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Wave groups and small scale variability of wave heights observed by altimeters

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Key Points:

- Wave groups contribute to small-scale fluctuations in altimeter wave heights, explaining 25% of the variance measured by CFOSAT in 80 km.
- For the same wave height, fluctuations are larger in the presence of long and narrow-banded waves, typical of swell-dominated conditions.
- Altimeters smooth out scales shorter than the square root of half the H_s times the altitude, and distort spatial patterns at that scale.

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Abstract

Recent satellite altimeter retracking and filtering methods have considerably reduced the noise level in estimates of the significant wave height (H_s), allowing to study processes with smaller spatial scales. In particular, previous studies have shown that wave-current interactions may explain most of the variability of H_s at scales 20 to 100 km. As the spatial scale of the measurement is reduced, random fluctuations emerge that should be associated to wave groups. Here we quantify the magnitude of this effect, and the contribution of wave groups to the uncertainty in H_s measurements by altimeters, with a particular focus on extreme extra-tropical storms. We take advantage of the low orbit altitude of the China-France Ocean Satellite (CFOSAT), and the low noise level of the nadir beam of the SWIM instrument. Our estimate of wave group effects uses directional wave spectra measured by off-nadir beams on SWIM, and signal processing theory that gives statistical properties of the wave envelope, and thus the local wave heights, from the shape of the wave spectrum. We find that the standard deviation of H_s associated to wave groups is a function of satellite altitude, wave height and spectral bandwidth. For CFOSAT these fluctuations generally account for about 25% of the variance measured over a 80 km distance. This fraction is largest in storms and in the presence of long swells. When the estimated effect of wave groups is subtracted from the variance of H_s measurements, the remaining variability is higher in regions of strong currents.

Plain Language Summary

Satellite altimeters routinely provide measurements of the height of ocean waves, and improved instruments or processing techniques have led to more precise and detailed measurements. Here we use a combination of simulations and data from the China France Ocean satellite (CFOSAT) to interpret the small scale fluctuations in wave height measurements as the effect of wave "groups" which are fluctuations of the heights of consecutive waves associated to random waves. Due to spatial averaging within the radar footprint, we find that the fluctuations of significant wave heights (H_s) associated to wave group are a function of satellite altitude, wave height and other properties of the ocean waves. For CFOSAT, wave groups give a standard deviation of H_s that is of the order of 3 to 5% of H_s , typically half of the standard deviation in H_s measurements.

1 Introduction

Wind-waves impact all activities at sea, air-sea interactions and remote sensing, and there is a general need for obtaining more accurate and higher resolution information about the sea state. Today, satellite radar altimetry is the most extensive source of measurements with a global coverage, providing routine estimates of the significant wave height H_s (Ardhuin et al., 2019). As these data are getting used for a wide range of applications, it is important to understand what can be measured with altimeters, at what scale and with what uncertainty.

The fundamental measurement of an altimeter is the echo power as a function of the travel time of radar pulses. This function is known as a "waveform". Time is converted to "range", i.e. the distance between the radar and the ocean surface, and this waveform is discretized in range gates with a resolution δ_R , with some examples shown in Appendix A. From the shape of the waveform one can estimate H_s . The horizontal scale of the measurement was particularly discussed by Chelton et al. (1989), who introduced the concept of *oceanographic footprint*. This footprint is the ocean area that contains the sea surface points that contribute to the measurement of sea level and wave height, and it is a disc of radius

$$r_C = \sqrt{\frac{2h(H_s + \delta_R)}{1 + h/R_E}} \quad (1)$$

where h is the satellite altitude, R_E is the Earth radius, H_s is the significant wave height and the range resolution $\delta_R = c/(2B)$ is defined by the radar bandwidth B and the speed of light c . We note that all Ku-band altimeters have used $B = 320$ MHz giving $\delta_R = 0.47$ m. With $B = 500$ MHz, SARAL-AltiKa uses $\delta_R = 0.32$ m. As a result, r_C is always larger than 1 km. That size of the oceanic footprint corresponds to a single radar pulse. The sea echoes detected by the radar come from facets of the sea surface that are horizontal and reflect the signal back to the radar. These facets are randomly distributed within the oceanic footprint, with ranges that vary over many times the electromagnetic wavelength. The signal recorded in any given range gate is thus the sum of a large number of echoes with random phases, giving rise to large fluctuation in measured power, generally known as Rayleigh fading. In the context of radar remote sensing, these fluctuations are called "speckle noise" and have generally been considered to be the dominant source of altimeter measurement noise (Quarty et al., 2001; Tourain et al., 2021). This noise is reduced by averaging echoes from many pulses over a fraction of a second,

76 and these averaged waveforms are processed to estimate H_s . Because low Earth orbit
 77 satellites fly over the ocean at a speed around 7 km/s, averaging altimeter data over 0.05 s
 78 corresponds to a spatial average over 350 m, which is much smaller than r_C and thus
 79 does not change much the effective footprint of the measurement.

80 Even with this averaging, H_s estimates are typically more noisy than buoy mea-
 81 surements, which has led users of altimetry data to take longer averages of H_s , typically
 82 1 to 10 s, corresponding to a distance that spans 7 to 70 km (Dodet et al., 2022). While
 83 it effectively reduces noise, such averaging also blurs potentially interesting features, in
 84 particular the peaks of storms, coastal gradients (Passaro et al., 2021), and the signa-
 85 ture of surface currents (Quilfen & Chapron, 2019). Away from surface current gradi-
 86 ents and coastlines, sea states are uniform over scales of the order of 70 km (Tournadre,
 87 1993). Still, within these scales, the random nature of the wave field is another source
 88 of expected geophysical variability. Theoretical analysis, in situ time series and remote
 89 sensing (Borge et al., 2004) show that small scale variations in wave height contain a sig-
 90 nature of wave groups that can be estimated from the Power Spectral Density (PSD) of
 91 the surface elevation, hereinafter simply called "wave spectrum" (Arhan & Ezraty, 1978;
 92 Tayfun & Lo, 1989). These groups are the result of the linear superposition of many in-
 93 dependent wave trains. Wave groups have typical time scales of a few tens of seconds
 94 to a few minutes, that translate to spatial patterns at scales of a few kilometers, hence
 95 around the possible resolution limit of altimeters, of the order of r_C . At larger scales,
 96 non-linear wave-wave interactions should contribute to fluctuations at scales 10 to 20 min-
 97 utes, with spatial scales around 10 km, that should be important for wave growth (Lavrenov,
 98 2001) and may contain information on the wave period (Badulin, 2014).

99 Co-location of altimeter, buoy and model data with wave heights from 1 to 8 m,
 100 has been used to estimate a typical uncertainty 7% for $H_s > 2$ m, in the case of 1 s av-
 101 eraged altimeter data (Dodet et al., 2022). Unfortunately, no estimate can be made for
 102 higher values due to the lack of co-located buoy and altimeter data. Understanding what
 103 makes up this uncertainty will help extrapolate uncertainties to higher values of H_s , pro-
 104 viding a better understanding of the climatology of sea state extremes. From the prin-
 105 ciple of the estimation of wave heights from radar waveforms (Brown, 1977), satellite al-
 106 timeters should be able to measure H_s values exceeding 30 m with a relative precision
 107 that gets better for higher values of H_s because the signal is spread out over a wider part
 108 of the waveform. At the same time, the higher winds that often occur with high waves

109 will lead to a lower signal recorded by the radar and a lower signal to noise ratio for the
 110 waveform. So far, only numerical models could be compared to the highest values of wave
 111 heights, and their random differences is usually lowest for the highest wave heights (Alday
 112 et al., 2021; Alday & Ardhuin, 2023). The highest values of H_s reported by altimeters,
 113 up to 20.1 m, are consistent with all other observations including the presence of swells
 114 with very long periods (Hanafin et al., 2012). There is thus no fundamental reason to
 115 doubt that altimeters can measure the highest possible wave heights, but there is not
 116 yet a clear understanding of biases and random errors for H_s above 8 m.

117 In the present paper we focus our analysis on the fluctuations of H_s associated to
 118 wave groups and its contribution to Delay-only altimeters that provide the existing refer-
 119 ence time series for wave climate analysis (Timmermans et al., 2020; Dodet et al., 2020).
 120 The main question that we wish to answer is: how much wave groups contribute to the
 121 variability in H_s measurements? For this we take advantage of the unique opportunity
 122 provided by the SWIM instrument onboard the China-France Ocean Satellite (CFOSAT).
 123 SWIM provides both directional wave spectra from which we compute the spectrum of
 124 the wave envelope that contains wave groups, and along-track nadir altimetry. Our anal-
 125 ysis uses SWIM data over the globe for two full years 2020 and 2021.

126 We start with two illustrative and contrasting examples in section 2, before pro-
 127 viding results for the globe in section 3. Discussions and conclusion follow in section 4.
 128 A side question that we had to address is: how does an altimeter measure H_s over a re-
 129 alistic surface that contains local perturbations associated with wave groups? For this
 130 we used a simplified simulated altimeter with numerical results shown in section 2 and
 131 an analytical derivation in Appendix A. Those results suggest that altimeters report a
 132 particular kind of average of wave heights over a radius that is close to $r_C/2$. When us-
 133 ing a least-square fit to radar waveforms, estimated H_s give a spurious amplification of
 134 true H_s perturbations located at a distance around $r_C/4$ from nadir, and are blind to
 135 perturbations located right at the nadir.

136 **2 One particular storm and a theory of wave groups**

137 From now on, we will use the notation \hat{H}_s for estimates of the significant wave heights
 138 provided by altimeter data, to clearly differentiate these from the true significant wave
 139 height H_s . \hat{H}_s exhibits spatial variability that may be related to a true spatial variation

140 of H_s . As we will be considering different sizes of variability, we introduce the notation
 141 $\text{std}(x, d)$ and $\text{var}(x, d)$ that represent respectively the standard deviation and the vari-
 142 ance of a variable x performed over a spatial distance d .

143 As described in Hauser et al. (2017, 2021), the instrument SWIM is a Ku-band wave
 144 scatterometer that illuminates successively 6 incidence angles (0° , 2° , 4° , 6° , 8° and 10°).
 145 The nadir beam (0°) works as all previous Poseidon radar altimeters and provides \widehat{H}_s
 146 measurements every 0.22 s, using an average over 0.055 s. As a result, the nadir beam
 147 data is expected to be similar to data from previous Ku-band altimeters, such as Poseidon-
 148 3B on Jason-3, with the specific difference given by a lower data rate (4.5 Hz instead of
 149 20 Hz) and a different measurement geometry associated to a rather low orbit. In prin-
 150 ciple, the low orbit altitude $h = 519$ km of CFOSAT makes it possible to resolve smaller
 151 scale variations of H_s as r_C is reduced by a factor 1.4 compared to the Jason satellites
 152 that orbit at 1340 km altitude. The low noise level of the satellite and specific process-
 153 ing of the SWIM instrument also contribute to the capability of SWIM to resolve smaller
 154 along-track scales in the variations of H_s compared to previous altimeter datasets (Tourain
 155 et al., 2021).

156 The off-nadir beams use the concept of the wave spectrometer (Jackson et al., 1985)
 157 based on a real-aperture radar and the normalized radar cross-section (NRCS) sensitiv-
 158 ity to local surface slope at near-nadir incidences, providing estimates of the directional
 159 wave spectra (with a 180° ambiguity in direction). The CNES mission center (or CFOSAT
 160 Wind and Waves Instrument Center - CWWIC) provides Level 2 products, hereafter called
 161 L2, both for the nadir beam and the off-nadir beams 6° to 10° . The off-nadir L2 prod-
 162 ucts consist of 2D wave spectra discretized into 12 directions evenly spaced from from
 163 0 to 165° , and 32 wavenumbers forming a geometric progression from 0.0125 to 0.28 m^{-1}
 164 with a common ratio of 1.105. These spectra are constructed from overlapping of antenna
 165 scans over 180° (on each side of the track) over boxes of about 70 km by 90 km. In or-
 166 der to allow comparison, the nadir product is resampled by averaging values of \widehat{H}_s over
 167 the box size (c. 80 km along track), its variation at this scale is quantified by taking its
 168 standard deviation over the same distance.

169 The Ifremer Waves and Wind Operational Center (IWWOC) is in charge of devel-
 170 oping and testing different processing and provides an alternative Level 2 product for
 171 off-nadir beams. This product is referred to as L2S product and consists of 1D wave mod-

172 ulation spectra, one for each measurement azimuth. Whereas the L2 product uses the
 173 nadir H_s to rescale the spectrum, the L2S product is based on a theoretical modulation
 174 transfer function to transform the NRCS spectra into surface elevation spectra (Jackson
 175 et al., 1985). Also the L2 product uses a maximum wavelength of 500 m in order to avoid
 176 amplifying noise, where the L2S product does not use such a fixed value for the max-
 177 imum wavelength. As a result, L2 spectra are often badly distorted in the most severe
 178 storms where the spectrum is dominated by waves longer than 500 m. Because we fo-
 179 cused on extreme storms, we have used the L2S product to generate spectra similar to
 180 those in the L2 product, using the same "boxes" as the L2 product and rescaling the spec-
 181 trum energy to correspond to the root mean square average of the significant wave height
 182 in that box. This rescaling is particularly motivated by our investigation of wave prop-
 183 erties along the nadir track, and it corrects an average bias of the order or 14% for the
 184 total energy of the spectrum. Alternative data processings are discussed in section 4. In
 185 practice we also used the homogeneously reprocessed Level 2+ (L2P) product provided
 186 by the Copernicus Marine Environment Monitoring System (CMEMS), in particular we
 187 used the quality flags specific to that product.

188 2.1 Significant wave height variability in Storm Dennis

189 On February 14th 2020, the European windstorm Dennis, which became one of the
 190 most intense extratropical cyclones ever recorded, underwent through its explosive in-
 191 tensification in the middle of the North Atlantic. Around 9:10 UTC that same day, Den-
 192 nis was sampled by CFOSAT, with \widehat{H}_s values up to 19.7 m for the native (4.5 Hz) sam-
 193 pling, and 17.9 m for the 1 Hz sampling (averaging over 1 second). Fig. 1.a shows a model
 194 snapshot of H_s in the north Atlantic and the corresponding descending track of CFOSAT,
 195 while Fig. 1.b shows the altimeter-estimated significant wave heights \widehat{H}_s for the three
 196 different samplings: native (4.5 Hz), 1 Hz and 80 km box averaged.

197 On the periphery of the storm, where the average H_s is around 10 m, we were struck
 198 by the factor two difference in $\text{std}(\widehat{H}_s)$ that spans more than 420 km (1 minute of data).
 199 Our working hypothesis is that this variability of \widehat{H}_s may be dominated by fluctuations
 200 associated to wave groups. These fluctuations have different magnitudes and spatial scales
 201 which can be estimated from the directional wave spectrum (Longuet-Higgins, 1984). Hence
 202 CFOSAT is a unique instrument for studying this effect as we have both \widehat{H}_s variability
 203 along the satellite track and directional wave spectra. In the following, we illustrate the

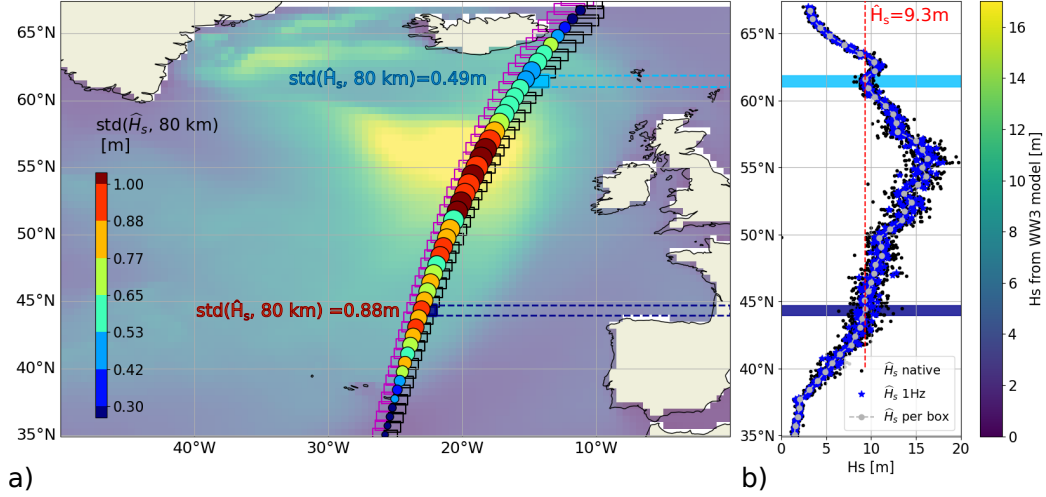


Figure 1. a) Map of significant wave heights in the North Atlantic at 09:00 on 14 February 2020, as provided by the model hindcast of (Alday et al., 2021), overlaid with circles located at the center of SWIM box estimates for the L2 wave spectra. Circles are sized by the L2 H_s estimate and color corresponds to $\text{std}(\hat{H}_s)$; b) corresponding \hat{H}_s values as a function of latitude (y-axis): black small dots represent native measurements at 4.5 Hz, blue stars represent the 1 Hz averaged and grey circles represent the \hat{H}_s averaged over a box. Two boxes are selected for the case study: box A — highlighted in light blue — is at 62°N, and box B — in dark blue — is at 44°N.

204 expected signature of wave groups for the two sea state conditions that correspond to
 205 the particular SWIM boxes highlighted in light and dark blue. It is worth noting that
 206 in these two examples, the \hat{H}_s values obtained from the sum of the L2S spectrum prior
 207 rescaling are around 7.5 m, which is lower than the 9 m given by the nadir beam and
 208 used in the L2 product and in this study to rescale the spectrum energy.

209 2.2 Variability of \hat{H}_s and envelope spectrum

210 The patterns of individual waves vary with the shape of the wave spectrum, as il-
 211 lustrated in Fig. 2. A key difference between the north-side (left column) and south-side
 212 (right column) of storm Dennis is that the south-side has a longer peak wavelength around
 213 600 m, and a narrower spectrum, in particular in directions. The smaller width in di-
 214 directions gives longer wave crests while the smaller width in wavenumber magnitude gives

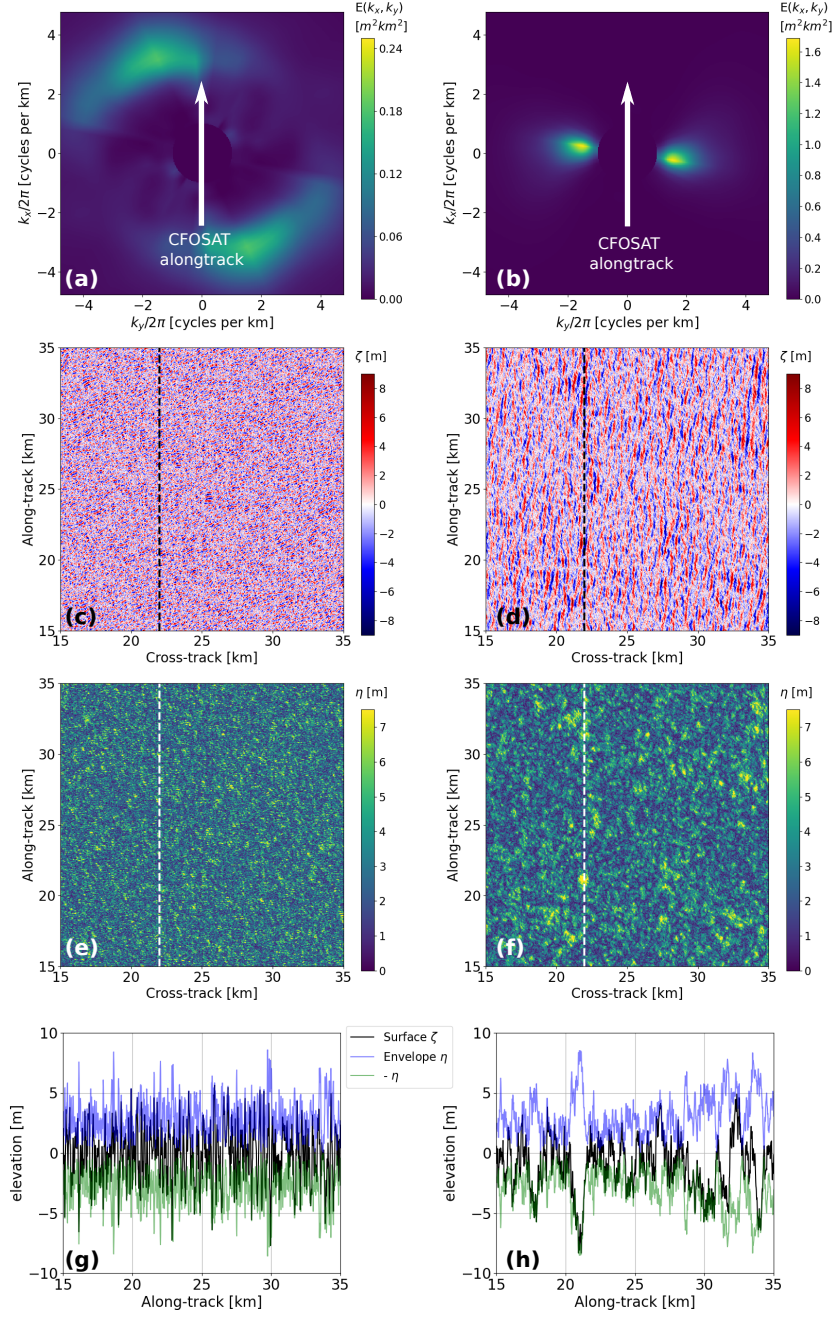


Figure 2. Left column corresponds to our chosen northern CFOSAT box, and right column to the southern box. From top to bottom, (a, b): L2S wave spectra $E(k_x, k_y)$ as provided by IWWOC. (c, d): simulated surface elevation maps generated from the wave spectra using random phases, (e, f): envelope η of the surface elevation, (g, h) along-track slices of elevation ζ and $\pm\eta$ at $x = 22$ km.

215 longer wave groups (see Longuet-Higgins, 1984, for a detailed definition of the length of
216 wave groups).

217 Although the surface elevations in Fig. 2.c or 2.d are realizations of a uniform sea
218 state each given by a well defined wave height H_s and spectrum shape, any measurement
219 that is sensitive to the surface elevation over a finite area will provide an estimate \hat{H}_s
220 that differs from H_s due to spatial fluctuations. We will now link this estimate \hat{H}_s in the
221 case of a radar altimeter, to the properties of the envelope. For this purpose we need to
222 define a local wave height.

223 *2.2.1 Definition of a local wave height*

224 Let ζ_c be the complex surface such that $\zeta = \text{Re}(\zeta_c)$ is the free surface. The en-
225 velope η of the signal is defined by $\eta = |\zeta_c|$. This defines a local amplitude of the sig-
226 nal, that does not contain the small scale crest-to-trough (positive to negative) varia-
227 tions of the original surface. From this envelope η we define the wave height H_r as a spa-
228 tial average over a disc of radius r

$$229 \quad H_r(x, y) = 4\sqrt{\frac{2}{\pi}}(\eta \otimes g_r)(x, y) \quad (2)$$

230 where \otimes is the convolution operator and g_r is a filtering kernel of radius r . Under the
231 Gaussian approximation of the distribution of sea surface elevations this spatial aver-
232 age actually converges to the usual significant wave height H_s .

233 The envelope of a signal is known to act as a low-pass filter and its fluctuations,
234 larger than those of an individual wave, can be related to wave groups, both in size and
235 amplitude (Arhan & Ezraty, 1978; Longuet-Higgins, 1984; Masson & Chandler, 1993).

236 Hence wave groups may contribute to the fluctuations of the estimated \hat{H}_s provided
237 by the nadir beam of SWIM, as indicated on Fig. 1.b. We will now attempt to quantify
238 that contribution. In order to understand how much wave groups may contribute to \hat{H}_s
239 fluctuations in satellite data, we have to address two questions: First, what are the scales
240 affected by wave groups?

241 And second, what are the scales of the H_r variation that are resolved by satellite
242 altimeters?

2.2.2 Surface elevation envelope and H_r spectrum

One simple way to quantify the different scales present in the envelope is to compute its spectrum. The most simple theoretical result comes directly from the theory of Fourier transforms that gives the spectrum of a product of functions as the convolution of the Fourier transforms. In our case, the envelope squared is the product of the elevation by its complex conjugate, and this is true for spectra in one or two dimensions. For waves in one dimension with wavenumber k , the spectrum of the envelope squared $\Psi_2(k)$ is the convolution of the spectrum of the single-sided surface elevation spectrum $E(k)$ by itself,

$$\Psi_2(k) = 8 \int_0^\infty E(u)E(u+k)du, \quad (3)$$

and we note that $\Psi_2(k)$ is also single-sided.

In practice people have rather studied the variations of H_s and not that of H_s^2 . Although the details of the theory are more complex, the important result is that, for low frequencies, the spectrum of the envelope $\Psi(k)$ has the same shape as the spectrum of the envelope squared $\Psi_2(k)$ (Rice, 1944). More specifically, Tayfun and Lo (1989) have showed that a good approximation for the spectrum of the envelope is given by

$$\Psi(k) = \frac{8 - 2\pi}{H_s^2} \Psi_2(k) \quad (4)$$

This same result is valid for spectra in two dimensions. We now consider the double-sided wave spectrum $E(k_x, k_y)$, defined for (k_x, k_y) in the entire wavenumber plane and centrally symmetric, the region of the envelope spectrum for $k \ll k_p$, with k_p the wavenumber peak, is proportional to

$$\Psi_2(k_x, k_y) = 8 \int_{-\infty}^\infty \int_{-\infty}^\infty E(u, v)E(u+k_x, v+k_y)du dv, \quad (5)$$

in which Ψ_2 is also double-sided.

From eq. (2), the spectrum of H_r is

$$\Psi_{H_r}(k_x, k_y) = \frac{32}{\pi} \Psi(k) G_r(k_x, k_y) \quad (6)$$

$$= \frac{32}{\pi} \underbrace{\frac{8 - 2\pi}{H_s^2} \Psi_2(k_x, k_y)}_{\Psi_{H_0}(k_x, k_y)} G_r(k_x, k_y) \quad (7)$$

with H_s the usual significant wave height and G_r the square of the Fourier transform of the filtering kernel g_r . We call $\Psi_{H_0}(k_x, k_y)$ the spectrum obtained before applying the filter $G_r(k_x, k_y)$.

272 Integrating this spectrum for $k_x > k_1$, with the x -axis taken in the along-track
 273 direction, amounts to integrating the expected variance up to the cut-off distance, $d_1 =$
 274 $2\pi/k_1$, giving $\text{var}(H_r, d_1)$. The group-induced variation of \widehat{H}_s is thus equal to

$$275 \quad \text{var}(H_r, d_1) = 2 \int_{-\infty}^{\infty} \int_{k_1}^{\infty} \Psi_{H_0}(k_x, k_y) G_r(k_x, k_y) dk_x dk_y \quad (8)$$

276 We now need to estimate the filter G_r associated with the SWIM altimeter.

277 **2.3 Altimeter measurements over varying wave heights H_r**

278 Going back to the fundamental altimeter measurement that is the waveform, Brown
 279 (1977) assumed a uniform ocean reflectivity and showed that the waveform is an area-
 280 weighted histogram of the radar echoes as a function of range R . Over a flat sea surface,
 281 this histogram is a Heaviside function because the part of the ocean surface with ranges
 282 between R and $R + \delta_R$ is an annulus of radius $r = \sqrt{R^2 - h^2}$ centered on the nadir
 283 point, with an area $2\pi R \delta_R$ that is almost constant as long as $R \approx h$. In the presence
 284 of waves, echoes from the sea surface at the elevation $z = \zeta$ and at the nadir (vertically
 285 under the satellite), will have a higher range when $\zeta < 0$. These echoes will have the
 286 same range as other echoes from $z = 0$ and horizontal locations away from nadir. Given
 287 the very small incidence angles, the change in range caused by waves is $\Delta_R = -\zeta$. For
 288 a Gaussian distribution of ζ with standard deviation $\sigma_H = H_s/4$, the presence of waves
 289 gives a smoothing of the histogram. Here we use the most simple theoretical waveform
 290 shape that is obtained in the limit of a very broad radar antenna pattern (Brown, 1977),

$$291 \quad w_B(R, \sigma_H) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{R - h}{\sqrt{2}\sigma_H} \right) \right]. \quad (9)$$

292 When "retracking" altimeter data, eq. (9) is inverted, giving the estimate \widehat{H}_s equal
 293 to 4 times the σ_H of the theoretical waveform that best fits the data. In practice the the-
 294 oretical waveform may also include effects of the antenna pattern and mispointing. Dif-
 295 ferent fitting methods have been developed to reduce the effect of noise or spurious echoes
 296 in the measured waveform (Rodriguez, 1988; Passaro et al., 2014; Tourain et al., 2021).
 297 Another important assumption needed to obtain the Brown waveform is that the sea state
 298 is homogeneous within the footprint. We thus have to discuss what sets the scale of the
 299 footprint, or more precisely where are the points on the sea surface that give the distinc-
 300 tive shape of the waveform and allow the fit to distinguish different values of H_s .

301 Compared to a flat sea surface, the elevation ζ at a distance r from the nadir point
 302 will change the range R of the surface point and make it look as if it was located at a
 303 different distance $r + \delta_r$, so that points from different locations on the sea surface will
 304 map to the same range R . This is the same "range bunching" or "overlay" or "surfboard
 305 effect" that is common to all radar systems (Peral et al., 2015). Following Chelton et al.
 306 (1989) we can estimate the apparent horizontal displacement. For a satellite altitude h
 307 and using $\zeta \ll R$, the calculation for a flat mean sea surface gives

$$308 \quad \delta_r \simeq \sqrt{r^2 - 2h\zeta} - r. \quad (10)$$

309 For a spherical Earth of radius R_E , ζ should be replaced by $\zeta/(1+h/R_E)$. In the par-
 310 ticular case where $\zeta = -H_s$ and $r = 0$, δ_r is the radius r_C of the oceanographic foot-
 311 print defined by Chelton et al. (1989), and given by eq. (1), when the range resolution
 312 δ_R is neglected compared to H_s . For $H_s = 10$ m, and $h = 519$ km, this gives $r_C =$
 313 3.3 km. Based on the Gaussian distribution of the sea surface elevation, there is only a
 314 0.003% probability that $\zeta > H_s$. Hence, we have the same negligible probability that
 315 points located at r_C from nadir (i. e. at the edge of the "Chelton footprint") contribute
 316 to the waveform at ranges $R < h$, i.e. in the first half of the rising part of the wave-
 317 form. We may thus give the following interpretation of r_C : points located at $r > r_C$
 318 are outside of the footprint and have a very limited impact on the estimated value \widehat{H}_s .

319 If wave heights H_r vary as a function of distance to nadir, then the waveform does
 320 not follow exactly the Brown form, as detailed in Appendix A. As different regions of
 321 the waveform contain different regions of the sea surface, one could imagine fitting dif-
 322 ferent parts of the waveform to measure variations in H_r as a function of distance from
 323 nadir. The theoretical limit to this capability is the blurring due to range bunching over
 324 a distance of the order $r_C/2$. Speckle noise is another limiting factor, and probably the
 325 main one in practice for small values of H_s .

326 Based on the analysis in Appendix A we expect that variations of H_r at scales much
 327 smaller than $r_C/4$ will be smoothed out in altimeter data, whereas variations at scales
 328 much larger than $r_C/2$ have no effect on the waveform that will follow the Brown shape
 329 for the local wave height. For our analysis of CFOSAT data we will define an "effective
 330 altimeter radius" r_a such that the variance associated to the random fluctuations of H_{r_a}
 331 — the envelope filtered with g_{r_a} , a Gaussian filter of standard deviation r_a — is the same

332 as that produced by an altimeter. The actual shape of the "altimeter filter" is discussed
 333 in Appendix A

334 2.4 Estimation of the equivalent r_a scale for an idealized altimeter

335 We have simulated the sampling of our simulated sea surface by a highly simpli-
 336 fied altimeter. We thus neglect radar noise, speckle and variations in ocean backscatter,
 337 and compute simulated waveforms as histograms of the number of discrete pixels as a
 338 function of range R discretized with the same resolution $\delta_R = 0.47$ m used in actual
 339 SWIM data. The histogram is computed for a finite region of size r_C by r_C centered at
 340 the nadir point. The value of \widehat{H}_s for each simulated histogram is given by the least square
 341 fit to the theoretical waveform in by eq. (9) for R varying from $h - H_s$ to $h + H_s$. As
 342 detailed in Appendix A, even this simplified altimeter makes a much more complex mea-
 343 surement than a simple Gaussian smoothing of the H_r field. We briefly tested that more
 344 realistic waveforms and different fitting procedures yield some quantitative differences.

345 Taking the simulated sea surface from Fig. 2, we compare a map of simulated alti-
 346 meter data \widehat{H}_s in Fig. 3 with maps of local wave heights H_r , smoothed on different scales.
 347 As expected, the patterns of the envelope with radiuses larger than 2 km, those that per-
 348 sist in Fig. 3.f., match the large scales of the simulated altimeter data. From a quanti-
 349 tative point of view, the standard deviation of the simulated altimeter data, here 0.705 m,
 350 is of the same order as the standard deviation of actual SWIM measurements over the
 351 same SWIM box (0.88 m). We also note that this value is very close to that obtained
 352 for a filtering of the envelope between a scale $r = r_C/5$ and $r = r_C/4.5$. As $r = r_C/4.5$
 353 gives the closest value, we define the effective altimeter radius r_a as $r_a = r_C/4.5$.

354 Looking at Fig. 3.a. it is clear that the map of \widehat{H}_s contains much smaller features
 355 than the envelope smoothed with $r_a = r_C/4.5$. All of these are spurious amplification
 356 of envelope perturbations that happen to be at the right distance from nadir, around $r_C/2$,
 357 as explained in Appendix A. As a result, maxima of \widehat{H}_s given by the altimeter are not
 358 located at the true wave height maxima but slightly displaced by a distance of the or-
 359 der of $r_C/2$. A striking example in Fig. 3 is the region of waves higher than 11 m around
 360 $x = 20$ km, $y = 19$ km. The altimeter gives a local minimum where the true wave height
 361 is maximum, and a round halo of maxima surrounding that point. Conversely a ring-
 362 shaped maximum in the envelope, such as around $x = 35$ km, $y = 45$ km, gives a lo-

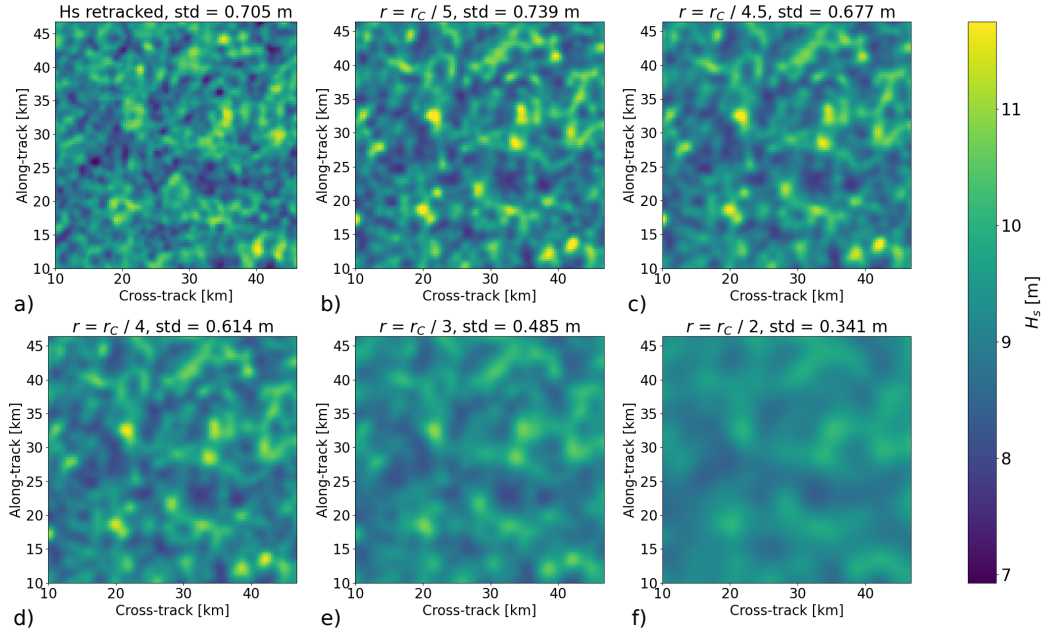


Figure 3. Maps of different estimates of the wave heights obtained by either (a) simulating altimeter processing or (b — f) computing the local average H_r using eq. (2) where the g_r kernel is a Gaussian filter of standard deviation r varied from $r_C/5$ to $r_C/2$. In this example, $r_C = 3063$ m thus r values are respectively 613, 681, 766, 1021, and 1531 m. In practice the smoothing is applied in a finite box of size $4r_C$ by $4r_C$.

363 cal maximum in the simulated altimeter data. We have found that different retracking
 364 methods produce the same large scale patterns but may differ in small scale details. These
 365 differences are beyond the scope of the present paper. Even though the patterns do not
 366 exactly coincide, we will now assume that the sampling of the sea surface by the altime-
 367 ter is equivalent, in terms of variability of wave height to filtering the envelope with a
 368 Gaussian of standard deviation $r_a = r_C/4.5$.

369 2.5 Predicting H_r variability and its contribution of \widehat{H}_s variability

370 Based on our analysis, we expect that SWIM measurements are contaminated by
 371 wave group structures at scales of a few kilometers, following the variation of r_C with
 372 wave height and satellite altitude. As illustrated by the two examples with different spec-
 373 tral widths, we note that for the same wave height, a wider spectrum leads to smaller
 374 scales of wave groups, part of which scales are smoothed away by the altimeter footprint
 375 and therefore not resolved. For a narrower spectrum, wave groups have larger scales and

376 amplitudes and a larger contribution to the variability of wave heights estimated by an
 377 altimeter.

378 For a quantitative analysis we first consider the simpler case of waves propagat-
 379 ing in only one direction, with a sea surface ζ distributed with the normal law $\mathcal{N}(0, \sigma_H =$
 380 $H_s/4)$ with a single-sided (defined for $k > 0$) Gaussian spectrum with mean value k_p
 381 and standard deviation σ_k

$$382 \quad E(k) = \frac{H_s^2}{16\sigma_k\sqrt{2\pi}} e^{-(k-k_p)^2/(2\sigma_k^2)}. \quad (11)$$

383 The spectrum of the envelope is also Gaussian and writes, using eq. (3) and (4), and af-
 384 ter computing the correlation (for a single-sided spectrum),

$$\begin{aligned} 385 \quad \Psi(k) &= \frac{8-2\pi}{H_s^2} \Psi_2(k), \\ 386 &= \frac{8-2\pi}{H_s^2} 8 \int_0^\infty E(u)E(u+k)du, \\ 387 &= \frac{16(4-\pi)}{H_s^2} \frac{H_s^4}{512\sqrt{\pi}\sigma_k} e^{-k^2/(4\sigma_k^2)}, \\ 388 &= \frac{(4-\pi)H_s^2}{32\sqrt{\pi}\sigma_k} e^{-k^2/(4\sigma_k^2)}. \end{aligned} \quad (12)$$

389 Using eq. (6), with a Gaussian filter of standard deviation r_a , the spectrum of H_{r_a}
 390 writes,

$$391 \quad \Psi_{H_{r_a}}(k) = \underbrace{\frac{(4-\pi)H_s^2}{\pi\sqrt{\pi}\sigma_k} e^{-k^2/(4\sigma_k^2)}}_{\Psi_{H_0}(k)} G_{r_a}(k) \quad (13)$$

392 Wave groups contain wavelengths larger than π/σ_k , with a constant spectral den-
 393 sity near $k = 0$. Around $k = 0$, the value of the H_{r_a} spectrum is $0.15H_s^2/\sigma_k \text{ m}^2/(\text{rad/m})$.

394 Fig. 4 presents one dimensional wave spectra — in solid lines — of two typical sea
 395 states with the same $H_s = 3.1 \text{ m}$ and their associated $\Psi_{H_0}(k)$ spectra — in dashed lines.
 396 The light blue spectrum is a JONSWAP spectrum (Hasselmann et al., 1973) with a peak
 397 period of 8 s and a peak enhancement factor $\gamma = 3.3$ that represents a moderate wind-
 398 sea. The dark blue spectrum is a narrow Gaussian spectrum with a peak period of 14 s
 399 and $\sigma_k = 0.002 \text{ rad/m}$, typical of swell conditions in the open ocean. The altimeter smooth-
 400 ing function $G_{r_a} = \exp(-k^2 r_a^2)$ allows to define a cut-off wave number $k_a = \sqrt{\pi}/(2r_a)$.
 401 As shown in Fig. 4 the wavelengths in altimeter-filtered envelopes, larger than the as-
 402 sociated wavelength cut-off $2\pi/k_a$ (in black dash-dotted line), are large compared to the
 403 typical wavelengths contained in the wave groups (of order $\pi\sqrt{2}/\sigma_k$ and represented by
 404 the dark and light blue vertical dash-dotted lines).

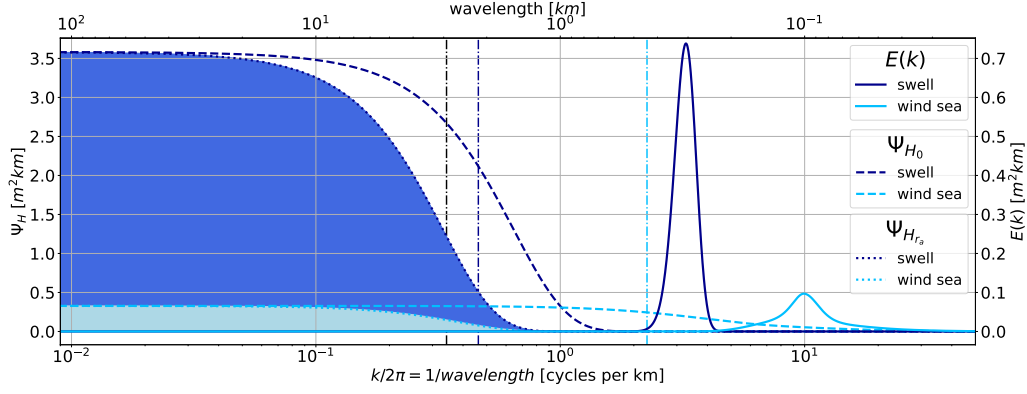


Figure 4. Example of two wave spectra — solid lines — in one dimension and the corresponding spectra of H_s — dashed lines —, for typical swell conditions in the open ocean in dark blue, and typical windsea in moderate wind conditions in light blue. Because the fluctuations of H_s are filtered by the altimeter with the function $G_{r_a}(k)$ — dotted lines —, the actual measured variance of H_s is the shaded area, in light blue for the windsea and dark blue for the swell. The vertical black line is the equivalent altimeter cut-off wavenumber at $k = k_a$, whereas the vertical dark and light blue lines represents the width of the H_s spectra.

405 Applying the one dimension version of eq. (8) gives the variance of altimeter-estimated
 406 \hat{H}_s as the shaded areas in Fig. 4. For a Gaussian spectrum, in cases where the altimeter
 407 filter scale is large enough not to be concerned about the shortest scales, this area
 408 is approximately k_a times $\Psi_{H_0}(k = 0)$ the PSD level at $k = 0$. This gives a standard
 409 deviation of \hat{H}_s of the order of $0.39\sqrt{k_a/\sigma_k}H_s$, which is $0.40 H_s$ for the one-dimensional
 410 swell example of Fig. 4.

411 For a generic one-dimensional wave spectrum $E(k)$, the reciprocal width $1/\sigma_k$ should
 412 be replaced by $(2\sqrt{\pi})Q_k$, with the peakedness parameter Q_k defined similarly to the re-
 413 reciprocal of the the usual frequency bandwidth (Saulnier et al., 2011),

$$414 \quad Q_k = \frac{(\int_0^\infty E(k)dk)^2}{\int_0^\infty E^2(k)dk}. \quad (14)$$

415 For a JONSWAP spectrum with a peak enhancement factor $\gamma = 3$, this definition gives
 416 $B_k = 1.3k_p$ and the standard deviation of H_r for the wind sea case above is $0.1H_s$.

417 For a realistic altimeter, we must consider waves in two dimensions, and the stan-
 418 dard deviation of H_{r_a} is the square root the variance as given by eq. (8). This variance
 419 is the integral of the H_{r_a} spectrum in the wavenumber plane. For large enough r_a , the

420 integral can be approximated as the value $\Psi_{H_{r_a}}$ of the spectrum at $(k_x = 0, k_y = 0)$ times
 421 an effective area in the wavenumber plane,

$$\begin{aligned}
 422 \quad \text{var}(H_{r_a}, d_1) &\simeq \Psi_{H_0}(k_x = 0, k_y = 0) \times \int_{k_y \in \mathbb{R}} \int_{k_x \in \mathbb{R} \setminus [-k_1, k_1]} G_{r_a}(k_x, k_y) dk_x dk_y \\
 423 &\simeq \Psi_{H_0}(k_x = 0, k_y = 0) \times (\pi/2) (2/r_a^2 - 4k_1/(\sqrt{\pi}r_a)) \\
 424 &\simeq \frac{32}{\pi} \frac{8 - 2\pi}{H_s^2} \Psi_2(k_x = 0, k_y = 0) \times (\pi/2) (2/r_a^2 - 4k_1/(\sqrt{\pi}r_a)) \\
 425 &\simeq Q_{kk}^2 H_s^2 (4 - \pi) (2/r_a^2 - 4k_1/(\sqrt{\pi}r_a)), \tag{15}
 \end{aligned}$$

426 where we have defined a two-dimensional spectral peakedness Q_{kk} which is measured in
 427 meters,

$$428 \quad Q_{kk}^2 = \frac{\iint_{\mathbb{R}^2} E^2(k_x, k_y) dk_x dk_y}{(\iint_{\mathbb{R}^2} E(k_x, k_y) dk_x dk_y)^2} = \frac{32\Psi_2(k_x = 0, k_y = 0)}{H_s^4}. \tag{16}$$

429 This expression gives the approximate value for the standard deviation,

$$430 \quad \text{std}(H_{r_a}, d_1) \simeq H_s Q_{kk} \sqrt{(4 - \pi) [2/r_a^2 - 4k_1/(\sqrt{\pi}r_a)]}. \tag{17}$$

431 This variability of H_{r_a} , which we have defined as the contribution of wave groups to the
 432 variability of measured wave heights \widehat{H}_s is thus the product of three factors: the signif-
 433 icant wave height H_s , the shape of the wave spectrum as quantified by Q_{kk} , and the ef-
 434 fective range of spatial scales over which the variance is integrated. That last factor is
 435 a function of the smoothing effect of the altimeter, represented by the scale r_a , and the
 436 distance $d_1 = 2\pi/k_1$ over which we consider the variability.

437 2.6 Practical implementation

438 In the previous subsections, various considerations have been made. Here, we sum-
 439 marize them to give a flow chart for estimating the part of the variability due to wave
 440 groups $\text{std}(H_{r_a}, d_1)$ from CFOSAT products.

- 441 1. Assemble the 1-dimensional L2S spectra for each azimuth to obtain an equivalent
 442 L2 spectrum, make it double-sided $E_{2S}(k, \theta)$ and rescale it with \widehat{H}_s from nadir mea-
 443 surement,
- 444 2. Interpolate $E_{2S}(k, \theta)$ over a regular (k_x, k_y) grid, with k_x the along track direc-
 445 tion, to obtain $E(k_x, k_y)$,

446 3. Compute the spectrum of the envelope squared $\Psi_2(k_x, k_y)$ from the discrete cor-
 447 relation

$$448 \quad \Psi_2(k_x, k_y) = 8 \sum_{k'_x} \sum_{k'_y} [E(k'_x, k'_y), E(k'_x + k_x, k'_y + k_y)] dk'_x dk'_y \quad (\text{Step 3})$$

449 4. Transform to a PSD of H_0 ,

$$450 \quad \Psi_{H_0}(k_x, k_y) = \frac{32}{\pi} \frac{8 - 2\pi}{H_s^2} \Psi_2(k_x, k_y) \quad (\text{Step 4})$$

451 where H_s is computed as $4\sqrt{E}$, with $E = \iint_{\mathbb{R}^2} E(k_x, k_y) dk_x dk_y$.

452 5. Compute the altimeter smoothing filter G_{r_a} as

$$453 \quad G_{r_a}(k_x, k_y) = |\mathcal{F}(g_{r_a})|^2 = e^{-(k_x^2 + k_y^2)r_a^2} \quad (\text{Step 5})$$

454 with $\mathcal{F}(\cdot)$ the Fourier transform and $r_a = r_C/4.5$.

455 6. Apply the filter to Ψ_{H_0} to obtain the PSD of altimeter H_{r_a} estimate,

$$456 \quad \Psi_{H_{r_a}}(k_x, k_y) = \Psi_{H_0}(k_x, k_y) G_{r_a}(k_x, k_y) \quad (\text{Step 6})$$

457 7. Integrate $\Psi_{H_{r_a}}(k_x, k_y)$, the spectrum of H_{r_a} , over all cross-track wavenumbers k_y
 458 and over along-track wavenumbers k_x with a magnitude larger than $2\pi/d_1$, as il-
 459 lustrated on Fig. 5.e. Because we compare our estimate to \widehat{H}_s variations within
 460 a SWIM L2 box size that is 80 km along-track, d_1 is taken as 80 km,

$$461 \quad \text{var}(H_{r_a}, d_1) = \int_{k_y \in \mathbb{R}} \int_{k_x \in \mathbb{R} \setminus [-k_1, k_1]} \Psi_{H_{r_a}}(k_x, k_y) dk_x dk_y \quad (\text{Step 7})$$

462 Fig. 5 shows the results of steps 3, 6 and 7 for the two selected boxes of our case
 463 study. The top line corresponds to the Ψ_2 spectra obtained from correlation, the mid-
 464 dle line shows $\Psi_{H_{r_a}}$, the PSD of our local H_{r_a} estimate. Note the different colour scales
 465 between the northern — A — and southern — B — boxes. The bottom line shows the
 466 one sided along-track k_x -spectra. The vertical line shows the lower integration limit over
 467 k_x that is used to obtain the variance of H_{r_a} .

468 Fig. 6 shows wave height and corresponding standard deviation, both observed and
 469 estimated from L2S spectrum, as a function of the sampling time (UTC) over storm Den-
 470 nis. For the northern part of the storm, where the spectrum is broader, around box A
 471 (light blue vertical line), the standard deviation due to wave groups is around half the
 472 measured standard deviation (i.e. wave groups represent a quarter of the measured vari-
 473 ance). On the other hand, for the southern part, around box B (dark blue), the wave

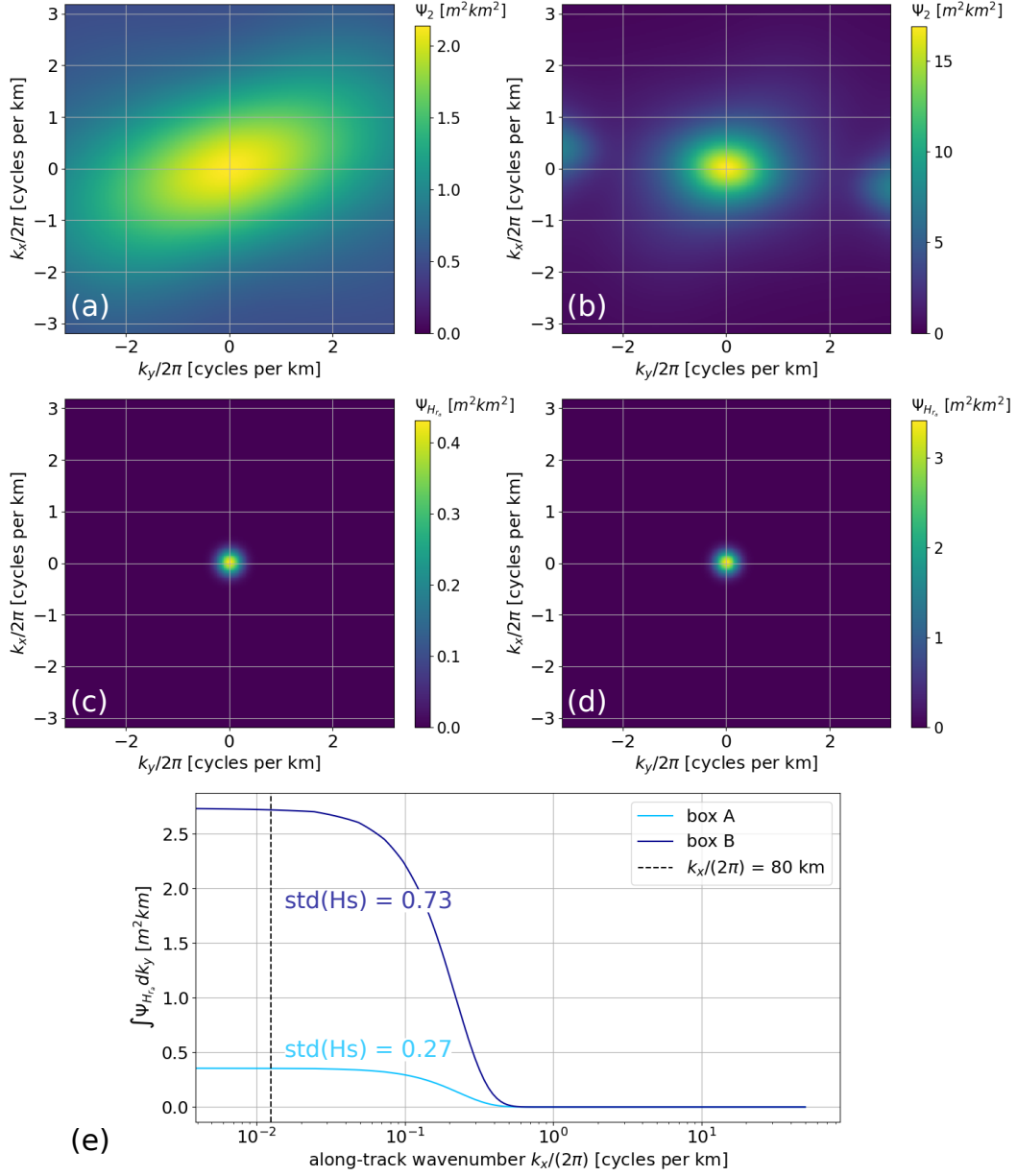


Figure 5. (a,c) corresponds to our chosen northern CFOSAT box, and (b,d) to the southern box. Top line: envelope squared spectrum $\Psi_2(k_x, k_y)$ from convolution. Middle line: $\Psi_{H_{ra}}(k_x, k_y)$, spectrum of H_{ra} (including the equivalent altimeter filtering). Bottom line: 1D along-track spectrum obtained by integrating over the cross-track axis, in light blue for the northern box and dark blue for the southern box.

474 height variability is strongly dominated by wave groups — more than half the observed
 475 variance is explained by wave groups.

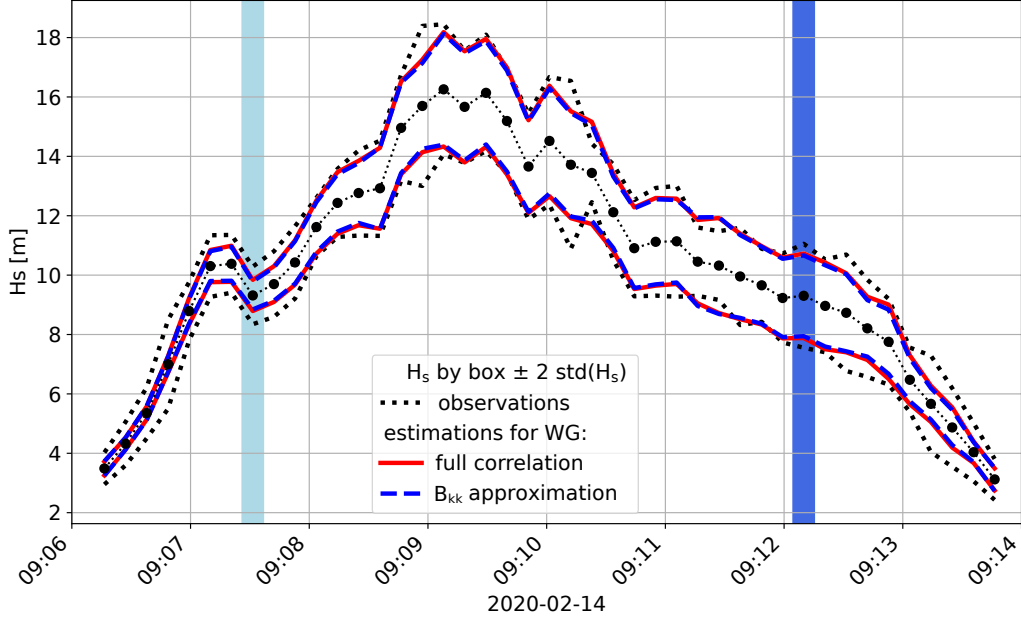


Figure 6. Values of measured \widehat{H}_s , averaged over boxes — black circles — , and corresponding $\text{std}(\widehat{H}_s, 80\text{km})$ — black dotted lines — ; as provided in the L2 as a function of sampling time (UTC), for the CFOSAT track shown in Fig. 1. Estimations of $\text{std}(H_{r_a}, 80\text{km})$ are also represented — in red and blue — using the two methods summarized at the end of section 2.

476 Alternatively we can approximate the full integral of the convolution by its value
 477 at $k = 0$ using the peakedness Q_{kk} , giving a faster estimate of the variability due to
 478 wave groups. The first two steps are the same, then

- 479 • Instead of the full convolution, compute only Q_{kk} defined from eq. (16), which can
 480 be re-written as,

$$481 \quad Q_{kk} = \frac{\sqrt{\iint_{\mathbb{R}^2} E^2(k_x, k_y) dk_x dk_y}}{\iint_{\mathbb{R}^2} E(k_x, k_y) dk_x dk_y} \quad (\text{Step 3bis})$$

- 482 • skip steps 4–6 to estimate $\text{std}(H_{r_a}, d_1)$ directly using eq. (17),

$$483 \quad \text{std}(H_{r_a}, d_1) \simeq Q_{kk} H_s \sqrt{(4 - \pi) \left[2/r_a^2 - 4k_1/(\sqrt{\pi} r_a) \right]}. \quad (\text{Step 7bis})$$

484 This $\text{std}(H_{r_a}, d_1)$ estimated through Q_{kk} is also shown in Fig. 6. The values are only slightly
 485 overestimated compared to the full correlation calculus, therefore, Q_{kk} could be a use-
 486 ful parameter when working with wave groups.

487 3 Results at the global scale

488 Beyond the particular case of storm Dennis, for which very large wavelength and
 489 narrow spectra lead to a dominant effect of wave groups in \hat{H}_s variability, one may wonder
 490 how important are wave groups in general, and how important can they be compared
 to other known sources of \hat{H}_s variability, including winds and currents. To answer this

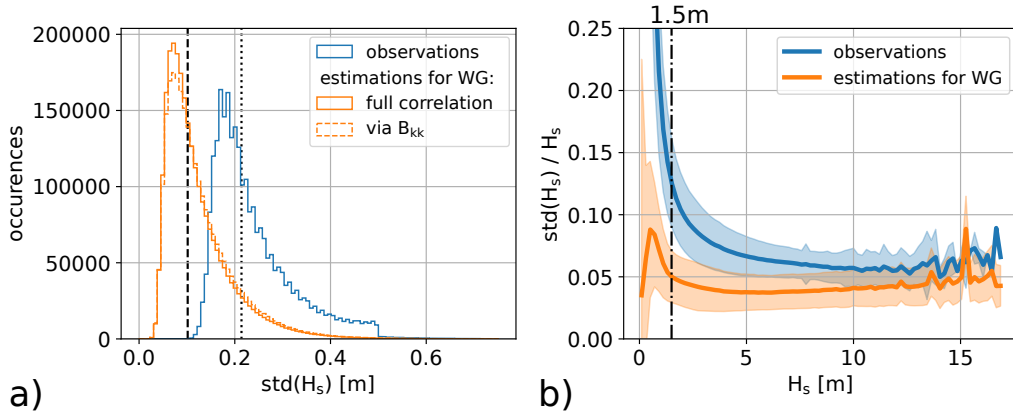


Figure 7. (a) Histograms of $\text{std}(\hat{H}_s, 80\text{km})$ measured at nadir in blue and our estimate of wave groups contribution $\text{std}(H_{r_a}, 80\text{km})$ in orange. (b) Mean — solid lines — and standard deviation — shaded areas — of $\text{std}(H)/\hat{H}_s$ over \hat{H}_s bins of 0.2 m, both for $\text{std}(\hat{H}_s, 80\text{km})$ measured at nadir in blue and our estimate of wave group effects $\text{std}(H_{r_a}, 80\text{km})$ in orange.

491 question, we have applied the methodology presented in section 2 for storm Dennis to
 492 the full SWIM L2S archive for the years 2020 and 2021, estimating for each of the 2.4
 493 million SWIM L2 box the expected value of $\text{std}(H_{r_a}, d_1)$ associated to wave groups as
 494 filtered by the altimeter. The distribution of these values is shown in Fig. 7.a, with a typ-
 495 ical value around 7 cm, and maximum values around 60 cm. This variability is typically
 496 half of the measured standard deviation of \hat{H}_s . We also computed $\text{std}(H_{r_a}, d_1)$ as esti-
 497 mated from the spectral peakedness parameter Q_{kk} for the same time period, giving re-
 498 sults that are highly correlated to the full convolution, with a Pearson’s linear correla-
 499 tion $R = 0.98$.
 500

501 In practice the group-induced variability of H_{r_a} that that should be present in SWIM
 502 data is strongly correlated with the mean value of H_s . In Fig. 7.b we show the statisti-
 503 cal distribution (mean and standard deviation) of $\text{std}(H_{r_a})/\text{mean}(\hat{H}_s)$ as a function
 504 of H_s . For H_s below 1.5 m the altimeter estimates of H_s are known to have the largest

505 relative errors (Dodet et al., 2022), which is partially due to a stronger effect of speckle
 506 noise, as discussed in Appendix A.3. In that range the wave group variability is 3 times
 507 smaller than the observed variability. Above 1.5 m, the relative variability that is expected
 508 from wave groups increases with H_s (from 3.5% to 5%), whereas the observed variabil-
 509 ity decreases from 11% to 5% between $H_s = 1.5$ m and $H_s = 10$ m. The share of the
 510 variability expected from wave groups dominates the observed variability for wave heights
 511 above 8 m, and probably explains the increase in observed $\text{std}(H_s)/\text{mean}(H_s)$ for H_s above
 512 12 m. Although there are very few data in that range, it is well known that dominant
 513 wave periods are generally higher for higher wave heights (Toba, 1973), corresponding
 514 to smaller bandwidths and thus an ever increasing variability due to wave groups.

515 We now consider the spatial distribution of $\text{std}(\widehat{H}_s)$, and in order to separate the
 516 possible effects of different sea states from the general trends associated to local aver-
 517 age value of H_s , we have chosen to show a map of the mean value of $\text{std}(\widehat{H}_s, 80 \text{ km})/\text{mean}(\widehat{H}_s, 80 \text{ km})$
 518 gridded at a resolution of 100 km. Before computing the local mean we have first removed
 519 all cases with $\widehat{H}_s < 1.5$ m. Fig. 8.a shows the distribution of \widehat{H}_s variability and com-
 520 pare it to the predicted variability of H_{r_a} in Fig. 8.b.

521 Note that the range of values are different for both panels because the contribu-
 522 tion of wave groups is, on average, half of the measured $\text{std}(\widehat{H}_s, 80 \text{ km})$. Both figures have
 523 some common patterns with a general increase from the west to the east of the ocean
 524 basins consistent with a dominance of swells in the east (Chen et al., 2002) with longer
 525 wavelengths and narrower spectra.

526 Now that we have quantified the variance of \widehat{H}_s associated to wave groups, we can
 527 subtract this contribution of wave groups from the total variance of \widehat{H}_s in order to look
 528 at the other sources of variability in \widehat{H}_s . As shown in Fig. 8.c, the remaining standard
 529 deviation of \widehat{H}_s after correction of the effects of wave groups (total variance minus vari-
 530 ance due to wave groups) contains a background level of 0.1 to 0.2 m, possibly associ-
 531 ated to known artificial effects that include the automatic gain control, speckle noise (Quartly
 532 et al., 2001) and true small scale wind variability. Larger localized values are up to 0.3 m.
 533 These larger values are co-located with regions of strong ocean circulation mesoscale vari-
 534 ability. These same regions match the location of strong H_s gradient in along-track 1
 535 Hz data from SARAL-AltiKa, Jason-2 and Cryosat-2 that have been denoised using an
 536 Empirical Mode Decomposition (EMD), by Quilfen and Chapron (2019). Here, we have

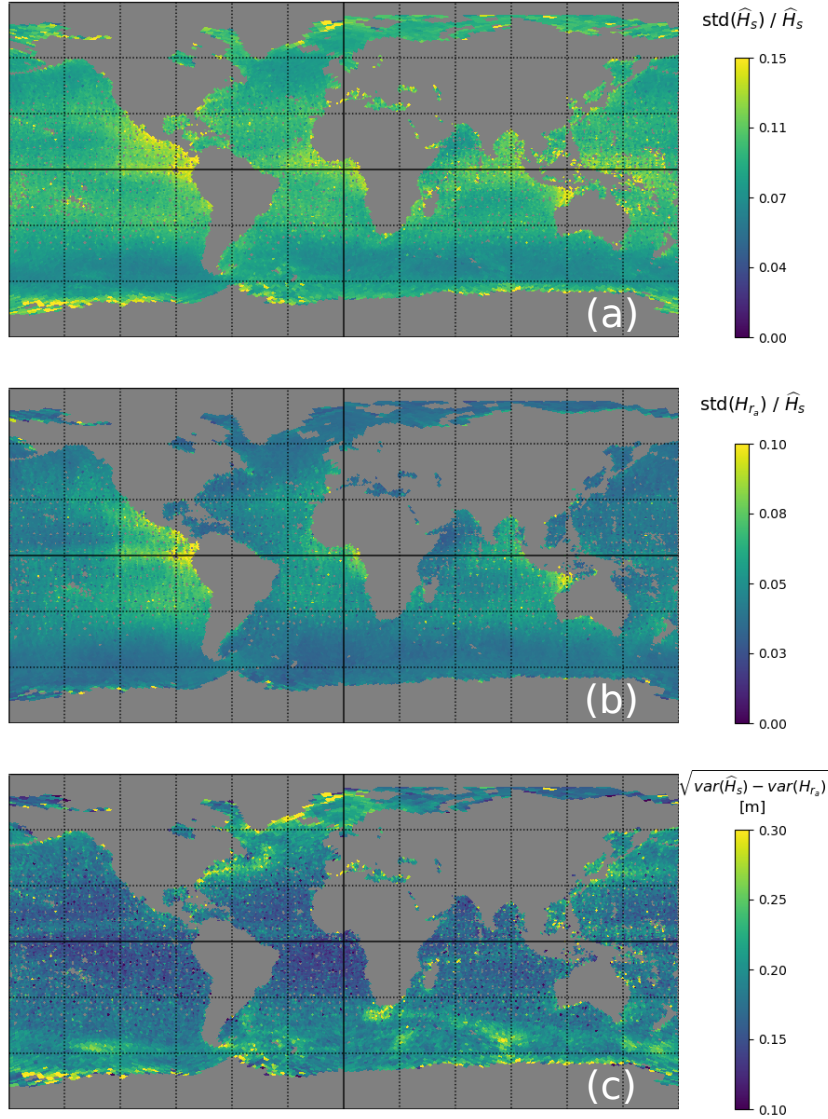


Figure 8. Map of the average of a) $\text{std}(\hat{H}_s, 80 \text{ km}) / \text{mean}(\hat{H}_s, 80 \text{ km})$ — upper panel —, b) $\text{std}(H_{r_s}, 80 \text{ km}) / \text{mean}(\hat{H}_s, 80 \text{ km})$ — middle panel — and c) residual standard deviation of \hat{H}_s , in meters, after removing the effect expected from wave groups — lower panel —, for the years 2020 and 2021 for all the SWIM L2boxes with a \hat{H}_s above 1.5 m. With the wave group contribution $\text{std}(H_{r_s}, 80 \text{ km})$ estimated from SWIM L2S spectra

537 applied the same EMD filtering to SWIM 4.5 Hz data, in order to remove small scale
 538 noise in \hat{H}_s , giving results shown in Fig. 9. The EMD filtered part Fig. 9.c is directly
 539 comparable to the wave group signature highlighted in Fig. 8. These maps were constructed
 540 using SWIM nadir data from SALP/CAWATAC experimental 4.5 Hz products available

541 on Aviso+ that include both the raw significant wave height estimates \widehat{H}_s and the de-
 542 noised values \widetilde{H}_s using the EMD method. The magnitude and distribution of the expected

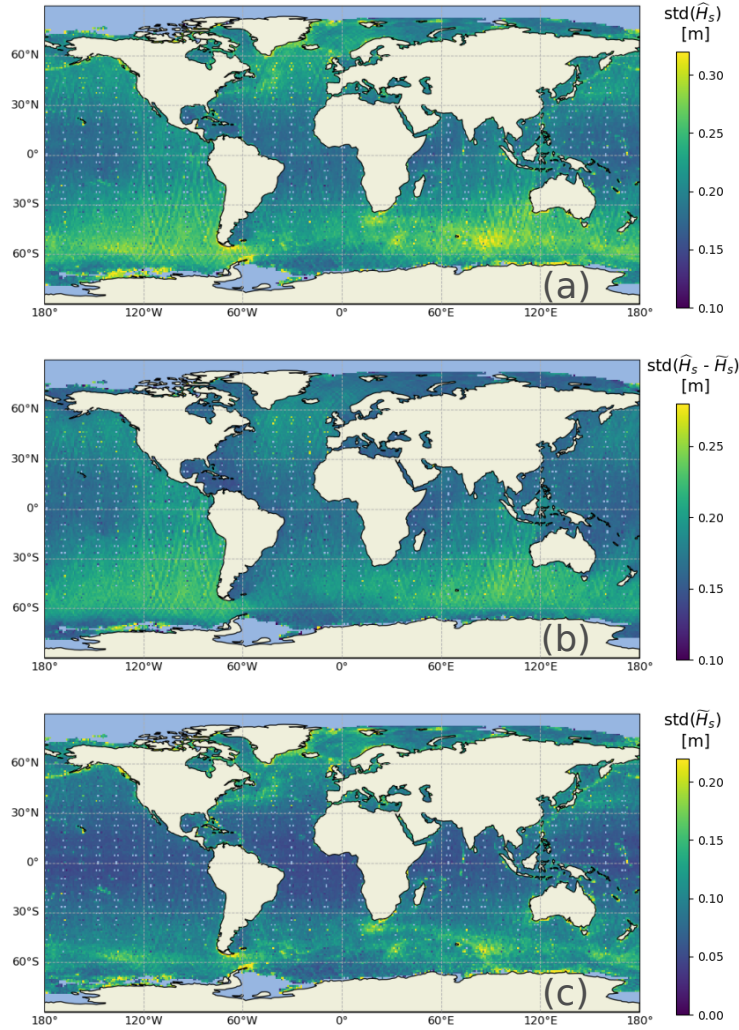


Figure 9. Maps of the average, for the year 2021, of (a) $\text{std}(\widehat{H}_s, 80\text{km})$ computed on original SWIM nadir native values (4.5 Hz), (b) standard deviation of the residual of H_s , defined as the difference between the original and the denoised wave height (c) $\text{std}(\widetilde{H}_s, 80\text{km})$ computed on EMD filtered data \widetilde{H}_s .

542 effect of wave groups apparently corresponds to the variability that is removed by EMD
 543 denoising, without using wave spectrum information. Thus, the uncertainty variable as-
 544 sociated to the nadir 4.5 Hz data in the SALP/CAWATAC products which is derived
 545 from the standard deviation of the fluctuations removed by EMD should be related to
 546 H_s and Q_{kk} . This estimation of the uncertainty may be useful for extrapolating uncer-
 547

548 tainty estimates based on triple-collocation methods to high values of H_s . For satellite
 549 missions other than CFOSAT, the EMD filtered data are available but we do not have
 550 measured wave spectra from which Q_{kk} could be estimated to verify this interpretation
 551 of the EMD filtering. The analysis of these other missions may use estimates of Q_{kk} from
 552 numerical wave models and/or co-location of data with SWIM.

553 4 Discussion

554 4.1 Effect of spectral shape

555 The accurate estimation of wave group contributions critically depends on the ac-
 556 curacy of the spectral shape, in particular the directional width and wavenumber width.
 557 Because of the hard wavelength cut-off in the L2 product we have chosen to work with
 558 the L2S spectra. Redoing the global analysis with the L2 product generally reduces the
 559 expected effect of wave groups. We note that a validation of spectral width from the L2
 560 product was performed by Le Merle et al. (2021), who found that SWIM L2 generally
 561 overestimate spectral width compared to buoy data. No such analysis has been performed
 562 for the L2S product. It would be also interesting to know how accurate could be the es-
 563 timation of $\text{std}(H_{r_a}, d)$ estimated from model spectra, for the application to other satel-
 564 lite mission that do not measure the wave directional spectrum. The minimum distance
 565 d that could be investigated will depend on the wave model resolution.

566 It should also be reminded that SWIM L2 spectra combine sparse measurements
 567 over a 70 km by 90 km box, as illustrated in Fig. 10. Because the wave field has gradi-
 568 ents, this combination generally produces a broader spectrum than a more local estimate
 569 of the wave spectrum, and this should produce a low bias in our estimate the effect of
 570 wave groups. In the example on Fig. 10 the assembly of the L2 spectrum combines data
 571 from two neighboring wave azimuths that are observed in distinct regions of the ocean
 572 separated by up to 72 km. We kept this assembly to be able to compare results from L2
 573 and L2S data, but it would be more logical, for the case of L2S data, to assemble a spec-
 574 trum with a spatial continuity of the footprints that correspond to azimuths around the
 575 spectral peaks.

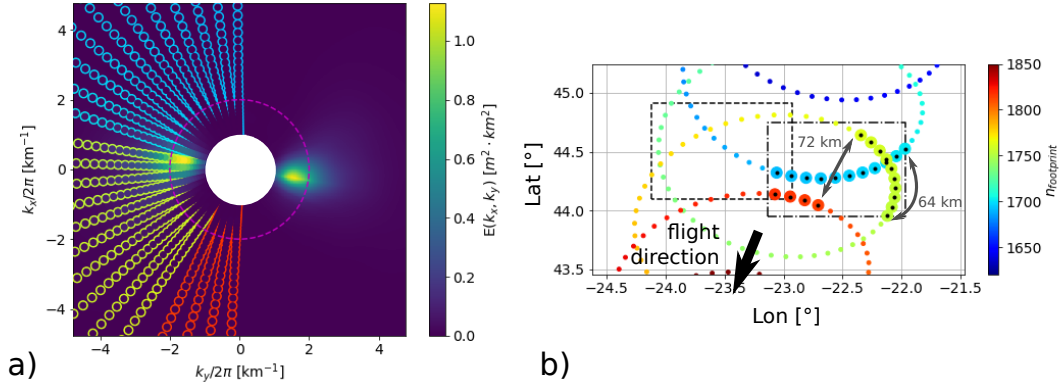


Figure 10. (a) Representation of a two-dimensional spectrum $E(k_x, k_y)$ with values given by the colorbar obtained from assembling L2 1-dimensional spectra $E(k)$ for all azimuths. The color of each circle corresponds to the index of the beam footprint in which the SWIM was making the measurement (b) Geographical layout of boxes — dashed and dash-dotted rectangles — and of centers of the beam footprints on the sea surface. the footprint diameter is about 20 km so that they actually overlap.

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4.2 Effects of satellite altitude

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The particularly low orbit of CFOSAT at 519 km gives a rather small oceanic footprint that allows for wave groups to be resolved in SWIM data. If we consider the higher altitudes used by other satellite missions, 891 km for the recently launched SWOT or 1340 km for the Topex-Poseidon / Jason / Sentinel 6 series, the oceanic footprint gets bigger and wave groups are more likely to be more smoothed out. In Fig. 11, we illustrate this effect with different altitudes following the method used for Fig. 3. Namely, for the same simulated ocean surface, the wave height is estimated by a least-square fit to the simplest Brown waveform given by eq. (9). As expected, the higher the satellite the lower the variability of \hat{H}_s . It is not clear that this effect of satellite altitude is noticeable in real data that are contaminated by speckle noise and that use different waveform fitting algorithms. More realistic simulations will be needed to compare the behaviour of different instruments and processing chains.

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4.3 Expected effect on delay doppler altimeters

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We have shown that the variability of \hat{H}_s at small scale contains some geophysical information and not just random noise related to the measurement. However, the noise

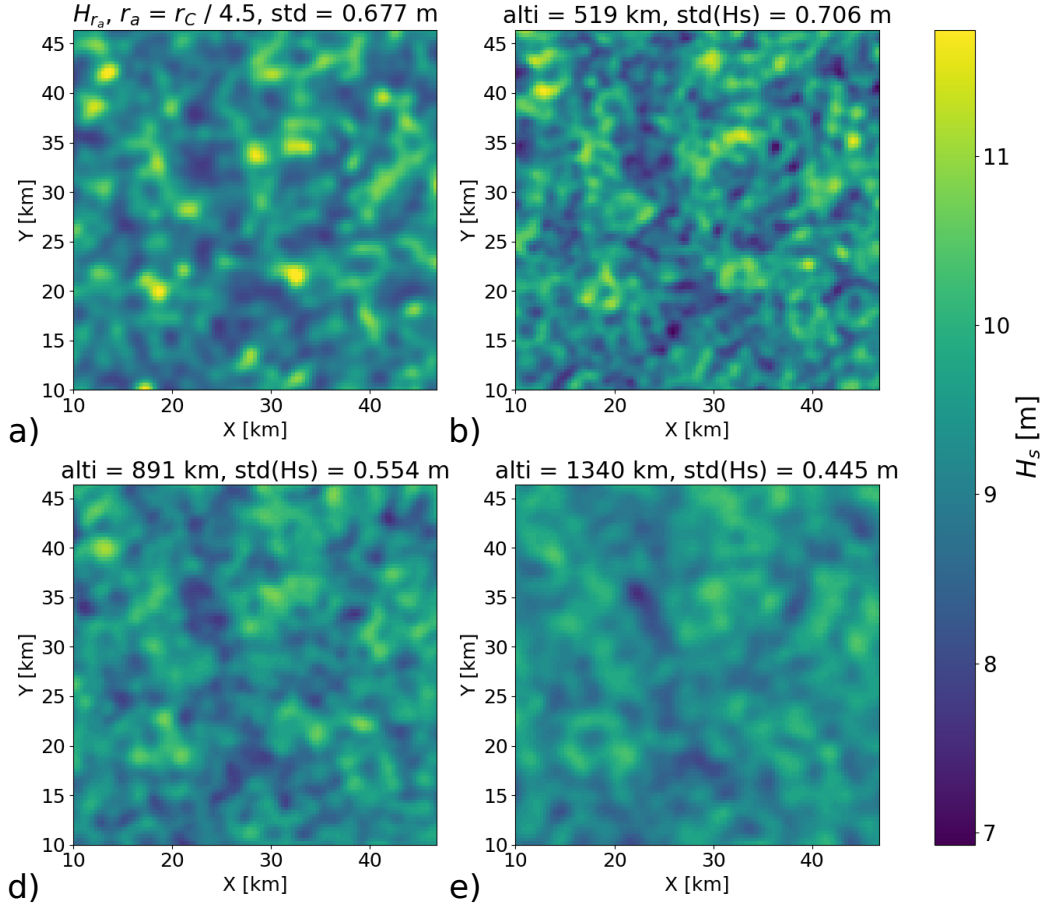


Figure 11. Maps of wave heights obtained a) by smoothing the envelope with Gaussian filter of scale $r_a = 619$ m; or by simulating altimeter waveforms without speckle and using the same least-squares fit (Appendix A), for different altitudes: b) 519 km, c) 891 km, d) 1,340 km, with corresponding r_a of 619 m, 790 m and 940 m.

592 for Delay-only altimeters is probably dominated by the speckle noise in the waveforms
 593 (Sandwell & Smith, 2005; Quartly et al., 2019). Doppler processing of recent altimeter
 594 instruments starting with Cryosat-2 and Sentinel-3 can strongly reduce this speckle noise
 595 by forming and combining independent looks of the same sea surface (Egido et al., 2021).
 596 It will therefore be interesting to study the effect of wave groups in these measurements
 597 of wave height and sea level. Waves can also be resolved directly in the sea level estimates
 598 when data is processed at very high resolution (Altiparmaki et al., 2022; Villas Bôas et
 599 al., 2022). If the Doppler induced by orbital velocities is neglected, the delay-Doppler
 600 measurement is similarly based on the convolution of a surface elevation distribution with
 601 a flat surface response (Ray et al., 2015). Only the flat surface response is different from

602 the Delay-only processing. We thus expect that wave groups will have similar distortion
 603 of the waveforms and contributions to estimates of wave heights and sea level. The blur-
 604 ring effect caused by range bunching will now be confined to the direction perpendic-
 605 ular to the track, with maximum effect of a H_s perturbation located off the satellite track
 606 (depending on fitting algorithm), possibly also at a distance of the order of $r_C/2$. Be-
 607 cause Delay-Doppler altimeters can actually resolve the along-track variability caused
 608 by wave groups instead of averaging it, we expect that \hat{H}_s fluctuations caused by wave
 609 groups are much larger in Delay-Doppler altimetry, together with their spurious effect
 610 on sea level estimates. This may explain the relative smaller reduction of $\text{std}(\hat{H}_s, 7\text{km})$
 611 at large H_s which is found when Doppler resolution is enhanced to reduce the speckle
 612 effect, and a typical value of $\text{std}(\hat{H}_s, 7\text{km})$ for Delay-Doppler Sentinel 3A data which that
 613 is around 0.7 m for $H_s = 7$ m (Egido et al., 2021), twice the typical value for SWIM
 614 data. This will be the topic of further studies.

615 4.4 Wave groups and satellite measurements uncertainties

616 Up to now, the uncertainty of satellite measurements has been determined by the
 617 triple-collocation method (Abdalla et al., 2011; Dodet et al., 2022), with the practical
 618 result that the uncertainty of altimeter data, either denoised or integrated along-track
 619 into super-observations, is of the order of 7% of H_s . However, that error contains rep-
 620 resentation errors (the co-located in situ data does not sample the same space and time
 621 frame), and cannot be extrapolated beyond the range of the co-located dataset, typically
 622 wave heights below 8 m. So what can we say about the largest measured wave heights
 623 of 20.1 m (Hanafin et al., 2012)? Can we use the measured variability of \hat{H}_s , for exam-
 624 ple the 4.5 Hz or 20 Hz data that is used to make a 1 Hz average, to refine our estimate
 625 of the uncertainty of this average? In the present paper we have shown that wave groups
 626 are responsible for random fluctuations in the estimates \hat{H}_s , that are generally propor-
 627 tional to H_s but with an effect that depends on the bandwidth of the spectrum, which
 628 is generally narrower for larger wave periods. As a result the variability associated to
 629 wave groups can be the dominant source of fluctuations in \hat{H}_s measurements for severe
 630 storm conditions. Even though the measurement fluctuations are weakly correlated to
 631 the actual wave height variations (as demonstrated in Fig. 3) their magnitudes are strongly
 632 correlated. Hence the measured fluctuation $\text{std}(\hat{H}_s, 7\text{km})$ contains both uncorrelated speckle
 633 noise effect, that can be expected to be reduced by $1/\sqrt{N}$ when averaged from N Hz to

634 1 Hz, and a true geophysical spatial variability associated to wave groups (and variable
 635 fetch, currents, etc.) that will only partially average out. We expect that an uncertainty
 636 model for averages of \widehat{H}_s measurements may take into account wave groups explicitly.

637 In the case of SWIM, directional wave spectra can be used to separate the actual vari-
 638 ability of the 4.5 Hz data into wave group effects and noise plus other geophysical effects.
 639 For other altimeters, one may use empirical correlations between spectral bandwidth,
 640 wave height and wind speed. For this information to be useful for a theoretically-based
 641 uncertainty estimate, which is much needed for wave heights above 8 m, one may extend
 642 the parameterization of speckle effects proposed in Appendix A.3, to the actual target
 643 waveform and cost function used in the retracking algorithm.

644 4.5 Considerations on the satellite resolution

645 The present work should be useful for the exploration of the resolution limits of
 646 satellite altimeters and other remote sensing systems that use radar or optical imagery
 647 (Kudryavtsev et al., 2017). As processing methods are refined to produce higher reso-
 648 lution near the coast (Passaro et al., 2021) and the ice edge (Collard et al., 2022), some
 649 of the high resolution data will be dominated by wave groups. The associated variance
 650 of H_s may provide some constraint on the shape of the directional wave spectrum, but
 651 the detailed fluctuations are probably of little value for most applications as groups will
 652 travel at speeds of the order of 10 m/s and persist for only a few minutes. The contri-
 653 bution of wave groups to the variability of wave heights measured by altimeters is thus
 654 a real effect that contains part of the true variability of wave heights at the scale of the
 655 altimeter footprint. Methods developed to remove noise in the data, such as the data-
 656 driven Empirical Mode Decomposition (EMD) used by (Quilfen & Chapron, 2019) ap-
 657 pear to remove the effect of wave groups. An investigation of the variability of wave heights
 658 at the smallest scales cannot be based on denoised data alone, because they miss a large
 659 part of the true variability.

660 In locations where H_s varies sharply such as over coral reefs, mud banks or across
 661 the sea ice edge, the high resolution wave heights will contain other effects, and these
 662 are particularly interesting. Some caution should be used when interpreting these sharp
 663 gradients. As we have found out, the maximum wave height will generally be displaced
 664 from the location of its true maximum. This displacement is smallest for the SWIM in-
 665 strument, thanks to the low orbit of CFOSAT, which makes it a particularly interest-

ing instrument for studying small scale wave height variations, in spite of its rather low rate for the nadir beam (4.5 Hz instead of 20 Hz for Jason), and the absence of Doppler processing.

5 Conclusion

In this paper, we took advantage of the low orbit altitude of CFOSAT, and the low noise level of the nadir beam of the SWIM instrument to study the along-track variability of wave height. The directional wave spectra measured by off-nadir beams on SWIM has been complementary to study the relationship between wave spectra and along-track H_s variability. After giving a theoretical estimate of the standard deviation of H_s associated to wave groups as a function of the wave spectrum and satellite altitude, we computed this estimate for 2 years of CFOSAT data using L2S products. We found that the standard deviation of H_s associated to wave groups is generally about half of the standard deviation of H_s measured over a 80 km distance, explaining 25% of measured H_s variance. This ratio of variances increases for storms and in the presence of long swells, it was found to be larger than 75% in 3% of the cases.

The residual variability after subtracting the estimated effect of wave groups from the measured H_s variance correlates positively with along-track variance of filtered signals, which brings out regions of strong currents. Studying these features is of major interest and requires some filtering out of the smaller scales. In turn, these smaller scales can be of interest, at least from a statistical point of view, as they are related to wave groups and the generation of infragravity waves and extreme sea level at the coast.

The main novelty of the present work was to expose the limitations of the theoretical Brown (1977) model that is used to retrieve wave heights and sea levels from altimeter data. The Brown model assumes a Gaussian sea level distribution that is spatially-uniform at the scale of the footprint, which is correct when averaged over long enough scales along-track. For an individual measurement, the footprint may not be large enough for the Brown model to be valid, and we have demonstrated that the effect of wave groups on the waveforms, is equivalent to introducing a range-dependent wave height. This limitation is common to all altimeters that provide estimates of wave height and sea level based on theoretical waveforms, including Low Rate Mode (LRM) and Synthetic Aperture Radar Mode (SARM) processing. The resulting waveforms have distorted shapes

697 that can be similar to the effect of non-Gaussian sea level statistics (Srokosz, 1986; Ro-
698 driguez, 1988), but with much larger distortions. To our knowledge this non-uniformity
699 effect had never been considered. We have shown that the wave group effect averages
700 out to zero over large scales when waveforms are fitted with a simple least square cost
701 function, but that is not necessarily the case for the more sophisticated methods that
702 are often used. We expect that further work will expand on our approach to consider
703 spurious effects on sea level estimates as well as applications to recent delay-Doppler al-
704 timeters.

Appendix A Non-homogeneous H_s and waveform retracking

In this analysis we keep the most simple model of altimeter measurement that is also used in section 2: we neglect antenna pattern, thermal noise and mispointing effects, and neglect the Earth sphericity. These assumptions are meant to simplify the algebra as much as possible while keeping the essential features of non-homogeneity in wave heights. Likewise we have used the most simple cost function when fitting the waveform, while maximum likelihood methods are generally used with real data (Rodriguez, 1988; Halimi, 2013). We also start by ignoring speckle noise. The analysis performed below is easily extended to consider the third parameter which is usually estimated in retracking waveforms, that is the Normalized Radar Cross Section.

A1 Wave groups and H_s estimate

We consider a small perturbation Δ_H of H_s over an area A , localized around a range $h+R_0$. The original normalized Brown waveform of eq. (9) corresponds to the histogram of the ocean area per unit range, divided by $2\pi h$ so that it varies between 0 and 1, with h the satellite altitude. The perturbation to the waveform is equivalent to removing the original Gaussian distribution of surface elevation with $\sigma_H = H_s/4$ over the area A , and replacing it by a new Gaussian with $\sigma' = (H_s+\Delta_H)/4$, over the same area A , and divide by the normalization factor $2\pi h$. We define the parameter $a = A\Delta_H/(8\pi h)$, which should be small compared to H_s^2 . For a small change in H_s , this change in waveform is proportional to the derivative of the Gaussian distribution with respect to σ_H and we find that the waveform is now

$$w(R) = w_B(R, \sigma_H) + a \frac{e^{-(R-h-R_0)^2/(2\sigma_H^2)}}{\sqrt{2\pi}} \frac{(R-h-R_0)^2 - \sigma_H^2}{\sigma_H^4} + O(a^2) \quad (\text{A1})$$

We note that a smaller change Δ_H over a larger area A changes the waveform in the same way as a larger change over a smaller area, provided that a is the same. For simplicity we redefine the Chelton footprint diameter as $r'_C = \sqrt{2H_s h}$, and we find that taking an area of radius $\alpha r'_C$ gives $a = \alpha^2 \Delta_H H_s/4$.

The shape of these simulated distorted waveforms is illustrated in Fig. A1.

With the distortion shown here, fitting a Brown waveform would give a wave height of $H_{s,\text{fit}} = 12.6$ m for $R_0 = 2.5$ m and $H_{s,\text{fit}} = 10$ m for $R_0 = 0$, which is a strange way to average the $H_s = 13$ m over part of the footprint and 10 m in the rest of the

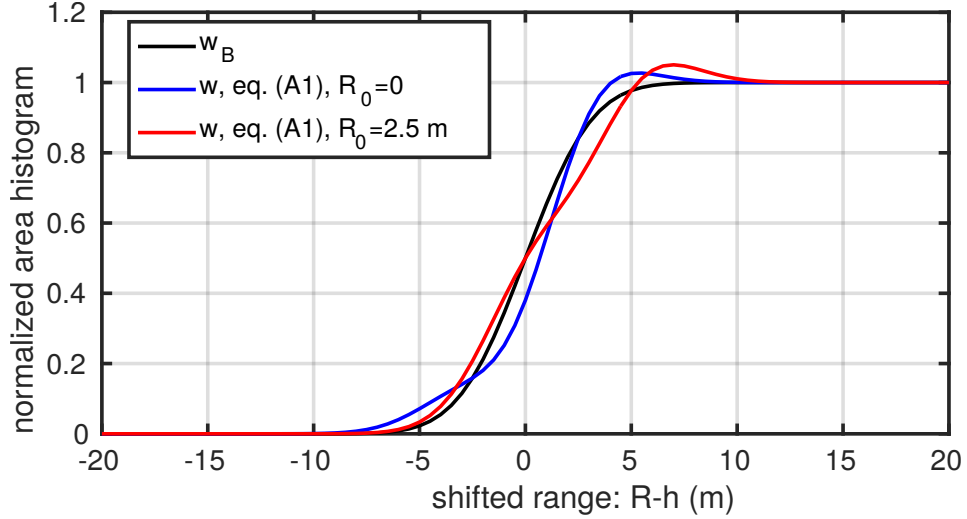


Figure A1. Example simulated waveforms in the presence of a localized change in H_s around the range $h + R_0$, for $H_s = 10$ m. The perturbations use $a = 1.875$ m², that would correspond to $\Delta_H = 3$ m over an area of radius $r'_C/4$, a perturbation that is neither small nor localized.

735 footprint. Fig. A2 shows that such perturbations are of the order of the deviations from
 736 the mean waveform in the case of the Box B SWIM waveforms, and are absent in Box
 737 A. The main difference between the simulated waveforms and the true waveforms is the
 speckle noise that is of the order of 10 % for both box A and box B.

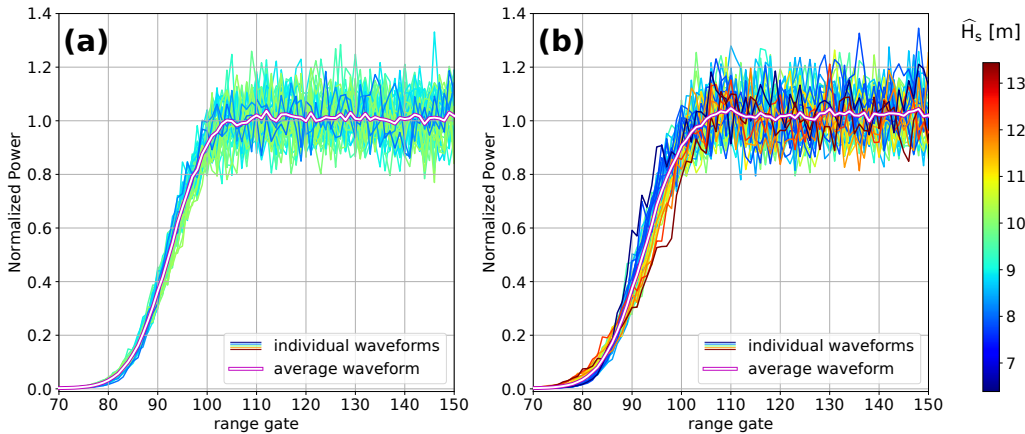


Figure A2. Ensembles of CFOSAT/SWIM waveforms in (a) box A and (b) box B. These are the L1A product, already corrected for the antenna pattern, and thus directly comparable to Fig. A1. Individual waveforms are color-coded with the estimated wave height. The white line represents the average waveform.

738 For small values of the perturbation a , the deviation in the fitted H_s can be com-
 739 puted analytically. For simplicity we will assume that the waveforms are defined for $-\infty <$
 740 $R < \infty$, and the sum of the difference squared between $w(R)$ and $w_B(R, \sigma'_H)$, when in-
 741 tegrated from $R = -\infty$ to $R = \infty$ is the following cost function,

$$\begin{aligned} C &= \int_{-\infty}^{\infty} \{[w_B(R, \sigma_H) - w_B(R, \sigma'_H)] + [w(R) - w_B(R, \sigma_H)]\}^2 dR \\ &\simeq \int_{-\infty}^{\infty} \left\{ (\sigma_H - \sigma'_H) \frac{\partial w_B(R, \sigma_H)}{\partial \sigma_H} + [w(R) - w_B(R, \sigma)] \right\}^2 dR \\ &= (\sigma'_H - \sigma_H)^2 \frac{1}{4\sqrt{\pi}\sigma_H} + (\sigma'_H - \sigma_H) \frac{aR_0}{8\sqrt{\pi}\sigma_H^5} e^{-R_0^2/(4\sigma_H^2)} (R_0^2 - 6\sigma_H^2) + \frac{3a^2}{8\sqrt{\pi}\sigma_H^3}. \end{aligned}$$

742 Fitting σ'_H corresponds to solving $\partial C / \partial (\sigma'_H - \sigma_H) = 0$. We note that error terms
 743 that are either not a function of $(\sigma'_H - \sigma_H)$ or odd functions of R have no impact on the
 744 fitted value. For example the a^2 term in eq. (A1) does not contribute any difference to
 745 the fit.

746 We find that the fitted value differs from the background value H_s by a factor pro-
 747 portional to a and function of R_0/H_s ,

$$748 \quad H_{s,\text{fit}} = H_s + \frac{A}{\pi h} \frac{\Delta_H}{H_s} \underbrace{\left[2 \frac{R_0}{H_s} \left(6 - \left(\frac{4R_0}{H_s} \right)^2 \right) e^{-4R_0^2/H_s^2} \right]}_{J(R_0/H_s)}, \quad (\text{A2})$$

749 with the function J in brackets having a maximum close to 2 for $R_0 \simeq H_s/4$, as shown
 750 in Fig. A3.

751 We note that this perturbation is zero for $R_0 = 0$, meaning that a localized change
 752 at the center of the footprint does not modify the estimated H_s . This lack of impact on
 753 $H_{s,\text{fit}}$ comes from the fact that the perturbation of the waveform (the second term in eq. (A1))
 754 is an odd function of range and thus orthogonal to the even functions that are the Brown
 755 waveforms with zero epoch $w_B(R, \sigma_H)$. The maximum perturbation of $H_{s,\text{fit}}$ occurs for
 756 H_s perturbations at a range R_0 close to σ_H , i.e. corresponding to a distance from nadir
 757 of $r'_C/4$. Eq. (A2) gives results that are fairly robust for finite values of a/H_s^2 , and would
 758 predict a wave height of 12.9 m in the case $R_0 = 2.5$ m shown in Fig. A1.

759 We now consider the average effect of the perturbation by computing the average
 760 over R_0 , taking all values of R_0 from 0 to nH_s , which corresponds to averaging over an

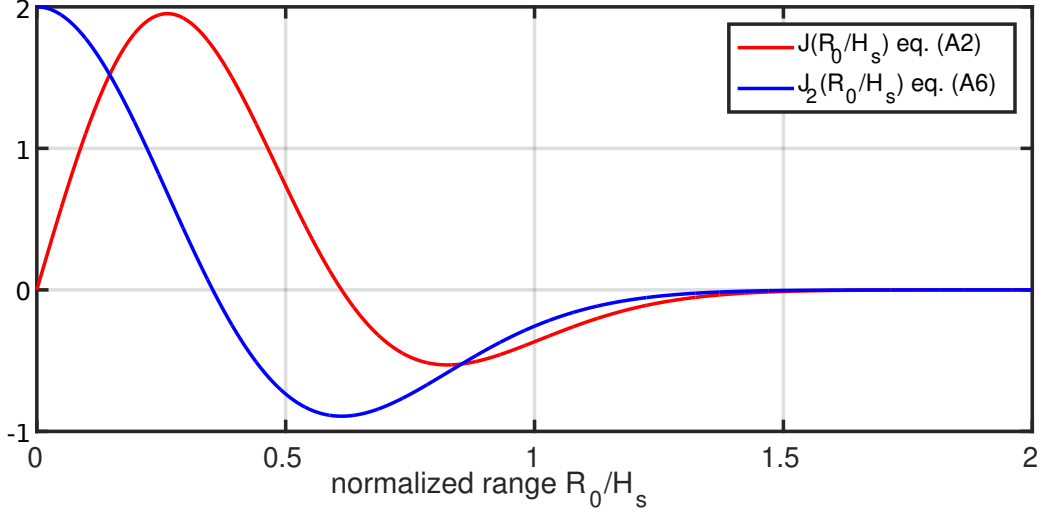


Figure A3. Functions $J(R_0/H_s)$ and $J_2(R_0/H_s)$ corresponding to the term in square brackets in eqs. (A2) and (A8). The maximum of J is at $R_0/H_s = 0.5\sqrt{0.5(3-\sqrt{6})} \simeq 0.26$, where J takes a value close to 1.96. This location corresponds to a distance from nadir approximately $\sqrt{0.26}r'_C \simeq r'_C/2$.

761 area $B = \pi n r'_C{}^2 = 2n\pi H_s h$. The integral of the function in brackets is

$$762 \quad I = \int_0^\infty 2 \frac{R_0}{H_s} \left(6 - 16 \frac{R_0^2}{H_s^2} \right) e^{-4R_0^2/H_s^2} dR_0 = 0.5H_s. \quad (\text{A3})$$

763 As a result, the average effect of a Δ_H change over an area $A = \pi\alpha^2 r'_C{}^2 = 2\pi\alpha^2 h H_s$

764 is, when n is large,

$$765 \quad \delta_{H,\text{alti}} = \frac{1}{nH_s} \int_0^{nH_s} (H_{s,\text{fit}} - H_s) dR_0 = \frac{1}{2n} \frac{A}{\pi h} \frac{\Delta_H}{H_s} = \frac{\alpha^2}{n} \Delta_H. \quad (\text{A4})$$

766 This average effect of the localized perturbation of H_s is the same as a true area aver-
767 age, which is the perturbation times the ratio of the areas A and B , namely $\delta_H = \Delta_H A/B$.

768 In other words, the perturbation is amplified if located at $0.15 < r/r'_C < 0.34$ from
769 nadir, by a factor J that is up to 2. Otherwise the perturbation is attenuated, so that
770 on average it is equal to the true perturbation. This averaging property and the unbi-
771 ased estimate of $H_{s,\text{fit}}$, with a perturbation that changes sign when Δ_H changes sign,
772 are specific to the simple least squares used here. For example, fitting the logarithm of
773 the waveform produces a biased estimator and a non-zero response for $R_0 = 0$. Hence
774 the results presented here are specific to the fitting method.

775 In practice, distributed anomalies of H_s are not only a function of the distance from
776 nadir, so that a local estimate of H_s will combine positive and negative anomalies Δ_H

777 that are located at the same distance from nadir, and will partially cancel. This explain
 778 that our best fit for r_0 is $r_C/4.5$, smaller than the $r_C/2$ which is a more typical scale of
 779 the footprint. Instead of retracking the simulated altimeter data, we can reproduce the
 780 H_s variability by first summing the Δ_H anomalies for a given r , compute the $H_{s,\text{fit}}$ anomaly
 781 for that r using eq. (A2) and then sum those anomalies for all r , as demonstrated in Fig. A4.
 782 This procedure is equivalent to a spatial filter $\mathcal{J}(r)$ that is built from the J function, con-
 783 verting the range $h + R_0$ to a horizontal distance from nadir $r = \sqrt{2hR_0}$,

$$784 \quad \mathcal{J}(r) = g_{r_C}(r) + J \otimes (Id - g_{r_C})(r) \quad (\text{A5})$$

785 where, g_{r_C} is a Gaussian filter with width r_C , and Id is the identity function. The es-
 786 timated H_s thus comes from

$$787 \quad H_s(x) = 4\sqrt{\frac{2}{\pi}}(\mathcal{J} \otimes \eta)(x) \quad (\text{A6})$$

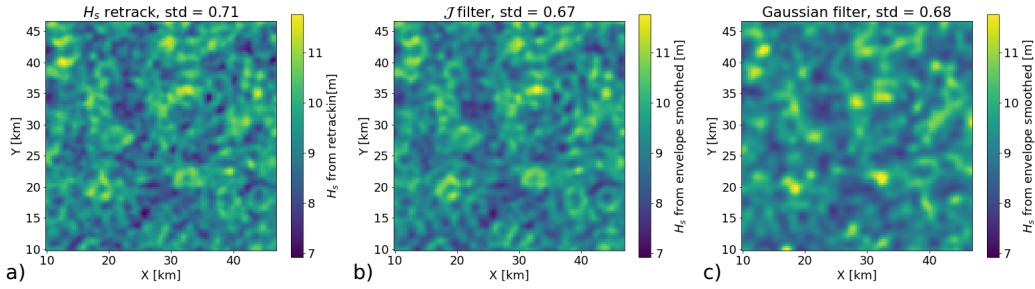


Figure A4. Equivalent to Fig. 3: (a) H_s surface obtained from retracking and H_s surfaces obtained from convoluting the envelope with (b) a spatial filter built from the J function, (c) a gaussian filter with $r_a = r_C/4.5$.

788 A2 Wave groups and sea level estimate

789 While perturbations at nadir do not change the H_s estimate, they would change
 790 the mean sea level z_e (the epoch is $-z_e$) when using a 2-parameter waveform

$$791 \quad w_{B2}(R, \sigma_H, z_e) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{(R + z_e) - h}{\sqrt{2}\sigma_H} \right) \right]. \quad (\text{A7})$$

792 In the case shown in Fig. A1 with $R_0 = 0$ the estimated mean sea level is $z = -37$ cm.
 793 We thus expect wave groups to contribute to fluctuations in the estimated sea level at
 794 the scale of groups. The estimation of that effect follows the same method used above.

895 Fitting $w_{B2}(R, \sigma'_H, z_e)$ to our waveform $w(R)$ given by eq. (A1) is obtained by minimiz-
 896 ing a modified cost function, that is the same as C but with one extra term $z_e \partial w_{B2} / \partial z_e$
 897 inside the curly brackets, giving two extra non-zero terms proportional to z_e^2 and z_e . We
 898 note that the cross-term proportional to $z_e(\sigma'_H - \sigma_H)$ is an odd function of R and thus
 899 integrates to zero. After integration over R we get the cost function,

$$800 \quad C_2 = C + \frac{z_e^2}{2\sqrt{\pi}\sigma_H} + \frac{az_e}{4\sqrt{\pi}\sigma_H^2} e^{-R_0^2/(4\sigma_H^2)} \left(\frac{R^2}{\sigma_H^2} - 2 \right).$$

801 Taking the derivative of C_2 with respect to z_e gives

$$802 \quad z_e = -\frac{A\Delta_H}{8\pi h H_s} \underbrace{\left[\left(2 - 16 \frac{R_0^2}{H_s^2} \right) e^{-4R_0^2/H_s^2} \right]}_{J_2(R_0/H_s)}. \quad (\text{A8})$$

803 The function J_2 is plotted in Fig. A3. Hence z_e has the strongest deviation when the
 804 wave height perturbation is centered at nadir, and the sign of the deviation is opposite
 805 to Δ_H : i.e. a wave group centered at the nadir would give a spurious lower sea level. On
 806 average the z_e deviation has a zero mean when R_0 is varied. As a result of the differ-
 807 ent shapes of J and J_2 , there is no simple correlation of the H_s and z_e perturbations,
 808 contrary to the correlations induced by speckle noise in the waveform measurement (Sandwell
 809 & Smith, 2005).

810 There is some correlation for R_0/H_s between 0.7 and 1.2 which may contribute to
 811 anti-correlation of sea level anomalies and wave heights at scales around r_C , and thus
 812 may persist in 1 Hz data. We insist that these are spurious sea level variations. In deep
 813 water these spurious oscillations are much larger than the fraction of a millimeter asso-
 814 ciated to true sea level variations with bound infragravity elevation that is anti-correlated
 815 with the envelope of kilometer-scale wave groups (Ardhuin et al., 2004). The spurious
 816 sea level oscillations described are also probably generally larger in amplitude than the
 817 larger scale (20-km wavelength) true sea level variations associated to free infragravity
 818 waves that have no phase correlation with the local envelope (Ardhuin et al., 2014). In
 819 shallow water, the real sea level fluctuations can be more important.

820 **A3 Speckle noise**

821 Random fluctuations in the electromagnetic power measured by the radar combine
 822 an additive thermal noise that can often be neglected and a multiplicative noise that is
 823 caused by the Rayleigh fading of the interfering reflections off a random sea surface (Quartly
 824 et al., 2001). In fact speckle is to the radar power what wave groups are to the wave en-

825 ergy. A good model for the speckle is a multiplicative random noise, so that the mea-
 826 sured waveform for each range is multiplied by a factor $(1+\varepsilon(R))$ with $\varepsilon(R)$ following
 827 a χ^2 distribution with $N(R)$ degrees of freedom depending on the number of pulses av-
 828 eraged and the pulse repetition frequency (Quarty et al., 2001).

829 For the retracking, the effect of this speckle perturbation is one additional term $\varepsilon(R)w(R)$
 830 inside the curly brackets of the cost function. Expanding the square and expressing the
 831 integral, it gives two terms, one proportional to $(\sigma'_H - \sigma_H)$ that is relevant to the H_s
 832 estimate and the other proportional to z_e fit, so that the cost function is now,

$$833 \quad C_3 \simeq C_2 - 2(\sigma'_H - \sigma_H) \int_{-\infty}^{\infty} \varepsilon(R)w(R) \frac{\partial w_{B2}}{\partial \sigma_H} dR - 2z_e \int_{-\infty}^{\infty} \varepsilon(R)w(R) \frac{\partial w_{B2}}{\partial z_e} dR, \quad (\text{A9})$$

834 with

$$835 \quad \frac{\partial w_{B2}}{\partial \sigma_H} = -\frac{R - h + z_e}{\sigma_H^2 \sqrt{2\pi}} e^{-(R-h+z_e)^2/(2\sigma_H^2)}, \quad (\text{A10})$$

836 and

$$837 \quad \frac{\partial w_{B2}}{\partial z_e} = \frac{1}{\sigma_H \sqrt{2\pi}} e^{-(R-h+z_e)^2/(2\sigma_H^2)}. \quad (\text{A11})$$

838 The estimated wave height that gives $\partial C_3/\partial(\sigma'_H - \sigma_H) = 0$ thus has an extra term
 839 induced by speckle noise,

$$840 \quad H_{s,\text{fit}} = H_s + \frac{A}{\pi h} \frac{\Delta_H}{H_s} J(R_0/H_s) + 16\sqrt{2}H_s \int_{-\infty}^{\infty} \varepsilon(u) \left(1 + \text{erf}(2\sqrt{2}u)\right) u e^{-8u^2} du, \quad (\text{A12})$$

841 with $u = (R - h + z_e)/H_s$. The speckle-induced perturbation of $H_{s,\text{fit}}$ is a weighted
 842 sum of random fluctuations with zero mean. In practice we can consider $\varepsilon(R)$ to be Gaus-
 843 sian, and the variance of the speckle perturbation is the sum of the variances associated
 844 to each range R times the weight squared. To get some useful order of magnitude we may
 845 take the variance of $\varepsilon(R)$, which is $1/N(R)$, to be constant at $1/N$. For large values of
 846 H_s , the discretized waveform is well approximated by the continuous form and the part
 847 of the variance of $H_{s,\text{fit}}$ induced by the speckle is approximately $5.0 H_s/N$, with a stan-
 848 dard deviation $2.24\sqrt{H_s/N}$. Using the value $N = 512$ for the number of pulses of the
 849 SWIM nadir beam that we may assume to be independent, and $H_s = 2$ m, this gives
 850 a standard deviation of 0.14 m, broadly consistent with the background level in Fig. 8.c.
 851 However, we note that the magnitude of the variability of $H_{s,\text{fit}}$ will depend on the method
 852 used to fit the waveform. In the case of the SWIM data, the adaptive method that is used
 853 is based on a maximum likelihood (Tourain et al., 2021). It is probably more robust to
 854 speckle noise perturbations than the least square estimate used here, in particular for
 855 this instrument that has a relatively high signal to noise ratio.

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862 **Open Research**

863 The L2 SWIM dataset used here corresponds to the files reprocessed by CNES (2020)
864 in version 5.1.2 and made available by CNES on the ftp server of AVISO+ (<ftp-access.aviso.altimetry.fr>,
865 directory `cfosat/swim_l2_op05`), accessible to anyone after registration.

866 The L2P SWIM dataset used here corresponds to the files reprocessed by CNES/CLS
867 (2021) in version 1.2 and made available by CNES on the ftp server of AVISO+ (<ftp-access.aviso.altimetry.fr>,
868 directory `cfosat/swim_l2p_box_nrt/`), accessible to anyone after registration.

869 The L2S SWIM dataset used in this paper corresponds to the files reprocessed by
870 Ifremer / CERSAT (2022) in version 1.0 and available at
871 https://data-cersat.ifremer.fr/projects/iwwoc/swi_l2s

872 The denoised CFOSAT data used for Fig. 9 were processed by Copernicus Marine
873 Service / Centre National D'Etudes Spatiales / Collecte Localisation Satellite (2022) and
874 distributed by AVISO+ with support from CNES ([https://doi.org/10.24400/527896/a01-](https://doi.org/10.24400/527896/a01-2022.013)
875 [2022.013](https://doi.org/10.24400/527896/a01-2022.013)). The data are available on the ftp server of AVISO+ (<ftp-access.aviso.altimetry.fr>)
876 after registration and selection of "Wave experimental products".

877 **References**

- 878 Abdalla, S., Janssen, P., & Bidlot, J.-R. (2011). Altimeter near real time wind and
879 wave products: Random error estimation. *Marine Geodesy*, *34*, 396–406. doi:
880 10.1080/01490419.2011.585113
- 881 Alday, M., Accensi, M., Ardhuin, F., & Dodet, G. (2021). A global wave parameter
882 database for geophysical applications. part 3: Improved forcing and spectral
883 resolution. *Ocean Modelling*, *166*, 101848. doi: 10.1016/j.ocemod.2021.101848
- 884 Alday, M., & Ardhuin, F. (2023). On consistent parameterizations for both dom-

- 885 inant wind-waves and spectral tail directionality. *J. Geophys. Res.*, *128*,
886 e2022JC019581. doi: 10.1029/2022JC019581
- 887 Altiparmaki, O., Kleinherenbrink, M., Naeije, M., Slobbe, C., & Visser, P. (2022).
888 Sar altimetry data as a new source for swell monitoring. *Geophys. Res. Lett.*
889 doi: 10.1029/2021GL096224
- 890 Ardhuin, F., Chapron, B., & Elfouhaily, T. (2004). Waves and the air-sea mo-
891 mentum budget, implications for ocean circulation modelling. *J. Phys.*
892 *Oceanogr.*, *34*, 1741–1755. doi: 10.1175/1520-0485%282004%29034%3C1741%
893 3AWATAMB%3E2.0.CO%3B2
- 894 Ardhuin, F., Rawat, A., & Aucan, J. (2014). A numerical model for free infragrav-
895 ity waves: Definition and validation at regional and global scales. *Ocean Mod-*
896 *elling*, *77*, 20–32. doi: 10.1016/j.ocemod.2014.02.006
- 897 Ardhuin, F., Stopa, J. E., Chapron, B., Collard, F., Husson, R., Jensen, R. E., ...
898 Young, I. (2019). Observing sea states. *Frontiers in Marine Sci.*, *6*, 124. doi:
899 10.3389/fmars.2019.00124
- 900 Arhan, M., & Ezraty, R. (1978). Statistical relations between successive wave
901 heights. *Oceanol. Acta*, *1*, 151–158.
- 902 Badulin, S. I. (2014). A physical model of sea wave period from altimeter data. *J.*
903 *Geophys. Res.*, *119*, 856–869. doi: 10.1002/2013JC009336
- 904 Borge, J. C. N., Lehner, S., Niedermeier, A., & Shulz-Stellenfleth, J. (2004). De-
905 tection of ocean wave groupiness from spaceborne synthetic aperture radar. *J.*
906 *Geophys. Res.*, *109*, C07005. doi: 10.1029/2004JC00298
- 907 Brown, G. S. (1977). The average impulse response of a rough surface and its appli-
908 cations. *IEEE J. Oceanic Eng.*, *2*(1), 67–63. doi: 10.1109/JOE.1977.1145328
- 909 Chelton, D. B., Walsh, E. J., & MacArthur, J. L. (1989). Pulse compression and sea
910 level tracking in satellite altimetry. *J. Atmos. Ocean Technol.*, *6*, 407–438. doi:
911 10.1175/1520-0426(1989)006<0407:pcaslt>2.0.co;2
- 912 Chen, G., Chapron, B., Ezraty, R., & Vandemark, D. (2002). A global view of swell
913 and wind sea climate in the ocean by satellite altimeter and scatterometer. *J.*
914 *Atmos. Ocean Technol.*, *19*, 1849–1859.
- 915 CNES. (2020). *Wave products from SWIM Level 2 (L2), version 5.1.2* [Dataset].
916 AVISO+ with support from CNES. Retrieved from [ftp://ftp-access.aviso](ftp://ftp-access.aviso.altimetry.fr/)
917 [.altimetry.fr/](ftp://ftp-access.aviso.altimetry.fr/) (after registration, directory: cfosat/swim_l2_op05)

- 918 CNES/CLS. (2021). *Wave products from SWIM Level 2+ Box Off Nadir NRT*
 919 *(L2P), version 1.2* [Dataset]. AVISO+ with support from CNES. Retrieved
 920 from <ftp://ftp-access.aviso.altimetry.fr/> (after registration, directory:
 921 [cfosat/swim_l2p_box_nrt](ftp://ftp-access.aviso.altimetry.fr/ftp-access/cfosat/swim_l2p_box_nrt))
- 922 Collard, F., Marié, L., Nougier, F., Kleinherenbrink, M., Ehlers, F., & Ardhuin,
 923 F. (2022). Wind-wave attenuation in arctic sea ice: A discussion of re-
 924 mote sensing capabilities. *J. Geophys. Res.*, *127*, e2022JC018654. doi:
 925 10.1029/2022JC018654
- 926 Copernicus Marine Service / Centre National D'Etudes Spatiales / Collecte Local-
 927 isation Satellite. (2022). *Wave experimental products: along-track significant*
 928 *wave height 5hz* [Dataset]. CNES. doi: "10.24400/527896/A01-2022.013"
- 929 Dodet, G., Abdalla, S., Alday, M., Accensi, M., Bidlot, J., & Ardhuin, F. (2022).
 930 Error characterization of significant wave heights in multidecadal satel-
 931 lite altimeter product, model hindcast, and in situ measurements using the
 932 triple collocation technique. *J. Atmos. Ocean Technol.*, *39*, 887–901. doi:
 933 10.1175/JTECH-D-21-0179.1
- 934 Dodet, G., Piolle, J.-F., Quilfen, Y., Abdalla, S., Accensi, M., Ardhuin, F., ... Don-
 935 lon, C. (2020). The sea state cci dataset v1: towards a sea state climate data
 936 record based on satellite observations. *Earth System Sci. Data*, *12*, 1929–1951.
 937 doi: 10.5194/essd-12-1929-2020
- 938 Egido, A., Dinardo, S., & Ray, C. (2021). The case for increasing the posting rate
 939 in delay/Doppler altimeters. *Adv. Space Res.*, *68*, 930–936. doi: 10.1016/j.asr
 940 .2020.03.014
- 941 Halimi, A. (2013). *From conventional to delay/doppler altimetry* (Doctoral disserta-
 942 tion, INP Toulouse, Toulouse, France). Retrieved from [https://theses.hal](https://theses.hal.science/tel-00951973)
 943 [.science/tel-00951973](https://theses.hal.science/tel-00951973)
- 944 Hanafin, J., Quilfen, Y., Ardhuin, F., Sienkiewicz, J., Queffeuilou, P., Obrebski, M.,
 945 ... Stutzmann, E. (2012). Phenomenal sea states and swell radiation: a com-
 946 prehensive analysis of the 12-16 February 2011 North Atlantic storms. *Bull.*
 947 *Amer. Meteorol. Soc.*, *93*, 1825–1832. doi: 10.1175/BAMS-D-11-00128.1
- 948 Hasselmann, K., Barnett, T. P., Bouws, E., Carlson, H., Cartwright, D. E., Enke,
 949 K., ... Walden, H. (1973). Measurements of wind-wave growth and swell de-
 950 cay during the Joint North Sea Wave Project. *Deut. Hydrogr. Z.*, *8*(12), 1–95.

- 951 (Suppl. A)
- 952 Hauser, D., Tison, C., Amiot, T., Delaye, L., Corcoral, N., & Castillan, P. (2017).
 953 SWIM: The first spaceborne wave scatterometer. *IEEE Trans. on Geosci. and*
 954 *Remote Sensing*, *55*(5), 3000–3014.
- 955 Hauser, D., Tourain, C., Hermozo, L., Alraddawi, D., Aouf, L., Chapron, B., . . .
 956 Tran, N. (2021). New observations from the SWIM radar on-board CFOSAT:
 957 Instrument validation and ocean wave measurement assessment. *IEEE Trans.*
 958 *on Geosci. and Remote Sensing*, *59*(1), 5–26. doi: 10.1109/tgrs.2020.2994372
- 959 Ifremer / CERSAT. (2022). *Global Ocean Directional Wave Parameters Level 2S*
 960 *from SWIM onboard CFOSAT for IWWOC project, version 1.0* [Dataset]. Ifre-
 961 mer, Plouzane, France. doi: 10.12770/12cfed8d-7645-442b-b8ef-a8d08decbaed
- 962 Jackson, F. C., Walton, W. T., & Peng, C. Y. (1985). A comparison of in situ and
 963 airborne radar observations of ocean wave directionality. *J. Geophys. Res.*,
 964 *90*(C1), 1005–1018.
- 965 Kudryavtsev, V., Yurovskaya, M., Chapron, B., Collard, F., & Donlon, C. (2017).
 966 Sun glitter imagery of surface waves. part 1: Directional spectrum retrieval
 967 and validation. *J. Geophys. Res.*, *122*. doi: 10.1002/2016JC012425
- 968 Lavrenov, I. V. (2001). Effect of wind wave parameter fluctuation on the nonlinear
 969 spectrum evolution. *J. Phys. Oceanogr.*, *31*, 861–873. Retrieved from [http://](http://ams.allenpress.com/archive/1520-0485/31/4/pdf/i1520-0485-31-4-861)
 970 ams.allenpress.com/archive/1520-0485/31/4/pdf/i1520-0485-31-4-861
- 971 Le Merle, E., Hauser, D., Peureux, C., Aouf, L., Schippers, P., Dufour, C., & Dal-
 972 phinet, A. (2021). Directional and frequency spread of surface ocean waves
 973 from swim measurements. *J. Geophys. Res.*, *126*(7), e2021JC017220. doi:
 974 10.1029/2021JC017220
- 975 Longuet-Higgins, M. S. (1984). New integral relations for gravity waves of finite am-
 976 plitude. *J. Fluid Mech.*, *149*, 205–215. (see also Yu and Wu, *J. Fluid Mech.*,
 977 1987)
- 978 Masson, D., & Chandler, P. (1993). Wave groups: a closer look at spectral methods.
 979 *Coastal Eng.*, *20*, 249–275.
- 980 Passaro, M., Cipollini, P., Vignudelli, S., Quartly, G. D., & Snaith, H. M. (2014).
 981 ALES: A multi-mission adaptive subwaveform retracker for coastal and open
 982 ocean altimetry. *Remote sensing of Environment*, *145*, 173–189.
- 983 Passaro, M., Hemer, M. A., Quartly, G. D., Schwatke, C., Dettmering, D., & Seitz,

- 984 F. (2021). Global coastal attenuation of wind-waves observed with radar al-
 985 timetry. *Nature communications*, *12*, 3812. doi: 10.1038/s41467-021-23982-4
- 986 Peral, E., Rodriguez, E., & Esteban-Fernandez, D. (2015). Impact of surface waves
 987 on SWOT's projected ocean accuracy. *Remote Sensing*, *7*(11), 14509–14529.
 988 doi: 10.3390/rs71114509
- 989 Quartly, G. D., Smith, W. H. F., & Passaro, M. (2019). Removing intra-1-Hz co-
 990 variant error to improve altimetric profiles of σ_0 and sea surface height. *IEEE*
 991 *Trans. on Geosci. and Remote Sensing*, *57*(6), 3741–3752. doi: 10.1109/TGRS
 992 .2018.2886998
- 993 Quartly, G. D., Srokosz, M. A., & McMillan, A. C. (2001). Analyzing altimeter arti-
 994 facts: Statistical properties of ocean waveforms. *J. Atmos. Ocean Technol.*, *18*,
 995 2074–2091. doi: 10.1175/1520-0426(2001)018<2074:AAASPO>2.0.CO;2
- 996 Quilfen, Y., & Chapron, B. (2019). Ocean surface wave-current signatures from
 997 satellite altimeter measurements. *Geophys. Res. Lett.*, *216*, 253–261. doi: 10
 998 .1029/2018GL081029
- 999 Ray, C., Martin-Puig, C., Clarizia, M. P., Ruffini, G., Dinardo, S., Gommengin-
 1000 ger, C., & Benveniste, J. (2015). SAR altimeter backscattered waveform
 1001 model. *IEEE Trans. on Geosci. and Remote Sensing*, *53*, 911–919. doi:
 1002 10.1109/TGRS.2014.2330423
- 1003 Rice, S. (1944). Mathematical analysis of random noise. In N. Wax (Ed.), *Noise and*
 1004 *stochastic processes* (pp. 133–294). New York: Dover Publications Inc. (pub-
 1005 lished 1954).
- 1006 Rodriguez, E. (1988). Altimetry for non-gaussian oceans: Height biases and es-
 1007 timation of parameters. *J. Geophys. Res.*, *93*, 14107–14120. doi: 10.1029/
 1008 JC093iC11p14107
- 1009 Sandwell, D. T., & Smith, W. H. F. (2005). Retracking ers-1 altimeter waveforms
 1010 for optimal gravity field recovery. *Geophys. J. Int.*, *163*, 79–89. doi: 10.1111/
 1011 j.1365-246X.2005.02724.x
- 1012 Saulnier, J.-B., Clément, A., de O. Falcão, A. F., Pontes, T., Prevosto, M., & Ricci,
 1013 P. (2011). Wave groupiness and spectral bandwidth as relevant parameters
 1014 for the performance assessment of wave energy converters. *Ocean Eng.*, *38*(1),
 1015 130–147.
- 1016 Srokosz, M. A. (1986). On the joint distribution of surface elevation and slopes for

- 1017 a non linear random sea, with an application to radar altimetry. *J. Geophys.*
1018 *Res.*, *91*, 995–1006.
- 1019 Tayfun, A., & Lo, J.-M. (1989). Wave envelope and related spectra. *J. of Waterway,*
1020 *Port Coast. Ocean Eng.*, *115*, 515–533. doi: 10.1061/(ASCE)0733-950X(1989)
1021 115:4(515)
- 1022 Timmermans, B. W., Gommenginger, C. P., Dodet, G., & Bidlot, J.-R. (2020).
1023 Global wave height trends and variability from new multimission satellite
1024 altimeter products, reanalyses, and wave buoys. *Geophys. Res. Lett.*, *47*,
1025 e2019GL086880. doi: 10.1029/2019GL086880
- 1026 Toba, Y. (1973). Local balance in the air-sea boundary processes. III on the spec-
1027 trum of wind waves. *J. Oceanogr. Soc. Japan*, *29*, 209–220. Retrieved from
1028 <http://www.terrapub.co.jp/journals/JO/JOSJ/pdf/2905/29050209.pdf>
- 1029 Tourain, C., Piras, F., Ollivier, A., Hauser, D., Poisson, J. C., Boy, F., ... Ti-
1030 son, C. (2021). Benefits of the adaptive algorithm for retracking altime-
1031 ter nadir echoes: Results from simulations and CFOSAT/SWIM observa-
1032 tions. *IEEE Trans. on Geosci. and Remote Sensing*, *59*, 9927–9940. doi:
1033 10.1109/TGRS.2021.3064236
- 1034 Tournadre, J. (1993). Time and space scales of significant wave heights. *J. Geophys.*
1035 *Res.*, *98*(C3), 4727–4738.
- 1036 Villas Bôas, A. B., Lenain, L., Cornuelle, B. D., Gille, S. T., & Mazloff, M. R.
1037 (2022). A broadband view of the sea surface height wavenumber spectrum.
1038 *Geophys. Res. Lett.*, *49*(e2021GL096699). doi: 10.1029/2021GL096699