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On ternary Dejean words avoiding 010

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Abstract: Thue has shown the existence of three types of infinite square-free words over $\{0, 1, 2\}$ avoiding the factor 010. They respectively avoid $\{010, 212\}$, $\{010, 101\}$, and $\{010, 020\}$. Also Dejean constructed an infinite $\left(\frac{7}{4}\right)$ -free ternary word. A word is d -directed if it does not contain both a factor of length d and its mirror image. We show that there exist exponentially many $\left(\frac{7}{4}\right)$ -free 180-directed ternary words avoiding 010. Moreover, there does not exist an infinite $\left(\frac{7}{4}\right)$ -free 179-directed ternary word avoiding 010.

Keywords: Word, Repetitions.

2020 Mathematics Subject Classification: 68R15.

1 Introduction

This note is about words avoiding repetitions, a well-studied area in combinatorics on words [5,6]. A *repetition* is a factor of the form $r = u^n v$ where u is non-empty and v is a prefix of u . Then $|u|$ is the *period* of the repetition r and its *exponent* is $|r|/|u|$. A word is α^+ -free (resp. α -free) if it contains no repetition with exponent β such that $\beta > \alpha$ (resp. $\beta \geq \alpha$).

We consider ternary square-free words (i.e., 2-free words) with additional avoidance constraints. Thue [7] has shown that there exist infinite square-free words avoiding the factors $\{010, 212\}$, $\{010, 101\}$, and $\{010, 020\}$ respectively. In each case, the avoiding word is essentially unique. Consider for example the fixed point of the morphism $0 \mapsto 012, 1 \mapsto 02, 2 \mapsto 1$, which is a



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