

Domain Wall Dynamics in Notched Ferromagnetic Nanowires

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ANR MOSICOF

**Conference in honour of Prof. Frédéric Hélein
"Analyse des EDP, géométrie et physique"
5-9 june 2023 - Toulouse**

1 Physical Context

2 Ferromagnetic Nanowires

3 DW in infinite nanowire of constant cross-section

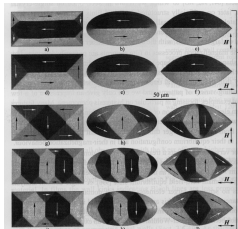
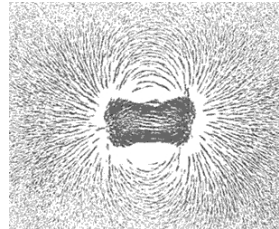
4 DW in infinite nanowire with one notch

- Without applied field
- Pinning and depinning
- After depinning

Ferromagnetic materials

Properties

- Spontaneous magnetization
- Magnetic field generated by the magnetization
- Formation of domains and domain walls



A. Aharoni, Introduction to the Theory of Ferromagnetism, Oxford University Press, 1996

Industrial Applications

- Electromagnets
- Transformers
- Radar absorbing paints
- Mobile phones
- Modern Recording Devices

Magnetic Moment

- Volume occupied by the ferromagnetic material: $\Omega \subset \mathbb{R}^3$
- Magnetization described by the magnetic moment $m : \Omega_x \rightarrow \mathbb{R}^3$

$$|m(x)| = 1 \text{ p. p.}$$

- Constitutive Relation: $B = H + m$
- Static configurations: critical points of \mathcal{E} under the constraint $|m| = 1$

Micromagnetism energy

$$\mathcal{E}(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(m)|^2 + \int_{\Omega} \Psi(m) - \int_{\Omega} m \cdot H_a$$

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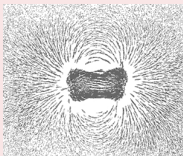
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$$\text{curl } h_d(m) = 0, \quad \text{div } (h_d(m) + \bar{m}) = 0$$

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Uniaxial Anisotropy: $\Psi(m) = -\frac{1}{2}(m \cdot \xi)^2$ or Planar Anisotropy: $\Psi(m) = +\frac{1}{2}(m \cdot \xi)^2$

Magnetic Moment

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Applied magnetic field: H_a

Magnetic Moment

- Magnetic domain : $\Omega \subset \mathbb{R}^3$
- Magnetization described by the magnetic moment $m : \mathbb{R}_t^+ \times \Omega_x \rightarrow \mathbb{R}^3$

$$|m(t, x)| = 1 \text{ p. p.}$$

- Constitutive Relation: $B = H + m$

Landau-Lifschitz Equation

$$\frac{\partial m}{\partial t} = -m \times \mathcal{H} - m \times (m \times \mathcal{H}), \quad \mathcal{H} = -\partial_m \mathcal{E}(m) = A \Delta m + h_d(m) + \nabla \Psi(m) + H_a$$

$$m(0, \cdot) = m_0 \text{ on } \Omega, \quad \frac{\partial m}{\partial n} = 0 \text{ on } \mathbb{R}^+ \times \partial \Omega,$$

$$\mathcal{E}(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(m)|^2 + \int_{\Omega} \Psi(m) - \int_{\Omega} m \cdot H_a$$

$$\mathcal{E}(m(t, \cdot)) + \int_0^t \left\| \frac{\partial m}{\partial t}(s, \cdot) \right\|_{L^2}^2 ds = \mathcal{E}(m_0)$$

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- Constitutive Relation: $B = H + m$

Landau-Lifschitz-Gilberg Equation

$$\frac{\partial m}{\partial t} - m \times \frac{\partial m}{\partial t} = -2m \times \mathcal{H} \quad m \times \Delta m = \sum_i \partial_{x_i} (m \times \partial_{x_i} m)$$

$$\mathcal{H} = -\partial_m \mathcal{E}(m) = A \Delta m + h_d(m) + \psi(m) + H_a$$

$$\mathcal{E}(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(m)|^2 + \int_{\Omega} \Psi(m) - \int_{\Omega} m \cdot H_a$$

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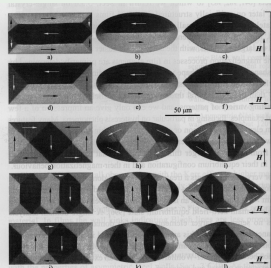
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Visintin (1985); C. (1997); Hardt, Kinderlherer (2000)

Desimone, Kohn, Müller, Otto (2002)

Rivière, Serfaty (2001)

Ignat, Moser (2022)



A. Aharoni, Introduction to the Theory of Ferromagnetism, Oxford University Press, 1996

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Existence Results

- Joly, Métivier, Rauch (LL-Maxwell without exchange field)
- Visintin; Alouges, Soyeur; C., Fabrie; Ding, Guo (Global in time weak solutions)
- C., Fabrie, Guès (Local in time strong solutions)

Numerical Simulations

- Joly, Haddar (without exchange field)
- Labbé; Garcia-Cervera; Alouges; Bartels, Pröhl; Praetorius;... (with exchange field)

1 Physical Context

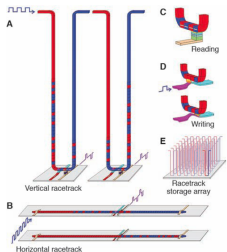
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Data storage in ferromagnetic nanowires

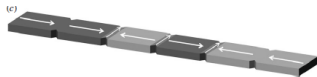


S. Parkin, M. Hayashi, L. Thomas, *Magnetic Domain-Wall Racetrack Memory*, *Science* **320** (2008)

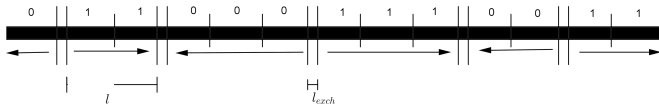
- Bits encoded by the sense of the magnetization
- Stabilization of walls positions by notches

Stuart S. P. Parkin, Masamitsu Hayashi, Luc Thomas, *Magnetic Domain-Wall Racetrack Memory*, *Science* **320** (2008)

- 3d devices for the storage of numerical data
- Fast access



G. Hrkac, J. Dean and D. A. Allwood, *Nanowire spintronics for storage class memories and logic*, *Phil. Trans. R. Soc. A* (2011) **369**



For data storage:

- DW are pinned by the notches (reliability of the storage)
- DW depinning by magnetic field: data recording
- DW depinning by application of an electric current: data reading

Goals

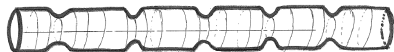
- Obtention and justification of equivalent simpler 1d-models for nanowires
- Description of walls configurations and walls dynamics
- Effects of notches on walls stability
- DW depinning

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$$\Omega_\eta = \{(x_1, x_2, x_3), x_1 \in \mathbf{I}, (x_2)^2 + (x_3)^2 < \eta^2(\rho(x_1))^2\}$$

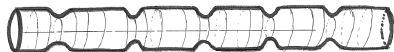
Weak Formulation:

$$\int_{[0, T] \times \Omega_\eta} \left(\frac{\partial m}{\partial t} - m \times \frac{\partial m}{\partial t} \right) \chi = 2 \int_{[0, T] \times \Omega_\eta} \sum_{i=1}^3 m \times \frac{\partial m}{\partial x_i} \cdot \frac{\partial \chi}{\partial x_i} - 2 \int_{[0, T] \times \Omega_\eta} m \times H_d(m) \cdot \chi.$$

- rescaling: fix domain $\Omega_1 = \{(x_1, \tilde{x}_2, \tilde{x}_3), x \in \mathbf{I}, (\tilde{x}_2)^2 + (\tilde{x}_3)^2 < (\rho(x_1))^2\}$
- $\eta \rightarrow 0$ in the weak formulation with $\chi(t, x_1, \tilde{x}_2, \tilde{x}_3) = \varphi(t, x_1)$

Radius of the cross section: ρ

- $\rho : \mathbf{I} \rightarrow \mathbb{R}^{*+}, \mathcal{C}^\infty,$
- $0 < \rho_{min} \leq \rho(x_1) \leq 1$
- $\mathbf{s}(x_1) = \pi(\rho(x_1))^2$



$$\Omega_\eta = \{(x_1, x_2, x_3), x_1 \in \mathbf{I}, (x_2)^2 + (x_3)^2 < \eta^2(\rho(x_1))^2\}$$

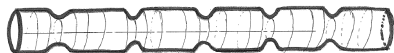
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$$m : \mathbb{R}_t^+ \times \mathbf{I}_x \rightarrow \mathcal{S}^2,$$

m is constant in the cross-section

$$\int_{[0, T] \times \mathbf{I}} \mathbf{s}(x) \left(\frac{\partial m}{\partial t} - m \times \frac{\partial m}{\partial t} \right) \varphi = 2 \int_{[0, T] \times \mathbf{I}} \mathbf{s}(x) m \times \frac{\partial m}{\partial x} \cdot \frac{\partial \varphi}{\partial x} - 2 \int_{[0, T] \times \Omega_\eta} \mathbf{s}(x) m \times (m_2 \mathbf{e}_2 + m_3 \mathbf{e}_3) \cdot \varphi.$$

$$\mathbf{s}(x) \frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$H_{\text{eff}} = \partial_x(\mathbf{s}(x) \partial_x m) - \mathbf{s}(x)(m_2 \vec{\mathbf{e}}_2 + m_3 \vec{\mathbf{e}}_3)$$

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1d-model of wire



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Model of ferromagnetic nanowire

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathcal{H}_{eff} - \mathbf{m} \times (\mathbf{m} \times \mathcal{H}_{eff})$$

$$\mathcal{H}_{eff} = \partial_{xx} \mathbf{m} + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x \mathbf{m} - (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

1d Ferromagnetism Energy

$$E(\mathbf{m}) = \frac{1}{2} \int_{\mathbf{I}} \mathbf{s}(x) |\partial_x \mathbf{m}|^2 dx + \frac{1}{2} \int_{\mathbf{I}} \mathbf{s}(x) (|m_2|^2 + |m_3|^2) dx$$

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- Pinning and depinning
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Infinite nanowire of constant cross-section

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathcal{S}^2, \mathbf{s} \equiv \pi$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$H_{\text{eff}} = m_{xx} - (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

- φ is constant
- $\theta'' + \cos \theta \sin \theta = 0$
- $\theta \rightarrow \pm\pi/2$ when $x \rightarrow \pm\infty$

Static Solutions: Walls

$$\bullet M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix} = R_{\varphi} \begin{pmatrix} \tanh x \\ \frac{1}{\cosh x} \\ 0 \end{pmatrix}.$$

$$R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$$

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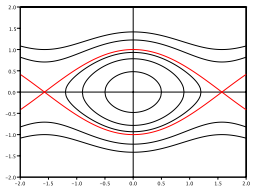
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Stability for Static Profiles

G.C. S. Labbé (2006):

Stability of the constant profiles but not asymptotic stability

$$m(t, x) \rightarrow R_{\varphi_{\infty}} M_0(x - \sigma_{\infty}) \text{ when } t \rightarrow \infty$$

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Moving DW

$$M_0(x) = \begin{pmatrix} \tanh x \\ 1/\cosh x \\ 0 \end{pmatrix}$$

$$R_\varphi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$$

$$m(t, x) = R_{h_a t} (M_0(x + h_a t))$$

Stability results

Moving DW with constant small applied field: K. Takasao (2011)

Stability threshold for constant applied field: R. Jizzini (2011)

Controlability results: G.C., S. Labbé, E. Trélat (2008); Y. Privat, E. Trélat (2015).

Non constant applied field: R. Côte, R. Ignat

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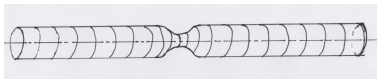
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Model of infinite wire with one notch



$$\Omega_\eta = \{(x, y, z), x \in \mathbb{R}, y^2 + z^2 < \eta^2(\rho(x))^2\}$$

Radius of the section: ρ

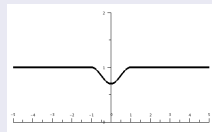
- $\rho : \mathbb{R} \rightarrow \mathbb{R}^+, C^\infty$,
- $\rho = 1$ outside $[-a, a]$,
- $\rho(-x) = \rho(x)$,
- ρ non decreasing on $[0, a]$
- $\mathbf{s}(x) = \pi(\rho(x))^2$

Model of infinite wire with notch

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow S^2$$

$$\frac{\partial m}{\partial t} = -m \times \mathcal{H}_{\text{eff}} - m \times (m \times \mathcal{H}_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = \partial_{xx} m + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x m - (m_2 \vec{e}_2 + m_3 \vec{e}_3) + h_a \vec{e}_1$$



Area of the cross section: $\mathbf{s}(x)$

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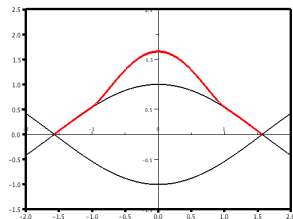
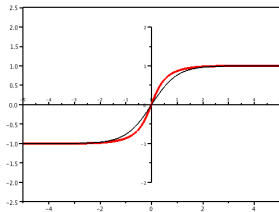
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Existence of profiles

- $M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \bar{\theta}(x) \\ \cos \bar{\theta}(x) \\ 0 \end{pmatrix}$
- $R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$
- $\bar{\theta} \rightarrow \pm\pi/2$ when $x \rightarrow \pm\infty$

- φ is constant
- $\bar{\theta}'' + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x \bar{\theta} + \cos \bar{\theta} \sin \bar{\theta} = 0$
- $\left((\bar{\theta}')^2 - \cos^2 \bar{\theta} \right)' = -2 \frac{\mathbf{s}'}{\mathbf{s}} (\bar{\theta}')^2$

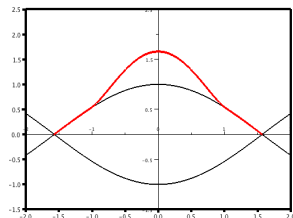
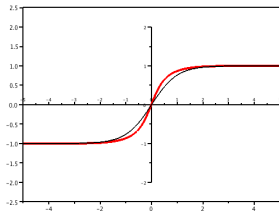


Shooting method to reach the separatrix after the notch

Existence of profiles

- $M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \bar{\theta}(x) \\ \cos \bar{\theta}(x) \\ 0 \end{pmatrix}$
- $R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$
- $\bar{\theta} \rightarrow \pm\pi/2$ when $x \rightarrow \pm\infty$

- φ is constant
- $\bar{\theta}'' + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x \bar{\theta} + \cos \bar{\theta} \sin \bar{\theta} = 0$
- $\left((\bar{\theta}')^2 - \cos^2 \bar{\theta} \right)' = -2 \frac{\mathbf{s}'}{\mathbf{s}} (\bar{\theta}')^2$



Shooting method to reach the separatrix after the notch

Infinite Nanowire with one notch

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathbb{S}^2$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3$$

$$m \rightarrow \pm e_1 \text{ when } x \rightarrow \pm\infty$$

Wall Profile

$$M_0(x) = \begin{pmatrix} \sin \bar{\theta}(x) \\ \cos \bar{\theta}(x) \\ 0 \end{pmatrix}$$

$$\bar{\theta}'' + \frac{\mathbf{s}'}{\mathbf{s}} \bar{\theta}' + \cos \bar{\theta} \sin \bar{\theta} = 0$$

$$\text{BC} : \bar{\theta} \rightarrow \pm\pi/2 \text{ when } x \rightarrow \pm\infty$$

Thm 1 (with D. Sanchez)

Existence and stability of M_0 , asymptotic stability for the position of the wall

If $\|m(t=0) - M_0\|_{H^1(\mathbb{R})}$ is small, then

- for all time, $\|m(t) - M_0\|_{H^1(\mathbb{R})}$ remains small,
- $m(t)$ tends to $R_{\theta_\infty}(M_0)$ when t tends to $+\infty$.

Mobile frame

Geometrical constraint $|m| = 1$

We describe the perturbations in the mobile frame

$$m(t, x) = \sqrt{1 - |r(t, x)|^2} M_0(x) + r_1(t, x) M_1(x) + r_2(t, x) M_2$$

$$M_0(x) = \begin{pmatrix} \sin \bar{\theta}(x) \\ \cos \bar{\theta}(x) \\ 0 \end{pmatrix} \quad M_1(x) = \begin{pmatrix} -\cos \bar{\theta}(x) \\ \sin \bar{\theta}(x) \\ 0 \end{pmatrix} \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

New Equations for r

$$r : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathbb{R}^2$$

$$\partial_t r_1 = -L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$\partial_t r_2 = L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$L_1(r) = -\partial_{xx} r - \frac{s'}{s} \partial_x r + (\sin^2 \bar{\theta} - \cos^2 \bar{\theta}) r$$

$$L_2(r) = -\partial_{xx} r - \frac{s'}{s} \partial_x r + (\sin^2 \bar{\theta} - (\bar{\theta}')^2) r$$

Exact Solution

M_0 for LL ~ 0 for the new system

Stability of 0 ?

Mobile frame

Geometrical constraint $|m| = 1$

We describe the perturbations in the mobile frame

$$m(t, x) = \sqrt{1 - |r(t, x)|^2} M_0(x) + r_1(t, x) M_1(x) + r_2(t, x) M_2$$

$$M_0(x) = \begin{pmatrix} \sin \bar{\theta}(x) \\ \cos \bar{\theta}(x) \\ 0 \end{pmatrix} \quad M_1(x) = \begin{pmatrix} -\cos \bar{\theta}(x) \\ \sin \bar{\theta}(x) \\ 0 \end{pmatrix} \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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$$\partial_t r_2 = L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$L_1(r) = -\partial_{xx} r - \frac{s'}{s} \partial_x r + (\sin^2 \bar{\theta} - \cos^2 \bar{\theta}) r$$

$$L_2(r) = -\partial_{xx} r - \frac{s'}{s} \partial_x r + (\sin^2 \bar{\theta} - (\bar{\theta}')^2) r$$

Exact Solution

M_0 for LL ~ 0 for the new system

Stability of 0 ?

Properties of the linearized: L_2

On $L^2(\mathbb{R})$, weighted scalar product:

$$\langle u|v \rangle_{\mathbf{s}} = \int_{\mathbb{R}} \mathbf{s}(x)u(x)v(x) dx,$$

$$L_2 = -\frac{1}{\mathbf{s}}\partial_x(\mathbf{s}\partial_x) + (\sin^2 \bar{\theta} - (\bar{\theta}')^2) \implies L_2 \text{ is self-adjoint for } \langle | \rangle_{\mathbf{s}}.$$

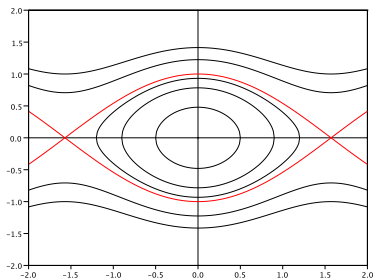
$$\langle L_2 u|v \rangle_{\mathbf{s}} = \langle \ell u|\ell v \rangle_{\mathbf{s}}, \quad \text{with } \ell = \partial_x + \bar{\theta}' \tan \bar{\theta}$$

$$L_2 \geq 0, \quad \text{Ker } L_2 = \mathbb{R}(\cos \bar{\theta})$$

- Valid whatever the configurations of the wire (several pinched zones)
- for one wall, on $(\cos \bar{\theta})^\perp$, $L_2 \geq \alpha_2 > 0$
- **Invariance by rotation** $\implies 0$ is an eigenvalue of the linearized

Properties of the linearized: L_1

$$L_1 = L_2 + (\bar{\theta}')^2 - \cos^2 \bar{\theta}$$

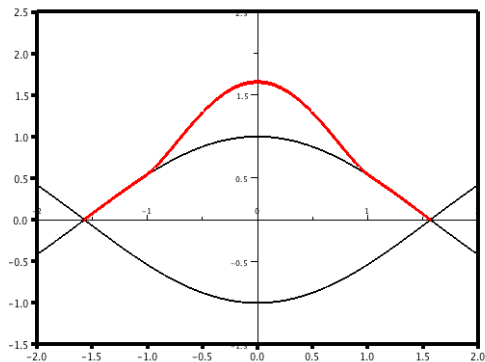


Key point : $L_1 \geq 0$?

Infinite nanowire without pinching: $L_1 = L_2$, $L_1 \geq 0$. **Stability**

Properties of the linearized: L_1

$$L_1 = L_2 + (\bar{\theta}')^2 - \cos^2 \bar{\theta}$$



$$(\bar{\theta}')^2 - \cos^2 \bar{\theta} > 0 \text{ on Ker } L_2 \implies L_1 > 0$$

1 Physical Context

2 Ferromagnetic Nanowires

3 DW in infinite nanowire of constant cross-section

4 DW in infinite nanowire with one notch

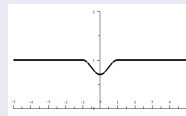
- Without applied field
- **Pinning and depinning**
- After depinning

Notched Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow S^2$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3 + h_a \vec{e}_1$$



Area of the cross section: $\mathbf{s}(x)$

Constant-cross-section wire: precessional regime

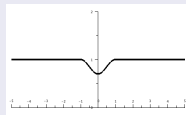
$$m(t, x) = R_{+h_a t} M_0(x + h_a t)$$

Notched Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathcal{S}^2$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3 + h_a \vec{e}_1$$



Area of the cross section: $\mathbf{s}(x)$

Notched wire:

Thm 2, with D. Sanchez

Pinning effects: for small h_a : the wall remains in the notch (static solution), its profile is planar.

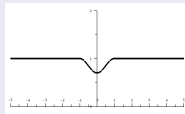
Depinning: for $|h_a| \geq \frac{\pi - \mathbf{s}(0)}{\pi + \mathbf{s}(0)}$, there is no static solution with one DW

Notched Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow S^2$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3 + h_a \vec{e}_1$$

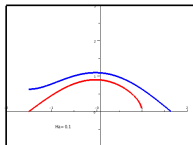


Area of the cross section: $\mathbf{s}(x)$

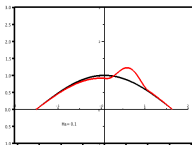
$$\theta'' + \frac{\mathbf{s}'}{\mathbf{s}} \theta' + \cos \theta \sin \theta + h_a \cos \theta = 0$$

Left side: $(\theta')^2 = -\sin^2 \theta - 2h_a \sin \theta + 1 - 2h_a$,

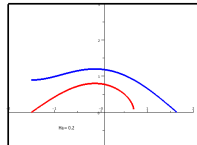
Right side: $(\theta')^2 = -\sin^2 \theta - 2h_a \sin \theta + 1 + 2h_a$



$h_a = 0.2$



$h_a = 0.2$



$h_a = 0.5$

1 Physical Context

2 Ferromagnetic Nanowires

3 DW in infinite nanowire of constant cross-section

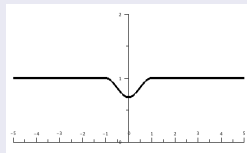
4 DW in infinite nanowire with one notch

- Without applied field
- Pinning and depinning
- **After depinning**

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathbb{S}^2$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3 - h_a \vec{e}_1$$



Area of the cross section: $\mathbf{s}(x)$

Solution without notch:

$$\mathcal{M}_{\sigma_0}(t, x) = R_{-H(t)} M_0(x - \sigma_0 - H(t)), \quad H(t) = \int_0^t h_a(s) ds.$$

Thm 3 (with M. Kassan)

There exists $0 < h^- < h^+$ such that if $h_a \in \mathcal{C}^0(\mathbb{R}^+)$ with $\forall t, 0 < h^- \leq h(t) \leq h^+$, then for all $\varepsilon > 0$, there exists σ^* and $\eta > 0$ such that:

for all $\sigma_0 \geq \sigma^*$, for all m , solution of LL, with $\|m(0, \cdot) - \mathcal{M}_{\sigma_0}(0, \cdot)\|_{H^1} \leq \eta$,

$$\forall t > 0, \|m(t, \cdot) - \mathcal{M}_{\sigma_0}(t, \cdot)\|_{H^1} \leq \varepsilon,$$

$$\exists \varphi_\infty, \exists \sigma_\infty, \|m(t, \cdot) - R_{\varphi_\infty} \mathcal{M}_{\sigma_0}(t, \cdot + \sigma_\infty)\|_{H^1} \rightarrow 0$$

First step: mobile frame

$$\mathcal{M}_{\sigma_0}(t, x) = R_{-H(t)} M_0(x - \sigma_0 - H(t)), \quad H(t) = \int_0^t h_a(s) ds.$$

We rewrite m as:

$$m(t, x) = R_{-H(t)} M(t, x - \sigma_0 - H(t)), \quad H(t) = \int_0^t h_a(s) ds.$$

$m(t, x)$ close to $\mathcal{M}_{\sigma_0}(t, x) \iff M(t, \tilde{x})$ close to $M_0(\tilde{x})$, with $\tilde{x} = x - \sigma_0 - H(t)$

Equation for M

$$\partial_t M = -M \times \tilde{H}_{\text{eff}}(M) - M \times (M \times \tilde{H}_{\text{eff}}(M)) + h_a(t)(\partial_x M + M \times (M \times \tilde{e}_1)),$$

$$\tilde{H}_{\text{eff}}(M) = \partial_{xx} M + \frac{\mathbf{s}'}{\mathbf{s}}(x + \sigma_0 + H(t)) \partial_x M + (M \cdot \tilde{e}_1) \tilde{e}_1.$$

$$M : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{S}^2$$

Mobile frame and new unknown

$$M(t, \tilde{x}) = r_1(t, \tilde{x}) M_1(\tilde{x}) + r_2(t, \tilde{x}) M_2 + \sqrt{1 - |r|^2} M_0(\tilde{x})$$

$$M_0 = \begin{pmatrix} \tanh \tilde{x} \\ \frac{1}{\cosh \tilde{x}} \\ 0 \end{pmatrix}, \quad M_1 = \begin{pmatrix} -\frac{1}{\cosh \tilde{x}} \\ \tanh \tilde{x} \\ 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

First step: mobile frame

$m(t, x)$ close to $\mathcal{M}_{\sigma_0}(t, x) \iff M(t, \tilde{x})$ close to $M_0(\tilde{x})$, with $\tilde{x} = x - \sigma_0 - H(t)$

Equation for M

$$\partial_t M = -M \times \tilde{H}_{\text{eff}}(M) - M \times (M \times \tilde{H}_{\text{eff}}(M)) + h_a(t)(\partial_x M + M \times (M \times \tilde{e}_1)),$$

$$\tilde{H}_{\text{eff}}(M) = \partial_{xx} M + \frac{\mathbf{s}'}{\mathbf{s}}(x + \sigma_0 + H(t))\partial_x M + (M \cdot \tilde{e}_1)\tilde{e}_1.$$

$$M : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{S}^2$$

Mobile frame and new unknown

$$M(t, \tilde{x}) = r_1(t, \tilde{x})M_1(\tilde{x}) + r_2(t, \tilde{x})M_2 + \sqrt{1 - |r|^2}M_0(\tilde{x})$$

$$M_0 = \begin{pmatrix} \tanh \tilde{x} \\ \frac{1}{\cosh \tilde{x}} \\ 0 \end{pmatrix}, \quad M_1 = \begin{pmatrix} -\frac{1}{\cosh \tilde{x}} \\ \tanh \tilde{x} \\ 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$M(t, \cdot)$ close to $M_0 \iff r$ close to 0

Equation in r

$$\partial_t r = \mathcal{V}_{\sigma_0}(t, x) + (\mathcal{J}\mathcal{L} + h_a(t)\ell)r + \text{non linear terms}$$

First step: mobile frame

Mobile frame and new unknown

$$M(t, \tilde{x}) = r_1(t, \tilde{x})M_1(\tilde{x}) + r_2(t, \tilde{x})M_2 + \sqrt{1 - |r|^2}M_0(\tilde{x})$$

$$M_0 = \begin{pmatrix} \tanh \tilde{x} \\ \frac{1}{\cosh \tilde{x}} \\ 0 \end{pmatrix}, \quad M_1 = \begin{pmatrix} -\frac{1}{\cosh \tilde{x}} \\ \tanh \tilde{x} \\ 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$M(t, \cdot)$ close to $M_0 \iff r$ close to 0

Equation in r

$$\partial_t r = \mathcal{V}_{\sigma_0}(t, x) + (J\mathcal{L} + h_a(t)\ell)r + \text{non linear terms}$$

$$\mathcal{V}(t, x) = \frac{1}{\cosh x} \frac{\mathbf{s}'}{\mathbf{s}} (x + \sigma_0 + H(t)) \vec{e}_1, \quad |\mathcal{V}(t, x)| \leq Ke^{-\gamma(\sigma_0 + H(t))},$$

$$J = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix},$$

$$\mathcal{L}r = -\partial_{xx}r - \frac{\mathbf{s}'}{\mathbf{s}}(x + \sigma_0 + H(t))\partial_x r + (2 \tanh^2 x - 1)r, \quad \ell r = \partial_x r + \tanh(x)r$$

Equation in r

$$\partial_t r = \mathcal{V}_{\sigma_0}(t, x) + (J\mathcal{L} + h_a(t)\ell)r + \text{non linear terms}$$

$$\mathcal{V}(t, x) = \frac{1}{\cosh x} \frac{\mathbf{s}'}{\mathbf{s}} (x + \sigma_0 + H(t)) \vec{e}_1, \quad |\mathcal{V}(t, x)| \leq Ke^{-\gamma(\sigma_0 + H(t))},$$

$$J = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix},$$

$$\mathcal{L}r = -\partial_{xx}r - \frac{\mathbf{s}'}{\mathbf{s}}(x + \sigma_0 + H(t))\partial_x r + (2 \tanh^2 x - 1)r, \quad \ell r = \partial_x r + \tanh(x)r$$

To be proved

If σ_0 is large enough, if $r(t=0)$ is small enough, then $r(t, \cdot)$ remains small.

LL without notch ($\mathbf{s} \equiv 1$)

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$H_{\text{eff}} = m_{xx} - (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

Invariance by rotation and translation

\implies existence of a 2-parameter family of solutions for LL:

$$(t, x) \mapsto R_\theta M_0(x + \sigma)$$

\implies existence of a 2-parameter family of solutions in the mobile frame

$\implies 0 \in \text{Ker } L$, with $L = -\partial_{xx} + (2 \tanh^2 x - 1)$ associated to $x \mapsto \frac{1}{\cosh x}$ and $L = \ell^* \circ \ell$.

LL with notch and applied field

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$H_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - (m_2 \vec{e}_2 + m_3 \vec{e}_3) - h_a \vec{e}_1$$

m close to

$$\mathcal{M}_{\sigma_0}(t, x) = R_{-H(t)} M_0(x - \sigma_0 - H(t))$$

Invariance by rotation, quasi-invariance by translation

2-parameter family of quasi-solutions
close to \mathcal{M}_{σ_0} :

$$(t, x) \mapsto R_{\theta} \mathcal{M}_{\sigma_0}(t, x + \sigma)$$

2-parameter family of quasi solutions
for Eq. in r :

$$(t, x) \mapsto \mathcal{R}_{\Lambda}(t, x), \quad \Lambda = (\theta, \sigma)$$

New unknowns: Λ and W

$$r(t, \cdot) = \mathcal{R}_{\Lambda(t)}(\cdot) + W(t, \cdot), \quad \text{with } \int_{\mathbb{R}} W_i(t, x) \frac{1}{\cosh x} dx = 0.$$

Equations in Λ and W

$$\partial_t W = \mathcal{V}_{\sigma_0}(t, x) + (J\mathcal{L} + h_a(t)\ell)W + F(t, x, \sigma_0, \Lambda, W)$$

$$\frac{d\Lambda}{dt} = \mathcal{G}(t, x, \sigma_0, \Lambda, W)$$

$$\|\mathcal{V}_{\sigma_0}(t, \cdot)\|_{L_t^2} \leq C \exp(-c(\sigma_0 + H(t))), \quad J = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix},$$

$$\mathcal{L}W = -\partial_{xx}W - \frac{\mathbf{s}'}{\mathbf{s}}(x + \sigma_0 + H(t))\partial_x W + (2 \tanh^2 x - 1)W, \quad \ell W = \partial_x W + \tanh(x)W$$

$$\|F\|_{L^2} \leq C\|W\|_{H^2} (\|\Lambda\| + \|W\|_{H^1}) + C \exp(-c(\sigma_0 + H(t)))$$

$$|G| \leq C\|W\|_{H^1} + C \exp(-c(\sigma_0 + H(t)))$$

To be proved:

If $W(0)$ and $\Lambda(0)$ are small, then

- $W(t)$ and $\Lambda(t)$ remain small
- $W \rightarrow 0$ in H^1 and $\Lambda(t) \rightarrow \Lambda_\infty$

$$\left\langle W_i \left| \frac{1}{\cosh x} \right. \right\rangle = 0 \implies W \in (\text{Ker } L)^\perp, \text{ with } L = -\partial_{xx} + 2 \tanh^2 x - 1.$$

$$\|W\|_{L^2}^2 \leq \langle LW | W \rangle = \|\ell W\|_{L^2}^2 \leq \|LW\|_{L^2}^2$$

We deal with:

$$\mathcal{L}W = -\partial_{xx}W - \frac{\mathbf{s}'}{\mathbf{s}}(x + \sigma_0 + H(t))\partial_x W + (2 \tanh^2 x - 1)W$$

We use a scalar product with time-dependent weight:

$$\langle u | v \rangle_t = \int_{\mathbb{R}} \mathbf{s}(x + \sigma_0 + H(t))u(x)v(x)dx$$

for which \mathcal{L} is self-adjoint and coercive, so that:

$$\|W\|_{H^1} \sim \langle \mathcal{L}W | W \rangle_t^{\frac{1}{2}} \text{ and } \|W\|_{H^2} \sim \|LW\|_{L_t^2}.$$

$$\partial_t W = \mathcal{V}_{\sigma_0}(t, x) + J\mathcal{L}W + h_a(t)\ell W + F(t, x, \sigma_0, \Lambda, W)$$

$$\|\mathcal{V}_{\sigma_0}(t, \cdot)\|_{L_t^2} \leq C \exp(-c(\sigma_0 + H(t))), \quad J = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix},$$

$$\mathcal{L}W = -\partial_{xx}W - \frac{\mathbf{s}'}{\mathbf{s}}(x + \sigma_0 + H(t))\partial_x W + (2 \tanh^2 x - 1)W, \quad \ell W = \partial_x W + \tanh(x)W$$

$$\|F\|_{L^2} \leq C\|W\|_{H^2} (|\Lambda| + \|W\|_{H^1}) + C \exp(-c(\sigma_0 + H(t)))$$

$$\frac{d}{dt} \langle \mathcal{L}W | W \rangle_t \leq -\|\mathcal{L}W\|_{L_t^2}^2 + c(|h| + |\Lambda| + \|W\|_{H^1})\|\mathcal{L}W\|_{L_t^2}^2 + C \exp(-c(\sigma_0 + H(t)))\|\mathcal{L}W\|_{L_t^2}$$

Assumption: $|h| \leq \frac{1}{8c}$

While $|\Lambda| \leq \frac{1}{8c}$ and $\|W\|_{H^1} \leq \frac{1}{8c}$, then

$$\frac{d}{dt} \langle \mathcal{L}W | W \rangle_t + \frac{1}{2}\|\mathcal{L}W\|_{L_t^2}^2 \leq C \exp(-2c(\sigma_0 + H(t)))$$

$$\frac{d}{dt} \langle \mathcal{L}W | W \rangle_t + \alpha \langle \mathcal{L}W | W \rangle_t \leq C \exp(-2c(\sigma_0 + H(t)))$$

Comparison lemma

Assumption: $c|h| \leq \frac{1}{8c}$

While $|\Lambda| \leq \frac{1}{8c}$ and $\|W\|_{H^1} \leq \frac{1}{8c}$, then

$$\frac{d}{dt} \langle \mathcal{L}W | W \rangle_t + \alpha \langle \mathcal{L}W | W \rangle_t \leq C \exp(-2c(\sigma_0 + H(t))),$$

so $\langle \mathcal{L}W | W \rangle_t \leq Y(t)$ where Y satisfies $Y' + \alpha Y = C \exp(-2c(\sigma_0 + H(t)))$.

$$Y(t) = Y_0 \exp(-\alpha t) + \exp(-2c\sigma_0) \int_0^t C \exp(-\alpha(t-s)) \exp(-2cH(s)) ds.$$

We need:

- $Y(t)$ small for all time if $Y(0)$ is small and σ_0 large enough,
- $Y(t) \rightarrow 0$
- to control Λ , $\sqrt{Y(t)}$ integrable and its integral can be as small as necessary.

Assumption on h

- $|h(t)|$ small enough
- $t \mapsto \left(\int_0^t \exp(-\alpha(t-s) - 2cH(s)) ds \right)^{\frac{1}{2}}$ tends to zero when $t \rightarrow +\infty$ and integrable on \mathbb{R}^+ .

Example: if $0 < h^- \leq h(t) \leq \frac{1}{8c}$

Done

- Static solution for small constant h_a
- Static solution for multiply notched wire
- Dynamics far from the notch

- Stability threshold for h_a ?
- Dynamics of the walls for wires with several notches?
- Dynamics induced by electric current?

Thank you for your attention

Done

- Static solution for small constant h_a
- Static solution for multiply notched wire
- Dynamics far from the notch

- Stability threshold for h_a ?
- Dynamics of the walls for wires with several notches?
- Dynamics induced by electric current?

Thank you for your attention