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Stability Analysis of Lure Systems under Aperiodic Sampled-Data Control

Mathias Giordani Titton¹, João Manoel Gomes da Silva Jr.*¹, Giorgio Valmorbidia², and
Marc Jungers³

¹PPGEE / DELAE, Universidade Federal do Rio Grande do Sul, Porto Alegre-RS,
Brazil

² Laboratoire des Signaux et Systèmes, CentraleSupélec, CNRS, Université Paris-Saclay,
Projet DISCO Inria-Saclay, Gif-sur-Yvette, France

³ Université de Lorraine, CNRS, CRAN, Nancy, France

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Abstract

This paper deals with the stability analysis of Lure type systems through aperiodic sampled-data control laws, where the nonlinearity is assumed to be both sector and slope restricted. The proposed method is based on the use of a new class of looped-functionals, which depends on the nonlinearity and its slope, and on a generalized Lure type function, that is quadratic on both the states and the nonlinearity and has a Lure-Postnikov integral term. On this basis, conditions in the form of linear matrix inequalities (LMIs) to certify global or regional asymptotic stability of the closed-loop system are obtained. These conditions are then used in optimization problems for computing the maximum intersampling interval or the maximum sector bounds for which the stability of the sampled-data closed-loop system is guaranteed. Numerical examples to illustrate the results are provided.

Keywords: Sampled-data control; Lure systems; stability analysis; sector bounded nonlinearities; looped-functional approach.

1 Introduction

The study of the stability and stabilization of systems evolving in continuous-time whereas the controller delivers inputs at discrete-time instants, *i.e.*, sampled-data control, has been the subject of many works in the literature. In the book from Åström[1] (and references therein) linear sampled-data control systems updated at constant sampling periods were extensively studied. In the last years, the sampled-data control problem, in particular considering aperiodic sampling [13], has presented a renewed interest mainly from the following facts:

- The spread of networked control, where the control loops are implemented over communication networks, brought some challenges regarding the operation of the closed-loop system in the presence of communication restrictions, which induces jitters, packet dropouts and delays, among others. These restrictions can be perceived by the control loop as an aperiodic sampling policy [13]. In this case, the exact linearization commonly applied to the analysis of linear systems with periodic sampled control, which leads to a discrete-time model for the closed-loop system cannot be longer used to conclude about stability and performance.
- For nonlinear systems, the formal analysis of sampled data controlled systems is more challenging, since even considering a periodic sampling policy, exact discretization is in general not possible. Furthermore, the use of numerical approximations of the system dynamics to design sampled-data control laws in a discrete-time framework does not guarantee that the actual continuous-time nonlinear system considering a sample and hold implementation will operate properly [22].

*Corresponding author. jmgomes@ufrgs.br

To conclude about the stability of nonlinear systems or even linear systems under aperiodic sampled-data control, the hybrid behavior of the closed-loop system has to be taken into account, that is, the continuous evolution of the state trajectories during the intersampling time and the impulsive update of the control signal at sampling instants. In this sense, different methods have been proposed in the literature to model sampled-data systems, such as: the lifting and discrete-time approaches [2, 8]; the impulsive modeling, in which a time-varying Lyapunov function is used [14, 21]; the modeling with a continuous-time system with time-varying delay on the plant control input [7, 6]; and the looped-functional approach [30, 31] that focuses on the behavior in the intersampling interval, ensuring that a positive definite function is strictly decreasing at the sampling instants. For a general overview of these approaches and others, the reader can refer to the survey by Hetel et al. [13].

Lure type systems are a particular class of nonlinear systems composed by the feedback connection of a linear time-invariant system with a sector-bounded nonlinearity. As many physical systems can be represented in this setup, the stability analysis of Lure systems has been attracting the attention of the control community over the years. The first studies by Lure-Postnikov[19] regarding the absolute stability problem were based on time-domain representation and considered only sector-bounded nonlinearities. Then, stability criteria based on the properties of the frequency response of the linear part of the system, leading to the conditions known as the circle and the Popov criteria [17], where different assumptions regarding the time-variation of the the sector-bounded nonlinearity are made, have been proposed. More recently, motivated by semi-definite programming advances, several Lyapunov and passivity based methods have been proposed to assess the stability and to design stabilizing control laws for Lure type systems in continuous [5, 35, 4, 37] and discrete-time [10, 36]. We can also mention works focusing on event-triggered control [40, 20], synchronization of chaotic systems [24] and network delays[12, 39]. In these contexts, different classes of Lyapunov Functions (LFs), *e.g.* quadratic ones [5] (associated to the Circle criterion), Lure-Postnikov LFs (associated to the Popov criterion) [38, 32, 25, 35, 37] and composite LFs [15] have been considered. To handle delays, Lyapunov-Krasovskii functionals are in general considered[12, 39, 11].

However, regarding Lure systems under sampled-data control, only few studies were carried out. The problem is addressed by Seifullaev and Fradkov [28, 27] considering the input delay approach proposed by Fridman [7, 6], which is based on Lyapunov-Krasovskii functionals. The functionals used in these references are required to be positive definite and do not depend on the nonlinearity. The design of stabilizing sampled data control laws, considering periodic sampling, is tackled by Gabriel and Geromel [9] through differential linear matrix inequalities obtained from a classical Lure type Lyapunov function, where a piecewise affine time-dependence is considered in the quadratic part. In this case, to obtain conditions in LMI form a suitable partition of the intersampling interval needs to be chosen. The sampled-data synchronization of Lure type systems is treated in Park et al. [24, 16] considering a fragmentation approach. Conditions to assess the stability of closed-loop systems when the sampled-data control law is designed from the Euler approximated discrete-time Lure system are provided in Louis et al.[18]. It should be pointed out that all the above papers are concerned only by the global stability of the origin. The case where the nonlinearity verifies sector conditions only locally is not considered.

This paper focuses on the stability analysis of aperiodic sampled-data closed-loop Lure systems with sector bounded and slope restricted nonlinearities.

The main contributions of the present paper can be summarized as follows:

1. A looped functional approach is considered to provide conditions to assess the stability of Lure type systems under sampled-data control. This approach is based on the use of two elements: a function V and a functional \mathcal{W}_0 . Differently from the linear case, treated in [30], and similar approaches considering Lyapunov-Krasovskii functionals [6, 27], in our case we consider that V and \mathcal{W}_0 depend also on the sector nonlinearity. In particular, the function V is considered as a generalized Lure-type function [37] composed by two terms: a generic quadratic term depending on the state and the nonlinearity; and an integral term. Differently from classical Lure-type functions [17], the quadratic part does not need to be defined by a positive definite matrix and the multipliers of the integral terms do not need to be nonnegative. Moreover, we also show that the functional \mathcal{W}_0 does not need to be positive definite for the assessment of global stability, which is an extra relaxation with respect to the previous results in [27] considering Lyapunov-Krasovskii functionals. All these new elements and relaxations lead to a significant conservatism reduction, as shown in the numerical examples.
2. Both the global and local stability cases are tackled. To the best of our knowledge, this is the first time that the local stability analysis of Lure type systems under sampled-data control is formally addressed. This is particularly relevant when the nonlinearity verifies the sector conditions only locally, which allows to provide stability certificates for a broad class of nonlinear systems by casting them in a Lure setup. Regarding this case, we also provide conditions that allow to characterize estimates of the region of attraction of the origin of the closed-loop systems as the level sets of the function V .

Furthermore, the proposed stability conditions are in the form of linear matrix inequalities (LMIs). We show therefore how to incorporate these conditions as constraints of optimization problems aiming at providing bounds on the maximal intersampling interval or on the nonlinearity sector, for which the stability of the Lure-type system under aperiodic sampled-data control can be certified.

It should be noticed that some preliminary results in this sense were presented in [33], considering only global results and without an in-depth discussion and a detailed mathematical development of the theoretical results.

This paper is organized as follows. The next section describes the problem formulation. Section 3 presents the looped-functional approach in a general way. In Section 4, considering a generalized Lure-Postnikov function and a looped-functional that depends on the nonlinearities, the conditions derived in Section 3 are cast as LMIs. Section 5 features some optimization problems and Section 6 gives numerical examples to highlight the potentialities of the method. Section 7 summarizes the main points covered in the paper and discuss some possible extensions.

Notation. The set of diagonal and symmetric matrices of dimension n are denoted, respectively, by \mathbb{D}^n and \mathbb{S}^n . $\mathbb{D}_{\succeq 0}^n$ ($\mathbb{S}_{\succeq 0}^n$) means that the matrix is diagonal (symmetric) and positive semi-definite. The set of positive real numbers is denoted by \mathbb{R}^+ . To express diagonal matrices in a compact form, we write ‘diag(e_1, \dots, e_n)’, where e_i , $i = 1, \dots, n$ are, respectively, the elements in position (i, i) . A^T denotes the transpose of matrix A . $\text{He}(A) \triangleq A + A^T$, I_m and 0_m are, respectively, the identity and zero square matrices of order m . For a given positive scalar \mathcal{J} , define $\mathcal{C}_{[0, \mathcal{J}]}$ the set of continuous functions from an interval $[0, \mathcal{J}]$ to \mathbb{R}^n and the union set of continuous functions with support in a certain range defined as $\mathbb{K}_{[\mathcal{T}_1, \mathcal{T}_2]}^n = \cup_{\mathcal{T} \in [\mathcal{T}_1, \mathcal{T}_2]} \{\mathcal{C}_{[0, \mathcal{T}]}\}$. The notations $|\cdot|$ and $\|\cdot\|$ represent the absolute value of a scalar and the Euclidean norm, respectively. The i th element of a vector v is denoted by v_i . $P \succ 0$ ($P \succeq 0$) means that P is positive (semi-)definite and $P \prec 0$ ($P \preceq 0$) means that P is negative (semi-)definite. Depending on the context, we use ∂ to denote the sub-differential operator or the boundary of a set. The symmetric terms in a symmetric matrix are denoted by \star .

2 Problem Statement

2.1 The Lure System

Consider the continuous-time plant described by the following Lure system [17, 5, 37]:

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_\phi \phi(y(t)) \quad (1a)$$

$$y(t) = Cx(t) + D_\phi \phi(y(t)), \quad (1b)$$

where $x \in \mathbb{R}^n$ represents the states of the plant, $u \in \mathbb{R}^q$ represents its inputs and $y \in \mathbb{R}^m$ its outputs (*i.e.*, the argument of the nonlinearity ϕ), $A \in \mathbb{R}^{n \times n}$, $B_u \in \mathbb{R}^{n \times q}$, $B_\phi \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{m \times n}$ and $D_\phi \in \mathbb{R}^{m \times m}$. The vector-valued nonlinearity $\phi : \mathbb{R}^m \rightarrow \mathbb{R}^m$, is assumed to be time-invariant, memoryless, Lipschitz on \mathbb{R}^m , decentralized, sector bounded and slope restricted, *i.e.*, $\phi(y) = [\phi_1(y_1), \dots, \phi_m(y_m)]^T$, with $\phi_i(y_i)$ satisfying, $\forall i = 1, \dots, m$:

$$\phi_i(0) = 0, \quad (2a)$$

$$\frac{\phi_i(y_i)}{y_i} \in [\underline{\delta}_i, \bar{\delta}_i] \quad \forall y \in \mathcal{Y}_0 \subseteq \mathbb{R}^m, \quad (2b)$$

$$\partial \phi_i(y_i) \in [\underline{\gamma}_i, \bar{\gamma}_i], \quad \forall y \in \mathcal{Y}_0 \subseteq \mathbb{R}^m, \quad (2c)$$

where $\underline{\delta}_i \in \mathbb{R}$, $\bar{\delta}_i \in \mathbb{R}$, $\underline{\delta}_i \leq \bar{\delta}_i$ are, respectively, the lower and upper sector bounds for the i th nonlinearity and $\underline{\gamma}_i \in \mathbb{R}$, $\bar{\gamma}_i \in \mathbb{R}$, $\underline{\gamma}_i \leq \bar{\gamma}_i$ are, respectively, the lower and upper slope bounds for the i th nonlinearity. From these bounds, denote $\underline{\Delta} \triangleq \text{diag}(\underline{\delta}_1, \dots, \underline{\delta}_m)$, $\bar{\Delta} \triangleq \text{diag}(\bar{\delta}_1, \dots, \bar{\delta}_m)$, $\underline{\Gamma} \triangleq \text{diag}(\underline{\gamma}_1, \dots, \underline{\gamma}_m)$ and $\bar{\Gamma} \triangleq \text{diag}(\bar{\gamma}_1, \dots, \bar{\gamma}_m)$. If $\mathcal{Y}_0 = \mathbb{R}^m$, the relations are said to be verified *globally* and otherwise *locally*. The Lipschitz assumption on ϕ implies that $\partial \phi_i(y_i) = \frac{d\phi_i}{dy_i}$ almost everywhere, relaxing the requirement for the nonlinearity to be continuously differentiable.

Taking into account that $\dot{\phi}_i(y_i) = \frac{d\phi_i(y_i)}{dt} = \partial \phi_i(y_i) \dot{y}_i$ and relations (2), the following Lemmas can be stated.

Lemma 1. [17] If $U_1 \in \mathbb{D}_{\succeq 0}^m$ and $\phi : \mathbb{R}^m \rightarrow \mathbb{R}^m$ satisfies (2), then

$$S_\Delta(U_1, \phi(y), y) \triangleq (\phi(y) - \underline{\Delta}y)^T U_1 (\bar{\Delta}y - \phi(y)) \geq 0, \quad (3)$$

for all $y \in \mathcal{Y}_0$.

Lemma 2. [37] If $U_2 \in \mathbb{D}_{\succeq 0}^m$ and $\phi : \mathbb{R}^m \rightarrow \mathbb{R}^m$ satisfies (2), then

$$S_\Gamma(U_2, \dot{\phi}(y), \dot{y}) \triangleq (\dot{\phi}(y) - \underline{\Gamma}\dot{y})^T U_2 (\bar{\Gamma}\dot{y} - \dot{\phi}(y)) \geq 0, \quad (4)$$

almost everywhere for $y \in \mathcal{Y}_0$.

It should be noted that if $D_\phi \neq 0$ equation (1b) is an implicit one.

In this case, the well-posedness of the algebraic loop (1b) is guaranteed if there exists a unique solution to the implicit equation $F(y) \triangleq y - D_\phi \phi(y) = Cx$, that is, a mapping $y(x)$ satisfying $F(y(x)) = Cx$. Applying the reasoning used in Valmorbidia and Drummond[37], a condition for the well-posedness of (1b) considering ϕ satisfying (2) is given by the following proposition.

Proposition 1. [37] *If there exists a matrix $N \in \mathbb{D}_{\geq 0}^m$ such that*

$$2N - \text{He}\{N(I_m - D_\phi \Gamma)^{-1} D_\phi (\bar{\Gamma} - \Gamma)\} \succ 0, \quad (5)$$

then matrix $(I_m - D_\phi \Gamma)$ is nonsingular for all matrices $\Gamma \in \Delta_\Gamma \triangleq \{\Gamma \in \mathbb{D}^m ; \Gamma = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_m), \gamma_i \in [\underline{\delta}_i, \bar{\delta}_i], i = 1, \dots, m\}$ and the algebraic-loop (1b) is well-posed.

Henceforth, we consider the following Assumption.

Assumption 1 (Well-posedness). *There exists a matrix $N \in \mathbb{D}_{\geq 0}^m$ such that (5) holds.*

Note that when $D_\phi = 0$ this assumption is trivially verified.

Provided that Assumption 1 is satisfied, the following set can be defined

$$\mathcal{X}_0 \triangleq \{x \in \mathbb{R}^n ; y \in \mathcal{Y}_0, y - D_\phi \phi(y) = Cx\}. \quad (6)$$

The set \mathcal{X}_0 corresponds to all the states such that $y = Cx + D_\phi \phi(y) \in \mathcal{Y}_0$. When the sector conditions and the slope restrictions are satisfied globally, i.e. $\mathcal{Y}_0 = \mathbb{R}^m$, we have that \mathcal{X}_0 is equal to \mathbb{R}^n .

2.2 The sampled-data control law

We suppose that the control signal u is updated at sampling instants denoted t_k and remains constant between two successive sampling instants through a zero-order holder (ZOH). Namely, we consider the following sampled-data control law:

$$u(t) = K_x x(t_k) + K_\phi \phi(y(t_k)), \quad \forall t \in [t_k, t_{k+1}), \quad (7)$$

where $K_x \in \mathbb{R}^{q \times n}$, $K_\phi \in \mathbb{R}^{q \times m}$. This generic control law allows to use information about the nonlinearity [5]. For saturating systems, a method for the design of such a control law using piecewise quadratic functions is presented in Queinnec et al. [26]. If the nonlinearity ϕ is not precisely known or $\phi(y)$ cannot be directly measured, it suffices to consider $K_\phi = 0$, i.e. a linear state feedback control law. We consider the case of aperiodic sampling. In this context, we assume that there exist two positive scalars $\mathcal{T}_1 \leq \mathcal{T}_2$ such that the difference between two successive sampling instants $T_k = t_{k+1} - t_k$ satisfies

$$0 < \mathcal{T}_1 \leq T_k \leq \mathcal{T}_2, \quad \forall k \in \mathbb{N}. \quad (8)$$

Thus, $\{t_k\}_{k \in \mathbb{N}}$ is an increasing sequence of positive scalars such that $\bigcup_{k \in \mathbb{N}} [t_k, t_{k+1}) = [0, +\infty)$. Note that the periodic sampling appears as a particular case in which T_k has the same value $\forall k \in \mathbb{N}$, i.e., $T_k = \mathcal{T}_1 = \mathcal{T}_2, \forall k$.

In this paper, we are interested in deriving conditions that allow to assess the asymptotic stability of the origin of system (1), where the nonlinearity ϕ satisfies the sector and slope restrictions given in (2), with a sampled-data control law given by (7) assuming an aperiodic sampling policy satisfying (8). With this aim, we consider a looped-functional approach, which is detailed in the next section.

3 Looped-Functional Approach

The looped-functional approach has originally been introduced by Seuret [30] to study linear systems under sampled-data control. It can be seen as a generalization of Lyapunov-Krasovskii approaches [6], in the sense that the functional does not need to be positive definite. In this section we extend these ideas to cope with Lure type systems.

As in the linear systems case, we formally show that for global stability results the positivity of the functional is not needed. On the other hand, for deriving local results some further assumptions are needed to ensure that the trajectories are bounded in the region where the sector conditions are valid.

To state the results, we consider the notation [30] $x_k(\tau) \triangleq x(t_k + \tau)$, $y_k(\tau) \triangleq y(t_k + \tau)$ and $\phi_k(\tau) \triangleq \phi(y_k(\tau))$. Hence, the closed-loop dynamics in the interval $[t_k, t_{k+1})$ is governed by the following equations:

$$\begin{cases} \dot{x}_k(\tau) = Ax_k(\tau) + B_\phi \phi_k(\tau) + B_u (K_x x_k(0) + K_\phi \phi_k(0)) \\ y_k(\tau) = Cx_k(\tau) + D_\phi \phi_k(\tau), \end{cases} \quad \forall \tau \in [0, T_k), \quad (9)$$

where $\dot{x}_k(\tau) = \frac{d}{d\tau}x_k(\tau)$.

Let $\mathcal{D}_x \subseteq \mathcal{X}_0 \subseteq \mathbb{R}^n$ be a compact domain containing the origin in its interior. In what follows we consider a differentiable function $V : \mathcal{D}_x \rightarrow \mathbb{R}^+$ satisfying

$$\mu_1 \|x\|^p \leq V(x) \leq \mu_2 \|x\|^p, \quad \forall x \in \mathcal{D}_x, \text{ with } \mu_1, \mu_2 > 0, p > 1. \quad (10)$$

Associated to $V(x)$, define the level sets

$$\mathcal{L}(V, \rho) = \{x \in \mathbb{R}^n | V(x) \leq \rho\}, \quad (11)$$

with $\rho > 0$.

In addition, consider a differentiable functional with respect to τ , called a *looped functional* [30], $\mathcal{W}_0 : [0, \mathcal{T}_2] \times \mathbb{K}_{[\mathcal{T}_1, \mathcal{T}_2]}^n \times [\mathcal{T}_1, \mathcal{T}_2] \rightarrow \mathbb{R}$ that satisfies:

$$\mathcal{W}_0(T_k, z, T_k) = \mathcal{W}_0(0, z, T_k), \quad \forall z \in \mathbb{K}_{[\mathcal{T}_1, \mathcal{T}_2]}^n \text{ and } \forall T_k \in [\mathcal{T}_1, \mathcal{T}_2]. \quad (12)$$

3.1 Global Stability

Assume that the nonlinearity ϕ satisfies (2b) and (2c) globally, that is, they are verified $\forall y \in \mathcal{Y}_0 = \mathbb{R}^m$, or equivalently, $\mathcal{X}_0 = \mathbb{R}^n$. In this case, the next theorem provides conditions to ensure the global asymptotic stability of the origin of the closed-loop system composed by (1) and (7) with $T_k \in [\mathcal{T}_1, \mathcal{T}_2]$.

Theorem 1. *Consider a differentiable function V and a differentiable looped-functional \mathcal{W}_0 satisfying (10) and (12), respectively, with $\mathcal{D}_x = \mathbb{R}^n$. If for all $k \in \mathbb{N}$, $T_k \in [\mathcal{T}_1, \mathcal{T}_2]$ and $\tau \in [0, T_k]$ it follows that*

$$\frac{d}{d\tau}[V(x_k(\tau)) + \mathcal{W}_0(\tau, x_k, T_k)] < 0 \quad (13)$$

along the trajectories of (9), then the origin of the closed-loop system composed by (1) and the sampled-data control law (7), with $T_k \in [\mathcal{T}_1, \mathcal{T}_2]$, is globally asymptotically stable (GAS), i.e. $x(t) \rightarrow 0$ as $t \rightarrow \infty$, $\forall x(0) \in \mathbb{R}^n$.

Proof. First, by continuity of the system trajectories, note that $x_k(T_k) = x_{k+1}(0)$. Integrating now (13) in the interval $[0, T_k]$ and using (12), it follows that

$$\begin{aligned} \Delta V(k) &= V(x_{k+1}(0)) - V(x_k(0)) \\ &= V(x(t_{k+1})) - V(x(t_k)) < 0, \quad \forall k \in \mathbb{N}. \end{aligned}$$

Thus, we can conclude that the sequence of samples $x_k(0) = x(t_k) \rightarrow 0$ as $k \rightarrow \infty$.

To conclude the proof, we need to show that the $x(t)$ is uniformly bounded $\forall t \in [t_k, t_{k+1})$, $\forall k$, and that the continuous-time trajectories also converge to the origin, i.e., $x(t) \rightarrow 0$ as $t \rightarrow \infty$. For this, note that as $\frac{\phi_i(y_i)}{y_i} \in [\underline{\delta}_i, \bar{\delta}_i]$, it follows that, $\forall t \geq 0$, there exist scalars $\alpha_i(t)$, $0 \leq \alpha_i(t) \leq 1$, $\forall i = 1, \dots, m$, such that

$$\phi_i(y_i(t)) = [\alpha_i(t)\underline{\delta}_i + (1 - \alpha_i(t))\bar{\delta}_i]y_i(t) = v_i(t)y_i(t), \quad \text{with } v_i(t) \in [\underline{\delta}_i, \bar{\delta}_i]. \quad (14)$$

Thus, we can write $\phi(y(t)) = \Upsilon(v(t))y(t)$, with $\Upsilon(v(t)) = \text{diag}(v_1(t), v_2(t), \dots, v_m(t))$, $v(t) \in \bar{\mathcal{U}}_v = \{v \in \mathbb{R}^m | \underline{\delta}_i \leq v_i \leq \bar{\delta}_i, i = 1, \dots, m\}$. In this case, we have that $y(t) = Cx(t) + D_\phi \Upsilon(v(t))y(t)$, and from Assumption 1 it follows that

$$y(t) = (I - D_\phi \Upsilon(v(t)))^{-1}Cx(t). \quad (15)$$

Hence, the closed-loop system can be represented by the following linear time-varying system, $\forall t \in [t_k, t_{k+1})$:

$$\dot{x}(t) = Ax(t) + B_u(K_x x(t_k) + K_\phi \phi(y(t_k))) + B_\phi \Upsilon(v(t))(I - D_\phi \Upsilon(v(t)))^{-1}Cx(t). \quad (16)$$

Equivalently, considering the lifted-variables one has

$$\dot{x}_k(\tau) = (A + B_\phi \Upsilon(v_k(\tau))(I - D_\phi \Upsilon(v_k(\tau)))^{-1}C)x_k(\tau) + B_u(K_x x_k(0) + K_\phi \phi_k(0)), \quad (17)$$

where $v_k(\tau) \triangleq v(t_k + \tau)$. For each admissible $v_k \in \mathbb{K}_{[\mathcal{T}_1, \mathcal{T}_2]}^n$, such that $v_k(\tau) \in \bar{\mathcal{U}}_v$, define the transition matrix for the system (16) without the constant term in B_u , as $\Psi_{v_k}(\tau, s) \triangleq \Psi_v(t_k + \tau, t_k + s)$. Thus, it follows that

$$x_k(\tau) = \Psi_{v_k}(\tau, 0)x_k(0) + \int_0^\tau \Psi_{v_k}(\tau, s)B_u \begin{bmatrix} K_x & K_\phi \end{bmatrix} \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix} ds. \quad (18)$$

Thus, we obtain the following upper bound for the trajectories:

$$\begin{aligned}
\|x_k(\tau)\| &\leq \|\Psi_{v_k}(\tau, 0)\| \|x_k(0)\| + \int_0^\tau \left\| \Psi_{v_k}(\tau, s) \right\| ds \left\| B_u \begin{bmatrix} K_x & K_\phi \end{bmatrix} \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix} \right\| \\
&\leq \left(\|\Psi_{v_k}(\tau, 0)\| + \int_0^\tau \left\| \Psi_{v_k}(\tau, s) \right\| ds (\|B_u K_x\| + \|B_u K_\phi\| \|\Upsilon(v_k(0))(I - D_\phi \Upsilon(v_k(0)))^{-1} C\|) \right) \|x_k(0)\| \\
&\leq \left(\|\Psi_{v_k}(\tau, 0)\| + \int_0^{\mathcal{T}_2} \left\| \Psi_{v_k}(\tau, s) \right\| ds (\|B_u K_x\| + \|B_u K_\phi\| \|\Upsilon(v_k(0))(I - D_\phi \Upsilon(v_k(0)))^{-1} C\|) \right) \|x_k(0)\|.
\end{aligned} \tag{19}$$

Then, since $v_k(\tau) \in \mathcal{U}_v, \forall \tau \in [0, \mathcal{T}_2]$, there exists a scalar μ_Ψ ,

$$\mu_\Psi = \sup_{\substack{v_k \in \mathcal{C}_{[0, \mathcal{T}_2]}^n \\ \text{s.t. } v_k(\tau) \in \mathcal{U}_v}} \left(\|\Psi_{v_k}(\tau, 0)\| + \int_0^{\mathcal{T}_2} \left\| \Psi_{v_k}(\tau, s) \right\| ds (\|B_u K_x\| + \|B_u K_\phi\| \mu_\nu) \right),$$

with $\mu_\nu = \max_{v \in \mathcal{U}_v} \|\Upsilon(v)(I - D_\phi \Upsilon(v))^{-1} C\|$, such that $\|x_k(\tau)\| \leq \mu_\Psi \|x_k(0)\|$. Hence, if $x_k(0) \rightarrow 0$ as $k \rightarrow \infty$, i.e., $x(t_k) \rightarrow 0$ as $k \rightarrow \infty$, then $x(t_k + \tau) = x_k(\tau) \rightarrow 0$ as $k \rightarrow \infty, \forall \tau \in [0, T_k]$, that is $x(t) = x(t_k + \tau) \rightarrow 0$ as $t \rightarrow \infty$. \square

It should be noticed that, as for the case of linear systems [30], in Theorem 1 no assumption regarding the positivity of the functional \mathcal{W}_0 is needed.

3.2 Regional Stability

Whenever the nonlinearity ϕ verifies (2b) and (2c) only locally, that is only for $y \in \mathcal{Y}_0$ (or, equivalently, $x \in \mathcal{X}_0$), stability results only in a local context can be derived. In this case, beyond the asymptotic stability of the origin, it is important to characterize an estimate of its region of attraction. Hence, provided that the initial state of system (1) belongs to this estimate, it is ensured that the trajectories of the closed-loop system (1)-(7) converge to the origin $\forall T_k \in [\mathcal{T}_1, \mathcal{T}_2]$. Similar to reference [23], general conditions to tackle this problem are presented in the next theorem.

Theorem 2. *Consider the closed-loop system composed by (1) and the sampled-data control law (7), with T_k satisfying (8) and (2) being satisfied $\forall y \in \mathcal{Y}_0 \subset \mathbb{R}^m$. Consider a compact domain $\mathcal{D}_x \subseteq \mathcal{X}_0 \subset \mathbb{R}^n$ containing the origin and let $V : \mathcal{D}_x \rightarrow \mathbb{R}^+$ be a differentiable function such that*

$$\mu_1 \|x\|^p \leq V(x) \leq \mu_2 \|x\|^p, \quad \forall x \in \mathcal{D}_x, \text{ with } \mu_1, \mu_2 > 0, p > 1. \tag{20}$$

Let $\mathcal{L}(V, \rho)$, with $\rho > 0$, be the level sets of V and a continuous-time functional $\mathcal{W}_0 : [0, \mathcal{T}_2] \times \mathbb{K}_{[\mathcal{T}_1, \mathcal{T}_2]}^n \times [\mathcal{T}_1, \mathcal{T}_2] \rightarrow \mathbb{R}^+$, such that

$$\mathcal{W}_0(0, x_k, T_k) = \mathcal{W}_0(T_k, x_k, T_k) = 0, \tag{21a}$$

$$\mathcal{W}_0(\tau, x_k, T_k) > 0, \forall \tau \in [0, T_k], \forall T_k \in [\mathcal{T}_1, \mathcal{T}_2], \forall x_k \in \mathbb{K}_{[\mathcal{T}_1, \mathcal{T}_2]}^n \text{ such that } x_k(\tau) \in \mathcal{D}_x - \{0\}. \tag{21b}$$

Define $W(\tau, x_k, T_k) = V(x_k(\tau)) + \mathcal{W}_0(\tau, x_k, T_k)$ and let $\dot{W}(\tau, x_k, T_k)$ be the derivative of $W(\tau, x_k, T_k)$ with respect to τ . If the inequality

$$\dot{W}(\tau, x_k, T_k) < 0 \tag{22}$$

is satisfied $\forall x_k \in \mathbb{K}_{[\mathcal{T}_1, \mathcal{T}_2]}^n$ such that $x_k(\tau) \in \mathcal{D}_x - \{0\} \subset \mathbb{R}^n, \forall \tau \in [0, T_k], \forall T_k \in [\mathcal{T}_1, \mathcal{T}_2], \forall k \in \mathbb{N}$, then for any initial condition $x(0) = x_0(0)$ lying inside any level set $\mathcal{L}(V, \rho) \subset \mathcal{D}_x$, it follows that:

- (i) $\Delta V(k) = V(x_{k+1}(0)) - V(x_k(0)) < 0, \forall k \in \mathbb{N}, x_k(0) \neq 0$;
- (ii) *The corresponding trajectories of the closed-loop system, formed by (1) and (7) with $T_k \in [\mathcal{T}_1, \mathcal{T}_2]$ satisfy $x(t) \in \mathcal{L}(V, \rho), \forall t$, and converge asymptotically to the origin.*

Proof. Suppose that $x(0) = x_0(0) \in \partial \mathcal{L}(V, \rho) \subset \mathcal{D}_x \subseteq \mathcal{X}_0$ and that (22) is satisfied. Then, it follows that

$$W(\tau, x_0, T_0) < W(0, x_0, T_0), \quad \forall \tau \in (0, T_0]. \tag{23}$$

Taking into consideration (23) and the properties of \mathcal{W}_0 stated in (21), it follows that $V(x_0(\tau)) < W(\tau, x_0, T_0) < W(0, x_0, T_0) = V(x_0(0)) = \rho, \forall \tau \in (0, T_0]$, implying that $x_0(\tau) \in \mathcal{L}(V, \rho) \subset \mathcal{D}_x, \forall \tau \in (0, T_0]$. Now, integrating (22) over the interval $[0, T_0]$ and considering (21), one obtains that $V(x_0(T_0)) - V(x_0(0)) < 0$. Since by continuity of the trajectories it follows that $x_0(T_0) = x_1(0)$, we conclude that $\Delta V(0) < 0$ and thus $V(x_1(0)) = \rho_1 < V(x_0(0)) = \rho$, which implies that $V(x_1(0)) < \rho$, i.e., $x_1(0) \in \partial \mathcal{L}(V, \rho_1) \subset \mathcal{L}(V, \rho)$. By repeating this reasoning for $k = 1, 2, \dots, \infty$, it follows that

$\Delta V(k) < 0, \forall k \in \mathbb{N}$, which proves item (i). As a consequence, it follows that $\lim_{k \rightarrow \infty} x_k(0) = \lim_{k \rightarrow \infty} x(t_k) = 0$. Moreover, it follows that $V(x_k(\tau)) < V(x_k(0)), \forall k \geq 0$, and we can conclude that $\mathcal{L}(V, \rho) \subset \mathcal{D}_x \subseteq \mathcal{X}_0$ is a positively invariant set and thus the continuous state trajectories never leave the set \mathcal{X}_0 , for which the conditions (2) are satisfied. Furthermore, since $\lim_{k \rightarrow \infty} V(x_k(0)) = 0$ and $V(x_k(\tau)) < V(x_k(0))$, we conclude that $\lim_{k \rightarrow \infty} V(x_k(\tau)) = 0$ and hence $x_k(\tau) \rightarrow 0$ for $k \rightarrow \infty$, which concludes the proof of item (ii). \square

Note that, differently from Theorem 1, here we have to consider a functional \mathcal{W}_0 that is positive definite and satisfies $\mathcal{W}_0(0, x_k, T_k) = \mathcal{W}_0(T_k, x_k, T_k) = 0$. This is necessary to ensure that the trajectories do not leave $\mathcal{D}_x \subseteq \mathcal{X}_0$ between two sampling instants. Note however that function V is not required to be strictly decreasing with respect to the continuous-time trajectory, which implies that $\mathcal{L}(V, \rho)$ is positively invariant but not continuously contractive.

Remark 1. *Theorems 1 and 2 are inspired by the results in [30]. It should however be pointed out that reference [30] regards only linear systems. In that paper, since the closed-loop dynamics is linear, the proof that $\Delta V(k) < 0$ implies the convergence of the continuous-time trajectories to the origin is straightforward. For nonlinear systems, in general, it is not possible to conclude that $\Delta V(k) < 0$ implies the convergence of the continuous-time trajectories to the origin without further conditions. However, for the class of Lure type nonlinear systems, we formally prove in Theorem 1 that $\Delta V(k) < 0$ indeed implies the convergence of the continuous-time trajectories to the origin. Then, as in reference [30], the considered functional \mathcal{W}_0 does not need to be positive definite. This is an original and important contribution of the present work. On the other hand, for the local stability results, it is necessary to impose the positivity of \mathcal{W}_0 , since we have to ensure that the trajectories of the system are confined in the region where the sector conditions are valid. This is formally proved in Theorem 2 and it is another contribution of the present work. Note that whenever the system is linear, as in reference [30], the stability of trajectories is a global property and therefore there is no need for a particular analysis of regional stability as for nonlinear systems.*

4 LMI Stability Conditions

Based on appropriate classes of candidates for the function V and the functional \mathcal{W}_0 , we show now how to cast the conditions in Theorems 1 and 2 as LMIs, which allow to assess the stability of the origin of the closed-loop system under the sampled-data control law through numerical tests. Moreover, these conditions can be added as constraints in optimization problems as it will be seen in Section 5.

4.1 Parametrization of the function V and the looped functional \mathcal{W}_0

We consider as candidates for the function V the class of generalized Lure functions $V : \mathbb{R}^n \rightarrow \mathbb{R}$ defined as follows

$$V(x) = V_{QG}(x) + \sum_{i=1}^m \lambda_i \int_0^{y_i(x)} (\phi_i(s) - \underline{\delta}_i s) ds, \quad (24)$$

where

$$V_{QG}(x) = \begin{bmatrix} x \\ \phi(y(x)) \end{bmatrix}^T \underbrace{\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}}_P \begin{bmatrix} x \\ \phi(y(x)) \end{bmatrix}, \quad (25)$$

and $y(x)$ is the solution to (1b).

This function generalizes the classic Lure-Postnikov [17] one, given by $x^T P_{11} x + \sum_{i=1}^m \lambda_i \int_0^{y_i(x)} (\phi_i(s) - \underline{\delta}_i s) ds$ with $P_{11} \succ 0$ and non-negative coefficients λ_i (also known as Lure-Postnikov terms), $i = 1, \dots, m$. Regarding (24) we do not require the positive-definiteness of P , nor the non-negativity of the coefficients λ_i . In this case, the positive-definiteness of V should be ensured as stated in the following lemma.

Lemma 3. [37] *Consider V in (24) where ϕ satisfies (2) and define $\Lambda \triangleq \text{diag}(\lambda_1, \dots, \lambda_m)$. If there exists a matrix $\tilde{\Lambda} \in \mathbb{D}_{\geq 0}^m$ such that*

$$\Lambda \geq -\tilde{\Lambda}, \quad (26)$$

$$V_{QG}(x) - \frac{1}{2} y^T(x) (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} y(x) > 0, \quad \forall x \in \mathcal{X}_0, \quad (27)$$

then $V(x) > 0, \forall x \in \mathcal{X}_0, x \neq 0$.

For the looped-functional \mathcal{W}_0 , we consider a generalized version of the one proposed in Seuret and Gomes da Silva Jr. [31], where terms depending on ϕ and $\dot{\phi}$ are added as follows:

$$\begin{aligned} \mathcal{W}_0(\tau, x_k, T_k) &= (T_k - \tau)\tau \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix}^T X \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix} \\ &+ (T_k - \tau) \left\{ \int_0^\tau \dot{x}_k^T(\theta) R_x \dot{x}_k(\theta) d\theta + (x_k(\tau) - x_k(0))^T [F_x(x_k(\tau) - x_k(0)) + 2G_x x_k(0)] \right\} \\ &+ (T_k - \tau) \left\{ \int_0^\tau \dot{\phi}_k^T(\theta) R_\phi \dot{\phi}_k(\theta) d\theta + (\phi_k(\tau) - \phi_k(0))^T [F_\phi(\phi_k(\tau) - \phi_k(0)) + 2G_\phi \phi_k(0)] \right\} \end{aligned} \quad (28)$$

with $F_x \in \mathbb{S}^n$, $G_x \in \mathbb{R}^{n \times n}$, $X \in \mathbb{S}^{n+m}$, $F_\phi \in \mathbb{S}^m$, $G_\phi \in \mathbb{R}^{m \times m}$, $R_x \in \mathbb{S}_{>0}^n$ and $R_\phi \in \mathbb{S}_{>0}^m$.

Note that the functional \mathcal{W}_0 given in (28) satisfies the conditions (12) and (21a). Since $(T_k - \tau) = 0$ when $\tau = T_k$ and $x_k(\tau) - x_k(0) = 0$ when $\tau = 0$, it follows that $\mathcal{W}_0(0, x_k, T_k) = \mathcal{W}_0(T_k, x_k, T_k) = 0$. Moreover, it is continuous and differentiable with respect to τ .

Remark 2. The function V and the looped-functional \mathcal{W}_0 can be seen as a generalized versions of the ones proposed by Seuret [30]. In that paper, to address the case of linear systems, it has been considered

$$V(x) = x^T P x \text{ with } P = P' > 0 \quad (29)$$

and

$$\begin{aligned} \mathcal{W}_0(\tau, x_k, T_k) &= (T_k - \tau)\tau x_k(0)^T X x_k(0) \\ &+ (T_k - \tau) \left\{ \int_0^\tau \dot{x}_k^T(\theta) R_x \dot{x}_k(\theta) d\theta + (x_k(\tau) - x_k(0))^T [F_x(x_k(\tau) - x_k(0)) + 2G_x x_k(0)] \right\} \end{aligned} \quad (30)$$

It should also be noticed that the Lyapunov-Krasovskii functional used by Seifullaev and Fradkov [27] to deal with Lure type sampled-data systems, corresponds (using our notation) to $V + \mathcal{W}_0$ with V as in (29) and \mathcal{W}_0 as in (31), but without the term $x_k(0)^T X x_k(0)$. On the other hand, note that V and \mathcal{W}_0 in our case, as defined in (24)-(25) and (28) respectively, present several terms that depend on the nonlinearity ϕ . Moreover, Lemma 3 allows to ensure the positive definiteness of function V without imposing the positive definiteness of matrices P and Λ . These aspects introduce extra degrees of freedom in the stability conditions, leading to a conservatism reduction at the expense of a higher computational complexity. As it will be seen in the next sections, since the considered functional has more terms, the stability conditions derived with (24)-(25) and (28) will present more decision variables when compared with the conditions in reference [27], which are obtained with a simpler functional.

4.2 Global case

Based on the general formulation of Theorem 1, the following theorem provides LMI conditions to certify the global asymptotic stability of the origin of the closed-loop system under aperiodic sampling, by using the class of functions V and functionals \mathcal{W}_0 defined in the previous section.

To present the LMIs in a more compact form, consider the definition of the following auxiliary matrices:¹

$$\begin{aligned} M_0 &= [A \ B_\phi \ -I \ 0 \ B_u K_x \ B_u K_\phi] \\ M_1 &= [I \ 0 \ 0 \ 0 \ 0 \ 0] \quad M_2 = [0 \ I \ 0 \ 0 \ 0 \ 0] \quad M_3 = [0 \ 0 \ I \ 0 \ 0 \ 0] \\ M_4 &= [0 \ 0 \ 0 \ I \ 0 \ 0] \quad M_5 = [0 \ 0 \ 0 \ 0 \ I \ 0] \quad M_6 = [0 \ 0 \ 0 \ 0 \ 0 \ I] \\ M_{15} &= M_1 - M_5 \quad M_{26} = M_2 - M_6 \\ M_{12} &= [M_1^T \ M_2^T]^T \quad M_{34} = [M_3^T \ M_4^T]^T \quad M_{56} = [M_5^T \ M_6^T]^T \\ M_{135} &= [M_1^T \ M_3^T \ M_5^T]^T \quad M_{246} = [M_2^T \ M_4^T \ M_6^T]^T. \end{aligned}$$

Theorem 3. For given \mathcal{J}_1 and \mathcal{J}_2 such that $0 < \mathcal{J}_1 \leq \mathcal{J}_2$, assume that there exist matrices $P \in \mathbb{S}^{n+m}$, $\Lambda \in \mathbb{D}^m$, $\tilde{\Lambda} \in \mathbb{D}_{\geq 0}^m$, U_j , $j = 0, \dots, 3 \in \mathbb{D}_{\geq 0}^m$, $F_x \in \mathbb{S}^n$, $G_x \in \mathbb{R}^{n \times n}$, $F_\phi \in \mathbb{S}^m$, $G_\phi \in \mathbb{R}^{m \times m}$, $R_x \in \mathbb{S}_{>0}^n$, $R_\phi \in \mathbb{S}_{>0}^m$, $Q_x \in \mathbb{R}^{(3n) \times n}$, $Q_\phi \in \mathbb{R}^{(3m) \times m}$, $X \in \mathbb{S}^{n+m}$, $L \in \mathbb{R}^{3(n+m) \times n}$ that satisfy, for $i = 1, 2$:

$$\Psi_1(\mathcal{J}_i) = \Pi_1 + \mathcal{J}_i \Pi_2 + \mathcal{J}_i \Pi_3 < 0 \quad (31)$$

$$\Psi_2(\mathcal{J}_i) = \begin{bmatrix} \Pi_1 - \mathcal{J}_i \Pi_3 & \mathcal{J}_i M_{135}^T Q_x & \mathcal{J}_i M_{246}^T Q_\phi \\ \star & -\mathcal{J}_i R_x & 0 \\ \star & \star & -\mathcal{J}_i R_\phi \end{bmatrix} < 0 \quad (32)$$

$$\Lambda \succeq -\tilde{\Lambda} \quad (33)$$

¹The matrices M_i are not of the same dimension. The notations 0 and I correspond to the zero and identity matrices of appropriate dimensions that will become clear in the context.

$$P - \frac{1}{2} \begin{bmatrix} C^T \\ D_\phi^T \end{bmatrix} (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} \begin{bmatrix} C & D_\phi \end{bmatrix} + \text{He} \left\{ \frac{1}{2} \begin{bmatrix} (\underline{\Delta} C)^T \\ (\underline{\Delta} D_\phi - I_m)^T \end{bmatrix} U_0 \begin{bmatrix} \bar{\Delta} C & (\bar{\Delta} D_\phi - I_m) \end{bmatrix} \right\} \succ 0, \quad (34)$$

with

$$\begin{aligned} \Pi_1 &= \text{He}\{M_{12}^T P M_{34}\} - M_{15}^T F_x M_{15} - M_{26}^T F_\phi M_{26} - \text{He}\{M_{26}^T G_\phi M_6\} - \text{He}\{M_{15}^T G_x M_5\} \\ &\quad - \text{He} \left\{ \frac{1}{2} (M_1^T (\underline{\Delta} C)^T + M_2^T (\underline{\Delta} D_\phi - I_m)^T) \Lambda C M_3 \right\} \\ &\quad - \text{He} \left\{ \frac{1}{2} (M_1^T (\underline{\Delta} C)^T + M_2^T (\underline{\Delta} D_\phi - I_m)^T) \Lambda D_\phi M_4 \right\} \\ &\quad + \text{He}\{M_2^T (I_m - (\underline{\Delta} D_\phi)^T) - M_1^T (\underline{\Delta} C)^T\} U_1 (\bar{\Delta} C M_1 + (\bar{\Delta} D_\phi - I_m) M_2) \\ &\quad + \text{He}\{M_4^T (I_m - (\underline{\Gamma} D_\phi)^T) - M_3^T (\underline{\Gamma} C)^T\} U_2 (\bar{\Gamma} C M_3 + (\bar{\Gamma} D_\phi - I_m) M_4) \\ &\quad + \text{He}\{M_6^T (I_m - (\underline{\Delta} D_\phi)^T) - M_5^T (\underline{\Delta} C)^T\} U_3 (\bar{\Delta} C M_5 + (\bar{\Delta} D_\phi - I_m) M_6) \\ &\quad - \text{He}\{M_{135}^T Q_x M_{15}\} - \text{He}\{M_{246}^T Q_\phi M_{26}\} + \text{He}\{L M_0\} \\ \Pi_2 &= M_3^T R_x M_3 + \text{He}\{M_3^T (F_x M_{15} + G_x M_5)\} + \text{He}\{M_4^T (F_\phi M_{26} + G_\phi M_6)\} + M_4^T R_\phi M_4 \\ \Pi_3 &= M_{56}^T X M_{56}, \end{aligned}$$

Then the origin of the sampled-data closed-loop system (1)-(7), with ϕ satisfying (2) globally and $T_k \in [\mathcal{J}_1, \mathcal{J}_2]$, is GAS.

Proof. Considering Theorem 1, we obtain the result by showing that (10), (12) and (13) are satisfied considering the generalized Lure type function (24)-(25) and W_0 as in (28).

By using Lemma 1 and Lemma 3, it is straightforward to prove that the positivity of V is ensured by the inequalities (34) and (35). Moreover, note that $\int_0^{y_i} (\phi_i(s) - \underline{\delta}_i s) ds = \frac{1}{2} y_i^T (\bar{\delta}_{(i)} - \underline{\delta}_{(i)}) y_i - \int_0^{y_i} (\bar{\delta}_i s - \phi_i(s)) ds$. Hence, if (34) holds with $\tilde{\Lambda} \in \mathbb{D}_{\geq 0}^m$, it follows that

$$V(x) \geq \begin{bmatrix} x \\ \phi \end{bmatrix}^T \underline{\Psi}(\tau) \begin{bmatrix} x \\ \phi \end{bmatrix} + \sum_{i=1}^m \tilde{\lambda}_i \int_0^{y_i} (\bar{\delta}_i s - \phi_i(s)) ds \geq \begin{bmatrix} x \\ \phi \end{bmatrix}^T \underline{\Psi}(\tau) \begin{bmatrix} x \\ \phi \end{bmatrix}, \quad (35)$$

with $\underline{\Psi} = P - \frac{1}{2} \begin{bmatrix} C^T \\ D_\phi^T \end{bmatrix} (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} \begin{bmatrix} C & D_\phi \end{bmatrix}$. Now if (35) holds, it follows that there exists a scalar $\mu_1 > 0$ such that

$$\begin{bmatrix} x \\ \phi \end{bmatrix}^T \underline{\Psi}(\tau) \begin{bmatrix} x \\ \phi \end{bmatrix} - S_\Delta(U_0, \phi, y) \geq \mu_1 \left\| \begin{bmatrix} x \\ \phi \end{bmatrix} \right\|^2 \geq \mu_1 \|x\|^2 \quad (36)$$

Hence, from Lemma 1, (36) and (37) we conclude that

$$V(x) \geq \mu_1 \|x\|^2.$$

On the other hand, recalling from the proof of Theorem 1 that $\phi(y) = \Upsilon(v)y$ and $y = (I - D_\phi \Upsilon(v))^{-1} Cx = \tilde{\Upsilon}(v)x$, with $v \in \mathcal{U}_v = \{v \in \mathbb{R}^m \mid \underline{\delta}_i \leq v_i \leq \bar{\delta}_i, i = 1, \dots, m\}$, and using the fact that $\lambda_i \int_0^{y_i} (\phi_i(s) - \underline{\delta}_i s) ds \leq |\lambda_i| \int_0^{y_i} (\bar{\delta}_i s - \underline{\delta}_i s) ds = \frac{1}{2} y_i^T |\lambda_i| (\bar{\delta}_{(i)} - \underline{\delta}_{(i)}) y_i$, it follows that:

$$V(x) \leq x^T P_{11} x + 2x^T \tilde{\Upsilon}^T(v) \Upsilon(v) P_{12} x + x^T \tilde{\Upsilon}^T(v) \Upsilon(v) P_{22} \Upsilon(v) \tilde{\Upsilon}(v) x + x^T \|\Lambda\| \tilde{\Upsilon}(v)^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Upsilon}(v) x \quad (37)$$

$$\leq \mu_2 \|x\|^2 \quad (38)$$

with $\mu_2 = \max_{v \in \mathcal{U}_v} (\|P_{11} + 2\tilde{\Upsilon}^T(v) \Upsilon(v) P_{12} + \tilde{\Upsilon}(v)^T \Upsilon(v) P_{22} \Upsilon(v) \tilde{\Upsilon}(v) + (\|\Lambda\| \tilde{\Upsilon}(v)^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Upsilon}(v))\|)$. Hence, we conclude that $V(x)$ defined in (24)-(25) verifies (10).

Since W_0 given in (28) satisfies the condition (12) by construction, the rest of the proof consists

in showing that (32)-(33) implies (13). With this aim, the expression of \dot{W} is given as follows:

$$\begin{aligned}
\dot{W}(\tau, x_k, T_k) = & 2 \begin{bmatrix} x_k(\tau) \\ \phi_k(\tau) \end{bmatrix}^T P \begin{bmatrix} \dot{x}_k(\tau) \\ \dot{\phi}_k(\tau) \end{bmatrix} \\
& - [(x_k^T(\tau)(\underline{\Delta}C)^T + \phi_k^T(\tau)(\underline{\Delta}D_\phi - I_m)^T)\Lambda C]\dot{x}_k(\tau) \\
& - [(x_k^T(\tau)(\underline{\Delta}C)^T + \phi_k^T(\tau)(\underline{\Delta}D_\phi - I_m)^T)\Lambda D_\phi]\dot{\phi}_k(\tau) \\
& - (x_k(\tau) - x_k(0))^T [F_x(x_k(\tau) - x_k(0)) + 2G_x x_k(0)] + (T_k - 2\tau) \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix}^T X \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix} \\
& + (T_k - \tau)\dot{x}_k^T(\tau)[R_x \dot{x}_k(\tau) + 2F_x(x_k(\tau) - x_k(0)) + 2G_x x_k(0)] - \int_0^\tau \dot{x}_k^T(\theta)R_x \dot{x}_k(\theta)d\theta \\
& - (\phi_k(\tau) - \phi_k(0))^T [F_\phi(\phi_k(\tau) - \phi_k(0)) + 2G_\phi \phi_k(0)] \\
& + (T_k - \tau)\dot{\phi}_k^T(\tau)[R_\phi \dot{\phi}_k(\tau) + 2F_\phi(\phi_k(\tau) - \phi_k(0)) + 2G_\phi \phi_k(0)] - \int_0^\tau \dot{\phi}_k^T(\theta)R_\phi \dot{\phi}_k(\theta)d\theta.
\end{aligned} \tag{39}$$

Using the sector conditions defined in Lemma 1 and Lemma 2, we have that

$$\dot{W}(\tau, x_k, T_k) < \dot{W}(\tau, x_k, T_k) + 2S_\Delta(U_1, \phi_k(\tau), y_k(\tau)) + 2S_\Gamma(U_2, \dot{\phi}_k(\tau), \dot{y}_k(\tau)) + 2S_\Delta(U_3, \phi_k(0), y_k(0)),$$

or equivalently

$$\begin{aligned}
\dot{W}(\tau, x_k, T_k) < & \dot{W}(\tau, x_k, T_k) + 2(\phi_k^T(\tau)(I_m - (\underline{\Delta}D_\phi)^T) - x_k^T(\tau)(\underline{\Delta}C)^T)U_1(\overline{\Delta}C x_k(\tau) + (\overline{\Delta}D_\phi - I_m)\phi_k(\tau)) \\
& + 2(\dot{\phi}_k^T(\tau)(I_m - (\underline{\Gamma}D_\phi)^T) - \dot{x}_k^T(\tau)(\underline{\Gamma}C)^T)U_2(\overline{\Gamma}C \dot{x}_k(\tau) + (\overline{\Gamma}D_\phi - I_m)\dot{\phi}_k(\tau)) \\
& + 2(\phi_k^T(0)(I_m - (\underline{\Delta}D_\phi)^T) - x_k^T(0)(\underline{\Delta}C)^T)U_3(\overline{\Delta}C x_k(0) + (\overline{\Delta}D_\phi - I_m)\phi_k(0)).
\end{aligned} \tag{40}$$

Consider the vector $\eta_k(\tau) = [x_k^T(\tau) \phi_k^T(\tau) \dot{x}_k^T(\tau) \dot{\phi}_k^T(\tau) x_k^T(0) \phi_k^T(0)]^T$, the vector $\zeta_k(\tau) = M_{135} \eta_k(\tau)$ and a matrix $Q_x \in \mathbb{R}^{(3n) \times n}$. Since R_x is assumed to be positive definite, it follows that $(\dot{x}_k(\theta) - R_x^{-1}Q_x^T \zeta_k(\tau))^T R_x (\dot{x}_k(\theta) - R_x^{-1}Q_x^T \zeta_k(\tau)) > 0$. Integrating this expression over $[0, \tau]$, the following inequality is obtained [3]

$$\int_0^\tau \dot{x}_k(\theta)^T R_x \dot{x}_k(\theta)d\theta - 2\zeta_k^T(\tau)Q_x(x_k(\tau) - x_k(0)) + \tau \zeta_k^T(\tau)Q_x R_x^{-1}Q_x^T \zeta_k(\tau) \geq 0. \tag{41}$$

Consider now a new vector $\psi_k(\tau) = M_{246} \eta_k(\tau)$ and a matrix $Q_\phi \in \mathbb{R}^{(3m) \times m}$. Following the same reasoning as above, we have that:

$$\int_0^\tau \dot{\phi}_k(\theta)^T R_\phi \dot{\phi}_k(\theta)d\theta - 2\psi_k^T(\tau)Q_\phi(\phi_k(\tau) - \phi_k(0)) + \tau \psi_k^T(\tau)Q_\phi R_\phi^{-1}Q_\phi^T \psi_k(\tau) \geq 0. \tag{42}$$

On the other hand, along the solutions of (9) there exists a coupling relation between the components of the vector $\eta_k(\tau)$. Hence, $\forall L \in \mathbb{R}^{3(n+m) \times n}$ the following equality is satisfied:

$$LM_0 \eta_k(\tau) = 0. \tag{43}$$

This null term can be added to (41). Hence, combining (41), (42), (43) and (44), one obtains that

$$\dot{W} \leq \eta_k^T(\tau)[\Pi_1 + (T_k - \tau)\Pi_2 + \tau(M_{135}^T Q_x R_x^{-1} Q_x^T M_{135} + M_{246}^T Q_\phi R_\phi^{-1} Q_\phi^T M_{246}) + (T_k - 2\tau)\Pi_3]\eta_k(\tau).$$

Thus, to prove that $\dot{W} < 0$, it suffices to guarantee that

$$\Pi_1 + (T_k - \tau)\Pi_2 + \tau(M_{135}^T Q_x R_x^{-1} Q_x^T M_{135} + M_{246}^T Q_\phi R_\phi^{-1} Q_\phi^T M_{246}) + (T_k - 2\tau)\Pi_3 < 0.$$

Since this matrix inequality is affine in τ , and $\tau \in [0, T_k]$, a necessary and sufficient condition for the inequality to hold is obtained by checking the inequality in the interval limits 0 and T_k , thus giving the following two inequalities:

$$\begin{cases} \Pi_1 + T_k(\Pi_2 + \Pi_3) < 0 \\ \Pi_1 - T_k \Pi_3 + T_k(M_{135}^T Q_x R_x^{-1} Q_x^T M_{135} + M_{246}^T Q_\phi R_\phi^{-1} Q_\phi^T M_{246}) < 0. \end{cases} \tag{44}$$

$$\tag{45}$$

Finally, since (45) and (46) are affine in T_k and $T_k \in [\mathcal{J}_1, \mathcal{J}_2]$, applying the same reasoning and the Schur complement to (46), we conclude that $\Psi_1(\mathcal{J}_i) < 0$ and $\Psi_2(\mathcal{J}_i) < 0$, $i = 1, 2$, are sufficient to ensure $\dot{W} < 0$. Hence, by virtue of Theorem 1 the satisfaction of conditions (32)-(35) ensures the global asymptotic convergence of the trajectories to the origin. \square

4.3 Regional case

Based on the result of Theorem 2, in this section we derive LMI conditions to assess the local stability of the origin of the closed-loop system. It should be recalled that in this case the functional \mathcal{W}_0 is required to be positive definite to ensure the boundedness of the trajectories in the region of validity of the sector conditions. For this, we consider V as defined in (24) and a functional \mathcal{W}_0 as defined in (28) with $G_x = 0$, $G_\phi = 0$ and $X \succ 0$, $F_x \succ 0$ and $F_\phi \succ 0$. Note that this particular structure will ensure the positivity of the crossed terms between $x_k(\tau)$ and $x_k(0)$ and between $\phi_k(\tau)$ and $\phi_k(0)$, guaranteeing therefore the positivity of the functional.

Theorem 4. *For given \mathcal{J}_1 and \mathcal{J}_2 such that $0 < \mathcal{J}_1 \leq \mathcal{J}_2$, assume that there exist matrices $P \in \mathbb{S}^{n+m}$, $\Lambda \in \mathbb{D}^m$, $\tilde{\Lambda} \in \mathbb{D}_{\geq 0}^m$, $U_j \in \mathbb{D}_{\geq 0}^m$, $j = 0, \dots, 3$, $F_x \in \mathbb{S}_{>0}^n$, $F_\phi \in \mathbb{S}_{>0}^m$, $R_x \in \mathbb{S}_{>0}^n$, $R_\phi \in \mathbb{S}_{>0}^m$, $Q_x \in \mathbb{R}^{(3n) \times n}$, $Q_\phi \in \mathbb{R}^{(3m) \times m}$, $X \in \mathbb{S}_{>0}^{n+m}$, $L \in \mathbb{R}^{3(n+m) \times n}$ that satisfy, for $i = 1, 2$:*

$$\Psi_1(\mathcal{J}_i) = \Pi_1 + \mathcal{J}_i \Pi_2 + \mathcal{J}_i \Pi_3 \prec 0 \quad (46)$$

$$\Psi_2(\mathcal{J}_i) = \begin{bmatrix} \Pi_1 - \mathcal{J}_i \Pi_3 & \mathcal{J}_i M_{135}^T Q_x & \mathcal{J}_i M_{246}^T Q_\phi \\ \star & -\mathcal{J}_i R_x & 0 \\ \star & \star & -\mathcal{J}_i R_\phi \end{bmatrix} \prec 0 \quad (47)$$

$$\Lambda \succeq -\tilde{\Lambda} \quad (48)$$

$$P - \frac{1}{2} \begin{bmatrix} C^T \\ D_\phi^T \end{bmatrix} (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} \begin{bmatrix} C & D_\phi \end{bmatrix} + \text{He} \left\{ \frac{1}{2} \begin{bmatrix} (\underline{\Delta} C)^T \\ (\underline{\Delta} D_\phi - I)^T \end{bmatrix} U_0 \begin{bmatrix} \bar{\Delta} C & (\bar{\Delta} D_\phi - I) \end{bmatrix} \right\} \succ 0, \quad (49)$$

with

$$\begin{aligned} \Pi_1 &= \text{He}\{M_{12}^T P M_{34}\} - M_{15}^T F_x M_{15} - M_{26}^T F_\phi M_{26} \\ &\quad - \text{He} \left\{ \frac{1}{2} (M_1^T (\underline{\Delta} C)^T + M_2^T (\underline{\Delta} D_\phi - I)^T) \Lambda C M_3 \right\} \\ &\quad - \text{He} \left\{ \frac{1}{2} (M_1^T (\underline{\Delta} C)^T + M_2^T (\underline{\Delta} D_\phi - I)^T) \Lambda D_\phi M_4 \right\} \\ &\quad + \text{He}\{M_2^T (I - (\underline{\Delta} D_\phi)^T) - M_1^T (\underline{\Delta} C)^T\} U_1 (\bar{\Delta} C M_1 + (\bar{\Delta} D_\phi - I) M_2) \\ &\quad + \text{He}\{M_4^T (I - (\underline{\Delta} D_\phi)^T) - M_3^T (\underline{\Delta} C)^T\} U_2 (\bar{\Gamma} C M_3 + (\bar{\Gamma} D_\phi - I) M_4) \\ &\quad + \text{He}\{M_6^T (I - (\underline{\Delta} D_\phi)^T) - M_5^T (\underline{\Delta} C)^T\} U_3 (\bar{\Delta} C M_5 + (\bar{\Delta} D_\phi - I) M_6) \\ &\quad - \text{He}\{M_{135}^T Q_x M_{15}\} - \text{He}\{M_{246}^T Q_\phi M_{26}\} + \text{He}\{L M_0\} \\ \Pi_2 &= M_3^T R_x M_3 + \text{He}\{M_3^T F_x M_{15}\} + \text{He}\{M_4^T F_\phi M_{26}\} + M_4^T R_\phi M_4 \\ \Pi_3 &= M_{56}^T X M_{56}, \end{aligned} \quad (50)$$

Then, for all initial conditions belonging to $\mathcal{L}(V, \rho) \subset \mathcal{X}_0$, the corresponding trajectories of the closed-loop system (1)-(7) with ϕ satisfying (2) and $T_k \in [\mathcal{J}_1, \mathcal{J}_2]$ converge asymptotically to the origin.

Proof. The proof follows the same steps of the one of Theorem 3. The inequalities (47)-(50) ensure the positivity of V and that $\dot{W}(\tau, x_k, T_k) < 0$, considering \mathcal{W}_0 with $G_x = 0$, $G_\phi = 0$, $X \succ 0$, $F_x \succ 0$ and $F_\phi \succ 0$. \square

Remark 3. *When the slope bounds (i.e. $\underline{\Gamma}$ and $\bar{\Gamma}$) are unknown or when we want to ensure stability for any nonlinearity in a given sector, that is, considering that only (2a) and (2b) are satisfied, Lemma 2 can no longer be used to deal with the terms depending on $\dot{\phi}$ in the derivative of $V + \mathcal{W}_0$. In this case, stability conditions can be obtained by applying Theorems 1 and 2 with a quadratic function $V(x) = x^T P_{11} x$ and the simplified functional*

$$\begin{aligned} \mathcal{W}_0(\tau, x_k, T_k) &= (T_k - \tau) \tau \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix}^T X \begin{bmatrix} x_k(0) \\ \phi_k(0) \end{bmatrix} \\ &\quad + (T_k - \tau) \left\{ \int_0^\tau \dot{x}_k^T(\theta) R_x \dot{x}_k(\theta) d\theta + (x_k(\tau) - x_k(0))^T [F_x (x_k(\tau) - x_k(0)) + 2G_x x_k(0)] \right\} \end{aligned} \quad (51)$$

with $F_x \in \mathbb{S}^n$, $G_x \in \mathbb{R}^{n \times n}$, $X \in \mathbb{S}^{n+m}$ and $R_x \in \mathbb{S}_{>0}^n$. Then, following the same developments of Theorems 3 and 4, LMI conditions can be straightforwardly derived for this case both in global and regional contexts.

5 Optimization Problems

From the conditions stated in Theorems 3 and 4, we propose three optimization problems with different goals as follows.

- P1. Given \mathcal{T}_1 , the sector and slope bounds, find the maximum value of \mathcal{T}_2 such that the asymptotic stability of the origin of closed-loop system (9) can be ensured.
- P2. Given a nominal sampling time \mathcal{T}_{nom} , the sector and slope bounds, find a bound on the maximum symmetrical allowable jitter, denoted by σ , (*i.e.*, $\mathcal{T}_1 = \mathcal{T}_{nom} - \sigma$, $\mathcal{T}_2 = \mathcal{T}_{nom} + \sigma$), such that the asymptotic stability of the origin of closed-loop system (9) can be ensured.
- P3. Given \mathcal{T}_1 and \mathcal{T}_2 , compute the maximum sector and slope bounds, such that the asymptotic stability of the origin of closed-loop system (9) can be ensured.

Provided \mathcal{T}_1 , \mathcal{T}_2 and the sector bounds (that is, matrices $\underline{\Delta}$, $\bar{\Delta}$, $\underline{\Gamma}$ and $\bar{\Gamma}$) are fixed, note that the matrix inequalities of Theorem 3 ((32)-(35)) and Theorem 4 ((47)-(50)) are LMIs. Hence, these optimization problems can be straightforwardly solved by considering feasibility LMI problems and bisection techniques, where we iteratively increase/decrease and test \mathcal{T}_2 , σ or a parameter defining the sector.

In the regional case, we are also interested in computing an estimate of the region of attraction of the origin given by the sublevel set $\mathcal{L}(V, \rho)$, with ρ as large as possible, such that $\mathcal{L}(V, \rho) \subset \mathcal{X}_0$. With this aim, consider that the region of validity of the sector and slope bounds in (2) are given by

$$\mathcal{Y}_0 = \{y \in \mathbb{R}^m \mid \underline{y}_j \leq y_j \leq \bar{y}_j \leq 0, j = 1, \dots, m\}. \quad (52)$$

Since y depends on the state x through the implicit equation (1b), the region \mathcal{Y}_0 can be mapped in space \mathbb{R}^n by \mathcal{X}_0 given in (6), which can be equivalently described as follows:

$$\mathcal{X}_0 = \{x \in \mathbb{R}^n \mid (y_j(x) - \underline{y}_j)(y_j(x) - \bar{y}_j) \leq 0, j = 1, \dots, m\}. \quad (53)$$

with $y_j(x) = C_j x + D_{\phi_j} \phi(y(x))$. In this case, if there exist scalars $\sigma_j > 0$ such that the following inequalities are satisfied [37].

$$-\sigma_j (y_j(x) - \underline{y}_j)(y_j(x) - \bar{y}_j) \geq (\rho - V(x)), \quad (54)$$

for $j = 1, \dots, m$, then it follows that $\mathcal{L}(V, \rho) \subset \mathcal{X}_0$. Note that for each $x \in \mathcal{L}(V, \rho)$ one has that $(\rho - V(x)) \geq 0$ and, from (55), we conclude that $(y_j(x) - \underline{y}_j)(y_j(x) - \bar{y}_j) \leq 0$.

Based on relations (55), the following proposition provides LMI conditions to check the inclusion of $\mathcal{L}(V, \rho)$ in the set given by (54).

Proposition 2. Consider function V as defined in (24) and the set \mathcal{X}_0 given by (54). If there exist matrices, $\tilde{\Lambda} \in \mathbb{D}_{\geq 0}^m$, $\tilde{\Lambda} \geq -\Lambda$, $S_{c,j} \in \mathbb{D}_{\geq 0}^m$ and positive scalars σ_j and ρ , such that the following LMI is satisfied

$$\begin{bmatrix} -(\sigma_j \underline{y}_j \bar{y}_j + \rho) & \sigma_j \frac{\underline{y}_j + \bar{y}_j}{2} C_j & \sigma_j \frac{\underline{y}_j + \bar{y}_j}{2} D_{\phi_j} \\ \star & P_{11} - \frac{1}{2} C^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} C & P_{12} - \frac{1}{2} C^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} D_{\phi} \\ \star & \star & P_{22} - \frac{1}{2} D_{\phi}^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} D_{\phi} \end{bmatrix} - \sigma_j \begin{bmatrix} 0 & 0 & 0 \\ \star & C_j^T C_j & C_j^T D_{\phi_j} \\ \star & \star & D_{\phi_j}^T D_{\phi_j} \end{bmatrix} + \text{He} \left\{ \frac{1}{2} \begin{bmatrix} 0 \\ (\underline{\Delta} C)^T \\ (\underline{\Delta} D_{\phi} - I)^T \end{bmatrix} S_{c,j} \begin{bmatrix} 0 & \bar{\Delta} C & (\bar{\Delta} D_{\phi} - I) \end{bmatrix} \right\} \succeq 0, \quad (55)$$

for $j = 1, \dots, m$, then $\mathcal{L}(V, \rho) \subset \mathcal{X}_0$.

Proof. From the definition of V in (24), inequalities (55) become

$$-\sigma_j \underline{y}_j \bar{y}_j - \rho + \sigma_j (\underline{y}_j + \bar{y}_j) y_j(x) - \sigma_j y_j^2(x) + V_{QG}(x) + \sum_{i=1}^m \lambda_i \int_0^{y_i} (\phi_i(s) - \underline{\delta}_i s) ds \geq 0, \quad (56)$$

$j = 1, \dots, m$. Since by assumption $\Lambda \geq -\tilde{\Lambda}_i$, we obtain from (27) the following lower bound for the Lure-Postnikov terms (see details in reference [37]):

$$\sum_{i=1}^m \lambda_i \int_0^{y_i} (\phi_i(s) - \underline{\delta}_i s) ds \geq -\frac{1}{2} y^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} y \geq 0. \quad (57)$$

Hence, provided the inequalities

$$-\sigma_j \underline{y}_j \bar{y}_j - \rho + \sigma_j (\underline{y}_j + \bar{y}_j) y_j(x) - \sigma_j y_j^2(x) + V_{QG}(x) - \frac{1}{2} y^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} y \geq 0 \quad (58)$$

hold, $\forall j = 1, \dots, m$, we have that (57) holds. Expanding the terms $y_j(x) = C_j x + D_{\phi_j} \phi(y(x))$, rewriting (59) using a matrix Ψ and relaxing for all x and ϕ which satisfies the sector conditions by adding the term $-S_{\Delta}(S_{c,j}, \phi, y)$ (as defined in Lemma 1) on the left side of (59), we have that the following inequalities provide sufficient conditions to verify (59):

$$\begin{bmatrix} 1 \\ x \\ \phi \end{bmatrix}^T \Psi \begin{bmatrix} 1 \\ x \\ \phi \end{bmatrix} - S_{\Delta}(S_{c,j}, \phi, y) = \begin{bmatrix} 1 \\ x \\ \phi \end{bmatrix}^T \tilde{\Psi} \begin{bmatrix} 1 \\ x \\ \phi \end{bmatrix} \geq 0, \quad (59)$$

with $\tilde{\Psi}$ given by

$$\tilde{\Psi} = \begin{bmatrix} -(\sigma_j \underline{y}_j \bar{y}_j + \rho) & \sigma_j \frac{\underline{y}_j + \bar{y}_j}{2} C_j & \sigma_j \frac{\underline{y}_j + \bar{y}_j}{2} D_{\phi_j} \\ \star & P_{11} - \frac{1}{2} C^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} C & P_{12} - \frac{1}{2} C^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} D_{\phi} \\ \star & \star & P_{22} - \frac{1}{2} D_{\phi}^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} D_{\phi} \end{bmatrix} - \sigma_j \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_j^T C_j & C_j^T D_{\phi_j} \\ 0 & \star & D_{\phi_j}^T D_{\phi_j} \end{bmatrix} + \text{He} \left\{ \frac{1}{2} \begin{bmatrix} 0 \\ (\underline{\Delta} C)^T \\ (\underline{\Delta} D_{\phi} - I)^T \end{bmatrix} S_{c,j} \begin{bmatrix} 0 & \bar{\Delta} C & (\bar{\Delta} D_{\phi} - I) \end{bmatrix} \right\}. \quad (60)$$

Hence, a sufficient condition to ensure (60) and thus (55) is that $\tilde{\Psi} \succeq 0$, which corresponds to (56). \square

If the validity region \mathcal{Y}_0 is symmetric, *i.e.*, $\underline{y}_j = \bar{y}_j$, then a simplified inclusion condition can be considered. In this case, we have that

$$\mathcal{X}_0 = \{x \in \mathbb{R}^n \mid |y_j(x)| \leq \bar{y}_j, j = 1, \dots, m\}, \quad (61)$$

and thus a condition to verify the inclusion $\mathcal{L}(V, \rho) \subset \mathcal{X}_0$ is given by

$$y_j(x)^T \rho (\bar{y}_j)^{-2} y_j(x) \leq V(x), \quad (62)$$

for $j = 1, \dots, m$. Note that if $x \in \mathcal{L}(V, \rho)$ then $V(x) \leq \rho$. Thus, from (63), one has $\rho \geq y_j(x)^T \rho (\bar{y}_j)^{-2} y_j(x)$, that is, $y_j(x)^T y_j(x) \leq \bar{y}_j^2$, which implies that $x \in \mathcal{X}_0$.

The following proposition provides a LMI test to verify relations (63).

Proposition 3. Consider function V as defined in (24) and the set \mathcal{X}_0 given by (62). If there exist matrices $\tilde{\Lambda} \in \mathbb{D}_{\succeq 0}^m$, $\tilde{\Lambda} \geq -\Lambda$, $S_{c,j} \in \mathbb{D}_{\succeq 0}^m$ and a positive scalar ρ such that the following LMIs are satisfied

$$\begin{bmatrix} P_{11} - \frac{1}{2} C^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} C & P_{12} - \frac{1}{2} C^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} D_{\phi} & \rho C_j^T \\ \star & P_{22} - \frac{1}{2} D_{\phi}^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} D_{\phi} & \rho D_{\phi_j}^T \\ \star & \star & \rho \bar{y}_j^2 \end{bmatrix} + \text{He} \left\{ \frac{1}{2} \begin{bmatrix} (\underline{\Delta} C)^T \\ (\underline{\Delta} D_{\phi} - I)^T \\ 0 \end{bmatrix} S_{c,j} \begin{bmatrix} \bar{\Delta} C & (\bar{\Delta} D_{\phi} - I) & 0 \end{bmatrix} \right\} \succeq 0, \quad (63)$$

for $j = 1, \dots, m$, then $\mathcal{L}(V, \rho) \subset \mathcal{X}_0$.

Proof. Applying Schur's Complement to the terms on the third row and third column of the matrix on the left of (64), then right and left multiplying the resulting inequality, respectively by $[x^T \ \phi^T]^T$ and its transpose, respectively, one obtains:

$$\begin{bmatrix} x \\ \phi \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ \star & P_{22} \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix} - \frac{1}{2} y^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} y - S_{\Delta}(S_{c,j}, \phi, y) \geq \rho (x^T C_j^T + \phi^T D_{\phi_j}^T) \rho^{-1} (\bar{y}_j)^{-2} (C_j x + D_{\phi_j} \phi) \rho. \quad (64)$$

As $S_{\Delta}(S_{c,j}, \phi, y) \geq 0$ for $x \in \mathcal{X}_0$, from (65) it follows that

$$\begin{bmatrix} x \\ \phi \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ \star & P_{22} \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix} - \frac{1}{2} y^T (\bar{\Delta} - \underline{\Delta}) \tilde{\Lambda} y \geq \rho (x^T C_j^T + \phi^T D_{\phi_j}^T) \rho^{-1} (\bar{y}_j)^{-2} (C_j x + D_{\phi_j} \phi) \rho. \quad (65)$$

Finally using (58) and the fact that $y_j(x) = C_j x + D_{\phi_j} \phi$, it follows that (63) is verified, which concludes the proof. \square

Hence, once the function V that solves one of the problems (P1, P2 or P3) above is determined, that is, the matrices P and Λ are obtained, the maximum sublevel set $\mathcal{L}(V, \rho)$ included in \mathcal{X}_0 can be determined through the following convex problem:

$$\text{maximize } \rho, \text{ subject to (56) (or (64)).} \quad (66)$$

6 Numerical Examples

Next four examples illustrate the application of the proposed results. In Example 1 we emphasize the conservatism reduction obtained thanks to the use of a generalized Lure type function V as defined in (24)- (25). Examples 2 and 3 present comparisons with the approach in reference [27], showing the conservatism reduction achieved with the use of our functional, which depends also on the nonlinearity. Finally, Example 4 illustrates the application of the conditions to assess local stability and to provide an estimate of the region of attraction of the origin. The associated Matlab routines for each one of the examples can be provided on demand to the authors.

Example 1: Consider system (1) given by the following matrices

$$A = \begin{bmatrix} -0.5 & -6.2 & -0.105 & -1.2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B_u = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_\phi = \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 0.2 \quad 0 \quad 0], D_\phi = [0],$$

the control law (7) with the following gains

$$K_x = [0.1 \quad 0.2 \quad 0.005 \quad 0.2], \quad K_\phi = [0.5],$$

and the bounds on ϕ given by $\underline{\delta} = 0$, $\bar{\delta} = \epsilon$, $\gamma = -\epsilon$, $\bar{\gamma} = \epsilon$. To show the potential advantage and the conservatism reduction induced by the generalized Lure type function, we consider different structures from the function V defined in (24). More specifically, we evaluate the feasibility of the LMIs such that they also hold for V_{QG} (*i.e.* V with $\Lambda = 0$), for a quadratic function $V_Q(x) = x^T P_{11} x$, which corresponds to V with $P_{12} = 0$, $P_{22} = 0$, $\Lambda = 0$ and for a classical Lure-Postnikov function

$$V_{LP}(x) = x^T P_{11} x + \sum_{i=1}^m \lambda_i \int_0^{y_i} (\phi_i(s) - \underline{\delta}_i s) ds, \quad (67)$$

which corresponds to V with $P_{12} = 0$, $P_{22} = 0$ and $\lambda_i \geq 0$, $i = 1, \dots, m$. Note that for V_Q and V_{LP} we consider $P_{11} \succ 0$ and for V_{QG} we consider $P \succ 0$.

The obtained results for problem P1 are detailed in Table 1, for values of $\mathcal{T}_1 = 0.1$ ms and $\epsilon = \frac{1}{\sqrt{2}}$. For problem P2, we fixed $\mathcal{T}_{nom} = 1.5$ s and $\epsilon = \frac{1}{\sqrt{2}}$, which results are presented in Table 2. Finally, for problem P3 we defined $\mathcal{T}_1 = 0.1$ ms and $\mathcal{T}_2 = 2.0$ s, which results are depicted in Table 3.

Table 1: Maximum value of \mathcal{T}_2 for different Lyapunov function structures.

	V_Q	V_{QG}	V_{LP}	V
\mathcal{T}_2	1.1151	1.1161	2.2121	2.6362

Table 2: Maximum value of σ for different Lyapunov function structures.

	V_Q	V_{QG}	V_{LP}	V
σ	1.0492	1.0496	1.0888	1.2260

Table 3: Maximum value of ϵ for different Lyapunov function structures.

	V_Q	V_{QG}	V_{LP}	V
ϵ	0.5955	0.5970	0.7842	1.1043

In the first problem, using the generalized Lure function V we obtain values for \mathcal{T}_2 that are 19.1%, 136.1% and 136.4% larger than the ones obtained with V_{LP} , V_{QG} and V_Q , respectively. For problem P2, we guarantee the stability for $\mathcal{T}_1 = 0.2740$ s and $\mathcal{T}_2 = 2.7260$ s, with σ 12.6%, 16.8 % and 16.8 % larger than the ones obtained with V_{LP} , V_{QG} and V_Q , respectively. For the last feasibility problem, the maximum sector bounds, represented by ϵ were 40.8%, 84.9% and 85.4% larger than the ones obtained with V_{LP} , V_{QG} and V_Q , respectively. We can conclude that a significant conservatism reduction is achieved with the use of a generalized Lure type function V in comparison with some classical and simpler functions.

For the result obtained with the proposed function V in Table 1, the following matrix P have been obtained:

$$P = \begin{bmatrix} 0.1942 & 0.0202 & 0.0892 & 0.0029 & -0.0068 \\ 0.0202 & 1.0496 & 0.0276 & 0.1909 & -0.2620 \\ 0.0892 & 0.0276 & 0.3330 & 0.0057 & -0.0089 \\ 0.0029 & 0.1909 & 0.0057 & 0.0876 & -0.0478 \\ -0.0068 & -0.2620 & -0.0089 & -0.0478 & 0.0546 \end{bmatrix}$$

whose eigenvalues are $-0.0102, 0.0512, 0.1506, 0.3750, 1.1525$. This illustrates that, with the proposed function V , matrix P does not need to be positive definite, thanks to the application of Lemma 3.

To show that the obtained results certify the global stabilization of the closed-loop system (9), we simulate the system considering the result from problem P1, *i.e.*, $T_k \in [0.0001, 2.6362]$, and the nonlinearity

$$\phi(y(t)) = 0.1\sin(5y(t)) + 0.15y(t), \quad (68)$$

that satisfies (2) with $\underline{\delta} = 0, \bar{\delta} = \frac{1}{\sqrt{2}}, \underline{\gamma} = -\frac{1}{\sqrt{2}}, \bar{\gamma} = \frac{1}{\sqrt{2}}$. The control action and the states of the closed-loop system are presented in Figures 1 and 2. To obtain the values of T_k , we used a pseudo-random algorithm that generates its numbers from the standard uniform distribution on the interval previously defined. We observe that the states and the control signal converge to the origin, as

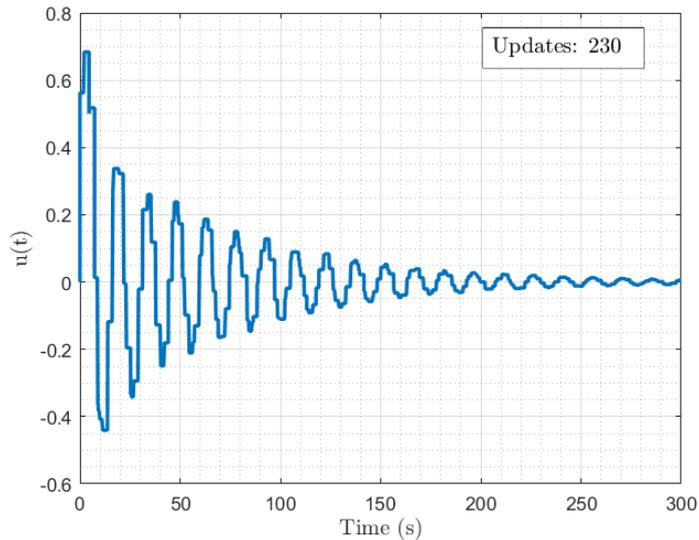


Figure 1: Control signal of the closed-loop system with nonlinearity (69).

expected.

Example 2: For the plant of Example 1, we consider now the control law given by:

$$K_x = [-2.8322 \quad -2.5547 \quad -6.6458 \quad -1.3226], \quad K_\phi = 0,$$

that is, a linear state feedback. Supposing that ϕ satisfies the sector and slope bounds given by given by $\underline{\delta} = 0, \bar{\delta} = \epsilon, \underline{\gamma} = -\epsilon, \bar{\gamma} = \epsilon$, with $\epsilon = \frac{1}{\sqrt{2}}$ and considering $\mathcal{T}_1 = 0.0001$, the maximal admissible bound \mathcal{T}_2 for which we can certify stability with the proposed results (*i.e.* by solving P1) was 0.60. On the other hand, by simulation, we obtain a diverging trajectory of the closed loop with $\mathcal{T}_2 = 0.61$. To illustrate this consider again the nonlinearity $\phi(y(t)) = 0.15y(t) + 0.1\sin(5y(t))$. In this case, considering the initial condition $x(0) = [1 \quad 1 \quad 1 \quad 1]'$, Figures 3 and 4 show the states response for a periodic sampling with $T_k = 0.6$ and $T_k = 0.61, \forall k$, respectively. Note that the bound on \mathcal{T}_2 obtained with the proposed conditions is close of the actual maximal admissible one.

For comparison purposes, the maximal \mathcal{T}_2 obtained with the results from Seifullaev and Fradkov [27], which consider a time-delay approach and a Lyapunov-Krasovskii functional proposed in [6] (see Remark 2), was 0.57. This shows the conservatism reduction achieved by the proposed method.

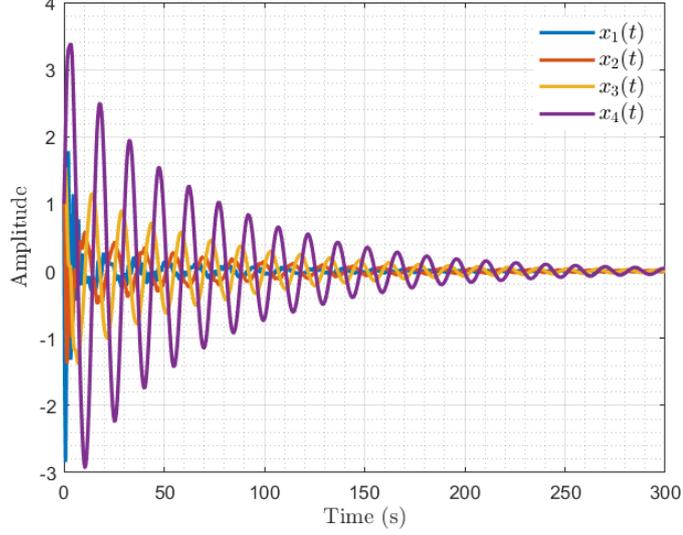


Figure 2: States of the closed-loop system with nonlinearity (69).

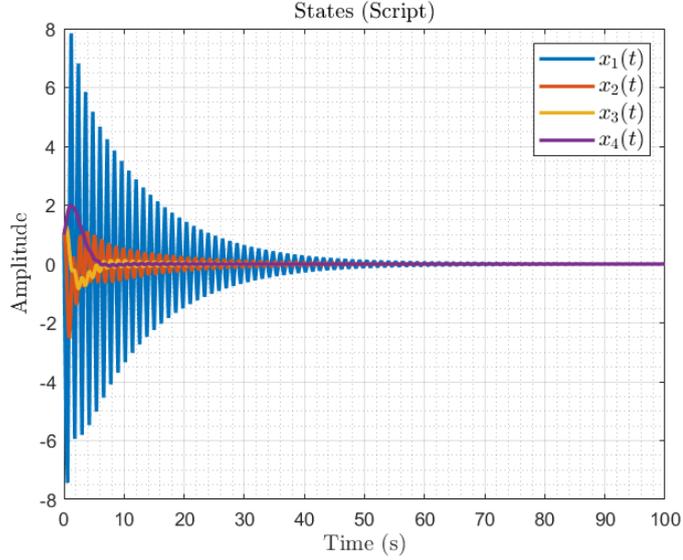


Figure 3: States response for $T_k = 0.6, \forall k$

Example 3: Consider the following system treated in Seifullaev and Fradkov [27]:

$$\begin{aligned}
 \dot{x}_1(t) &= -2x_1(t) + \sin(x_2(t)), \\
 \dot{x}_2(t) &= x_1(t) - x_2(t) + 2\sin(x_2(t)) + u(t), \\
 y(t) &= x_2(t), \\
 u(t) &= -Ky(t_k), \quad \forall t \in [t_k, t_{k+1}), \quad t_{k+1} - t_k \leq \mathcal{T}_2.
 \end{aligned} \tag{69}$$

System (70) can be rewritten in form (1)-(7) with the following matrices:

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_\phi = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, C = [0 \quad 1], D_\phi = [0], K_x = [0 \quad -K], K_\phi = [0],$$

with nonlinearity $\phi(y(t)) = \sin(y(t)) - y(t)$, satisfying (2) with $\underline{\delta} = -1.2173, \bar{\delta} = 0, \underline{\gamma} = -2, \bar{\gamma} = 0$. The values of the maximum \mathcal{T}_2 obtained with Theorem 3, by considering V and $\mathcal{T}_1 = 0.1$ ms and with Theorem 3 from reference [27] are given in Table 4 for different gains.

From Table 4, we observe that the proposed conditions lead to larger values of \mathcal{T}_2 than the ones obtained with the conditions of reference [27]. The dependence of the upper bound of the sampling intervals \mathcal{T}_2 obtained by both approaches as a function of the controller gain K is shown in Figure

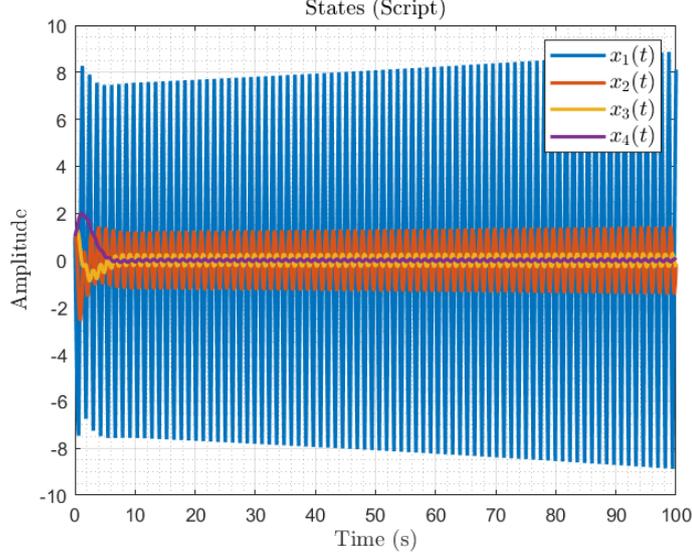


Figure 4: States response for $T_k = 0.61, \forall k$

Table 4: Upper bounds for aperiodic sampling.

	Theorem 2	Theorem 3 [27]
$K = 2$	$\mathcal{T}_2 = 1.0197$	$\mathcal{T}_2 = 0.6800$
$K = 3$	$\mathcal{T}_2 = 0.6740$	$\mathcal{T}_2 = 0.5300$
$K = 5$	$\mathcal{T}_2 = 0.4018$	$\mathcal{T}_2 = 0.3500$
$K = 10$	$\mathcal{T}_2 = 0.2002$	$\mathcal{T}_2 = 0.1870$

5. It can be observed that our approach provides less conservative results in terms of the maximum admissible bound \mathcal{T}_2 , in particular for small values of K . The maximum value of \mathcal{T}_2 obtained was $\mathcal{T}_2 = 1.3602$ s, with $K = 1.51$. As discussed in Remark 2, this conservatism reduction comes from the fact that our function V and functional \mathcal{W}_0 are more generic than the ones used to derive the results in reference [27]. Moreover, as discussed in Remark 1, in our conditions \mathcal{W}_0 is not required to be positive definite.

Example 4: Consider system (1) given by the following matrices:

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}, B_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_\phi = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D_\phi = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

and the control law (7) with the following gains

$$K_x = \begin{bmatrix} -1 & -2 \\ -0.5 & -2 \end{bmatrix}, K_\phi = \begin{bmatrix} 0 & 1 \\ -0.5 & 1 \end{bmatrix}.$$

Suppose that this system is fed back by the nonlinearities $\phi(y_1) = \sin(y_1)$, $\phi(y_2) = \sin(y_2)$. These nonlinearities lie in a sector given by $\underline{\delta}_1 = \underline{\delta}_2 = 0$, $\bar{\delta}_1 = \bar{\delta}_2 = 1$, with slope bounds $\underline{\gamma}_1 = \underline{\gamma}_2 = -1$ and $\bar{\gamma}_1 = \bar{\gamma}_2 = 1$, provided that $y_1, y_2 \in [-\pi, \pi]$. In this case, the obtained results for problems P1, P2 and P3 are detailed in Table 5 by considering different structures of V . For problem P1, we fixed $\mathcal{T}_1 = 0.1$ ms. For problem P2, we defined $\mathcal{T}_{nom} = 0.25$ s and for problem P3 we defined $\mathcal{T}_1 = 0.1$ ms and $\mathcal{T}_2 = 0.2$ s, but we change the sector conditions to $\underline{\delta}_i = 0$, $\underline{\gamma}_i = -\Omega$, $\bar{\delta}_i = \bar{\gamma}_i = \Omega$, for $i = 1, 2$ (\underline{y}_i and \bar{y}_i were kept the same).

Table 5: Regional analysis, Ex. 3 - Results for problems P1-P3.

Problem	Parameter	V_Q	V_{QG}	V_L	V
P1	\mathcal{T}_2	0.3403	0.3464	0.3437	0.4052
P2	σ	0.0904	0.0965	0.1513	0.1553
P3	Ω	1.2902	1.3063	1.3750	1.4397

Using the generalized Lure function V , for problem P1 we achieve values for \mathcal{T}_2 that are 16.9%, 17.8% and 19.0% larger than the ones obtained with V_{QG} , V_L and V_Q , respectively. For problem

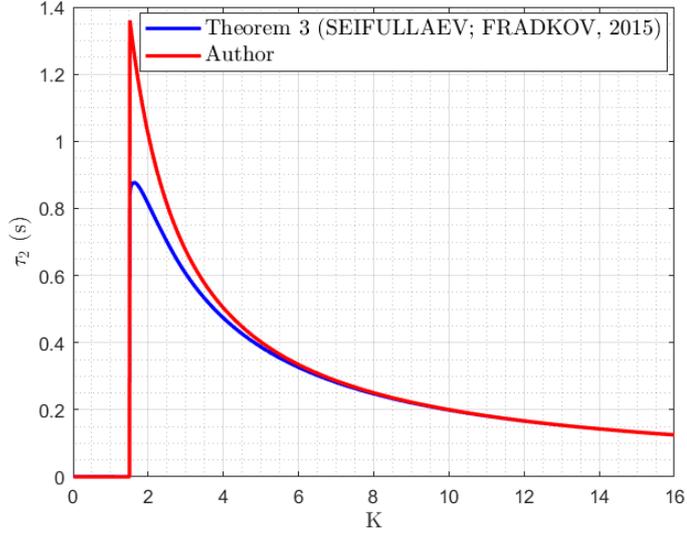
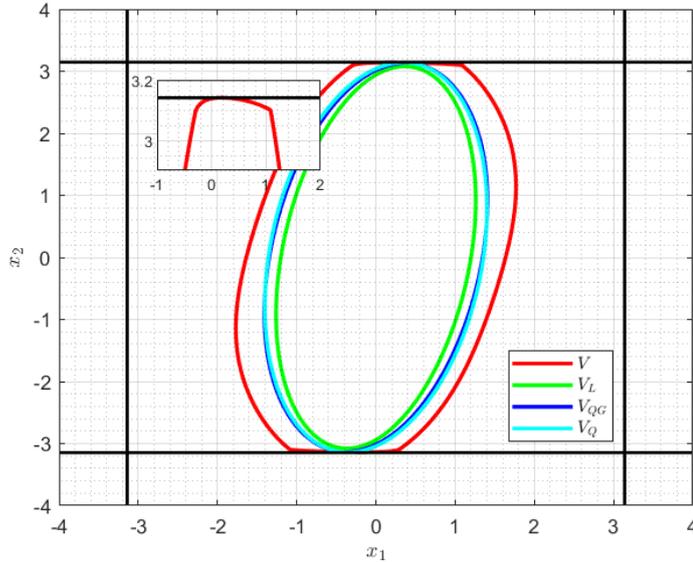


Figure 5: Upper bound \mathcal{T}_2 as a function of gain K .

Figure 6: Estimates of the region of attraction for P3 with different functions.



P2, we guaranteed the stability for $\mathcal{T}_1 = 0.0947$ s and $\mathcal{T}_2 = 0.4053$ s, with σ 2.6%, 60.9% and 71.7% larger than the ones obtained with V_L , V_{QG} and V_Q , respectively. For the last problem, the sector and slope bounds represented by Ω were 4.7%, 10.2% and 11.5% larger than the ones obtained with V_L , V_{QG} and V_Q , respectively. The estimates of the region of attraction corresponding to the solution of problem P3 for the different functions are presented in Fig. 6, along with the boundaries of the region of validity $\mathcal{X}_0 = \{x \in \mathbb{R}^2 \mid -\pi \leq x_i \leq \pi, i = 1, 2\}$. In this case, the obtained generalized Lure function V is defined by the matrices

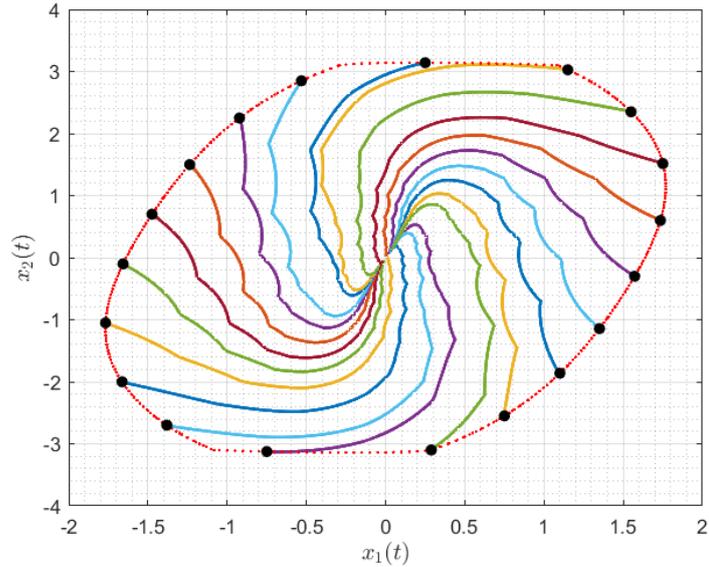
$$P = \begin{bmatrix} 0.1233 & -0.0040 & -0.0161 & 0.0175 \\ -0.0040 & 0.0478 & -0.0013 & -0.0227 \\ -0.0161 & -0.0013 & 0.0022 & 0.0070 \\ 0.0175 & -0.0227 & 0.0070 & 0.0140 \end{bmatrix}, \text{ and } \Lambda = \begin{bmatrix} -0.0340 & 0 \\ 0 & 0.0501 \end{bmatrix}.$$

and solving (67) we obtain $\rho = 0.4550$.

Observe that with the function V , we enlarge the estimate of the region of attraction in comparison to the other functions. Furthermore, we increase the sector and slope admissible bounds.

To show that stability is guaranteed for the obtained region, we simulate the closed-loop system by considering the solution of problem P3 with V , $T_k \in [0.0001, 0.2]$ and the nonlinearities $\phi(y_1) = 1.4 \sin(y_1)$ and $\phi(y_2) = 1.4 \sin(y_2)$, which satisfies the sector conditions for $\underline{\delta}_1 = \underline{\delta}_2 = 0$, $\bar{\delta}_1 = \bar{\delta}_2 = 1.4$,

Figure 7: Trajectories of the closed-loop system



$\underline{\gamma}_1 = \underline{\gamma}_2 = -1.4$ and $\bar{\gamma}_1 = \bar{\gamma}_2 = 1.4$. The resulting trajectories for initial conditions on the boundary of the obtained estimate of the region of attraction are presented in Fig. 7. We considered in the simulation the same sampling sequence for all trajectories. As expected, they all converge to the origin.

7 Conclusions

In this work, new convex conditions for the asymptotic stability assessment of Lure-type aperiodic sampled-data controlled systems, subject to sector and slope-restricted nonlinearities have been proposed. The approach is based on the use of a generalized Lure type function, in the sense that the quadratic part depends also on the nonlinearity and the coefficient of the Lure-Postnikov terms are not necessarily non-negative, and on a looped-functional that also depends on the system nonlinearities. From these elements, LMI conditions to certify the global as well as the regional stability of the origin have been formulated. In particular, for the global case it has been shown that the positivity of the functional can also be relaxed, as in the linear systems case. For the regional case, sets of admissible initial states (which can be seen as estimates of the region of attraction of the origin) have been formally characterized as level sets of the generalized Lure type function.

Finally, it has been discussed how the proposed LMI conditions can be used to find a bound on the maximum admissible intersampling time, on the jitter around a nominal sampling period, or on the maximum sector for which the stability can be ensured. Several numerical examples illustrate the application and effectiveness of the proposed conditions, which are based on a more general function and functional than the ones considered in previous literature, to reduce the conservatism of the results.

As future work, other classes of functionals and the use of other relaxations to bound integral terms instead of the equivalent Jensen's inequality [3], such as Wirtinger or Bessel ones [29], can be considered at the expense of obtaining numerical formulations of increasing complexity. The extension of the results using matrices depending on τ is also relatively straightforward, but it will lead to even more complex conditions. If this dependence is polynomial, the problem can be indeed addressed in a sum of squares (SOS) framework. The extension of the proposed results to address the stabilization problem, i.e. to provide conditions allowing the synthesis of the control law gains, is another problem that should be investigated. The main issue regards the determination of linear (or quasi-linear) matrix inequalities due to several products between variables. Even performing some suitable congruence transformations and changes of variables some products will still remain. In this case, relaxation schemes have to be used by fixing some variables and solving LMI problems in an iterative way. A preliminary result in this sense has been published in [34].

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