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# Meeting the biocalculus challenges: a reflection on didactic transposition processes in a cross-disciplinary context

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## Introduction

Two special issues have recently appeared in *Primus* on the theme of “Mathematics and the Life Sciences” (Robeva et al., 2022). Despite various institutional injunctions since the early 2000s in the USA, the desired reform of a greater integration of mathematics and biology in the undergraduate curricula seems slow to materialize. In this context, the special issue presents approaches undertaken by different academic communities to meet the challenges of biocalculus.

Understanding how the contents of university mathematics education (UME) came to be what they currently are, as well as how they evolve (or are submitted to inertia) under various sorts of institutional conditions and constraints is an endeavor undertaken by the Anthropological Theory of the Didactic (ATD; Chevallard & Bosch, 2020) under the name *didactic transposition* (Bosch et al., 2021). In this paper, we present a reflexive analysis of transposition processes that we participated in when the first author (a mathematician and UME researcher) was given the responsibility of teaching and orchestrating a biocalculus course for more than 700 first-year biology students tutored by 10 mathematics instructors. The course material was developed on the basis of the official syllabus and materials provided by the second author, an evolutionary anthropologist familiar with quantitative approaches in biology. In an overall context of stability of mathematics curricula (Bosch et al., 2021), this reform of the mathematics course for biology students has been experienced as a “revolution” by mathematics teachers, a paradigm shift.

Didactic transposition processes will be modeled and discussed using ATD theoretical constructs in order to provide answers to the following research questions: What set of conditions and constraints allowed the emergence of a biocalculus course at the University of Montpellier (UM, France)? What are the characteristics of the transposition phenomena that took place, as compared to a more classical calculus course (such as the one that preceded it in Montpellier)?

## Theoretical framework

In the ATD model, 3 types of institutions intervene in didactic transposition processes: the *scholarly institutions* of knowledge producers, the *school institutions* (universities), and in between the *noosphere* (curriculum designers, policymakers, ...) whose agents elaborate the *knowledge to be taught* from the scholarly knowledge. This first step, called *external didactic transposition* (EDT), produces the curricula and syllabi. The subsequent step towards the *actually taught knowledge* is called *internal didactic transposition* (IDT), which is achieved by the course teachers. The two processes are intertwined since a teacher can take a noospherian position (as a member of a program committee) to update the syllabus of the course he or she is responsible for.

Let us note  $I_{BT}$  and  $I_{MT}$  the disciplinary school institutions corresponding to biology and mathematics teaching, and  $I_B$  and  $I_M$  the scholarly institutions, respectively. Classical EDT

processes, e.g. calculus transposition by the mathematics noosphere, rely on series of *textualizations of knowledge* that generate a whole posterity of textbooks, rather than drawing on scholarly sources. This immediately brings out a fundamental difference with biocalculus, which has a much more recent and reduced textualization (e.g. Schreiber et al., 2014) produced by members of the scholarly interface between mathematics and biology ( $I_{BM}$  for biomathematics) or originating from the collaboration between mathematicians and biologists.

In ATD, pieces of knowledge are modeled in terms of *praxeologies* (Chevallard & Bosch, 2020). Although it remains in the hands of  $I_{MT}$ , biocalculus teaching is inevitably influenced by  $I_{BT}$  and finds its epistemological investiture in the scholarly praxeologies developed by  $I_{BM}$ . It is no coincidence that the biocalculus assessment instrument (Taylor et al., 2020) that aims to assess biocalculus comprehension in various modalities of integration of biological contexts in calculus teaching was developed by mathematics education researchers in collaboration with  $I_{BM}$  members. ATD aims at elucidating the sources of biocalculus praxeologies, in particular how standard calculus praxeologies are modified when applied to or developed within biological contexts.

### **External didactic transposition**

Our main data for analyzing the biocalculus EDT at the UM are the documents produced by the noospherian institutions. Strikingly, the process was initiated by the biology noosphere which launched a working group coordinated by a quantitative geneticist and an epidemiologist (thus  $I_{BM}$  members) with the mission to elaborate the curriculum and content of the mathematics, statistics and computer science courses of the entire biology degree in a coherent and interdisciplinary manner. The first-year first-semester mandatory biocalculus course was entitled “computational methods”, with essential content “elementary algebra and analysis”, considered as pertaining to both mathematics and informatics (due to the importance of data manipulation in biology, requiring the use of computer software) as disciplines, and entrusted to  $I_{MT}$  for its teaching. Mainly based on economic criteria, 12 h of lectures and 21 h of tutorials in groups of 40 students has been allocated.

The working group observed that biology students had a great weakness in mastering the basics of mathematics (going back to lower high school) and for some a sort of phobia of mathematics, which impacted, for example, on their ability to calculate dilutions and concentrations. Moreover, the recent reform of the French upper high school was going to reinforce the heterogeneity of the students’ profiles. In other words, the biology noosphere was well aware of 2 crucial issues identified by UME research: the secondary/tertiary transition and the isolation of mathematics courses in non-mathematics majors. In such a context, minimal requirements expected from biology students to be successful in biology studies were identified (e.g. “reading an exponential or logistic curve”, “understanding of the notion of derivative”,...) and the teaching method was proposed to “start as much as possible with biological problems, then move on to formulas and calculations without demonstrations”. As a final document, a complete syllabus has been written, organized in two sectors (“elementary algebra” and “real univariate analysis”) declined under different competences (e.g. “know how to study a function”), which were associated with contents (“domain of a function, variations,...”), computational aspects (“code the Newton-Raphson algorithm to approximate a root to a given precision”) and biological applications (“optimum tolerance curves, enzymatic activity”).

This syllabus has been extensively revised by the noospherian members of  $I_{MT}$ , first the colleague in charge of the former calculus module for biology students, then the head of the program committee. The division into two sectors has been preserved (the first one renamed “basic mathematical techniques”), but the competencies have disappeared in favor of themes and topics, bringing back the classic headings (limits and continuity, intermediate value theorem, etc.). Some of the biological contextualization has been retained, under the heading “example illustrations” which reflects well the function of the latter in the mind of the noospherian mathematicians. The coherence of a pyramidal construction of calculus concepts has determined the organization of the syllabus while the examples that go beyond the framework of a shared scientific culture have been eliminated.

### **Internal didactic transposition**

Although the mathematical noosphere has pulled in the direction of standardizing the syllabus to  $I_{MT}$  standards to allow for teaching by its members, the challenge of authentically bringing the biological contextualization to life to meet the new philosophy driven by the biology noosphere remains for those in charge of the IDT. The institutional issue, in the background, is the conservation of the teaching volume under the responsibility of  $I_{MT}$ . This motivated the first author (Thomas), a mathematician and didactician, to take over the teaching of biocalculus and to collaborate with the second author (Bernard), a biologist who had already taught a mathematical remediation module for first-year biology students, jointly with a physicist and a mathematician.

The documents and sources (e.g. Milo & Philipps, 2015) passed by Bernard to Thomas do not come from the existing biocalculus textbooks (not known to Bernard, perhaps because these TDE products are still too young to spread beyond the Anglo-Saxon world) but from quantitative biology textbooks and Bernard’s own scholarly biomathematical practice, transposed into his biology teaching as well as into the remediation module. Thomas also looked up primary sources (e.g. Bigelow, 1921) to collect data and their representations by biologists in order to build authentic biological contexts bearing the rationale of calculus concepts (on this example, the scope of an exponential decay in connection with applications in predictive microbiology).

In this endeavor, informed by ATD constructs, new praxeologies specific to biomathematics have emerged. For example, model fitting praxeologies (exponential and allometric in particular), using instrumented techniques (spreadsheet and regression), occupy an important place. They require a good mastery of the properties of logarithms and the articulation between graphical and algebraic registers, in order to correctly interpret standard representations in biocalculus: on logarithmic scales, the biologist notes the measured values  $x$  where the mathematician would note the logarithm  $X$  of these values, with regular graduations. This results in the need to teach a “yoga” of  $x$ - $X$  conversion, so that students may compute the slope properly or determine a law like Bigelow’s from the graphical representation of a fit of the data. Similarly, a proportionality relation of coefficient  $k$  between two quantities that admit a wide spectrum of variation is identified after passage in log-log coordinates by a linear regression line of slope 1 and y-intercept  $\log k$ . For a learner, this praxeology confuses the issue by interfering with the classical technique of drawing a regression line of slope  $k$  passing through the origin. Such examples illustrate a common phenomenon: although biomathematical praxeologies are built on classical calculus praxeologies, learned at the end of high school and completed by a more formal logoi in classical calculus courses, their application in authentic biological contextualizations often requires an adaptation of

known techniques together with an additional part of the technology of the praxeology, which establishes a link with the particularities of the biological context and allows its proper application to the latter. In the eyes of the ATD, then, it is not surprising that the biocalculus assessment instrument identified log-log graphs as a source of persistent student difficulty (Taylor et al., 2020).

## Conclusion and outlook

This reflective study of didactic transposition carried out at the University of Montpellier in the context of calculus for biology students, based on the theoretical constructs of ATD, underlines the essential role played by the biology noosphere, imposing the constraint of authentic biology contextualizations and the processing of data on computers (even if largely reduced to homework and its presentation in class), in the emergence of a biocalculus course that offers a better integration of calculus knowledge with biology. A favorable (even necessary) condition for the good realization of the project was the cooperation between a mathematician and a biologist whose own research is related to the field of biomathematics where the epistemological legitimacy of biocalculus lies. Without relying on the few existing Anglo-Saxon biocalculus textbooks, the interactions have converged towards classical biomathematical themes (allometry, basal metabolism, Verhulst logistic model, etc.) which are conducive to articulating mathematical work with biological stakes that meet the combined requirements of mathematics and biology teaching institutions, and which can be found in the biocalculus textbooks. This shows the existence of an attractive pole of the didactic transposition processes, which tend to converge towards a body of knowledge whose didactic efficiency is still to be shown and reinforced, due to the emergence of biomathematics specific praxeologies that need to be investigated and analyzed in relation to classical calculus praxeologies, but which bases its coherence in the scholarly practice of reference, at the interface between mathematics and biology.

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