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► **To cite this version:**

Pascal Ochem. On ternary Dejean words avoiding 010. Notes on Number Theory and Discrete Mathematics, 2023, 29 (3), pp.545-548. 10.7546/nntdm.2023.29.3.545-548 . hal-04241523

**HAL Id: hal-04241523**

**<https://hal.science/hal-04241523v1>**

Submitted on 14 Oct 2023

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# On ternary Dejean words avoiding 010

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**Received:** 3 March 2023

**Revised:** 17 July 2023

**Accepted:** 25 July 2023

**Online First:** 27 July 2023

**Abstract:** Thue has shown the existence of three types of infinite square-free words over  $\{0, 1, 2\}$  avoiding the factor 010. They respectively avoid  $\{010, 212\}$ ,  $\{010, 101\}$ , and  $\{010, 020\}$ . Also Dejean constructed an infinite  $\left(\frac{7}{4}\right)$ -free ternary word. A word is  $d$ -directed if it does not contain both a factor of length  $d$  and its mirror image. We show that there exist exponentially many  $\left(\frac{7}{4}\right)$ -free 180-directed ternary words avoiding 010. Moreover, there does not exist an infinite  $\left(\frac{7}{4}\right)$ -free 179-directed ternary word avoiding 010.

**Keywords:** Word, Repetitions.

**2020 Mathematics Subject Classification:** 68R15.

## 1 Introduction

This note is about words avoiding repetitions, a well-studied area in combinatorics on words [5,6]. A *repetition* is a factor of the form  $r = u^n v$  where  $u$  is non-empty and  $v$  is a prefix of  $u$ . Then  $|u|$  is the *period* of the repetition  $r$  and its *exponent* is  $|r|/|u|$ . A word is  $\alpha^+$ -free (resp.  $\alpha$ -free) if it contains no repetition with exponent  $\beta$  such that  $\beta > \alpha$  (resp.  $\beta \geq \alpha$ ).

We consider ternary square-free words (i.e., 2-free words) with additional avoidance constraints. Thue [7] has shown that there exist infinite square-free words avoiding the factors  $\{010, 212\}$ ,  $\{010, 101\}$ , and  $\{010, 020\}$  respectively. In each case, the avoiding word is essentially unique. Consider for example the fixed point of the morphism  $0 \mapsto 012, 1 \mapsto 02, 2 \mapsto 1$ , which is a







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