



HAL
open science

MA-BBOB: Many-Affine Combinations of BBOB Functions for Evaluating AutoML Approaches in Noiseless Numerical Black-Box Optimization Contexts

Diederick Vermetten, Furong Ye, Thomas Bäck, Carola Doerr

► **To cite this version:**

Diederick Vermetten, Furong Ye, Thomas Bäck, Carola Doerr. MA-BBOB: Many-Affine Combinations of BBOB Functions for Evaluating AutoML Approaches in Noiseless Numerical Black-Box Optimization Contexts. International Conference on Automated Machine Learning (AutoML 2023), Sep 2023, Potsdam, Germany. hal-04242054

HAL Id: hal-04242054

<https://hal.science/hal-04242054v1>

Submitted on 14 Oct 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

MA-BBOB: Many-Affine Combinations of BBOB Functions for Evaluating AutoML Approaches in Noiseless Numerical Black-Box Optimization Contexts

Diederick Vermetten¹ Furong Ye¹ Thomas Bäck¹ Carola Doerr²

¹Leiden Institute for Advanced Computer Science (LIACS), Leiden University, The Netherlands

²Sorbonne Université, CNRS, LIP6, Paris, France

Abstract Extending a recent suggestion to generate new instances for numerical black-box optimization benchmarking by interpolating pairs of the well-established BBOB functions from the COmparing COntinuous Optimizers (COCO) platform, we propose in this work a further generalization that allows multiple affine combinations of the original instances and arbitrarily chosen locations of the global optima.

We demonstrate that the MA-BBOB generator can help fill the instance space, while overall patterns in algorithm performance are preserved. By combining the landscape features of the problems with the performance data, we pose the question of whether these features are as useful for algorithm selection as previous studies suggested.

MA-BBOB is built on the publicly available IOHprofiler platform, which facilitates standardized experimentation routines, provides access to the interactive IOHanalyzer module for performance analysis and visualization, and enables comparisons with the rich and growing data collection available for the (MA-)BBOB functions.

1 Introduction

Black-box optimization deals with algorithms that are capable of finding high-quality solutions for problems that are not necessarily explicitly given, but for which the quality of solution candidates can be obtained through some process such as numerical simulations, physical experiments, or user studies. Black-box optimization algorithms work in an iterative fashion, alternating between creating new solution candidates and using the obtained quality information to update the procedure to generate the next set of candidates. Black-box optimization algorithms are the method of choice whenever the problem is truly black-box or when no problem-tailored solvers are available. Among the most prominent application domains of black-box optimization algorithms are engineering problems, biomedical problems, and computer science itself, represented via problems such as hyperparameter optimization.

Many different black-box optimization methods exist. In addition, they typically have hyperparameters to help customize the behavior of the algorithm to the problem at hand. From the user perspective, these design choices create the very critical meta-optimization problem of selecting which algorithm to use for a problem at hand, and how to configure it. This is where automated Machine Learning (AutoML [12]) comes into play. However, despite a long tradition of developing automated Machine Learning (AutoML) approaches for numerical black-box optimization contexts [3, 14, 30], empirical evaluations are heavily centered around very few benchmark collections. One of the most popular collections is the BBOB suite [11] of the COmparing COntinuous Optimizers (COCO) platform [10]. The BBOB suite was originally designed to help researchers analyze the behavior of black-numerical black-box algorithms in different optimization contexts. Over time, however, BBOB has been used for many other purposes, including evaluating AutoML methods, even though the problems were never designed to be suitable for this task.

With the increasing popularity of the BBOB benchmarks, wide availability of shared performance data enabled the application of, e.g., per-instance algorithm selection [14] and configuration [3] methods. A common class of state-of-the-art meta-optimization techniques is feature-based, and relies on suitable representations of the problem spaces to predict which algorithm (configuration) performs best on a given problem. In the case of BBOB, the most commonly used representation makes use of Exploratory Landscape Analysis (ELA [22]), which has been shown to be able to accurately distinguish between BBOB problems [29].

A key problem of algorithm selection based on BBOB problems lies in the ability to test how well the results generalize. One approach is to use a leave-one-function-out method [25], where the selector is trained on 23 functions and tested on the remaining one. This generally leads to poor performance, as each problem has been specifically designed to have different global function properties. As such, another common method is to leave out a set of problem instances for testing. This way, the selector is trained on all types of problems. However, this has a high potential to overfit the particular biases of the BBOB problems [15], an often overlooked risk.

To remedy these potential issues, the ability to construct new functions which fill the spaces between existing BBOB functions could be critical. If the instance space can be filled with new problems, these could be used to not only test the generalizability of algorithm selection methods, but also more generally to gain insights into the relation between the ELA representation of a problem and the behavior of optimization algorithms.

Filling the instance space is a topic of rising interest within the optimization community [1, 21, 24, 36]. While some work has been conducted to create problem instances that reflect the properties of real-world applications or obtain similar characteristics of the existing problems, other work is trying to generate diverse instances. For example, symbolic regression and simulation of Gaussian processes have been applied to generate benchmarks reflecting real-world problem behaviours in [37] and [19, 31]. On the other hand, research in generating diverse instances of combinatorial optimization has been conducted in [4, 5, 18, 21]. Regarding black-box numerical optimization, approaches based on Genetic Programming (GP) have succeeded in generating novel problem instances with controllable characteristics defined by their ELA features in [23], in which the authors used ELA features of BBOB instances as a baseline to regenerate similar instances and design diverse instances. However, to obtain problems with desired characteristics, the GP needs to be executed for each dimension. In recent work, Dietrich and Mersmann [8] propose a different perspective on generating new problem instances for numerical optimization through weighted combinations of BBOB problems. By creating these affine combinations of existing problems, it seems that the ELA features can transition smoothly between the two component functions. Moreover, in our recent work [34] we observed that algorithms' performance can alter substantially along the weights of two combined problems.

In this paper, we extend upon the modified version of the affine BBOB combinations proposed in [34] by generalizing it to combinations between an arbitrary number of BBOB functions. Through doing this, we address the concerns regarding the scaling of the component functions and the impact of the location of the global optimum. We also propose a modified mechanism to sample weights to avoid potential biases resulting from including too many problems.

From the proposed many-affine problem generation method, we sample 1 000 instances, for which we perform both an ELA-based analysis as well as an analysis of the performance of a set of algorithms. By combining these results in a simple algorithm selection model, we raise the question of whether or not the ELA features are sufficiently representative to create a generalizable algorithm selection model.

In summary, our **key contributions and findings** are:

1. We introduce MA-BBOB, a generator of arbitrary affine combinations of the 24 BBOB functions. We explain the rationales behind the various design choices, which include the location of the

optimum, the scaling used for interpolating the different functions, and the way of sampling in functions from this space. The resulting generator is build on the IOHprofiler platform, which enables equivalent benchmarking setups to the original BBOB problems.

2. We analyze 1 000 randomly sampled instances in $2d$ and in $5d$ via Exploratory Landscape Analysis (ELA [22]) and show that the combined MA-BBOB functions cover the space between the original ‘pure’ BBOB functions quite well, with the exception of some of problems like the linear slope and ellipsoid problem, which are essentially only available in the ‘pure’ BBOB functions, but disappear in the MA-BBOB instances with non-trivial weights.
3. We compare the performance of five black-box optimization algorithms on the original BBOB and the 1 000 randomly sampled MA-BBOB instances and show that the rank distribution changes slightly in favour of the CMA-ES algorithms and to the disadvantage of RCobyta.
4. Finally, we also perform per-instance algorithm performance prediction studies on MA-BBOB. The results confirm that the regression accuracy is better when the training set includes generalized BBOB functions. However, we also observe a considerable performance gap between ELA-based regression models and those trained with full knowledge of the weights that are used to construct the test instances. These results indicate that the current set of ELA features fail to capture some instance properties that are crucial for algorithm performance, a shortcoming that we expect to motivate future research on the design of features for numerical black-box optimization.

2 Background

The BBOB Problem Suite. The BBOB collection [11] is one of the main components of the COCO benchmarking framework [10]. The BBOB suite is heavily used in the black-box optimization community for evaluating derivative-free numerical optimization techniques. To date, hundreds of different optimization algorithms have been tested on the original BBOB suite of 24 single-objective, noiseless optimization problems [2].

One key reason for the popularity of the BBOB suite is the ability to create independent instances of the same problem, which are generated by applying transformations in the domain and the objective space. These transformations include rotation, scaling of objective value, and moving the location of the global optimum. They allow researchers to evaluate possible bias in their algorithms, and are hence an important component of algorithm benchmarking.

The availability of many instances are also a key enabler for the evaluation of AutoML approaches in black-box optimization contexts. Since not all instances are easily accessible via the original COCO implementation, we have made direct access to the instances available in our IOHprofiler benchmarking environment [9, 7, 35].

Affine Function Combinations. While the availability of numerous instances per each BBOB function facilitates AutoML studies, it has been observed that the generalization ability of models trained on BBOB and tested on independent problems is disappointing [15, 17]. This motivated the design of new problems to extend the existing BBOB suite. One such approach was proposed in [8]. It suggests to consider affine combinations of two different problem instances. The resulting problems were analyzed with respect to their fitness landscapes, as seen via exploratory landscape analysis (ELA [22]). They have been shown to smoothly connect their component functions in a reduced-dimensionality ELA space. This seems to imply that we can use these problems to connect any pair of existing problems, which would significantly add to the instance space.

In our follow-up study [34] we recently proposed a modified version of creating these affine function combinations, see Section 3.1 for details. We used these functions to compare the performance of five selected black-box optimization algorithms and showed that the behavior differences are not as smooth as the differences in ELA space. In several cases, combinations of two functions are best solved by a different algorithm than the one which solved the component problems.

Function ID	1	2	3	4	5	6	7	8	9	10	11	12
Scale Factor	11.0	17.5	12.3	12.6	11.5	15.3	12.1	15.3	15.2	17.4	13.4	20.4
Function ID	13	14	15	16	17	18	19	20	21	22	23	24
Scale Factor	12.9	10.4	12.3	10.3	9.8	10.6	10.0	14.7	10.7	10.8	9.0	12.1

Table 1: Final scale factors used to generate MA-BBOB problems.

3 The MA-BBOB Benchmark Suite

3.1 Scaling of Function Values

When combining multiple functions to create a new benchmark problem, one key factor which impacts the landscape is the scaling of the combined functions. Since we are interested in taking affine combinations of existing functions, a difference in scale might lead one function to dominate all others, leading to limited coverage of the feature space.

The original affine BBOB functions proposed in [8] make use of a tuning procedure for finding useable weights. While this allows for selecting suitable problems, it makes it more challenging to just randomly sample a set of new problems. We therefore suggested an alternative way to generate the affine combinations in [34]. This change is two-fold: each component problem f is first transformed by subtracting the global optimum value $\min f$. This way, we know that each component functions optimum function value is set to 0. Then, instead of arithmetic weighting, a logarithmic combination is used to limit the impact of scale differences. While this simplifies the procedure of generating random function combinations, BBOB functions can sometimes differ by multiple orders of magnitude, which still produces some bias in this procedure.

To address this shortcoming in MA-BBOB, we have investigated different scaling procedures. We still scale the global optima and perform a logarithmic transform, but we now add a normalization step. This transforms the log-precision values into an approximation of $[0, 1]$, and then maps this back to the commonly used BBOB domain $[10^{-8}, 10^2]$. This is achieved by taking the log-transformed precision (capped at -8), adding 8 so the minimum is at 0 and dividing by a *scale factor*. The aim of this procedure is to make sure that the target precision of 10^2 is similarly easy to achieve on all problems.

In order to select appropriate scale factors, we need to determine practical limits of the function value for each BBOB function. We do this by considering a set of 50 000 random samples and aggregating the corresponding function values. We consider the following aggregation methods (based on the log-scaled precision): min, mean, max, $(\max + \min)/2$. Figure 1 illustrates the differences between these methods, for a $2d$ problem. Note that because we use log-scaled precision, the differences between instances are rather small, so we opted to only do the sampling for one instance of each BBOB problem. Based on visual interpretation of the contour plots in Figure 1, we (somewhat subjectively) select the $(\max + \min)/2$ scaling as the most promising method.

To avoid having to constantly repeat this random sampling procedure, we also investigate the way in which the scales of the random factors, and thus the scale factors, differ across dimensions. The results are shown in Figure 2. With exception of the smallest dimensions, the values remain quite stable. As such, we decide to implement them as hard-coded values based on the median of the shown values, rounded to the nearest decimal. The resulting factors are shown in Table 1.

3.2 Instance Creation

A second aspect to consider when combining multiple functions is the placement of the global optimum. In the previous two papers [8, 34] on affine BBOB functions, this was done based on the instance of one of the two component functions. However, the original BBOB instance creation process can be considered somewhat biased, as not all functions make use of the same

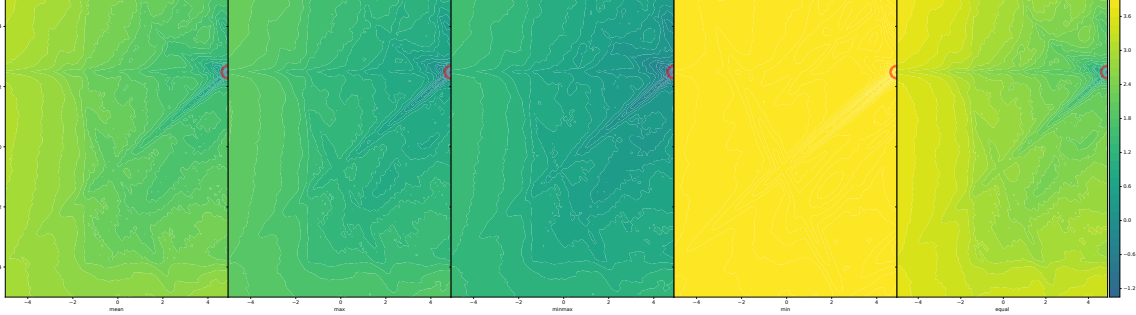


Figure 1: Log-scaled fitness values of an example of a single many-affine function with 5 different ways of scaling. The first 4 are taking the mean, max, $(\max + \min)/2$ and min of 50 000 random samples to create the scale factor, while the fifth ('equal') option does not make use of this scaling.

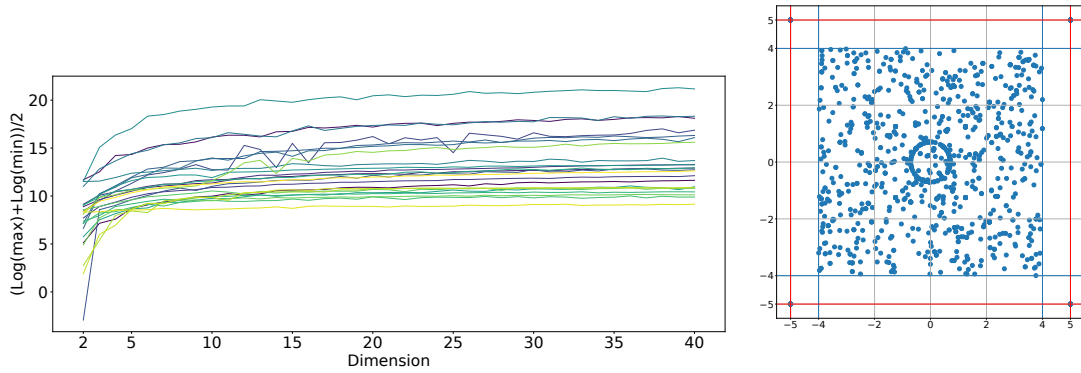


Figure 2: Evolution of the log-scaled $(\max + \min)/2$ scaling factor, relative to the problem dimension. The values are based on 50 000 samples. Each line corresponds to one of the 24 BBOB functions.

rel-Figure 3: Location of optima of the 24 2d BBOB functions. The red lines mark the commonly used box-constraints of $[-5, 5]^D$.

transformations [11, 20]. As such, if we extend the process of using the optimum of one of the used component functions, the optimum would be distributed as in Figure 3. To avoid this issue, we decided to generate the optimum location separately, uniformly at random in the full domain $[-5, 5]^d$. Figure 4 shows how a $2d$ -function changes when moving the optimum location.

3.3 Sampling random functions

As a final factor impacting the types of problems generated, we consider the way in which weights are sampled. While this can indeed be done uniformly at random (with a normalization afterwards), this might not lead to the most useful set of benchmark problems. When the weights for each function are generated this way, the probability of having a weight of 0 for any component is 0. This means that every function will contribute to some extent to the newly generated problem. As such, it would be almost impossible for this procedure to result in a unimodal problem.

One way to address this bias in function generation is to adapt how many functions are part of the newly created problem. Indeed, the combinations of two problems already lead to a vast space of interesting landscapes. We opt for a different approach: we make use of a threshold value which determines which functions contribute to the problem. The procedure for generating weights is

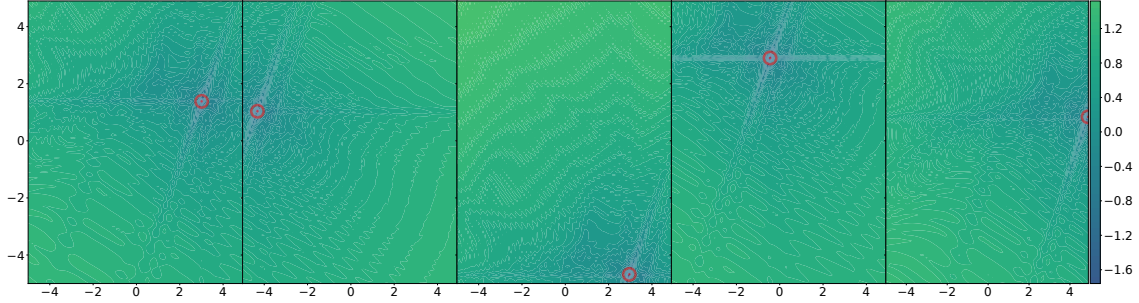


Figure 4: Log-scaled fitness values of an example of a single many-affine function with changed location of optimum.

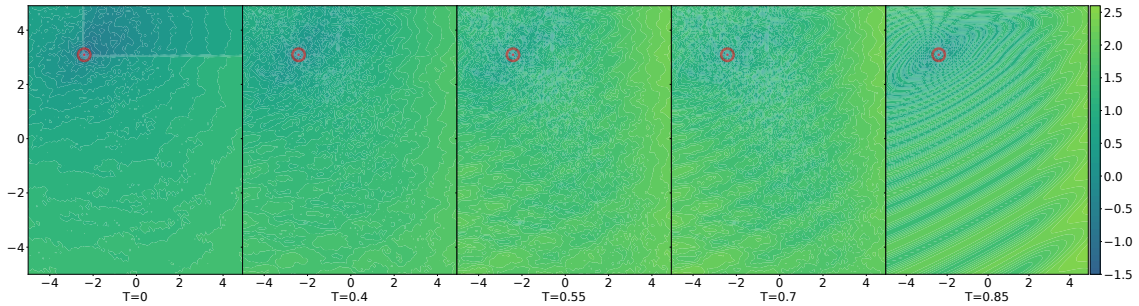


Figure 5: Log-scaled fitness values of an example of a 'single' many-affine function with 5 different sampling thresholds.

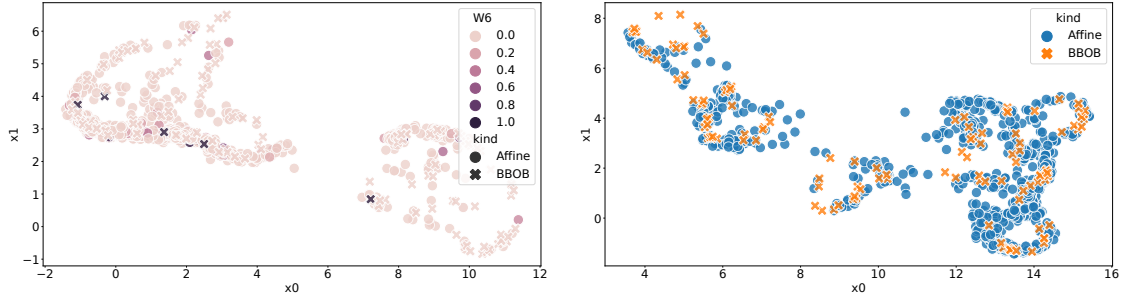
thus as follows: (1) Generate initial weight uniformly at random, (2) adapt the threshold to be the minimum of the user-specified threshold and the third-highest weight, (3) this threshold is subtracted from the weights, all negative values are set to 0. The second step is to ensure that at least two problems always contribute to the new problem. Figure 5 provides an example of a problem generated with different threshold values. We decide to set the default value at $T = 0.85$, such that on average 3.6 problems will have a non-zero weight.

4 Experimental Setup

In the remainder of this paper, we will make use of 1 000 functions, with weights sampled according to Section 3.3 with $T = 0.85$. Each problem uses instances uniformly selected between 1 and 100 for each of the component functions, and uniformly sampled locations of the global optimum. We use the same set of weights, instances and optima locations in both 5 and 2 dimensions.

Comparing this set of generated problems with the pure BBOB functions is a key aspect of this work. To remove biases in terms of scaling, we apply the same scale factors to the BBOB functions. Practically, this means we use the all-zero weights with a 1 for the selected function to collect the BBOB data (with the location of the optima set as original). We use 5 instances of each BBOB function for our comparisons. We refer to these 'pure' BBOB functions as 'BBOB', while we refer to the MA-BBOB instances as 'affine'.

Reproducibility: The code used during this project, as well as all resulting data, is available at [33]. The repository also contains additional versions of the figures which could not be included here because of the page limit. We are actively working towards a data repository for MA-BBOB performance data which will also allow automated annotation via the OPTION ontology [16], for FAIR data sharing [13].



(a) Points are colored according to the weights used for BBOB function F7. (b) Points are colored according to the function type: BBOB of affine combination.

Figure 6: UMAP-reduction of the 24 BBOB functions (5 instances each) and 1000 affine combinations for $5d$ (a) and $2d$ (b). The projection is created based on the BBOB only.

5 Landscape Analysis

To analyze the landscapes of the created affine problems, we make use of the `pflacco` package [26] to compute ELA features. We use 5 sets of $1000d$ points from a scrambled Sobol' sequence. We then evaluate these points and follow the advice of [27] and use min-max normalization on these function values. We finally remove all features which are constant across all problems or contain NAN values, resulting in a total of 44 remaining ELA features. For each of these features, we then take the mean value among the 5 samples.

To gain insight into the differences between the BBOB and affine functions, we reduce the original 44-dimensional space into $2d$. To achieve this, we make use of the Uniform Manifold Approximation Projection (UMAP). To focus on the parts of the instance space covered by the newly generated problems, we create the mapping based only on the BBOB problems. The result of applying this mapping to all $2d$ problems is visualized in Figure 6b.

From Figure 6b, we observe that many of the affine problems are clustered together. While some regions between existing BBOB problems are filled, it seems that the function generation process is not able to find solutions close to every BBOB problem. This might be caused by the fact that by combining an average of 3.6 functions, it is highly unlikely that we find functions similar to e.g., a linear slope or a function with low global structure.

In addition to the dimensionality reduction, we can also investigate the distributions of individual ELA feature values. By comparing the distributions on the BBOB functions with the ones on the affine problems, we can gain some insight into the most common types of problems generated. In Figure 7, we show these distributions for the min-max normalized ELA features. From this figure, we can see that for many features, the feature values of the affine problems are much more concentrated than the BBOB ones.

6 Algorithm Performance

While the ELA-based analysis gives us some insight into the low-level characteristics of the generated problems, it does not directly give insight into the power of these problems to differentiate between algorithms. We therefore complement the ELA-centered analyses from the previous section by investigating algorithm performance. To this end, we run a set of 5 different algorithms on each problem instance:

- (1) Diagonal CMA-ES from the Nevergrad platform [28] (dCMA),
- (2) RCobyala from the Nevergrad platform (Cobyala),
- (3) Differential Evolution from the Nevergrad platform (DE),

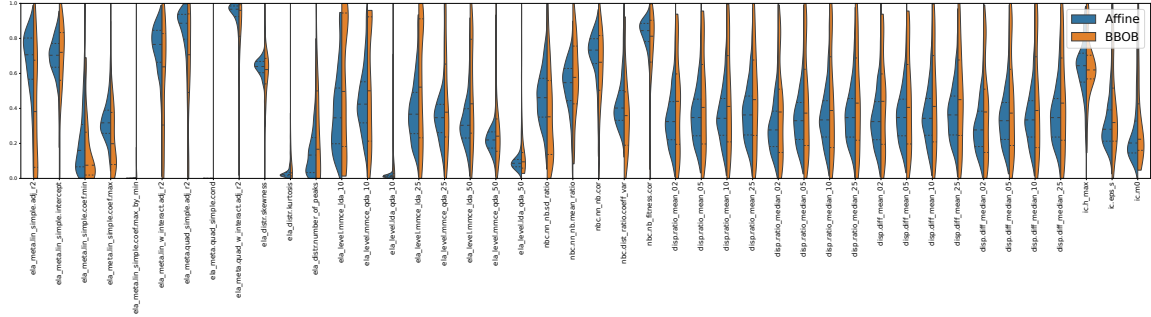
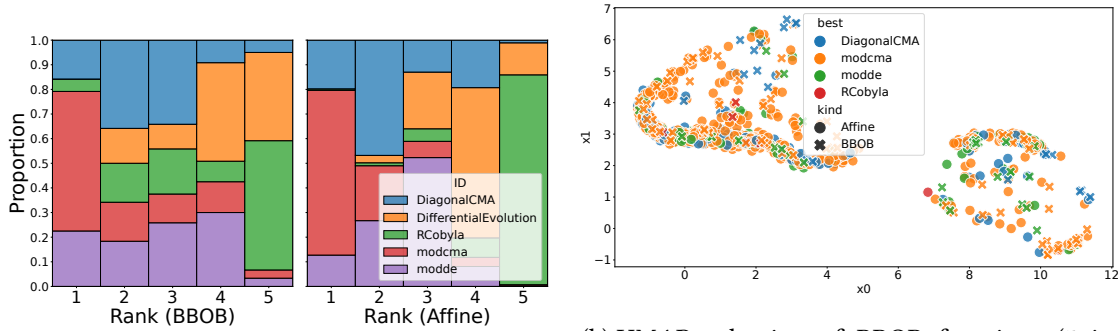


Figure 7: Distribution of (normalized) ELA feature values on the 5d version of the problems.



(a) Distribution of ranks based on per-function AUC after 10 000 evaluations.

(b) UMAP-reduction of BBOB functions (5 instances) and 1000 affine combinations. Projection created based on BBOB only. Color based on the algorithm with the largest AUC.

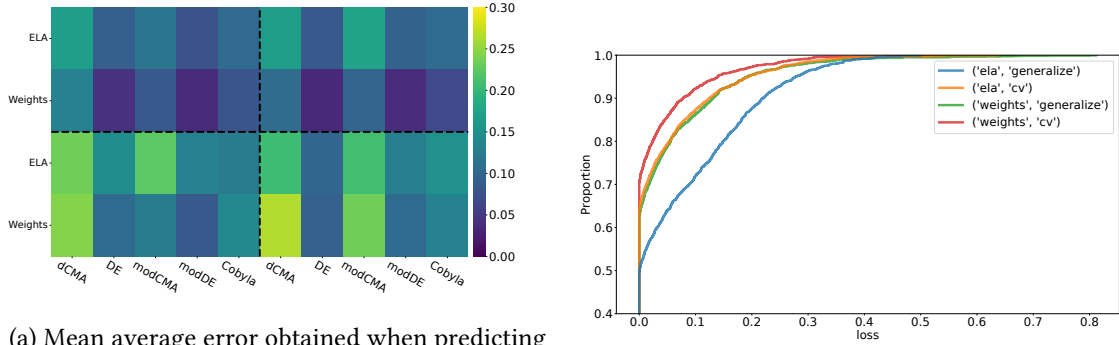
Figure 8: Results of ranking the 5 algorithms on the 5d problems, based on AUC after 10 000 evaluations.

- (4) CMA-ES from the modular CMA-ES package [6] (modCMA), and
- (5) L-SHADE, implemented using the modular DE package [32] (modDE).

For each of these algorithms, we perform 50 independent runs on each of the 1 000 affine functions as well as on the 5 instances from each of the 24 BBOB problems. It is important to note that the BBOB functions make use of the same scale factors as used to generate the affine functions in order to further reduce the impact of scale differences. These experiments are performed on both the 2d and 5d versions of these problems.

To analyze the differences in algorithm performance between the two sets of problems, we consider the normalized area under the curve (AUC) of the empirical cumulative distribution function (ECDF) as the performance metric. For the ECDF, we follow the default setting suggested in COCO [10] and use a set of 51 logarithmically spaced targets from 10^{-8} to 10^2 . Based on the AUC values, we then rank the set of 5 algorithms on each problem. The distribution of these ranks is shown in Figure 8a. We observe that the overall patterns between the BBOB and affine problems are preserved. There are some notable differences, particularly with regard to the performance of Cobyla. Even though this algorithm often performs poorly on BBOB, for the affine problems it is ranked worst in more than 80% of cases. This suggests that problems where this algorithm performs well (mostly unimodal problems) are not as well-represented in the MA-BBOB functions.

In addition to this ranking, we can also link the ELA features to the algorithm performance. To explore whether the used features might correlate with the problem’s difficulty from the algorithm’s perspective, we link the dimensionality reduction with the best algorithm from the portfolio. This is visualized for the 5d problems in Figure 8b.



(a) Mean average error obtained when predicting the AUC of each of the 5 algorithms based on either the ELA features or the used weights. Top: model trained on mixture of BBOB and Affine functions using 10-fold cross-validation. Bottom: model trained on BBOB only and predicting performance on affine problems. Left: $2d$ problems, right: $5d$ problems.

(b) Cumulative distribution of loss (AUC) of the random forest models predicting the best algorithm ($2d$ and $5d$ problems combined), based on either the ELA features or weights-representation of the problems.

Figure 9: Performance of the random forest model predicting algorithm performance (a) or the best algorithm for each problem (b).

7 Algorithm Selection

As a final experiment, we now use the generated problems in an algorithm selection context. For each of the 5 algorithms, we train a random forest regression model to predict the AUC on each problem. The input variables for this model are either the ELA features, as is commonly done, or the weights used to generate the functions. By contrasting these approaches, we obtain an intuition for how well the ELA features capture the algorithm-relevant properties of the function.

While we can train our models in a common cross-validation manner, we can also use the same setup to test the generalizability of models trained on the original BBOB problems only. The resulting mean absolute errors MAE of these models are plotted in Figure 9a.

We observe that the ELA representation is often worse than the weights-based one. This suggests that the used ELA features might not be sufficient to achieve generalization of an AS model. This is especially clear for the generalizability scenario, where we would have expected ELA to perform better. This poor performance seems to suggest that the ELA features might not fully capture all instance properties that determine the behavior of the algorithms.

When training a very basic AS model (predicting the best algorithm) in the same manner (training on BBOB and evaluating on Affine), we achieve similar performance differences as suggested by Figure 9a: the weighted F1-score based on ELA is 0.67, while the score based on weights is 0.70. The corresponding loss in terms of AUC values is plotted in Figure 9b. This figure confirms the previous observation that the ELA features are not sufficiently representative to accurately represent the problems in a way which is relevant for ranking optimization algorithms.

8 Conclusions and Future Work

The proposed procedure for generating new problems as an affine combination of the 24 BBOB problems can serve as a function generator to help fill the instance space spanned by the BBOB functions. By applying a scaling step before combining the problems, we make sure that the resulting problems all have an equivalent range of objective values, regardless of the used weights. In addition, the uniform location of the global optima in the full domain avoids some of the bias of the original BBOB problems.

By analyzing the ELA features of 1 000 of these many-affine MA-BBOB problems, we observed that they do indeed fill a part of the instance space. There are still some inherent limitations arising from the fact that the building blocks are fixed. For example, it is impossible to generate a problem similar to the linear slope. Similarly, it is highly unlikely that new problems have specific properties such as low global structure. Nevertheless, the overall ranking of optimization algorithms on these problems remains similar to that on the original BBOB problems, suggesting that the algorithmic challenges might be similar.

The results presented above had as primary focus a first analysis of the generated MA-BBOB instances, and how they compare to the BBOB functions. For this purpose, we have considered randomly sampled instances. The selection of ‘representative’ instance collections still remains to be done. Another important step for future work is to test the generalization ability of AutoML systems that are trained on MA-BBOB functions and tested on numerical black-box optimization problems that do not originate from the BBOB family. In this context, our basic Random Forest-based algorithm selector indicates that the ELA features might not be as suitable for this generalization task as expected, motivating further research on feature engineering for numerical black-box optimization.

Acknowledgements. Our work is financially supported by ANR-22-ERCS-0003-01 project VARIATION, by the CNRS INS2I project IOHprofiler, and by the NWO DACCOMPLI project (628.011.002). This work was performed using the ALICE compute resources provided by Leiden University.

References

- [1] Hossein Alipour, Mario Andrés Muñoz, and Kate Smith-Miles. 2023. Enhanced instance space analysis for the maximum flow problem. *Eur. J. Oper. Res.* 304, 2 (2023), 411–428. <https://doi.org/10.1016/j.ejor.2022.04.012>
- [2] Anne Auger and Nikolaus Hansen. 2020. A SIGEVO Impact Award for a Paper Arising from the COCO Platform: A Summary and Beyond. <https://evolution.sigevo.org/issues/HTML/sigevoevolution-13-4/home.html>. Issue 3.
- [3] Nacim Belkhir, Johann Dréo, Pierre Savéant, and Marc Schoenauer. 2017. Per instance algorithm configuration of CMA-ES with limited budget. In *Proc. of Genetic and Evolutionary Computation (GECCO’17)*. ACM, 681–688. <https://doi.org/10.1145/3071178.3071343>
- [4] Jakob Bossek, Pascal Kerschke, Aneta Neumann, Markus Wagner, Frank Neumann, and Heike Trautmann. 2019. Evolving diverse TSP instances by means of novel and creative mutation operators. In *Proc. of Conference on Foundations of Genetic Algorithms (FOGA’19)*, Tobias Friedrich, Carola Doerr, and Dirk V. Arnold (Eds.). ACM, 58–71. <https://doi.org/10.1145/3299904.3340307>
- [5] Jakob Bossek and Markus Wagner. 2021. Generating instances with performance differences for more than just two algorithms. In *Proc. of Genetic and Evolutionary Computation Conference (GECCO’21, Companion material)*, Krzysztof Krawiec (Ed.). ACM, 1423–1432. <https://doi.org/10.1145/3449726.3463165>
- [6] Jacob de Nobel, Diederick Vermetten, Hao Wang, Carola Doerr, and Thomas Bäck. 2021. Tuning as a means of assessing the benefits of new ideas in interplay with existing algorithmic modules. In *Proc. of Genetic and Evolutionary Computation Conference (GECCO’21, Companion material)*. ACM, 1375–1384. <https://doi.org/10.1145/3449726.3463167>
- [7] Jacob de Nobel, Furong Ye, Diederick Vermetten, Hao Wang, Carola Doerr, and Thomas Bäck. 2021. IOHexperimenter: Benchmarking Platform for Iterative Optimization Heuristics. *CoRR* abs/2111.04077 (2021). arXiv:2111.04077 <https://arxiv.org/abs/2111.04077>

- [8] Konstantin Dietrich and Olaf Mersmann. 2022. Increasing the Diversity of Benchmark Function Sets Through Affine Recombination. In *Proc. of Parallel Problem Solving from Nature (PPSN'22) (LNCS, Vol. 13398)*, Günter Rudolph, Anna V. Kononova, Hernán E. Aguirre, Pascal Kerschke, Gabriela Ochoa, and Tea Tusar (Eds.). Springer, 590–602. https://doi.org/10.1007/978-3-031-14714-2_41
- [9] Carola Doerr, Hao Wang, Furong Ye, Sander van Rijn, and Thomas Bäck. 2018. IOHprofiler: A Benchmarking and Profiling Tool for Iterative Optimization Heuristics. *CoRR* abs/1810.05281 (2018). arXiv:1810.05281 <http://arxiv.org/abs/1810.05281>
- [10] Nikolaus Hansen, Anne Auger, Raymond Ros, Olaf Mersmann, Tea Tušar, and Dimo Brockhoff. 2021. COCO: A platform for comparing continuous optimizers in a black-box setting. *Optim. Methods Softw.* 36, 1 (2021), 114–144.
- [11] Nikolaus Hansen, Steffen Finck, Raymond Ros, and Anne Auger. 2009. *Real-Parameter Black-Box Optimization Benchmarking 2009: Noiseless Functions Definitions*. Technical Report RR-6829. INRIA. <https://hal.inria.fr/inria-00362633/document>
- [12] Frank Hutter, Lars Kotthoff, and Joaquin Vanschoren (Eds.). 2019. *Automated Machine Learning - Methods, Systems, Challenges*. Springer. <https://doi.org/10.1007/978-3-030-05318-5>
- [13] Annika Jacobsen, Ricardo de Miranda Azevedo, Nick S. Juty, Dominique Batista, Simon J. Coles, Ronald Cornet, Mélanie Courtot, Mercè Crosas, Michel Dumontier, Chris T. A. Evelo, Carole A. Goble, Giancarlo Guizzardi, Karsten Kryger Hansen, Ali Hasnain, Kristina M. Hettne, Jaap Heringa, Rob W. W. Hooft, Melanie Imming, Keith G. Jeffery, Rajaram Kaliyaperumal, Martijn G. Kersloot, Christine R. Kirkpatrick, Tobias Kuhn, Ignasi Labastida, Barbara Magagna, Peter McQuilton, Natalie Meyers, Annalisa Montesanti, Mirjam van Reisen, Philippe Rocca-Serra, Robert Pergl, Susanna-Assunta Sansone, Luiz Olavo Bonino da Silva Santos, Juliane Schneider, George O. Strawn, Mark Thompson, Andra Waagmeester, Tobias Weigel, Mark D. Wilkinson, Egon L. Willighagen, Peter Wittenburg, Marco Roos, Barend Mons, and Erik Schultes. 2020. FAIR Principles: Interpretations and Implementation Considerations. *Data Intell.* 2, 1-2 (2020), 10–29. https://doi.org/10.1162/dint_r_00024
- [14] Pascal Kerschke, Holger H. Hoos, Frank Neumann, and Heike Trautmann. 2019. Automated Algorithm Selection: Survey and Perspectives. *Evol. Comput.* 27, 1 (2019), 3–45. https://doi.org/10.1162/evco_a_00242
- [15] Ana Kostovska, Anja Jankovic, Diederick Vermetten, Jacob de Nobel, Hao Wang, Tome Eftimov, and Carola Doerr. 2022. Per-run Algorithm Selection with Warm-starting using Trajectory-based Features. In *Proc. of Parallel Problem Solving from Nature (PPSN'22) (LNCS, Vol. 13398)*. Springer, 46–60. https://doi.org/10.1007/978-3-031-14714-2_4 Free version available at <https://arxiv.org/abs/2204.09483>.
- [16] Ana Kostovska, Diederick Vermetten, Carola Doerr, Sašo Džeroski, Panče Panov, and Tome Eftimov. 2022. OPTION: OPTImization Algorithm Benchmarking ONtology. *IEEE Trans. Evol. Comput.* (2022). <https://doi.org/10.1109/TEVC.2022.3232844> To appear. Free version available at <https://arxiv.org/abs/2211.11332>.
- [17] Benjamin Lacroix and John McCall. 2019. Limitations of Benchmark Sets and Landscape Features for Algorithm Selection and Performance Prediction. In *Proc. of Genetic and Evolutionary Computation (GECCO'19)* (Prague, Czech Republic). ACM, New York, NY, USA, 261–262. <https://doi.org/10.1145/3319619.3322051>

- [18] Thibault Lechien, Jorik Jooken, and Patrick De Causmaecker. 2023. Evolving test instances of the Hamiltonian completion problem. *Comput. Oper. Res.* 149 (2023), 106019. <https://doi.org/10.1016/j.cor.2022.106019>
- [19] Fu Xing Long, Bas van Stein, Moritz Frenzel, Peter Krause, Markus Gitterle, and Thomas Bäck. 2022. Learning the characteristics of engineering optimization problems with applications in automotive crash. In *Proc. of Genetic and Evolutionary Computation (GECCO'22)*, Jonathan E. Fieldsend and Markus Wagner (Eds.). ACM, 1227–1236. <https://doi.org/10.1145/3512290.3528712>
- [20] Fu Xing Long, Diederick Vermetten, Bas van Stein, and Anna V. Kononova. 2022. BBOB Instance Analysis: Landscape Properties and Algorithm Performance across Problem Instances. *CoRR* abs/2211.16318 (2022). <https://doi.org/10.48550/arXiv.2211.16318> arXiv:2211.16318
- [21] Alejandro Marrero, Eduardo Segredo, Coromoto León, and Emma Hart. 2022. A Novelty-Search Approach to Filling an Instance-Space with Diverse and Discriminatory Instances for the Knapsack Problem. In *Proc. of Parallel Problem Solving from Nature (PPSN'22) (LNCS, Vol. 13398)*. Springer, 223–236. https://doi.org/10.1007/978-3-031-14714-2_16
- [22] Olaf Mersmann, Bernd Bischl, Heike Trautmann, Mike Preuss, Claus Weihs, and Günter Rudolph. 2011. Exploratory landscape analysis. In *Proc. of Genetic and Evolutionary Computation (GECCO'11)*. ACM, 829–836.
- [23] Mario A. Muñoz and Kate Smith-Miles. 2020. Generating New Space-Filling Test Instances for Continuous Black-Box Optimization. *Evol. Comput.* 28, 3 (2020), 379–404. https://doi.org/10.1162/evco_a_00262
- [24] Mario Andrés Muñoz, Tao Yan, Matheus R. Leal, Kate Smith-Miles, Ana Carolina Lorena, Gisele L. Pappa, and Rômulo Madureira Rodrigues. 2021. An Instance Space Analysis of Regression Problems. *ACM Trans. Knowl. Discov. Data* 15, 2 (2021), 28:1–28:25. <https://doi.org/10.1145/3436893>
- [25] Ana Nikolikj, Carola Doerr, and Tome Eftimov. 2023. RF+ clust for Leave-One-Problem-Out Performance Prediction. In *Proc. of Applications of Evolutionary Computation (Evo Applications'23)*. Springer, 285–301.
- [26] Raphael Patrick Prager. 2022. pFlacco. <https://pypi.org/project/pflacco/>.
- [27] Raphael Patrick Prager and Heike Trautmann. 2023. Nullifying the Inherent Bias of Non-invariant Exploratory Landscape Analysis Features. In *Proc. of Applications of Evolutionary Computation (Evo Applications'23)*. Springer, 411–425.
- [28] Jérémy Rapin and Olivier Teytaud. 2018. Nevergrad - A gradient-free optimization platform. <https://GitHub.com/FacebookResearch/Nevergrad>.
- [29] Quentin Renau, Johann Dreö, Carola Doerr, and Benjamin Doerr. 2019. Expressiveness and Robustness of Landscape Features. In *Proc. of Genetic and Evolutionary Computation (GECCO'19)* (Prague, Czech Republic). ACM, 2048–2051. <https://doi.org/10.1145/3319619.3326913>
- [30] Gresa Shala, André Biedenkapp, Noor H. Awad, Steven Adriaensen, Marius Lindauer, and Frank Hutter. 2020. Learning Step-Size Adaptation in CMA-ES. In *Proc. of Parallel Problem Solving from Nature (PPSN'20) (LNCS, Vol. 12269)*. Springer, 691–706. https://doi.org/10.1007/978-3-030-58112-1_48

- [31] Ye Tian, Shichen Peng, Xingyi Zhang, Tobias Rodemann, Kay Chen Tan, and Yaochu Jin. 2020. A Recommender System for Metaheuristic Algorithms for Continuous Optimization Based on Deep Recurrent Neural Networks. *IEEE Trans. Artif. Intell.* 1, 1 (2020), 5–18. <https://doi.org/10.1109/TAI.2020.3022339>
- [32] Diederick Vermetten, Fabio Caraffini, Anna V Kononova, and Thomas Bäck. 2023. Modular Differential Evolution. In *Proc. of Genetic and Evolutionary Computation (GECCO'23)*. ACM. <https://doi.org/10.1145/3583131.3590417> To appear. Code available at <https://github.com/Dvermetten/ModDE>.
- [33] Diederick Vermetten, Furong Ye, Thomas Bäck, and Carola Doerr. 2023. Reproducibility files and additional figures. Code repository: <https://github.com/Dvermetten/Many-affine-BBOB> Data and figure repository: <https://doi.org/10.5281/zenodo.7826036>.
- [34] Diederick Vermetten, Furong Ye, and Carola Doerr. 2023. Using Affine Combinations of BBOB Problems for Performance Assessment. In *Proc. of Genetic and Evolutionary Computation Conference (GECCO'23)*, Vol. abs/2303.04573. ACM. <https://doi.org/10.1145/3583131.3590412>
- [35] Hao Wang, Diederick Vermetten, Furong Ye, Carola Doerr, and Thomas Bäck. 2022. IOH-analyzer: Detailed Performance Analysis for Iterative Optimization Heuristic. *ACM Trans. Evol. Learn. Optim.* 2, 1 (2022), 3:1–3:29. <https://doi.org/10.1145/3510426> IOHanalyzer is available at CRAN, on GitHub, and as web-based GUI, see <https://iohprofiler.github.io/IOHanalyzer/> for links.
- [36] Estefania Yap, Mario Andrés Muñoz, and Kate Smith-Miles. 2022. Informing Multiobjective Optimization Benchmark Construction Through Instance Space Analysis. *IEEE Trans. Evol. Comput.* 26, 6 (2022), 1246–1260. <https://doi.org/10.1109/TEVC.2022.3205165>
- [37] Martin Zaefferer and Frederik Rehbach. 2020. Continuous Optimization Benchmarks by Simulation. In *Proc. of Parallel Problem Solving from Nature (PPSN'20) (LNCS, Vol. 12269)*, Thomas Bäck, Mike Preuss, André H. Deutz, Hao Wang, Carola Doerr, Michael T. M. Emmerich, and Heike Trautmann (Eds.). Springer, 273–286. https://doi.org/10.1007/978-3-030-58112-1_19