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Reinforcement learning for patient-specific optimal stenting of intracranial aneurysms

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Developing new capabilities to predict the risk of intracranial aneurysm rupture and to improve treatment outcomes in the follow-up of endovascular repair is of tremendous medical and societal interest, both to support decision-making and assessment of treatment options by medical doctors, and to improve the life quality and expectancy of patients. This study aims at identifying and characterizing novel flow-deviator stent devices through a high-fidelity computational framework that combines state-of-the-art numerical methods to accurately describe the mechanical exchanges between the blood flow, the aneurysm, and the flow-deviator and deep reinforcement learning algorithms to identify a new stent concepts enabling patient-specific treatment via accurate adjustment of the functional parameters in the implanted state.

Keywords: Deep Reinforcement Learning; Proximal Policy Optimization; Neural Networks; Computational fluid dynamics; Open-loop flow control; Adjoint method

INTRODUCTION

An estimated 2-3 % of the population harbour intracranial aneurysms (IAs) [1, 2], a pathological, localized sac-like outpouching of the arterial wall, whose rupture is the leading cause of nontraumatic subarachnoid haemorrhage, associated with a high rate of morbidity and mortality and a significant economic burden [3]. The increased frequency at which unruptured IAs are being diagnosed, due to the widespread use of cross-sectional neuroimaging in routine clinical practice, poses a persistent dilemma for physicians. This is imputable to the lack of definitive guidelines for optimal management, which is due to the high prevalence of aneurysms along with low rupture rates (with the annual occurrence of subarachnoid haemorrhage being about 10 per 100.000 persons [4]), and preventive treatment carrying risks of adverse complications [5].

Aberrant vascular remodelling occurring through abnormal hemodynamic stress on blood vessels is believed to be a major factor in intracranial aneurysms pathophysiology, i.e., formation, growth, and stabilization or rupture [6]. The stress distribution, as determined by the blood flow and aneurysm geometry, elicits vascular remodelling via cell-mediated biologic pathways. This modifies the geometry, the stress, and drives further biologic processes, with rupture occurring when the stress on the aneurysm wall exceeds the yield strength of the material [7, 8]. This biomechanical approach has proven relevant in assessing rupture risk, with hemodynamic indices such as flow-induced pressure (the stress normal to the vascular wall) and wall shear stress (WSS, the viscous frictional force exerted parallel to the blood flow) identified as potentially significant determinants of aneurysm natural history [9-12].

Preventive treatment of unruptured intracranial aneurysms consists in occluding the sac to prevent blood from flowing directly into the aneurysm, which in turn helps reduce the stress on its wall. The two main options have long been surgical clipping and coiling [13]. Clipping is invasive, as it requires performing a craniotomy and exposing the aneurysm before placing surgical clips across the neck. While highly effective, clipping is constrained to easily accessible aneurysms and operations generally bear a substantial complication risk. Endovascular procedures, such as stenting and coiling, minimize the operational risk by avoiding open skull surgery. The latter approach involves filling the aneurysm sac with flexible platinum wires that dampen out ingoing blood jets and contribute towards the occlusion of the bulge. Since the wires are contained by the sac, wide

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neck IA or fusiform IA cannot be treated this way, due to the high risk of embolic disease and coil detachment.

In recent years, the implantation of flow-diverter (FD) stents has gained increasing acceptance among the interventional and neurosurgical communities as an effective alternative treatment option [14]. Such an approach consists in the endovascular deployment of flexible, highly conforming braided mesh devices along the parent artery and across the neck. The blood flow into the aneurysm is damped and redirected by the low porosity layer of FD wires covering the neck, reducing the overall circulation in the sac. The blood stagnation that follows a successful deployment causes a thrombus formation in the aneurysm cavity and a subsequent endothelialization of the neck [15]. In some cases, the completely occluded aneurysm is progressively reabsorbed by the parent vessel, precluding regrowth by hemodynamic mechanisms. Flow diverter stents have started a breakthrough in the endovascular management of intracranial aneurysms (including many wide-necked and fusiform aneurysms that were previously considered untreatable) but their mechanism of action is not thoroughly understood, as about 5 to 25 % of aneurysms remain with circulation even after multiple-layer implantations [16].

A substantial body of work is ongoing to improve aneurysm treatment outcomes by increasing the flow-diversion effect of the implanted stent [17, 18]. The functional performance is largely dependent on implantation (e.g., sizing, landing zone) and geometrical features (e.g., braid angle, wire density, wire diameter) with wire material properties also being an important contributor. Hemodynamically, we believe that the pore density is a key parameter, as it must be high enough to occlude the aneurysm sac satisfactorily [15], but not so high that it would trigger inflammatory remodelling associated with low-WSS values [19]. Nonetheless, there is currently a lack of empirical evidence supporting the superiority of one design over the others, meaning that the type of stent used for each patient is often based on the length of the lesion and the personal preference of the physician (even availability of stock). Therefore, the ability to design novel stent concepts from fast and accurate identification of patient-specific functional parameters is of utmost importance to provide clinical insight, optimize treatment decision-making, and improve prognosis. This has never been done before.

In order to make progress towards this objective, the present study combines multi-physics computational fluid dynamics (CFD) and deep reinforcement learning (DRL) to prove the applicability of such an optimization workflow for patient-specific stent design. On the one hand, CFD has risen to a prominent position in the endovascular research community due to its potential for rupture risk prediction via objective, quantitative, and mechanism-based parameters [20, 21], and its contribution to the design, development and evaluation of endovascular management methods [22, 23]. On the other hand, DRL has been shown to perform with unprecedented efficiency in several areas, e.g., language processing [24], robotics [25, 26], autonomous driving [27], finance [28] or healthcare management [29, 30], including recent inroads in computational biomechanics [31].

The efforts for coupling CFD and DRL are developing rapidly, with a handful of pioneering studies providing insight into the performance improvements to be delivered in shape optimization [32–34] and flow control [35–37]; see [38] for a review. This is largely ascribed to the sustained efforts and commitment of the machine learning community, which has allowed expanding the scope from computationally inexpensive, low-dimensional model reductions [39–41] to complex two- and three-dimensional Navier–Stokes systems [42–50]. Nonetheless, DRL has never been applied to hemodynamics computations (let alone biomedical flow computations in patient-specific geometries), even though we believe the field has matured up to the point where a breakthrough may be in reach for targeted control of unruptured intracranial aneurysms.

RESULTS

Pre-stent hemodynamics. Direct numerical simulations of the three-dimensional, incompressible Navier–Stokes equations performed with the Carreau–Yasuda rheological model of blood are used to investigate two patient-specific models of unruptured intracranial aneurysm. In the first place, the focus is drawn on the geometry labelled A, whose vascular information is provided in Figure 1. It is a side-wall, wide-neck aneurysm of the supraclinoid internal carotid artery (ICA), proximal to the ICA bifurcation into the anterior cerebral artery (ACA) and the middle cerebral artery (MCA); see Figure 1 for provision of the detailed vascular information. The posterior communicating artery (PCoMA) is neglected, as it branches off well past the aneurysm. The ophthalmic

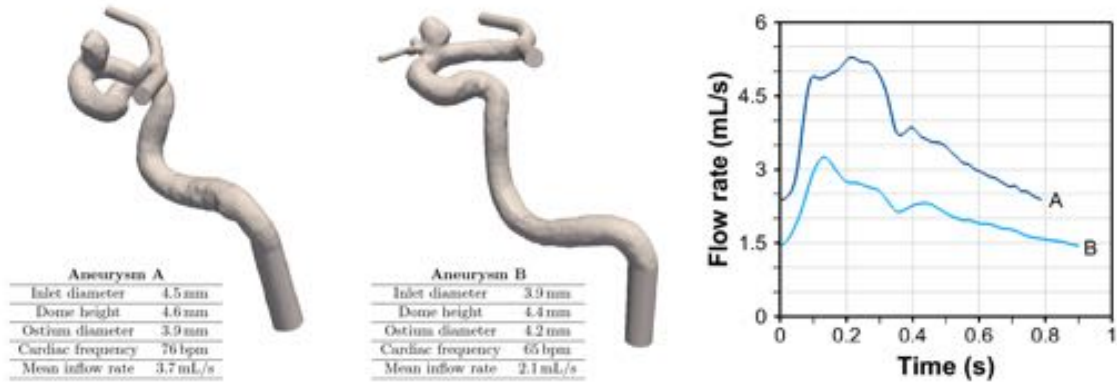


Figure 1. **Patient-specific geometries and datasets for the two ICA aneurysms investigated in this study.** The rightmost plot provides one period of the patient-specific inflow pulses used for the CFD simulations. The range of Reynolds numbers in the parent vessel (based on the inlet diameter, inlet velocity and infinite-shear rate viscosity) is 220-490 for patient A, and 160-370 for patient B (resp. at diastole and systole).

96 artery (OA) is also neglected, although it branches off at this section of the ICA, in the vicinity
 97 of the aneurysm. Nonetheless, we do not anticipate any significant effect on the hemodynamics
 98 given its patient-specific smallness (~ 0.5 mm in diameter), plus this outflow is often neglected in
 99 numerical simulations as it is a common clinical practice to let flow diverters occlude it if no other
 100 viable option is present [51]. The simplified model therefore ultimately features a single source of
 101 inflow (the ICA) and two outflows (ACA/MCA).

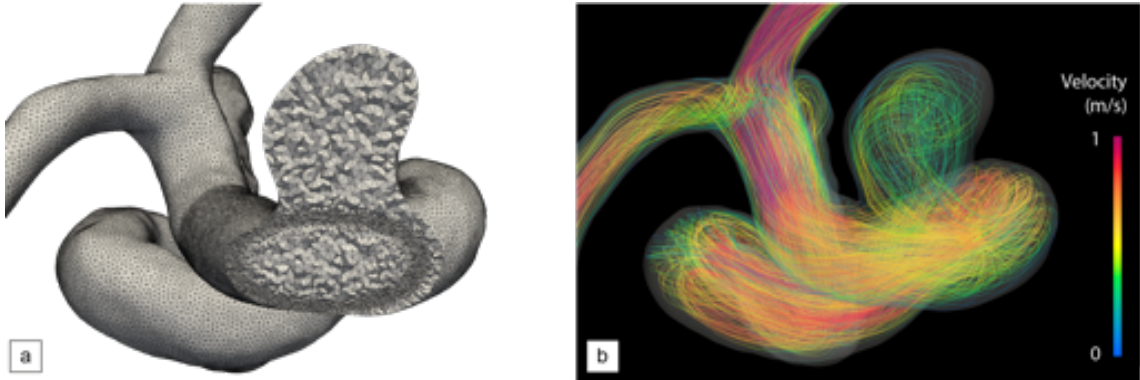


Figure 2. **Pre-operative hemodynamics of aneurysm A.** (a) Cut view of the employed anisotropic tetrahedral mesh. (b) Velocity streamlines at systole.

102 The vessel walls, taken to be impermeable and rigid, are treated with body-fitted, unstructured
 103 adapted grids (see Figure 2a). The numerical solutions are customized to the patient (also labelled
 104 A) specific physiology using vascular geometries reconstructed from three-dimensional rotational
 105 angiography (3D-RA) images and pulsatile volumetric inflow rates adjusted to two-dimensional
 106 phase-contrast magnetic resonance imaging (2D-PCMRI) measurements. All quantities of interest,
 107 including velocity, WSS (the local, instantaneous, intra-saccular wall shear stress of pivotal impor-
 108 tance in this context), SAWSS (the instantaneous WSS spatially averaged over all intra-saccular
 109 positions), and TAWSS (the local WSS averaged over a cardiac cycle), are computed from parallel
 110 hemodynamics simulations run over two cardiac cycles (representing approximately 1.6 s of physi-
 111 cal time) with a reference inflow sequence of consecutive pulses starting in the end-diastolic state.
 112 This is because the solution has settled into regular, sinusoidal oscillations by the end of the first
 113 cycle, as obtained from preliminary comparison of WSS data over up to ten cardiac cycle.

114 The peak-systolic streamlines in Figures 2 and 3 show that the flow remains close to parabolic
 115 in the inflow segment (hence reminiscent of Hagen-Poiseuille flow), but quickly becomes helical

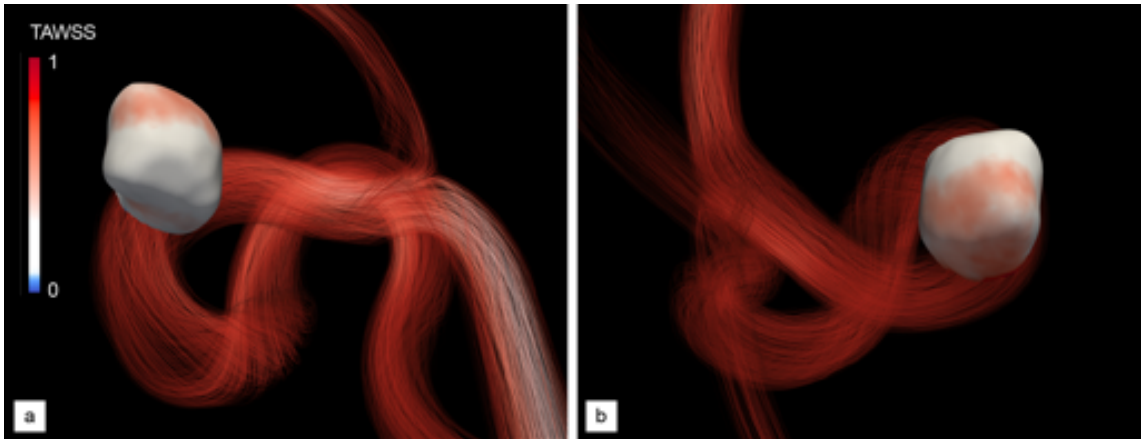


Figure 3. **Pre-operative normalized TAWSS of aneurysm A.** (a) Proximal view. (b) Distal view. The color scale has been adjusted to emphasize low/high WSS areas associated with remodelling and rupture risk [52].

116 because the curved vessel geometry acts as a source of flow instability, as recently assessed in
 117 patient-specific geometries [53]. Most of the blood enters the aneurysm at the proximal part of the
 118 neck in the form of a high-speed jet (~ 0.73 m/s in velocity magnitude), impinging on and reflecting
 119 off the aneurysm wall, rolling up into complex vortical structures and finally swirling out of the
 120 bulge and to the outflow segments. This creates a strongly heterogeneous WSS pattern, with most
 121 of the distal part of the dome sustaining high WSS classically associated with aneurysm growth
 122 and rupture, as clearly illustrated in Figures 3a,b. Maximum local, instantaneous WSS values of
 123 more than ~ 169 dyne/cm² have been measured near the impaction zone, which is about 8 times
 124 normal WSS in cerebral arteries [52].

125 **Virtual stenting.** Endovascular treatment is modeled by wrapping a distribution of identi-
 126 cal, cylindrical wires around a toroidal envelope inscribed in the arterial segment containing the
 127 aneurysm (half of them clockwise and the other counter-clockwise). In order to achieve hetero-
 128 geneous functional parameters (in the sense that the pre-deployment stent structure must have
 129 variable pore density and porosity), the proximal end section of the envelope is divided into four
 130 quadrants, each of which with a specific (possibly different) number of uniformly distributed wires.

131 The parametrization foresees the modification of six design variables: the number of wires in each
 132 group, their radius and a winding factor (the same for all wires) that controls the local braiding
 133 angle between wires. Given the difference in scales between the vascular vessels (about a few mm
 134 in diameter) and the stent strut thickness (about a few ten μm), we rely on a hybrid meshing
 135 approach wherein the stents are embedded in the body-fitted vascular grid [54, 55]. Anisotropic
 136 adaptation in the vicinity of the stent envelope, as shown in Figure 2a. We then use the monolithic
 137 immersed volume method (IVM [56]) together with anisotropic mesh adaptation in the vicinity of
 138 the stent envelope to solve the interaction between the blood flow and the stent material. This
 139 allows easy handling of any complex device whose struts may be in contact with, or form very
 140 small gaps with the vessel walls, without additionally conforming the vascular grid to the stent
 141 geometry.

142 The proposed approach is first used to assess the ability of the method to capture numerically the
 143 flow diversion effect using a standard homogeneous stent strut distribution made of 24 wires (12 in
 144 each braiding direction, 3 in each group) with a radius set to $60 \mu\text{m}$. We believe this is a reasonable
 145 compromise between desirability and feasibility, as thinner wires mirroring more accurately those
 146 of real medical stents (whose radii are in a range from about 15 to $30 \mu\text{m}$) would escalate the CPU
 147 time and memory requirements (due to the need to embed large stent meshes and to additionally
 148 refine the vascular grids). All wires are braided with winding factor 25, that yields a braiding angle
 149 of 75° and a porosity of about 68.5%, (pore density of 3.1 pores/mm²), all values estimated from
 150 a pre-deployment, cylindrical stent structure.

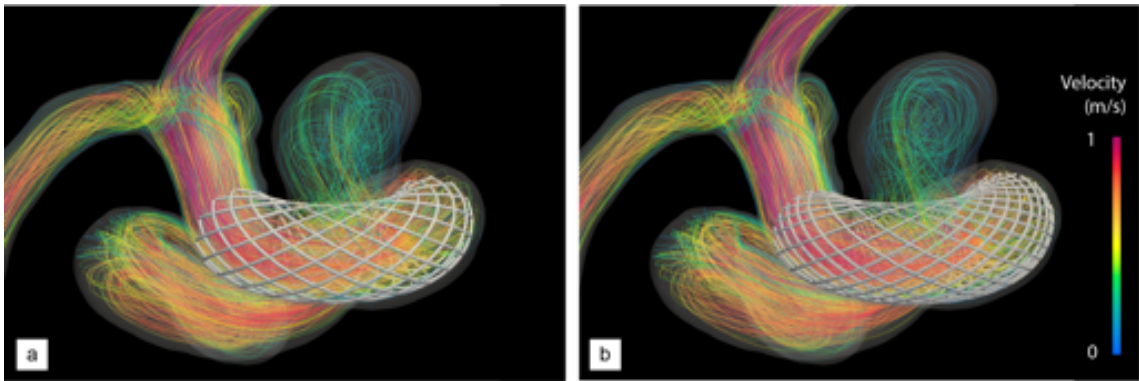


Figure 4. **Post-operative, peak-systolic velocity streamlines of aneurysm A.** (a) After treatment with a homogeneous stent made of 24 wires. (b) After treatment with a 34 wire, non-uniformly braided stent. The stent design is the result of the optimization provided in the following sections (see DRL optimization).

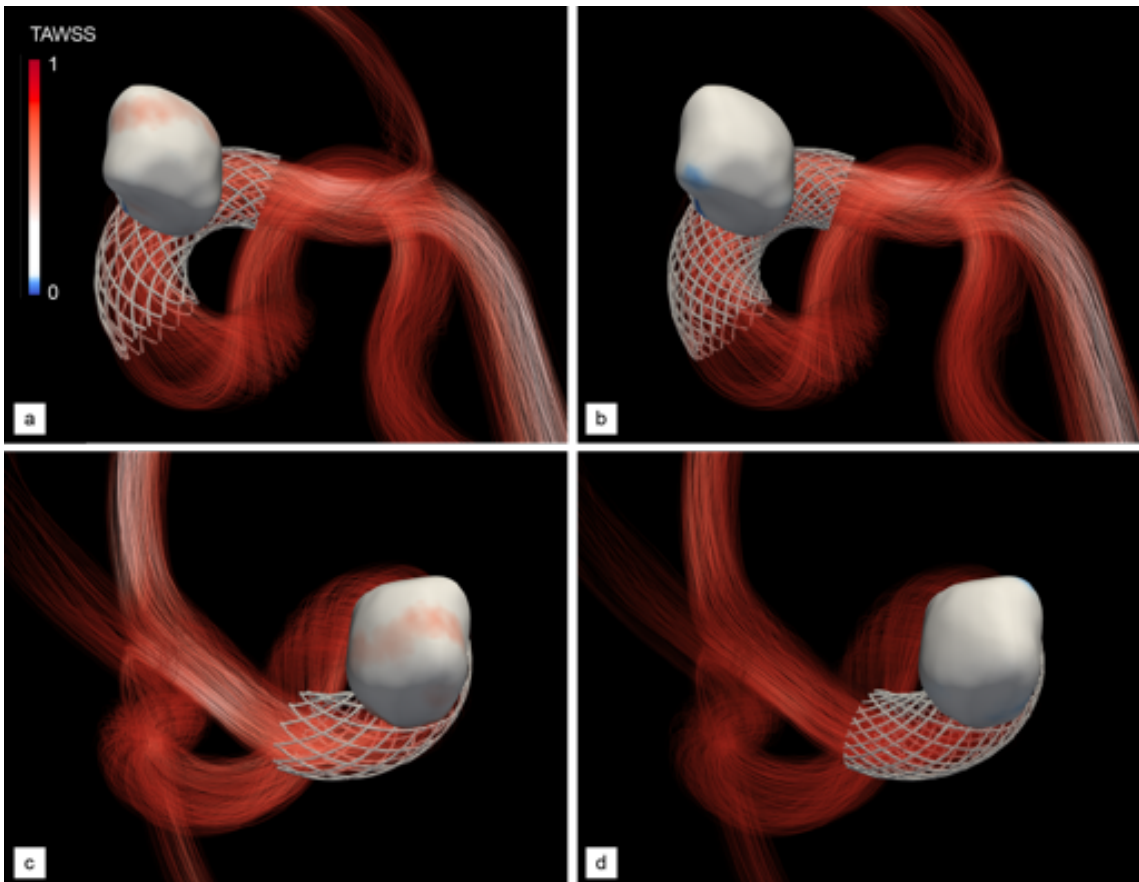


Figure 5. **Post-operative, normalized TAWSS of aneurysm A.** (a,c) Proximal and distal views after treatment with a homogeneous stent made of 24 wires. (b,d) Same as (a,c) after treatment with the optimal, non-homogeneous stent provided by our DRL framework, made of 34 non-uniformly braided wires (see Figure 6d). Values are normalized based on the maximum TAWSS encountered in the pre-operative configuration (see Figure 3).

151 Visualizations of the inflow jet, intra-saccular flow pattern (Figure 4a) show that the stent sig-
 152 nificantly disrupts the blood flow, more of which is diverted away from the aneurysm and flows
 153 directly to the outflow segments. Nonetheless, the flow organization inside the aneurysm is essen-

154 tially reminiscent of its unstented counterpart, with blood entering at the proximal part of the neck
 155 and swirling in and out after impinging on the distal wall. The key difference lies in the inflow jet
 156 having lower velocity (about 0.60 m/s in velocity magnitude) and weaker shear, hence less vorticity,
 157 and ultimately less shear stress on the aneurysm wall. This is further illustrated by the TAWSS
 158 distributions in Figures 5a,c, where the instantaneous WSS peaks at about ~ 120 dyne/cm². This
 159 represents a reduction of about 30% with respect to the unstented case illustrated in Figure 3,
 160 although we notice the persistence of a heterogeneous WSS pattern over the distal part of the
 161 dome.

162 **DRL optimization.** The optimization objective considered herein consists of bringing back
 163 the post-operative value of MWSS (defined as the maximum of SAWSS over a full cardiac cycle)
 164 to a setpoint of half the pre-operative value, hence the reward

$$r = -|\text{MWSS} - \text{MWSS}_{\text{ref}}| \quad \text{with} \quad \text{MWSS}_{\text{ref}} = \frac{\text{MWSS}_0}{2}, \quad (1)$$

165 where the 0 subscript denotes a pre-stent quantity. This choice is intended to reduce high WSS
 166 associated with aneurysm growth and rupture, while avoiding low WSS conditions that might
 167 initiate apoptotic pathways via undesired vascular remodelling [19]. In practice, the MWSS₀ for
 168 patient A is 76.6 dyne/cm² (roughly half the maximum local, instantaneous value reported above),
 169 which yields a setpoint of 38.8 dyne/cm².

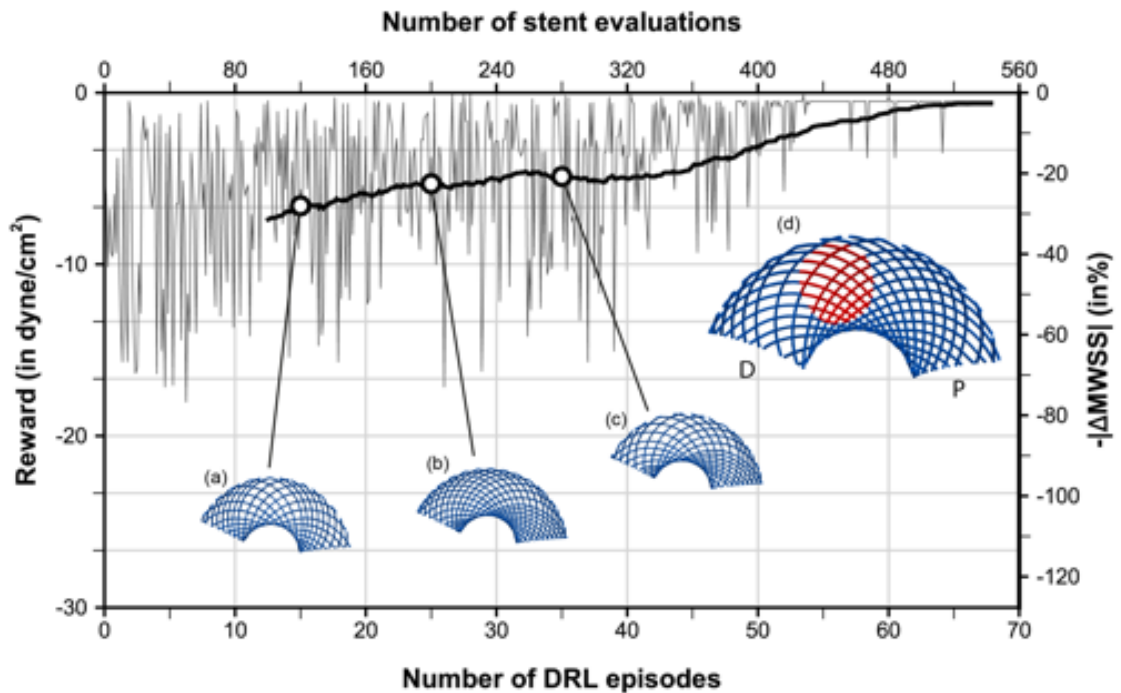


Figure 6. **Stent optimization along with deep reinforcement learning for aneurysm A.** The fine line represents the evolution per episode of the instant reward, while the thick line is the moving average reward computed over the 100 latest values. The right vertical axis presents the relative variations of MWSS with respect to the setpoint of half the pre-operative value. Representative stents generated over the course of optimization are superimposed, with the stent generated at episodes 15, 25 and 35 shown in (a-c) and the optimal stent predominantly generated after episode 55 shown in (d). The P/D labels indicate the proximal and distal end sections of the stent, whose upper half is displayed from below (longitudinal cut) to single out the region covering the aneurysm neck (shown by the red patch in (d)).

170 Single-step PPO, a reinforcement algorithm intended for situations where the optimal policy is
 171 independent of state [48], is used to evolve five (out of the six) design parameters, as the wire radius
 172 has been set to 60 μm to keep the computational cost affordable, although it is a free parameter
 173 that could also be learnt from data. For each learning episode, the DRL agent (a deep neural

network) therefore outputs five discrete values: four values $\{n_{j \in \{1..4\}}\}$ between 3 and 7 in step of 1 for the numbers of wires (one per set of equally spaced wires) and one value k between 20 and 35 in steps of 5 for the winding factor, hence 2500 parameter combinations comprising between 24 and 56 wires in total. The generated stent configurations have nominal braiding angles in a range from 65° to 95° and porosities in a range from 30.5% to 68.5% (pore densities between 3.1 and 22.7 pores/mm²). all values estimated from pre-deployment, cylindrical stent structures. The reward evaluation proceeds from hemodynamics simulations run over two cardiac cycles, with MWSS computed over the second cycle, after which the network is updated for 32 epochs using 8 environments and 2 steps mini-batches.

A total of 68 episodes have been run for this case, which represents 544 simulations, each of which lasts 20 min using 32 cores, hence 5,760 h of total CPU cost (equivalently, 45 h of resolution time). A moving average reward is also computed as the sliding average over the 100 latest values to assess convergence a posteriori (see Figure 6). The reward convergence history in Figure 6 evidences the successful convergence of the PPO algorithm coupled with patient-specific hemodynamic simulations. After 50 episodes (representing 400 simulations, hence 400 out of the 2500 possible designs), the DRL agent indeed starts to systematically pick the specific stent shown in Figure 6d, whose red patch singles out the region of interest located in front of the aneurysm neck. The latter is made of 34 wires (17 in each braiding direction, distributed into four groups of 5, 3, 5 and 4 wires, respectively) braided with winding factor 25. This yields in a nominal average porosity of 55%, with pore densities (in deployed state) ranging between 2.3 to 16.0/mm² in the neck region facing the aneurysm.

Such a design is meant to be optimal for the patient specific aneurysm geometry and pulse, which is assessed now by comparing numerically the post-operative hemodynamics treated with the optimal stent designed by DRL (that earns a MWSS of 37.8 dyne/cm² differing from the intended setpoint by 1%) and with the standard homogeneous stent considered so far (that earns a MWSS of 54.7 and is clearly inferior for the chosen reward). The peak-systolic streamlines in Figure 4 illustrates the different flow deviation effect of the optimal stent. The latter successfully and adequately cuts down the inflow jet (that was inducing high WSS values on the distal part of the bulge, whose velocity magnitude is now about 0.47 m/s) while substantially altering the intrasaccular flow organization, found to involve fewer vortex structures, more parallel streamlines, and less swirling. We note that the optimal stent also substantially reduces the blood velocity at the exit of the aneurysm: 0.26 m/s, to be compared to 0.35 m/s without stent and using the standard, homogeneous stent. The result on the WSS distribution is even more patent, as Figure 5 shows that the DRL stent has completely eliminated the unstented area of maximum WSS (the local, instantaneous WSS now peaks at about 91 dyne/cm², which represents a reduction of about 45% with respect to the unstented case.) while restoring an almost homogeneous WSS pattern, which the homogeneous stent had failed to achieve.

Generalizability study. For the sake of generalization (and in order to assess suitability for various aneurysm configurations), we apply now the DRL framework to a second patient-specific model of untreated, unruptured intracranial aneurysm (labelled B) whose vascular information is provided in Figure 1. It is a saccular, multilobulated aneurysm located on the ICA, at the junction with the ophthalmic artery (OA). The latter is thus retained in the model (as it cannot be cleanly removed), yet not occluded numerically, as we voluntarily let blood flow from the ICA into the OA across the stent to explore the model ability to handle more complex intra-aneurysmal flow conditions.

Following the same steps as for the previous case, the pre-operative hemodynamics of this patient has been analyzed from numerical simulations customized to his/her physiology (both in terms of vascular geometry and inflow pulse) carried out over two cardiac cycles (representing 1.8s of physical time). The peak-systolic streamlines shown in Figures 7a show that the case has many similarities to that of patient A, as the flow quickly becomes helical, and most of the blood enters the aneurysm at the proximal part of the neck in the form of a high-speed jet (0.72 m/s in velocity magnitude), that traverses the entirety of the primary bulge, impinges on its distal wall, rolls up into complex vortical structures and finally swirls out to the outflow segments (including the OA). As illustrated in Figures 8a,c, this again yields a strongly heterogeneous WSS pattern, with high WSS values up to ~ 146 dyne/cm² in the vicinity of the neck and in most of the distal part of the dome, but low WSS in the daughter sac, that turns to be barely exposed to the blood flow environment.

The DRL optimization has then been run in similar fashion over a total of 58 episodes (464 simulations) using a setpoint of 36.3 dyne/cm² (MWSS₀ for this patient is 72.6 dyne/cm²,) and

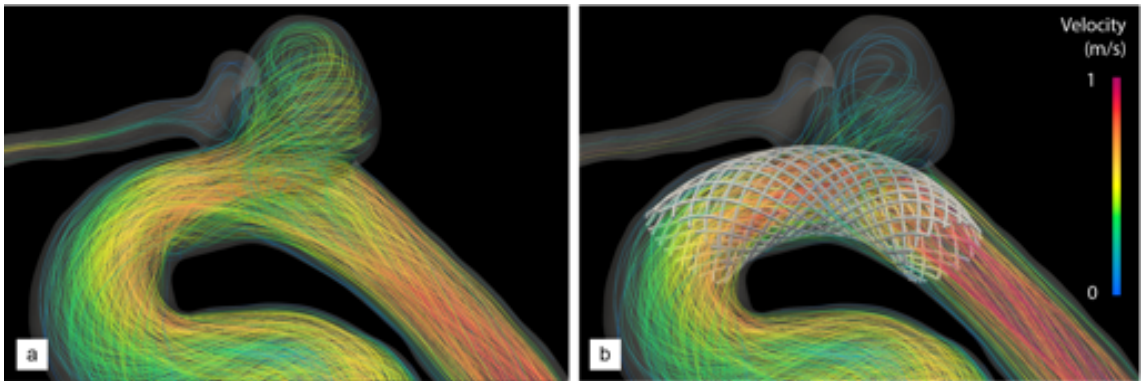


Figure 7. **Velocity streamlines of aneurysm B without and with DRL-optimized stent.** (a) Pre-operative state. (b) Post-operative flow after insertion of the optimal stent (see Figure 9f).

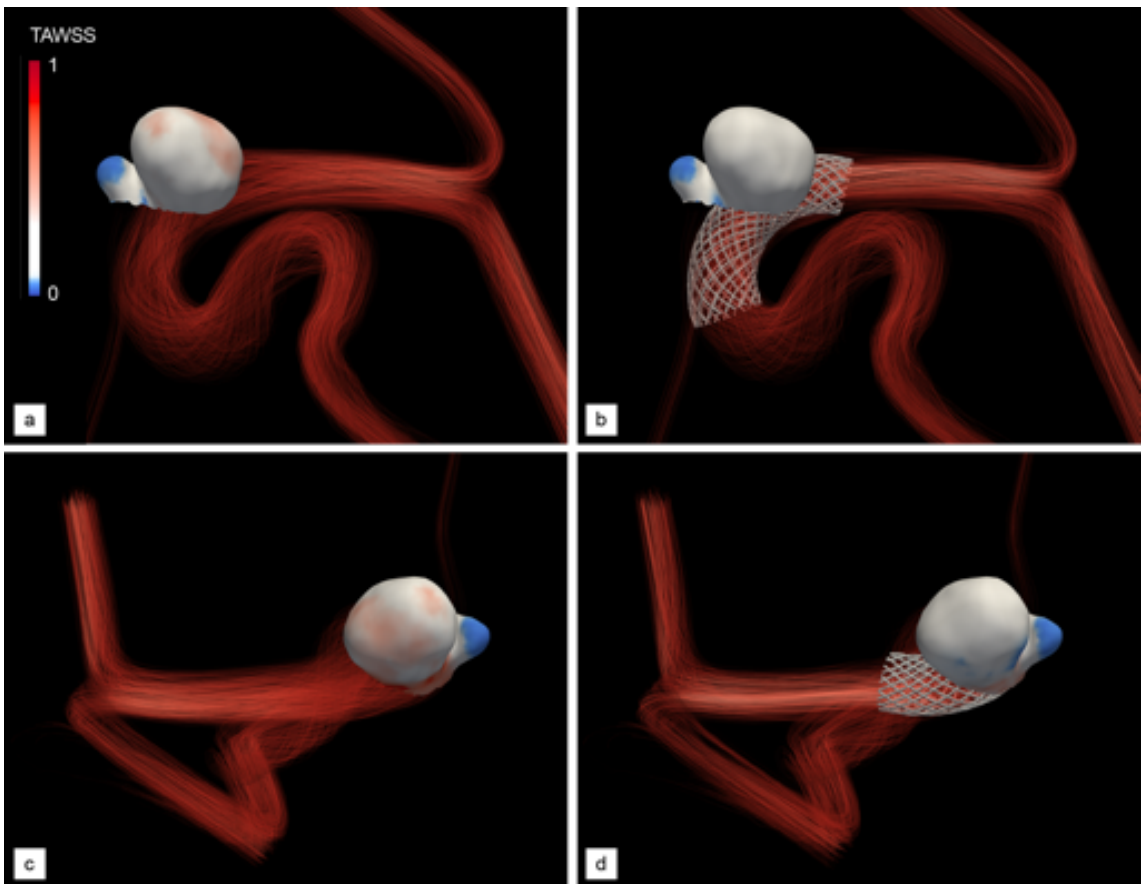


Figure 8. **Normalized TAWSS of aneurysm B without and with DRL-optimized stent.** (a,c) Proximal and distal views in the pre-operative state. (b,d) Same as (a,c) after treatment with the optimal, non-homogeneous stent provided by our DRL framework, made of 30 non-uniformly braided wires (see Figure 9h). Values are normalized based on the maximum TAWSS encountered in the pre-operative configuration (see Figure 3).

232 evaluated after convergence (accounted for when the agent outputs a majority of one specific design
 233 over several episodes). The reward convergence history in Figure 9 evidences good convergence
 234 after 45 episodes (representing 360 simulations, hence 360 out of the 2500 possible designs). The
 235 DRL agent then starts to sample the specific stent shown in Figure 9d, whose red patch singles
 236 out the region of interest located in front of the aneurysm neck. The latter earns a MWSS value

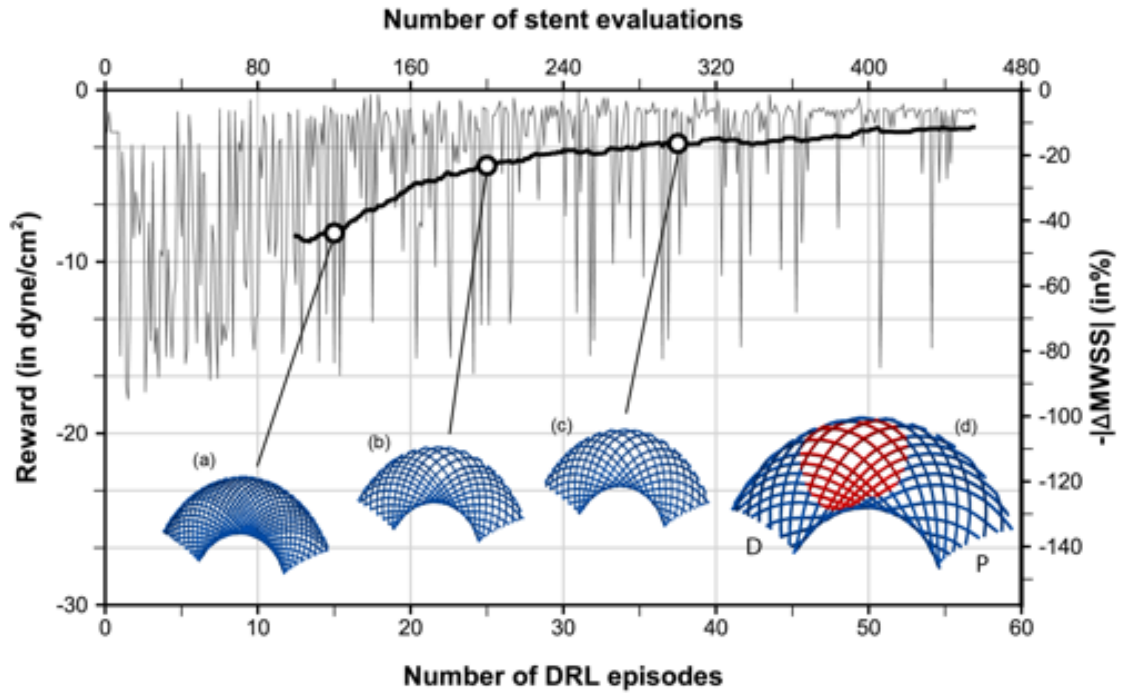


Figure 9. **Stent optimization along deep reinforcement learning for aneurysm B.** The fine line represents the evolution per episode of the instant reward, while the thick line is the moving average reward computed over the 100 latest values. The right vertical axis presents the relative variations of MWSS with respect to the setpoint of half the pre-operative value. Representative stents generated over the course of optimization are superimposed, with the stent generated at episodes 15, 25 and 35 shown in (a-c) and the optimal stent predominantly generated after episode 45 shown in (d). P and D annotations indicate the proximal and distal end sections of the stent, respectively.

237 is 37.2 dyne/cm^2 , which differs from the intended setpoint by 3%. It is made of 30 wires (15 in
 238 each braiding direction, distributed into four groups of 5, 3, 4 and 3 wires, respectively) braided
 239 with winding factor 25. This yields in a nominal average porosity of 61%, with pore densities (in
 240 deployed state) ranging between 4.1 to $11.2/\text{mm}^2$ in the neck region facing the aneurysm. We note
 241 that the convergence is slightly less good than for patient A, which may be because the neck of
 242 aneurysm B is larger, but the same ranges of design parameters have been used for both patients.
 243 This offers more leeway by allowing more threads to fit into the region of interest of patient B,
 244 which, combined to the fact that only a small number of discrete winding factors are evaluated,
 245 increases the sharpness of the reward, known to be detrimental to the conservative policy updates
 246 of the PPO algorithm [48].

247 Finally, the efficiency of this design (in fact different from the optimal determined for patient
 248 A) has been assessed by comparison of the pre- and post-operative hemodynamics computed after
 249 treatment with the optimal stent. The peak-systolic streamlines in Figure 7b show that the blood
 250 flow is adequately diverted away from the neck and into the parent vessel. The blood velocity is
 251 reduced at the entry (0.38 m/s in magnitude) but also at the exit of the aneurysm (0.23 m/s , to
 252 be compared to 0.43 m/s without stent), while swirling is essentially suppressed, as the inflow jet
 253 now merely slides along the distal wall. This considerably reduces the high distal values of WSS,
 254 as the maximum local, instantaneous WSS is now about 80 dyne/cm^2 (again a reduction by about
 255 45% with respect to the unstented case). We also note an almost homogeneous WSS pattern is
 256 restored in Figures 8b,d, save for the low WSS region in the daughter sac, meaning that the latter
 257 could still grow on its own via inflammatory and apoptotic remodelling.

DISCUSSION

258

259 **Optimal stents feature a gradient of porosity.** Reported results highlight the potential
 260 of DRL shape optimization for endovascular stenting of intracranial aneurysms. It should be
 261 emphasized the optimal designed generated by our DRL agent, all relative to the chosen reward
 262 functions, feature varying porosities in the region of interest (ROI) facing the aneurysm neck. This,
 263 by itself, could represent a breakthrough in the stent manufacturing industry, where such designs
 264 do not yet exist (to date, variable porosity can be achieved locally only by superposition of several
 265 flow-diverters, which increases the risk and the cost of the surgery). More importantly, the optimal
 266 porosity gradient differs for both aneurysms, which paves the way for developing novel devices
 267 tailored to the patient specific aneurysm, including (but not limited to) its geometry and pulse.

268 The results show that the optimal stents successfully cut down the blood velocity at the entry and
 269 the exit of the aneurysm while also altering the swirling flow inside the aneurysm, either by subtly
 270 modifying the swirling direction (patient A), or by suppressing swirling altogether (patient B).
 271 Nonetheless, the complexity of the correlation between local porosity distribution, flow deviation
 272 and hemodynamics makes it difficult to unravel the exact physical mechanism behind the efficiency
 273 of this or that design, although it is common knowledge that the stent must allow blood in and out to
 274 avoid the occurrence of too-low WSS values, while sufficiently impeding the blood flow associated
 275 with the highest values of WSS in the aneurysm. From this perspective, the DRL approach is
 276 beneficial in two important respects: first, it is efficient, even though the parameter spaces are large
 277 and it may be costly to identify optimal designs from simple parametric searches. Second, and more
 278 significantly, it succeeds in discovering optimal designs from unforeseen parameter combinations,
 279 without any priori knowledge or assumptions about hemodynamics concepts.

280 **CFD modelling assumptions and limitations.** Computational blood flow modelling in
 281 intracranial aneurysms has tremendous potential, yet limited applicability in a clinical context
 282 because of the simplifying assumptions that are traditionally (and often implicitly) made [57,
 283 58]. Chief among them is the fact that walls are almost always assumed rigid, while arteries
 284 are compliant vessels, i.e., they deform under the shear stress of blood flow, with possibly large
 285 displacements impacting the WSS estimates (the authors in [59] report 10-30 % WSS reductions
 286 compared to rigid wall simulations). A two-way coupled fluid-structure interaction (FSI) analysis
 287 is thus necessary to solve accurately the mechanical exchanges between the blood flow and the
 288 arterial tissue, while also encompassing the stent deformation occurring under load conditions (by
 289 the blood flow and/or the arterial tissue), another important factor that may alter the porosity at
 290 the neck and impair the long-term efficiency [60, 61]. One ongoing debate regarding the need to
 291 include the effects of compliance comes down to whether the uncertainties or inaccuracies in the
 292 data needed to model its effects may mask any perceived benefit of doing so: on the one hand, it
 293 has been acknowledged that improved computational models should incorporate patient-specific,
 294 spatially varying wall thicknesses, as uniform wall properties and thicknesses based on literature
 295 values will fail to represent inter- and intra-individual variations [62, 63]. On the other hand, it
 296 is feasible to measure individual wall properties by imaging and inverse modelling techniques, but
 297 such non-linear analyses introduce substantial uncertainties, for instance imaging can distort wall
 298 thickness measurements in a way that can be difficult to detect or correct [64]. In this regards, it is
 299 reasonable and expedient to use a rigid wall model for the present purpose of showcasing the use of
 300 DRL techniques for image-based CFD hemodynamics optimization (without any consideration of
 301 being directly applicable real medical cases), while leaving to future research to more fully address
 302 this issue and close the methodological gap of providing high-fidelity hemodynamic data. Finally,
 303 achieving the fine deployment of the stent in the arterial vessel was not in the scope of this study. A
 304 realistic deployment, along with a more versatile stent parametrization would expand the possible
 305 configurations and surely lead to fascinating results, which could drastically impact the medical
 306 community.

307 **DRL reward function.** There are two main aspects worth discussing regarding the DRL
 308 reward function used herein. First, the design of a feasible reward function is one of the challenges
 309 in reinforcement learning problems, but one that is barely discussed in the available literature. In
 310 the absence of best practice guidelines, it is essentially a trial-and-error exercise, with a human
 311 expert defining an initial reward function based on his/her knowledge of the problem, observing
 312 how the agent performs, then tweaking the reward function to achieve greater performance. We
 313 use here a reward function aligned with the objective function, meaning that when the agent

314 is learning to maximise this reward, it is also learning to minimize the distance between the
 315 post-stent maximum value of MWSS over a cardiac cycle and the setpoint of half the pre-stent
 316 value. This allows reducing WSS while preventing the occurrence of very low WSS values, which is
 317 consistent with the expected outcome of a stenting operation (in the absence of further quantitative
 318 information or reduction objectives). A more sophisticated approach to pursue in future work could
 319 be to force the WSS to remain in a physiological range (that could be defined from patient-specific
 320 data) at every point in the bulge, using for instance a local reward function defined as

$$r = \int_S r_{loc}(\mathbf{x}) ds \quad \text{with} \quad r_{loc}(\mathbf{x}) = \begin{cases} \text{WSS}(\mathbf{x}) - \text{WSS}_{inf} & \text{if } \text{WSS}(\mathbf{x}) < \text{WSS}_{inf}, \\ 0 & \text{if } \text{WSS}_{inf} \leq \text{WSS}(\mathbf{x}) \leq \text{WSS}_{sup}, \\ \text{WSS}_{sup} - \text{WSS}(\mathbf{x}) & \text{if } \text{WSS}_{sup} < \text{WSS}(\mathbf{x}). \end{cases} \quad (2)$$

321 Second, the reward uses MWSS as the sole predictor of aneurysm rupture, which implicitly
 322 assumes that a brief exposure to extended regions of high WSS is key towards predisposing the
 323 aneurysm wall to weakening and rupture. On the one hand, this suffices to lay the foundation for
 324 future research in this field, given the wide acceptance of WSS as a key factor in the physiological
 325 and pathological response of cerebral arteries. On the other hand, a gap of knowledge remains
 326 on this issue (for instance, both high or low WSS have been separately correlated to aneurysmal
 327 formation and growth [9, 19, 20, 65, 66]), and enriched reward functions (encompassing the time
 328 and space-dependent influence of blood dragging at the aneurysm wall, both in magnitude and in
 329 direction) are likely needed to improve clinical relevance. In this regards, it is worth insisting that
 330 the presented framework is highly generalizable, in the sense that it can assess new concepts of flow-
 331 deviator stents with respect to any or any combination of the markers of disturbed blood flow that
 332 have surfaced in recent publications (WSS gradient, oscillatory shear index, relative residence time,
 333 to name a few), that reflect different assumptions being made about the hemodynamic conditions
 334 driving the progression of intracranial aneurysms toward rupture [53, 67-70]. This falls under the
 335 scope of multi-objective DRL for which there are two main approaches. The most common way is
 336 to use a linear function to transform the multi-objective problem into a standard single-objective
 337 problem. Another interesting (but very costly) strategy is to explicitly separate the individual
 338 components of the reward function, in order to better understand the policy trade-off (the related
 339 methods, based on the Pareto optimum, are not yet frequently applied to DRL problems).

340 **DRL algorithm.** Future work should aim at further improving the flexibility of the proposed
 341 framework by allowing more realistic stent geometries (in terms of wire radius, number of wires),
 342 thus increasing the number of possible stent designs. Having a more continuous optimization space
 343 (by increasing the number of winding factors) will also undoubtedly improve the convergence of
 344 the PPO algorithm. From this standpoint, it should be emphasized that this is a proof-of-
 345 concept study and that convergence and efficiency (i.e., the number of stent designs that need to
 346 be evaluated to reach convergence) could be accelerated by hyper-parameter tuning or using pre-
 347 trained deep learning models (as is done for instance in transfer learning). Generally, the rather
 348 simplistic PPO framework could be substituted by a more elaborated algorithm, for instance Policy-
 349 based Optimization (PBO) [71], another single-step reinforcement algorithm that samples actions
 350 from full covariance matrices, and is theoretically better suited to represent higher order logic and
 351 to handle complex parameter interactions.

352 **Future research directions.** The purpose of this study is to lay out the foundation for future
 353 research in this field. We anticipate that large-scale studies with long-term follow-up will allow
 354 developing more reliable risk-prediction models. As envisioned by Meng *et al.* [19], we picture that
 355 intracranial aneurysms could be sorted into different categories associated with different predictors
 356 reflecting different growth and rupture mechanisms (say, high WSS and positive WSS gradient
 357 for narrow-necked aneurysms vs. low WSS and high fluctuations of WSS orientation for wide-
 358 necked aneurysms), at which point a high-fidelity DRL-CFD hemodynamics framework accurately
 359 modelling the elastic deformation of the parent artery will be instrumental in providing clinically
 360 relevant, patient-specific stent designs (except for the unpredictable delivery manipulations and
 361 variations of vessel geometry occurring during the intervention that still might impact the stent
 362 implantation). By then, it is reasonable to expect that further developments in the fast-moving field
 363 of deep reinforcement learning will allow for faster convergence and lesser execution load (using,
 364 e.g., auto-encoders and systematic state compression, or on-the-fly generation of surrogate models
 365 with uncertainty level prediction). This should set up a framework fast enough to inform design in a

366 matter of hours rather than days, which in turn will reliably augment the current clinical diagnostics
 367 capabilities. Another reason to push DRL forward in this context is the ability of neural networks
 368 to transfer knowledge from previous experiences, to quickly adapt to different environments (i.e.,
 369 different patient-specific numerical models of intracranial aneurysms, corresponding to new patients
 370 in practical applications) and effectively learn new tasks (i.e., different rewards, to achieve further
 371 refinement of risk prediction). We expect that this will be a key feature to reduce learning time and
 372 improved neural network performance, as progress are made towards realizing the clinical utility
 373 of CFD for assessment of intracranial aneurysm rupture.

374 METHODS

375 **Clinical and Imaging data.** Images obtained from 3 Tesla MRI (magnetic resonance imag-
 376 ing) and 3D-DSA (digital subtractiob angiography) to create the adequate geometry for the simu-
 377 lation and the pulse to impose the flow. All images have been acquired at the University Hospital
 378 - LMU Munich.

379 **Stent model.** Virtual stenting relies on a naive stent generator inspired by [72], in which $2n$
 380 wires are wrapped around the toroidal envelope parametrized by

$$((r \cos \theta + R) \cos \frac{s}{R}, (r \cos \theta + R) \sin \frac{s}{R}, r \sin \theta), \quad (\theta, s) \in [0; 2\pi] \times [0; l], \quad (3)$$

381 where r and R are the minor and major radii of the torus, and l is its centerline length. The
 382 wire centerlines follow hhelical curves generated from a circular basis, that in turn provides the
 383 scaffold for the struts. Circular profiles are then extruded along the splines to generate the final
 384 wires with diameter $d = 60 \mu m$. Nominal heterogeneous functional parameters (yet homogeneous
 385 within a given group of wires) functional parameters, e.g., braiding angle, porosity (the percentage
 386 ratio of the wire-free surface area) and pore density (the number of pores per unit surface area)
 387 are obtained by braiding two by two parallel wires from n_j initial positions uniformly distributed
 388 in each quadrant of the proximal end section (labelled counter-clockwise, adjusting the origin of
 389 azimuthal angle for the first quadrant of the cylinder to be mapped into the upper outer quadrant
 390 of the torus). In practice, all wire paths are actually computed under a slowly varying envelope
 391 approximation using

$$r(s) = r_{prox} \left(1 - \frac{s}{l}\right) + r_{dist} \frac{s}{l}, \quad (4)$$

392 to fit the weak variations in the minor radius caused by the irregular patient-specific vascular
 393 geometry. This is because the variations for the cases documented herein (by about 11% relative
 394 to the average value) have been found to be well modelled by affine transformations, but more
 395 complex analytical functions can be specified as well. Examples of generated stents are given on
 396 Figure 10.

397 **Computational domain and mesh.** The medical imaging data of both patients is segmented
 398 using the 3D Slicer software. The entire proximal portion of the parent artery visible in the
 399 images is reconstructed, for which 3D Slicer outputs point coordinates and connectivity of the
 400 centerline, together with the corresponding vessel radius (as defined by the minimal distance from
 401 the centerline to the vessel boundary) and curvature. A non-shrinking filter [73] is used as an
 402 additional step of shape regularization to obtain smooth surface triangulation of the aneurysm
 403 lumen and connected vessel walls (which helps mitigate the effect of inner surface roughness).
 404 All vessels are truncated at some distance from the aneurysm bulge and extended with straight
 405 cylindrical pipes closed perpendicularly to their axis, to allow for flow development and ease the
 406 subsequent application of inflow/outflow boundary conditions.

407 For each patient, three-dimensional unstructured isotropic meshes of the vascular domain and
 408 stent devices are generated with the Gmsh software [74], after which the vascular grid is finely and
 409 anisotropically refined with the method described in [75]. This allows evaluating any stent design
 410 sampled by DRL on the same vascular mesh, and thus yields considerable saving in the overall
 411 computational time. For the stent devices, 4 mesh points are allocated across any wire diameter,
 412 which is a reasonable compromise to assess feasibility while producing qualitative results to build
 413 on, as it yields a number of mesh elements that stands well below the few ten million elements

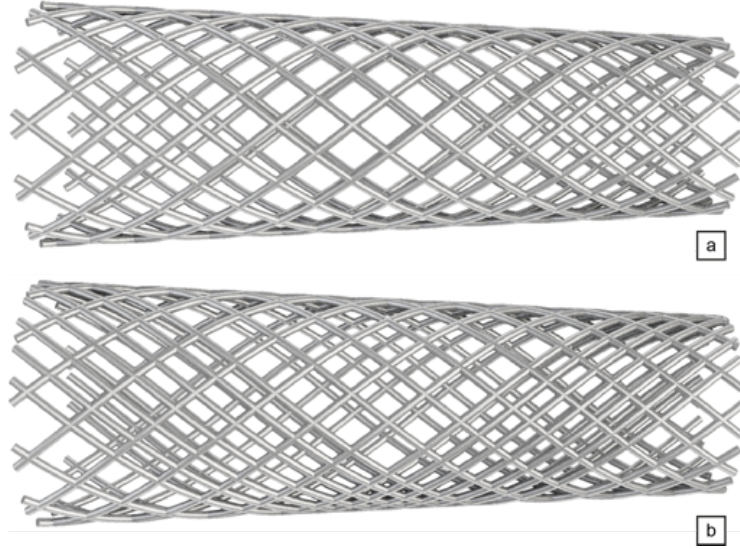


Figure 10. **Stent generation examples.** Homogeneous (a) and heterogeneous (b) braided structures computed under a slowly varying envelope approximation for aneurysm A.

414 reported in previous studies [54, 55]. This in turn keeps the computational cost affordable, which
 415 is mandatory given that optimization requires evaluating the performance of hundreds of stent
 416 designs.

417 **Computational hemodynamics framework.** The blood flow is mathematically modelled
 418 after the three-dimensional incompressible Navier–Stokes equations

$$\nabla \cdot \mathbf{u} = 0, \quad \rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = \nabla \cdot (-p\mathbf{I} + 2\mu\boldsymbol{\varepsilon}(\mathbf{u})), \quad (5)$$

419 where \mathbf{u} is the velocity field, p is the pressure, $\boldsymbol{\varepsilon}(\mathbf{u})$ is the rate-of-deformation tensor, $\rho = 1050$
 420 kg/m^3 is the constant blood density, and μ is the non-Newtonian blood viscosity evaluated from
 421 the Carreau–Yasuda law using zero-shear rate viscosity $\mu_0 = 0.0456$ Pa.s, infinite-shear rate viscosity
 422 $\mu_\infty = 0.00320$ Pa.s, relaxation time $\tau = 10.03$ s, power law index $n = 0.344$ and transition parameter
 423 $a = 1.25$ (all values for a hematocrit of 40 % and a temperature of 37 °C). The instantaneous wall
 424 shear stress whose peak value over a cardiac cycle is used for reward evaluation is computed as

$$\text{WSS} = \frac{3n+1}{4n} \mu \dot{\gamma} \delta_{\text{sac}}, \quad (6)$$

425 where $\dot{\gamma} = (2\boldsymbol{\varepsilon}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{u}))^{1/2}$ is the wall shear rate defined as the second invariant of the rate-of-
 426 deformation tensor, δ_{sac} is a boolean representation of the aneurysm surface (obtained by embed-
 427 ding a portion of the adapted body-fitted grid truncated to remove the extra-aneurysmal domain),
 428 and the prefactor is the Weissenberg–Rabinowitsch correction for shear-thinning effects [76]. Since
 429 the elastic motion of the arterial wall is overlooked as a first approximation, simple open flow con-
 430 ditions are used, that consist of no-slip conditions at the solid nodes, zero-stress outflow conditions,
 431 and pulsatile, parabolic inflow condition

$$\mathbf{u} = \frac{2Q(t)}{\pi r^2} \left(1 - \frac{\|\mathbf{x}\|^2}{r^2} \right) \mathbf{n}, \quad (7)$$

432 where $\|\mathbf{x}\|$ and \mathbf{n} are respectively the distance to the centerline and the normal vector in the inlet
 433 section of the parent artery, and Q is the time-dependent, volumetric flow rate adjusted at each
 434 time step to 2D-PCMRI measurements of the patients cross-sectionally averaged blood velocity
 435 (using linear regression from the two closest data points whenever the simulation and acquisition
 436 times do not coincide).

437 **Variational multiscale modeling.** A stabilized weak form of Eq. (5) is solved with a finite
 438 element variational multiscale method (VMS [77-79]). Such an approach consists in splitting the
 439 solution into coarse and fine-scale components, each corresponding to a different level of resolution.
 440 Only the large scales are fully represented and resolved at the discrete level. The fine scales are
 441 approximated in a way such that their effect into the large-scale equations is modelled after consis-
 442 tently derived source terms proportional to the residual of the resolved scale solution. Exhaustive
 443 details in [80] regarding the derivation of the stabilized formulations lead to the following weak
 444 form for the large scale

$$(\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}), \mathbf{w}) + (2\mu \boldsymbol{\varepsilon}(\mathbf{u}), \boldsymbol{\varepsilon}(\mathbf{w})) - (p, \nabla \cdot \mathbf{w}) + (\nabla \cdot \mathbf{u}, q) = \sum_{K \in \mathcal{T}_h} [(\tau_M \mathcal{R}_M, \mathbf{u} \cdot \nabla \mathbf{w} + \nabla q)_K + (\tau_C \mathcal{R}_C, \nabla \cdot \mathbf{w})_K], \quad (8)$$

445 where (\cdot, \cdot) is the L^2 inner product on the computational domain, $(\cdot, \cdot)_K$ is the inner product on
 446 element K , \mathbf{w} and q are relevant test functions for velocity and pressure, $\mathcal{R}_{C,M}$ are the governing
 447 equations residuals

$$-\mathcal{R}_C = \nabla \cdot \mathbf{u}, \quad -\mathcal{R}_M = \rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) + \nabla p \quad (9)$$

448 and $\tau_{C,M}$ are ad-hoc mesh-dependent stabilization parameters (comparable to local coefficients of
 449 proportionality) defined in [81, 82].

450 We solve Eq. (8) with an in-house VMS solver whose accuracy and reliability is assessed in
 451 a series of previous papers, see [82, 83] for a detailed mathematical formulation of the IVM in
 452 the context of finite element VMS methods, and [84, 85] for applications to non-Newtonian flows
 453 in complex geometry. Equal order, linear interpolation is used for spatial discretization of the
 454 velocity and pressure variables (as the inf-sup condition does not need to be satisfied due to the
 455 additional stabilization terms). Time-stepping is first-order accurate and combines explicit (for the
 456 VMS stabilization parameters), implicit (for the viscous, pressure and divergence terms), and semi-
 457 implicit integration schemes (for the time derivatives, convection terms and VMS source terms,
 458 using backward differentiation formula and Newton–Gregory backward polynomial). The time-step
 459 is set to 0.02 s, which allows distributing 40 and 46 points per cardiac cycle for aneurysm A and B,
 460 respectively. All linear systems are preconditioned with a block Jacobi method supplemented by
 461 an incomplete LU factorization, and solved with the GMRES algorithm, with tolerance threshold
 462 set to 10^{-6} .

463 **Computational hemodynamics framework with deep reinforcement learning.** The
 464 stent design is optimized solving a decision-making problem with reinforcement learning (RL),
 465 a process by which an agent learns to earn rewards through trial-and-error interaction with its
 466 environment. At each turn, the agent observes the state s_t of the environment and takes an
 467 action a_t , that prompts both the transition to the next state s_{t+1} and the reward received r_t .
 468 This repeats until the agent has learnt the succession of actions maximizing its cumulative reward
 469 over an episode (i.e., the reference unit for agent update, best understood as one instance of the
 470 scenario in which it takes actions). In the present context, the environment is a patient-specific
 471 CFD simulation of aneurysm hemodynamics after implantation of flow-diverting stent, that uses
 472 the computational hemodynamics framework described above. The agent is a policy represented
 473 by a deep neural network (a collection of artificial neurons that learns to represent a non-linear
 474 relation between input and output spaces, hence deep RL or DRL) trained with a RL algorithm, as
 475 reviewed in the next sections. The environment and the agent are coupled two-way, as illustrated
 476 in Figure 11: on the one hand, the actions sampled by the DRL agent (a set of five variables
 477 corresponding to four number of wires and a winding factor) are used to generate the stent meshes
 478 immersed in the CFD simulation. On the other hand, the reward function needed by the agent to
 479 learn (here, the maximum value of MWSS) is obtained by post-processing of the CFD data.

480 **DRL agent.** A fully connected neural network is used, whose neurons are stacked in layers,
 481 each of which maps the biased weighted sum of their inputs through an activation function to
 482 produce their outputs and propagate the information forward from the input to the output layer via
 483 “hidden” layers (we use here 2 such hidden layers, each with 4 neurons feeding hyperbolic tangent
 484 activation functions). The network is trained with the single-step PPO algorithm, that learns
 485 a five-dimensional (four numbers of wires plus a winding factor) multivariate normal distribution
 486 whose mean and variance depend on the network weights and biases. Single-step PPO is a variation

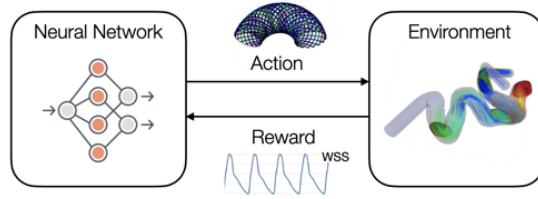


Figure 11. **Sketch of the present DRL-CFD action-loop.** The CFD hemodynamics environment and the DRL agent are coupled two-way through actions and rewards.

487 of the proximal policy optimization algorithm (PPO [25]) intended for situations where the optimal
 488 policy is independent of state, whose relevance for open-loop flow control is assessed in [48]. Just
 489 like PPO, it uses gradient ascent to maximize the surrogate loss

$$\mathbb{E}_{a \sim \pi_\theta} \left[\min \left(\frac{\pi_\theta(a)}{\pi_{\theta_{old}}(a)}, 1 + \epsilon \operatorname{sgn}(\widehat{A}^{\pi_\theta}(a)) \right) \widehat{A}^{\pi_\theta}(a) \right], \quad (10)$$

490 where $\pi_\theta(a)$ is the policy, i.e., a probability distribution of actions $\pi_\theta(a)$ parameterized by a set
 491 of free parameters θ (here the weight and biases of the deep neural network) that determines the
 492 agent behaviour, \widehat{A}^{π_θ} is a biased estimator of the advantage function A^{π_θ} measuring the gain of
 493 taking action a over the average value (here its normalization to zero mean and unit variance),
 494 and ϵ is a clipping range defining how far away the new policy is allowed to go from the old. A
 495 positive (resp. negative) advantage increases (resp. decreases) the probability of taking action a ,
 496 but always by a proportion smaller than ϵ , otherwise the min kicks in (10) and its argument hits
 497 a ceiling of $1 + \epsilon$ (resp. a floor of $1 - \epsilon$). This conservatism inherited from the parent algorithm
 498 ensures that the current and new policies behave similarly (which prevents the agent from falling
 499 off a cliff and restarting with a locally bad policy, in which case the performance may collapse
 500 drastically and never recover). Another trait shared by the two algorithms is the lack of necessity
 501 for assumptions regarding the optimization problem to be solved and for fine-tuning of the network
 502 hyper-parameters (i.e., those parameters not learnt from data).

503 Where the two methods differ is that PPO seeks the optimal set of actions a^* earning the
 504 largest possible reward, while single-step PPO seeks the optimal state-action mapping f_{θ^*} such
 505 that $a^* = f_{\theta^*}(s_0)$, where s_0 denotes some input state consistently fed to the agent for the optimal
 506 policy to eventually embody the transformation from s_0 to a^* . Starting from a random mapping f_{θ_0}
 507 from s_0 to the policy determined by the free parameters initialization, the agent gets one attempt
 508 per episode at finding the optimal (i.e., it interacts with the environment only once per episode)
 509 before updating the policy. Another subtle difference is that PPO is actor-critic, i.e., it features an
 510 actor network that learns the policy, and a critic network that learns to estimate the advantage.
 511 Single-step PPO works without knowledge of the critic evaluations (and is thus not actor-critic)
 512 because the trajectory of state and actions consists of a single pair. The discount factor adjusting
 513 the trade-off between immediate and future rewards can thus be set to $\gamma = 1$, in which case the
 514 advantage reduces to the whitened reward [48].

515 Our single step PPO method is based on the default open-source implementation of Stable
 516 Baselines (<https://github.com/openai/baselines/tree/master/baselines/ppo2>), for which a custom
 517 OpenAI environment has been designed with the Gym library [86]. We have updated and connected
 518 the original code with our CFD library for simple reading and writing of the results (the code is
 519 shared publicly using the following link : <https://github.com/jviquerat/pbo>). The convergence
 520 properties are illustrated in Figure 12 for a minimization test problem of two- and five-dimensional
 521 Rosenbrock functions, whose global minimum is notoriously difficult to catch for optimization
 522 algorithms and two-dimensional Branin function, that has two identical global minima. For this
 523 case, the single-step PPO-1 algorithm is benchmarked against classical (μ - λ)-ES and CMA-ES
 524 evolutionary methods, all implemented in in-house production codes. To ensure a fair comparison,
 525 the initial parameters and starting points are identical for all methods. All runs are afforded
 526 the same budget, namely 500 evaluations (20 episodes with 5 parallel environments in PPO-1,
 527 20 generations with 5 individuals per generation in evolutionary algorithms) for Rosenbrock and
 528 50 evaluations for Branin (10 episodes/generations with 5 parallel environments/individuals per
 529 generation). A large initial standard deviation is used by default, to ensure a good exploration of

530 the optimization domain. Finally, in order to emphasize flexibility and generalizability, all PPO
 531 runs are tackled without fine-tuning of the algorithm, i.e., all runs use the same meta-parameters
 532 as in Table I. Performances are averaged over 10 runs, with standard deviations shown as the light
 533 shade around. As could have been expected, the search efficiency of CMA-ES yields the best overall
 534 performance, which reflects the benefit of efficiently elongating the research area with respect to the
 535 local shape of the cost function. Among isotropic exploration methods, PPO-1 achieves final cost
 536 levels similar to $(\mu-\lambda)$ -ES, with faster convergence and better performance at intermediate stages
 537 (the final performance level ultimately saturates for the Rosenbrock function because the minimum
 538 is in a long, narrow valley, and PPO-1/ $(\mu-\lambda)$ -ES use isotropically sampled approximations of the
 539 descent direction). The general picture to be drawn is that (i) PPO-1 exhibits strong performance
 540 compared to methods relying on similar isotropic search distributions, and (ii) anisotropic search
 541 distributions are mandatory to outperform more advanced methods on a consistent basis, an issue
 542 that is being addressed in current research efforts by the authors [87].

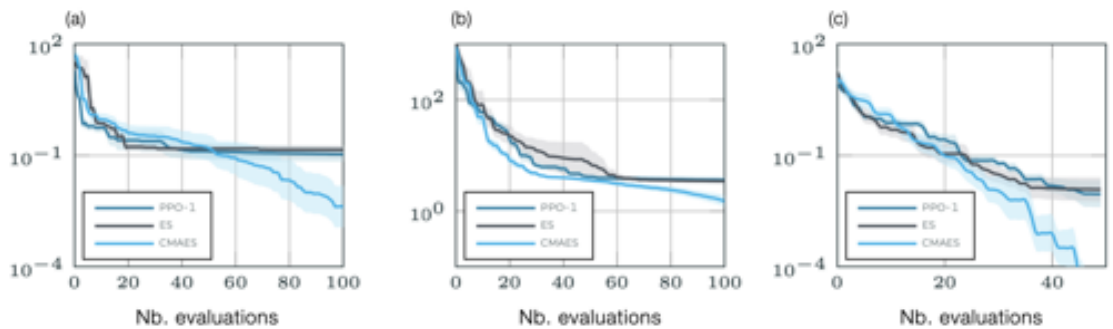


Figure 12. **Benchmark minimization problems** for the (a) two- and (b) five-dimensional Rosenbrock functions, and (c) the two-dimensional Branin function, using the present PPO-1 algorithm and reference $(\mu-\lambda)$ -ES and CMA-ES evolutionary algorithms.

543 **Parallel data collection.** In practice, actions are distributed to several environments running
 544 in parallel, each of which executes a self-contained MPI-parallel CFD simulation and feeds data to
 545 the DRL algorithm (hence, two levels of parallelism related to the environment and the computing
 546 architecture). All simulations are performed on a workstation of AMD EPYC 7502 processors.
 547 The algorithm waits for the simulations running in all parallel environments to complete, then
 548 shuffles and splits the rewards data set collected from all environments into several buffers (or
 549 mini-batches) used sequentially to compute the loss and perform a network update. This repeats
 550 for several epochs, i.e., several full passes of the training algorithm over the entire data set (which
 551 ultimately makes the algorithm slightly off-policy, since the policy network ends up being trained
 552 on samples generated by older policies). This simple parallelization technique is key to using DRL
 553 in the context of flow control applications, as estimating accurately the policy gradient requires
 554 assessing a sufficient number of actions drawn from the current policy, hence a large computational
 555 burden associated to reward computations for high-dimensional fluid dynamics problems (typically,
 556 the cost of a single call to the CFD solver times the number of evaluations required). In the
 557 same vein, it should be noted that the common practice in DRL studies to gain insight into the
 558 performances of the selected algorithm by averaging results over multiple independent training
 559 runs with different random seeds is not tractable, as it would trigger a prohibitively large CPU
 560 cost. The same random seeds have thus been deliberately used over the whole course of the study
 561 to ensure a minimal level of performance comparison between the two cases.

562

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 564 expressed are however those of the author(s) only and do not necessarily reflect those of the
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 566 authority can be held responsible for them.

32	Nb. epochs
8	Nb. environments
2	Size of mini-batches
5×10^{-3}	Learning rate
0.3	Clipping range

Table I. **PPO hyper parameters.**

567

DATA AVAILABILITY

568 The datasets generated and analysed during the current study are not publicly available due the
569 fact that they constitute an excerpt of research in progress but are available from the corresponding
570 author on reasonable request.

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