between standard and single-step PPO can be summed up as 181 follows: where standard PPO seeks the optimal set of parameters θ^{\star} leading to the largest possible 182 cumulative reward over one episode, single-step PPO seeks the optimal parameters θ^* such that 183 $a^{\star} = \pi_{\theta^{\star}}(s_0)$, where s_0 is some input state (usually a constant vector of zeros) consistently fed 184 to the agent for the optimal policy to eventually embody the transformation from s_0 to a^* . The 185 agent initially implements a random initial policy determined by the free parameters θ_0 , after 186 which it gets only one attempt per learning episode at finding the optimal (i.e., it interacts with 187 the environment only once per episode). This is illustrated in figure $\frac{3}{3}$ showing the agent draw a 188 population of actions a_t from the current policy, and being returned incentives from the associated 189 rewards to update the free parameters for the next population of actions $a_{t+1} = \pi_{\theta_{t+1}}(s_0)$ to yield 190 larger rewards. 191

In practice, the agent outputs a policy parameterized by the mean and variance of the proba-192 bility density function of a d-dimensional multivariate normal distribution, with d the dimension 193 of the action required by the environment. Actions drawn in $[-1, 1]^d$ are then mapped into rele-194 vant physical ranges, a step deferred to the environment as being problem-specific. The resolution 195 essentially follows the process described in section [3,3], only a normalized averaged reward substi-196 tutes for the advantage function. This is because classical PPO is actor-critic, i.e., it improves the 197 learning performance by updating two different networks, a first one called actor that controls the 198 actions taken by the agent, and a second one called critic, that learns to estimate the advantage 199 from the value function as 200

$$A(s_t, a_t) = r_t + \gamma V(s_{t+1}) - V(s_t).$$
(15)

In single-step PPO, the trajectory consists of a single state-action pair, so the discount factor can be set to $\gamma = 1$ with no loss of generality. In return, the advantage reduces to the whitened reward since the two rightmost terms cancel each other out in (15). This means that the approach can do without the value-function evaluations of the critic network, i.e., it is not actually actor-critic.

¹There is also a PPO-Penalty variant which uses a penalization on the average Kullback–Leibler divergence between the current and new policies, but PPO-clip performs better in practice.



Figure 3: Action loop for single-step PPO. At each episode, the input state s_0 is provided to the agent, which in turn provides n actions to n parallel environments. The latter return n rewards, that evaluate the quality of each action taken. Once all the rewards are collected, an update of the agent parameters is made using the PPO loss (13).



Figure 4: (a) Schematic of the 2D forced convection set-up. (b) Sensors positions in the solid domain.

²⁰⁵ 4. Control of forced convection heating in a 2D open cavity

206 4.1. Case description

This test case is based on the second test case of this paper 24 with a few twists. Here we 207 address the control of conjugate heat transfer for the heating of a piece by injection of a hot fluid 208 in the chamber. We use a Cartesian coordinate system with origin at the center of the chamber. 209 The solid has a rectangular shape with height h and aspect ratio 2:1, and is initially at the cold 210 temperature T_c . It can be fixed or move around the chamber according to the parameters we wish to 211 control. Its density, thermal conductivity and heat capacity can take two values, depending on the 212 type of solid we are trying to emulate in order two compare the two : brick or steel. The chamber 213 itself has a rectangular shape with height H and aspect ratio 4:1, and its walls are isothermal at 214 temperature T_w . The north wall of the chamber has three identical inlets of width e, each of which 215 models the exit plane of an injector blowing hot air with constant temperature T_h and velocities 216 $V_i \in [0.01, 0.99]$ subjected to 217

$$\sum_{i=1}^{3} V_i = 1, \qquad (16)$$

to emulate a constant input to the chamber. The fluid is released through two identical outlets on each side of the chamber with height e_0 , and positioned against the south wall.

In the absence of buoyancy, temperature evolves as a passive scalar to the Navier–Stokes equations. All parameters named above are provided in 1 along with the material properties used to model the composite fluid, that yield fluid values of the Reynolds and Prandtl numbers

Re =
$$\frac{\rho e \max_{i \in \{1,2,3\}} V_i}{\mu} \in [67, 200],$$
 Pr = $\frac{c_p \mu}{\lambda} = 2.$ (17)

²²³ Note the very high value of the solid to fluid viscosity ratio, meant to ensure that the velocity ²²⁴ inside the solid is zero and that the no-slip condition on the boundary is satisfied. Thus, only ²²⁵ pure conduction occurs in the solid. The governing equations are solved with no-slip isothermal ²²⁶ conditions $\mathbf{u} = \mathbf{0}$ and $T = T_w$ on $\partial\Omega$, except at the inlets where $\mathbf{u} = -V_i \mathbf{e}_y$ and $T = T_h$, and ²²⁷ at the exhausts where a zero-pressure condition is imposed : $p = \partial_x u = \partial_x T = 0$. No thermal ²²⁸ condition is imposed at the interface, where heat exchange is implicitly driven by the difference

Н	h	e	T_w	T_c	T_h	μ	ρ	λ	c_p	
						0.001	1	0.5	1000	Fluid
1	0.2	0.2	10	10	300	1000	1800	1	600	Brick
						1000	7800	50	500	Steel

Table 1: Geometric and numerical parameters used in the 2-D forced convection setup. All values in SI units, with the exception of temperature given in Celsius.

²²⁹ in the individual material properties, as intended in the Immersed Volume Method. The mesh on ²³⁰ which the computation is performed is defined at the beginning only around the interface to ensure ²³¹ its good definition, then remeshing is performed every 5 time steps based on velocity gradients ²³² to also capture accurate velocity profiles. We aim at 20000 elements, with $h_{min} = 0.0001$. More ²³³ about our remeshing technique here [37].

234 4.2. Control

The quantities subjected to optimization are the three inflow velocities $V_{i \in \{1,2,3\}}$, plus the insertion angle of the piece being heated, and the position of its center of mass, hence six control parameters (three for the inflow distribution, one for the angle and two for the position). We could have used two parameters to control the inflow distribution since the last one is constrained by the total inflow, but in order to avoid asymmetry in the learning process we decided to control each of the injectors in the same way. In practice, each injector is given a value between 0.01 and 0.99 which is then scaled in order to obtain their speed while following (16).

Just like in [24], we distribute 15 probes uniformly in the workpiece to compute the reward used by the DRL algorithm. The probes are arranged in an array of $n_x = 5$ columns and $n_y = 3$ rows with resolutions $\Delta x = 0.09$ and $\Delta y = 0.075$, respectively; see figure [4(b)]. The following formula gives an estimate of the tangential heat flux by averaging the norm of the temperature gradient across rows and columns respectively :

$$\langle ||\nabla_{\parallel}T||\rangle_{i} = \frac{2}{n_{y}-1} |\sum_{j\neq 0} \operatorname{sgn}(j)||\nabla T||_{ij}|, \qquad \langle ||\nabla_{\parallel}T||\rangle_{j} = \frac{2}{n_{x}-1} |\sum_{i\neq 0} \operatorname{sgn}(i)||\nabla T||_{ij}|, \qquad (18)$$

where subscripts i, j and ij denote quantities evaluated at $x = i\Delta x$, $y = j\Delta y$ and $(x, y) = (i\Delta x, j\Delta y)$, respectively, and symmetrical numbering is used for the center probe to sit at the intersection of the zero-th column and row. The reward $r_t = -\langle ||\nabla_{\parallel}T|| \rangle$ fed to the DRL agent is given by the average of the quantities calculated before

$$r_t = -\frac{1}{n_x + n_y} \sum_{i,j} \langle ||\nabla_{\parallel} T|| \rangle_i + \langle ||\nabla_{\parallel} T|| \rangle_j , \qquad (19)$$

which especially yields $r_t = 0$ for a perfectly homogeneous heating.

A second reward is also tested in this paper to assess the feasibility of controlling both the homogeneity and efficiency of the furnace. This is expressed as

$$\psi_t(w_1, w_2) = \frac{w_1}{n_x n_y} \sum_{i,j} T_{ij} + w_2 r_t \,, \tag{20}$$

where the first term is the right-hand side measures the solid temperature averaged across all sensors, and $w_{1,2}$ are scalar-valued factors weighing the priority given to each objective. In practice, a single point concurrently minimizing both objectives usually does not exist. The optimal solutions are thus to be understood as Pareto-efficient solutions 38 that best manage trade-offs between the two criteria, in the sense that further optimizing one objective decreases the performance of the other one (after which the final decision is made by the practitioner based on subjective preferences).

The agent is a fully-connected network with two hidden layers, each holding 2 neurons. The resolution process uses 8 environments and 2 steps mini-batches to update the network for 32 epochs, with learning rate set to 5×10^{-3} , and PBO loss clipping range to $\epsilon = 0.3$.



Figure 5: Control of heating homogeneity for the brick-like workpiece. (a) Optimal temperature distribution found by the DRL algorithm by controlling (from top to bottom) : the inflow velocities under constant workpiece angle and position, the inflow velocities and workpiece angle under constant position, and the inflow velocities, workpiece angle and position. (b) Corresponding streamlines colored by the magnitude of velocity.

264 4.3. Results

265 4.3.1. Control of heating homogeneity

We evaluate first the performance of the algorithm in several scenarios of increasing complexity, using the homogeneity-based reward r_t . Two different workpieces are used, one that has the properties of a brick-like material, and one that has the properties of a steel-like material. For each workpiece, three different cases are considered, in which the DRL agent is tasked with optimizing either the inflow velocities (under constant workpiece angle and position), or the inflow velocities and the workpiece angle (under constant position), or the inflow velocities, angle and position (the most general case).

For each case, 150 episodes have been run (1200 simulations), each of which performs 2000 273 iterations with time step $\Delta t = 0.1$ (hence a heating time of 200), starting from an initial condition 274 consisting of zero velocity and uniform temperature (except in the solid domain), and using the 275 level set, velocity and temperature as multiple-component criterion to adapt the mesh (initially 276 pre-adapted using the sole level set) every 5 time steps under the constraint of a fixed number of 277 elements $n_{el} = 15000$. This represents 1200 simulations, each of which is performed on 8 cores and 278 lasts 10mn, hence 200h of total CPU cost. We can clearly see in figures 5 and 6 the flow patterns 279 that develop when the blown fluid travels through the cavity. Moreover, it clearly depends on the 280 inflow distribution and position of the piece, and features complex rebound phenomena (either 281 fluid/solid, when a jet impinges on the workpiece, or fluid/fluid, when a deflected jet meets the 282 crossflow of another jet), leading to the formation of multiple recirculations varying in number, 283 position and size. 284

The results of the various optimization scenarios are shown in figure 5 for the brick-like material 285 and figure 6 for the steel-like material. The use of the reward r_t has a great effect on the aspect 286 of these results in any control configuration. Even though they are all different, we can spot some 287 common features : the workpiece is kept in colder areas of the chamber, which allows for lower 288 temperature gradients as prescribed by the reward, and it is also generally well surrounded by 289 streamlines that ensure symmetry in the heating. In the simplest case where the agent has to 290 control only the inflow velocities, it has no problem finding the best solution possible, which is true 291 also when we add the workpiece angle (not shown here for conciseness), and they both present the 292 features we mentioned. 293

When tasked with simultaneously optimizing all six parameters (inflow velocities, angle and position), we show in figure 7 that the algorithm learns quite fast up to episode 60, but pursues



Figure 6: Control of heating homogeneity for the steel-like workpiece. (a) Optimal temperature distribution found by the DRL algorithm by controlling (from top to bottom) : the inflow velocities under constant workpiece angle and position, the inflow velocities and workpiece angle under constant position, and the inflow velocities, workpiece angle and position. (b) Corresponding streamlines colored by the magnitude of velocity.

the iteration to handle the position on the y axis and the angle, as it restarts an exploration phase 296 around episode 85, which allows to reach even better rewards in the end (see also figure 8 for 297 an illustration of temperature distributions at the final step randomly sampled over the course of 298 optimization). Indeed, with a larger space to explore, some parameters may be optimized more 299 easily than others, for example if they have a greater impact on the reward. In this case, this may 300 be a good idea to optimize the last parameters individually, since the first reward chosen may not 301 be the best fitted to their optimization. The reward obtained from the original simulation (centered 302 piece, same speed at each inlet) has a mean value of -0.450, figure 7 indicates that PPO converged 303 around a reward of -0.050 which yields an improvement of 89% of the temperature homogeneity. 304

305 4.3.2. Control of heating efficiency

As explained in 4.2, we introduce here a second reward ψ_{t,w_1,w_2} to assess the feasibility of controlling the efficiency of heating of the piece while keeping the gradient homogeneous. The PPO algorithm should consider a new solution space in which minima are decided by both the temperature and the gradient. This should lead to different solutions than the ones before, especially concerning the average of the temperature values at the probes.

Only the steel-like workpiece is considered here (as it allows for a better conduction and there-311 fore larger temperature differences) under the third optimization scenario, *i.e.*, inflow velocities, 312 workpiece angle and position as free parameters). We tested two sets of weight, $(w_1, w_2) = (1, 1)$ 313 and (10,1), with the second one giving more priority to the averaged temperature component, 314 and compare in figure 9 the obtained average temperature at the final time step, to those ob-315 tained under the three scenario presented earlier for pure homogeneous control (that corresponds 316 to $(w_1, w_2) = (0, 1)$. For the simplest homogeneous control cases presented in figures 9(a) and 317 (b), the temperature is constrained by the position of the workpiece, and the algorithm quickly 318 converges. Adding in the workpiece position as free parameter increases the complexity, (the pos-319 sibility to move the workpiece anywhere in the chamber yields much higher variance in the space 320 of achievable temperature distributions). In return, the average temperature in figure 9(c) drops 321 to a lower value slightly above 26° , to be understood as an indirect consequence of the op-322 timization of the homogeneity reward r_t . The scenario shown in figure 9(d) corresponds to the 323 optimization of the compound reward ψ_t using all six control parameters $(w_1, w_2) = (1, 1)$, for 324 which the temperature again drops, similar to the previous case. Finally, by using the reward 325 $\psi_{t,10,1}$ with $(w_1, w_2) = (10, 1)$, the algorithm reaches a temperature of 26.5°. Consistently, the 326 optimal temperature distributions for this two cases are somewhat similar to those obtained by 327



Figure 7: Control of heating homogeneity for the steel-like workpiece using the inflow velocities, workpiece angle and position as free parameters, corresponding to the scenario at the bottom of figure 6 Evolution per episode of the (a) reward, and (b-g) control parameters. Black curves are the moving averages.

controlling the heating homogeneity, but with a workpiece that is closer to the heat sources as the value of w_1 in the reward ψ_t increases, to give more priority to lowering the averaged temperature.

330 4.4. Discussion

Whether it be for the brick-like or the steel-like workpiece, the algorithm finds, in each scenario, 331 a relevant local minimum that satisfies the conditions we imposed on it. One point worth noticing 332 is the wide variety of solutions the algorithm comes up with, best illustrated by comparing figures 333 5, 6 and 10. This may be because the complexity of this case gives room for a lot of equivalent 334 solutions, and the algorithm struggles to find the global minimum and always ends up in local 335 minima. Numerical approximations can also create noise and give an information too imprecise to 336 be processed by the PPO method, especially since the reward is calculated from point-wise data 337 interpolated from the simulation (similar to experimental measurements). By running the same 338 simulation 1000 times, we noticed a fluctuation in the reward, with estimated relative standard 339 deviation by 4.1%. 340



Figure 8: Control of heating homogeneity for the steel-like workpiece using the inflow velocities, workpiece angle and position as free parameters, corresponding to the scenario at the bottom of figure 6 Temperature distribution at the final time step for randomly sampled episodes marked by the arrows in figure 7(a).

This has been confirmed by running 8 times the same learning experiment (steel workpiece, 341 homogeneous reward r_t , control of the inflow velocities, workpiece angle and position), for which 342 the algorithm found local minima more or less sensitive to perturbations, and the relative standard 343 deviation computed for the moving average over the 50 latest values is 9.9%. As interpreted before, 344 the algorithm seems to struggle finding a global maximum of our reward function r_t , likely because 345 there exists a sensitivity to noise and system uncertainty. There are a few possible reactions to 346 that. Firstly, it could be useful to improve the balance between exploration and exploitation, 347 using improved search distributions to effectively encourage the policy to explore more on the 348 potential valuable actions, no matter whether they were preferred by the previous policies or not 349 (for instance, using the recently introduced PBO algorithm [39], that uses three separate neural 350 networks to learn the mean, variance and correlation parameters of a multivariate normal search 351 distribution, while single-step PPO updates the mean and variance, the same for all variables, 352 from a single neural network). Another possibility is to fine-tune the architecture of the neural 353 network. As the number of parameters to control increases, the number of neurons that model 354 the policy should increase too, but there is no guideline for that. Finally, the reward itself could 355 be changed, along with the constraints on the parameters to control. These aspects have a great 356 impact on the shape of the solution space, and therefore on the ease for the algorithm to find a 357 global maximum. In real world applications, a lot of constraints can come into consideration when 358 choosing the reward, or the parameters to operate on. Finding the right way to communicate our 359 needs to the algorithm is a whole topic in itself. We leave the exploration of these reactions to 360 future works. 361

³⁶² 5. Single-step control of a 3D serpentine heater

363 5.1. Case description

We propose in this test case to apply the same DRL-CFD framework to a three-dimensional case of industrial interest: the serpentine heater. It consists of a 3D simulation of two fluids, a liquid which is cold at first, and the hot gas that is distributed inside the chamber, and should come out colder due to the transfer of heat from the gas to the liquid (this resembles a heat exchanger but with a configuration closer to a gas burner).

The control objective here is to find the best flow distribution between an array of gas burners to heat a liquid in a pipe. We use Cartesian coordinate system with origin at the center of the chamber. The chamber is a simple parallelepiped with size H on x and z axes and h on the y axis. The pipe is made from the extrusion of a circle of radius R and a center at (-H/2, 0, -H/4), with three 180 degrees bends around the y axis. It has two planes of symmetry Oxz and Oxy. The longer straight part of the pipe has a length of L, the second straight part has a length of l and



Figure 9: Evolution per episode of the final temperature averaged across all sensor positions in the steel-like workpiece, using (a-c) the homogeneity reward and (d-e) the compound homogeneity/efficiency reward with (d) $(w_1, w_2) = (1, 1)$ and (e) $(w_1, w_2) = (10, 1)$. The free parameters subjected to optimization are (a) the inflow velocities under constant workpiece angle and position, (b) the inflow velocities and workpiece angle under constant position, and (c-e) the inflow velocities, workpiece angle and position.

the rest of the pipe can be deducted from symmetry with the condition that all three bends have exactly the same radius. The hot gas comes out of 6 circular inlets of radius r, 4 on the y^- side and two on the y^+ side. They are disposed in an array aligned on the center of the pipe at each of its bends, see figure 12 (a) et (c). Since this case has no moving parts, remeshing is not needed, hence a unique mesh is used, as seen in figure 12 (d). It is composed of 35021 nodes and 197949 tetrahedral elements. The elements aspect ratio is shown in the bar chart 13.

The fluid in the pipe comes in through the lower hole with a parabolic speed profile corresponding to a flow rate D, and a temperature T_c . At t = 0 the temperature in the chamber is T_c , and hot gas comes through the 6 inlets at temperature T_h , with speed $V_i \in [0.1, 0.9]$. In the same manner



Figure 10: Control of efficiency for the steel-like workpiece. (a) Optimal temperature distribution found by the DRL algorithm by controlling the inflow velocities, workpiece angle and position, using reward ψ_t with weighs (w_1, w_2) set to (from top to bottom) (1,1) and (10,1). (b) Corresponding streamlines colored by the magnitude of velocity.



Figure 11: Temperature distribution at the final time step for randomly sampled episodes of heating efficiency control for the steel-like workpiece. The free parameters subjected to optimization are the inflow velocities, workpiece angle and position, corresponding to the scenario at the bottom of figure 10.

as for the previous case, we emulate a constant input by imposing

$$\sum_{i=1}^{6} V_i = 1.$$
 (21)

The exhaust is positioned in the center of the top face of the chamber. It has a rectangular shape 385 and is half the size of the top face. We consider in this case that there is no solid material separating 386 the inside of the pipe from the gas. Only a no slip and isothermal condition is applied on the walls 387 of the chamber (with the exception of the inlets and exhausts), with $T = T_w$. A no slip condition 388 is also applied on the 2D interface representing the pipe. At the exhausts a zero-pressure condition 389 is imposed : $p = \partial_x u = \partial_x T = 0$. No thermal condition is imposed at the interface, where heat 390 exchange is implicitly driven by the difference in the individual material properties. Again, in the 391 absence of buoyancy, the temperature evolves as a passive scalar for the Navier–Stokes equations. 392 This allows us to solve the Navier–Stokes equations separately for the gas and the fluid in the 393 pipe and then find the temperature field in the whole domain. Using the parameters for the liquid 394 and the gas provided in Table 2, the Reynolds and Prandtl numbers in the hot gas domain are 395 estimated to be $\text{Re} \in [5, 30]$ and Pr = 20, and $\text{Re} \in [320]$ and Pr = 400 in the liquid. 396



Figure 12: (a) Schematic of the pipe with positions of the gas inlets projected along the y axis. Fully opaque circles are inlets in front of the pipe, i.e. on the y^+ side of the chamber, and transparent circles are inlets on the other side. (b) Position of the sensors on the outlet of the pipe. (c) Schematic of the whole setup with arrows representing the direction of the gas flow. (d) Representation of the mesh used.



Figure 13: Bar chart of the aspect ratio of tetrahedral elements.

Η	h	R	r	L	l	D	T_w	T_c	T_h	μ	ρ	λ	c_p	
4	2	0.2	0.15	2	1	0.05	150	20	300	0.01	1	0.5	1000	Gas
4	4	0.2	0.15	5	T	0.00	100	20	300	0.1	100	1	4000	Pipe

Table 2: Geometric and numerical parameters used in the 3D serpentine heater setup. All values in SI units, with the exception of temperature given in Celsius.



Figure 14: Representative temperature fields sampled on the course of optimization on the surface of the pipe together with streamlines colored by the magnitude of velocity, both in logarithmic scales.

³⁹⁷ 5.2. Control strategy

The quantity being optimized is the distribution of the inflow between the 6 injectors $V_{i \in [\![1,6]\!]}$. In order to avoid asymmetry in the learning process we decided to control each of the injectors in the same way, that is to assign each injector a value between 0.1 and 0.9, scaled a posteriori to recover a valid velocity satisfying (21). In order to compute the reward r_t we distribute 29 probes uniformly on the outlet of the pipe each at a distance $\Delta = 0.06$ of their closest neighbour, see figure 12 (b). These probes allow us to monitor the temperature out of the pipe, and the reward given the PPO algorithm (computed at the final simulation time) is defined as

$$r_t = \frac{1}{29} \sum_{i=1}^{29} T_i \,, \tag{22}$$

405 with T_i being the temperature at probe *i*.

In a second stage, we add a strong constraint to this reward. By looking at the highest temperature in the pipe across all time steps, we penalize the solutions that reach a temperature above a limit $T_l = 200$. We chose the limit by looking at the solutions found by the PPO algorithm with the first reward and their highest temperature inside the pipe. The new reward is thus

$$\rho_t = \begin{cases} r_t, & \text{if } \max_{\Pi} T < T_l, \\ 0, & \text{if } \max_{\Pi} T \ge T_l, \end{cases}$$
(23)

where Π is the inner pipe domain. This is meant to avoid concentrating too much energy in the same spot and thus protecting the materials used in the heating process.

412 5.3. Results



Figure 15: Evolution per episode of the (a) reward r_t , and (b) control velocity at each gas burner, and (c) maximum temperature $\max_{\Pi} T$ in the pipe. Black curves are the moving averages. The color code of the inlets is the same as in fig. 12

For this case, 150 episodes have been run, each composed of 8 environment, each of which performs 1300 iterations with time step $\Delta t = 0.1$ to march in time the same initial condition (consisting of zero velocity and uniform temperature, except in the pipe domain). This represents ⁴¹⁶ 1200 simulations, each of which is performed on 8 cores and lasts 40mn, hence 800h of total CPU
⁴¹⁷ cost; see figure 14 for representative temperature and velocity distributions sampled of the course
⁴¹⁸ of optimization.

Figure 15 shows the evolution of the reward r_t and the values given to each inlet at each 419 individual. The colors correspond to each inlet and are kept also in figure 16. The trend of these 420 curves demonstrate a fast convergence of PBO in this context, with a transitional phase during 421 The solution found by the algorithm puts all the which it tries to explore other possibilities. 422 energy in a single, well positioned hot gas inlet (which is rather intuitive except that the location 423 of said may not be), as the last 200 episodes all yield the same distribution among the inlets : all 424 for the top left inlet and nothing for the rest. As can be seen in figure 12 (a), this particular inlet is 425 near the end of the tube, right in front of large area thanks to the bend in the tube. This position 426 allows for a large part of the heat to be transferred to the tube without too much energy being lost 427 in perturbations. The results of the learning process under the constrained reward ρ_t are shown 428 in figure 16, where the red line marks the limit temperature T_l that we try not to top (which the 429 single-step PPO algorithm successfully achieves, as the temperature converges to a value slighly 430 below). No moving average is given here since the hard constraint forces the reward to be zero 431 when the limit temperature is topped. A trend is however visible since the reward still goes up in 432 general and the frequency of constraint violation reduces along the training. Colored curves can 433 be linked to the figures 12 (a) and 15. After convergence, the DRL still puts most of the weight 434 on the top left inlet, with the rest of energy mainly given to the top right inlet, right next to the 435 other, and closer to the output of the pipe. This shows the ability of the method to find optimal 436 solutions under the constraint of safe operating conditions (here the pipe maximum temperature). 437



Figure 16: Evolution per episode of the (a) reward ρ_t , (b) control velocity at each gas burner, and (c) maximum temperature max_{II} T in the pipe. Black curves are the moving averages. The color code of the inlets is the same as in fig. 12

438 5.4. Discussion

The main interest behind this comparison is to assess how the algorithm performs when a hard 439 constrained is applied directly to the reward. The first control experiment are somehow consistent 440 with intuition, but the second one provides with more relevant and exploitable results in a real 441 context where the ability to not exceed a certain temperature can be essential for the industrial 442 process. On another note, for this case, it could be more useful for this case to investigate the 443 feasibility to learn active control strategies (by adjusting dynamically the control velocities to 444 appropriate sensing of flow variables). As the temperature of one spot approches the maximum 445 authorized value, this would allow redistributing some of the power to the other spots to balance 446 everything properly, and to converge rapidly to an optimal configuration. This would also allow 447 for some evolution of the input flow of the pipe, and a reaction to it. 448

6. General discussion

Several points deserve further consideration to keep pushing forward the development of DRL 450 in the context of such real-life applications. First, improving efficiency and convergence (by fine-451 tuning the hyper parameters and comparing with different DRL algorithms allowing for increased 452 exploration, as PPO prevents by design large updates of the policy to avoid performance collapse). 453 Second, designing improved reward construction strategies, as approximating the reward from 454 point-wise temperature data has been shown to yield a certain sensitivity to system and numerical 455 uncertainty, which in turn may trap the algorithm in local optimal. Third, enriching the description 456 of the test cases using multi-physics modeling, e.g., radiative heat transfer and thermo-mechanical 457 coupling to encompass the solid deformations. Finally, investigating the case of active flow control 458 in which the control parameters are dynamically adjusted from measurements in the workpiece 459 and the furnace, and has often proved to be of great importance in industrial contexts. 460

461 7. Conclusion

⁴⁶² Optimization of heating processes is achieved here training a fully connected network with ⁴⁶³ the PBO deep reinforcement algorithm, in which it gets only one attempt per learning episode ⁴⁶⁴ at finding the optimal. The numerical reward fed to the network is computed with a stabilized ⁴⁶⁵ finite elements CFD environment solving the coupled Navier–Stokes and heat equations, using a ⁴⁶⁶ combination of variational multi-scale modeling, immerse volume method, and multi-component ⁴⁶⁷ anisotropic mesh adaptation.

The approach succeeds at improving the homogeneity of temperature (or a blend of homo-468 geneity and absolute temperature) across the surface of two-dimensional cold workpieces under 469 jet impingement heating. Several control scenarios have been considered (that can be considered 470 different levels of design constraint), from the simple case where only the inflow velocity of the 471 hot air injectors is optimized relative to a fixed workpiece position, up to the most complex case 472 where the DRL agent also optimizes the position and insertion angle of the workpiece itself. The 473 potential of the approach for industrial configurations of engineering interest is also showcased by 474 475 optimizing the inflow of multiple gas burners in a three-dimensional serpentine heater.

The present results highlight the capabilities of coupling DRL and computational fluid dynamics in the context of industrial manufacturing processes in general, and heating processes inside industrial furnaces in particular.

479 **References**

480	[1]	S. Dequan, G. Guili, G. Zhiwei, X. Peng, Application of Expert Fuzzy PID Metho	od for
481	1	Temperature Control of Heating Furnace, Procedia Engineering 29 (2012) 257–261.	doi:
482		10.1016/j.proeng.2011.12.703	
483		URL https://www.sciencedirect.com/science/article/pii/S1877705811065404	

- [2] M. Tóthová, M. Balara, J. Dubják, Simulation Model of Cascade Control of the Heating
 System, International Journal of Engineering Research in Africa 18 (2015) 20–27, conference
 Name: International Journal of Engineering Research in Africa Vol. 18 ISBN: 9783038356790
 Publisher: Trans Tech Publications Ltd. doi:10.4028/www.scientific.net/JERA.18.20
 URL https://www.scientific.net/JERA.18.20
- [3] N. Philip, S. Sahlan, A. P. I. D. N. Wahab, Application of Auto-Tuner Fuzzy PID Controller
 on Industrial Cascade Control, ELEKTRIKA- Journal of Electrical Engineering 19 (2020)
 61–65.
- [4] E. Rafajłowicz, W. Rafajłowicz, Image-Driven Decision Making with Application to Control
 Gas Burners, in: K. Saeed, W. Homenda, R. Chaki (Eds.), Computer Information Systems
 and Industrial Management, Lecture Notes in Computer Science, Springer International Publishing, Cham, 2017, pp. 436–446. doi:10.1007/978-3-319-59105-6_37.
- E. Rafajłowicz, H. Pawlak-Kruczek, W. Rafajłowicz, Statistical Classifier with Ordered Decisions as an Image Based Controller with Application to Gas Burners, in: L. Rutkowski,
 M. Korytkowski, R. Scherer, R. Tadeusiewicz, L. A. Zadeh, J. M. Zurada (Eds.), Artificial
 Intelligence and Soft Computing, Lecture Notes in Computer Science, Springer International
 Publishing, Cham, 2014, pp. 586–597. doi:10.1007/978-3-319-07173-2_50.
- [6] N. Nguyen, D. Tran, Neural Network based Model Reference Control for Electric Heating
 Furnace with Input Saturation, 2019, pp. 111–114. doi:10.1109/ICA-SYMP.2019.8646052.
- ⁵⁰³ [7] D.-D. Wen, Decoupling control of electric heating furnace temperature based on DRNN neural ⁵⁰⁴ network, 2010, pp. 261–264. doi:10.1109/ICECTECH.2010.5479934.
- [8] V. R. Radhakrishnan, A. R. Mohamed, Neural networks for the identification and control of blast furnace hot metal quality, Journal of Process Control 10 (6) (2000) 509-524. doi: 10.1016/S0959-1524(99)00052-9.
- URL https://www.sciencedirect.com/science/article/pii/S0959152499000529
- [9] L. Zhang, Y. Xue, Q. Xie, Z. Ren, Analysis and neural network prediction of combustion stability for industrial gases, Fuel 287 (2021) 119507. doi:10.1016/j.fuel.2020.119507. URL https://www.sciencedirect.com/science/article/pii/S0016236120325035
- J. Matthes, P. Waibel, M. Vogelbacher, H. J. Gehrmann, H. B. Keller, A new camera-based method for measuring the flame stability of non-oscillating and oscillating combustions, Experimental Thermal and Fluid Science 105 (2019) 27–34. doi:10.1016/j.expthermflusci.
 2019.03.008
- URL https://www.sciencedirect.com/science/article/pii/S0894177718317503
- [11] J. Matthes, P. Waibel, M. Vogelbacher, H. B. Keller, H. J. Gehrmann, D. Stapf, A camerabased flame stability controller for non-oscillating and forced-oscillating combustion, in: Proceedings of the 13TH EUROPEAN CONFERENCE ON INDUSTRIAL FURNACES AND BOILERS, 2022.
- J. Tao, Z. Yu, R. Zhang, F. Gao, RBF neural network modeling approach using PCA
 based LM-GA optimization for coke furnace system, Applied Soft Computing 111 (2021)
 107691. doi:10.1016/j.asoc.2021.107691.
- URL https://www.sciencedirect.com/science/article/pii/S1568494621006128
- [13] D. Zhao, Z. Lu, H. Zhao, X. Y. Li, B. Wang, P. Liu, A review of active control approaches
 in stabilizing combustion systems in aerospace industry, Progress in Aerospace Sciences 97
 (2018) 35–60. doi:10.1016/j.paerosci.2018.01.002.
- URL https://www.sciencedirect.com/science/article/pii/S0376042117300878
- [14] D. Silver, J. Schrittwieser, K. Simonyan, I. Antonoglou, A. Huang, A. Guez, T. Hubert,
 L. Baker, M. Lai, A. Bolton, Y. Chen, T. Lillicrap, F. Hui, L. Sifre, G. van den Driessche, T. Graepel, D. Hassabis, Mastering the game of Go without human knowledge, Nature 550 (7676) (2017) 354–359, number: 7676 Publisher: Nature Publishing Group. doi:

533 534		10.1038/nature24270. URL https://www.nature.com/articles/nature24270
535 536 537 538	[15]	V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller, Playing Atari with Deep Reinforcement Learning, arXiv:1312.5602 [cs]ArXiv: 1312.5602 (Dec. 2013). URL http://arxiv.org/abs/1312.5602
539 540 541	[16]	Google just gave control over data center cooling to an AI.URLhttps://www.technologyreview.com/2018/08/17/140987/google-just-gave-control-over-data-center-cooling-to-an-ai/
542 543 544	[17]	A. Gupta, A. Khwaja, L. Guan, B. Venkatesh, Policy-Gradient and Actor-Critic Based State Representation Learning for Safe Driving of Autonomous Vehicles, Sensors 20 (2020) 5991. doi:10.3390/s20215991.
545 546 547	[18]	F. Wang, L. P. Casalino, D. Khullar, Deep Learning in Medicine-Promise, Progress, and Challenges, JAMA internal medicine 179 (3) (2019) 293–294. doi:10.1001/jamainternmed. 2018.7117.
548 549 550 551	[19]	D. Skrobek, J. Krzywanski, M. Sosnowski, A. Kulakowska, A. Zylka, K. Grabowska, K. Ciesielska, W. Nowak, Prediction of sorption processes using the deep learning methods (long short-term memory), Energies 13 (24) (2020). doi:10.3390/en13246601. URL https://www.mdpi.com/1996-1073/13/24/6601
552 553 554 555 556	[20]	J. Krzywanski, K. Sztekler, M. Szubel, T. Siwek, W. Nowak, Mika, A comprehensive three- dimensional analysis of a large-scale multi-fuel cfb boiler burning coal and syngas. part 1. the cfd model of a large-scale multi-fuel cfb combustion, Entropy 22 (9) (2020). doi:10.3390/ e22090964. URL https://www.mdpi.com/1099-4300/22/9/964
557 558 559 560	[21]	P. Garnier, J. Viquerat, J. Rabault, A. Larcher, A. Kuhnle, E. Hachem, A review on deep reinforcement learning for fluid mechanics, Computers & Fluids 225 (2021) 104973. doi: 10.1016/j.compfluid.2021.104973. URL https://www.sciencedirect.com/science/article/pii/S0045793021001407
561 562	[22]	J. Viquerat, P. Meliga, A. Larcher, E. Hachem, A review on deep reinforcement learning for fluid mechanics : an update, Phys. Fluids 34 (2022) 111301.
563 564	[23]	G. Beintema, A. Corbetta, L. Biferale, F. Toschi, Controlling Rayleigh-B\'enard convection via reinforcement learning, arXiv preprint arXiv:2003.14358 (2020).
565 566 567 568	[24]	E. Hachem, H. Ghraieb, J. Viquerat, A. Larcher, P. Meliga, Deep reinforcement learning for the control of conjugate heat transfer, Journal of Computational Physics 436 (2021) 110317. doi:10.1016/j.jcp.2021.110317. URL https://www.sciencedirect.com/science/article/pii/S0021999121002126
569 570 571	[25]	J. Schulman, F. Wolski, P. Dhariwal, A. Radford, O. Klimov, Proximal Policy Optimization Algorithms, arXiv:1707.06347 [cs]ArXiv: 1707.06347 (Aug. 2017). URL http://arxiv.org/abs/1707.06347
572 573 574 575	[26]	J. Viquerat, J. Rabault, A. Kuhnle, H. Ghraieb, A. Larcher, E. Hachem, Direct shape opti- mization through deep reinforcement learning, arXiv:1908.09885 [cs]ArXiv: 1908.09885 (Dec. 2020). URL http://arxiv.org/abs/1908.09885
576 577	[27]	H. Ghraieb, J. Viquerat, A. Larcher, P. Meliga, E. Hachem, Single-step deep reinforcement learning for open-loop control of laminar and turbulent flows, Phys. Rev. Fluids 6 (2021)

053902.

- E. Hachem, H. Digonnet, E. Massoni, T. Coupez, Immersed volume method for solving natural convection, conduction and radiation of a hat-shaped disk inside a 3d enclosure, International Journal of numerical methods for heat & fluid flow (2012).
- [29] E. Hachem, B. Rivaux, T. Kloczko, H. Digonnet, T. Coupez, Stabilized finite element method
 for incompressible flows with high reynolds number, Journal of Computational Physics 229 (23)
 (2010) 8643-8665. doi:https://doi.org/10.1016/j.jcp.2010.07.030
- 585 URL https://www.sciencedirect.com/science/article/pii/S0021999110004237
- [30] E. Hachem, T. Kloczko, H. Digonnet, T. Coupez, Stabilized finite element solution to han dle complex heat and fluid flows in industrial furnaces using the immersed volume method,
 International Journal for Numerical Methods in Fluids 68 (1) (2012) 99–121. arXiv:
- 589 https://onlinelibrary.wiley.com/doi/pdf/10.1002/fld.2498, doi:https://doi.org/ 590 10.1002/fld.2498.
- ⁵⁹¹ URL https://onlinelibrary.wiley.com/doi/abs/10.1002/fld.2498
- ⁵⁹² [31] I. Goodfellow, Y. Bengio, A. Courville, The Deep Learning Book, MIT Press, 2017.
- ⁵⁹³ [32] R. S. Sutton, A. G. Barto, Reinforcement learning: An introduction, MIT press, 2018.
- [33] D. Rumelhart, G. Hinton, R. Williams, Learning representations by back-propagating errors, Nature 323 (1986) 533-536.
- ⁵⁹⁶ [34] A. Kakade, A natural policy gradient, Adv. Neural Inf. Process Syst. 14 (2001) 1531–1538.
- J. Schulman, S. Levine, P. Moritz, M. I. Jordan, P. Abbeel, Trust Region Policy Optimization, arXiv e-prints (Feb. 2015). arXiv:1502.05477.
- [36] J. Schulman, F. Wolski, P. Dhariwal, A. Radford, O. Klimov, Proximal Policy Optimization
 Algorithms, arXiv e-prints (Jul. 2017). arXiv:1707.06347.
- [37] T. Coupez, L. Silva, E. Hachem, Implicit Boundary and Adaptive Anisotropic
 Meshing, Springer International Publishing, Cham, 2015, pp. 1–18. doi:10.1007/
 978-3-319-06053-8_1.
- ⁶⁰⁴ URL https://doi.org/10.1007/978-3-319-06053-8_1
- [38] T. W. Athan, P. Y. Papalambros, A note on weighted criteria methods for compromise solutions in multi-objective optimization, Eng. Optim. 27 (1996) 155–176.
- ⁶⁰⁷ [39] J. Viquerat, R. Duvigneau, P. Meliga, A. Kuhnle, E. Hachem, Policy-based optimization: ⁶⁰⁸ single-step policy gradient method seen as an evolution strategy, Neural Comput. Appl. (2022).