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Francesco d'Alessandro, Hugo Festjens, Gael Chevallier, Scott Cogan

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F. D'Alessandro [1,2], H. Festjens[1], G. Chevallier[2], S. Cogan[2]

[1] SAFRAN AIRCRAFT ENGINES - Villaroche , Rond-Point Rene Ravaud , 77550 Moissy-Cramayel, FR

[2] Univ. Bourgogne Franche-Comte , FEMTO-ST Institute Department of Applied Mechanics, 24, rue de l'Epitaphe, 25000 Besancon, FR

e-mail: **francesco.alessandr@femto-st.fr**

Abstract

A mathematical model is presented that expresses the viscous and hysteretic damping forces, in the frequency domain, on rotating axisymmetric elements such as beams. The effect of the hysteretic damping on a rotating system will be discussed for the cases of forward and backward whirling modes and in both inertial and rotating reference frames.

An original post-processing diagram highlights the dynamic effect of the damping force on the rotor. A presentation of the main components of viscous and hysteretic damping forces will allow the two models to be quantitatively compared based on an academic illustration.

Keywords : Rotordynamics, Viscous Damping , Hysteretic Damping , Instabilities , Whirling.

1 Introduction

A hysteretic damping model was studied for the first time in 1935 by Robertson [Rob35] and is now commonly used for non-rotating dynamic systems. However, in the past its application to rotordynamic behaviour has led to misunderstandings in several papers [ZN77] [MZ98]. Hysteretic damping, like viscous damping, can lead to both stabilizing and destabilizing effects in rotordynamics, depending on the regime, while in several papers it is stated that it always produces a destabilizing effect. A first clarification regarding the hysteretic damping effect was published in 1970 by S.H. Crandall [Cra70] and then in 2004 by G.Genta [Gen04].

This paper re-examines the effect of viscous and hysteretic damping forces on a rotating axisymmetrical system. The approach is based on the assumption of a circular centered trajectory of the whirling motion and the definition of a hysteretic damping matrix equivalent to the viscous damping matrix [GA10]. The damping forces that participate in the whirling motion are presented and their effect on the stability of the structure are discussed.

2 Study case

In general, the viscous damping force in a rotating system can be represented in the time domain in the following form:

$$\left\{\boldsymbol{f}_v\right\}_{fr} = \boldsymbol{B}_v \left\{\dot{\boldsymbol{x}}\right\}_{fr} + \Omega \boldsymbol{B}_v \boldsymbol{J} \left\{\boldsymbol{x}\right\}_{fr} \tag{1}$$

Where $\left\{\boldsymbol{x}\right\}_{fr}$ represents the lateral displacement of the rotor's center expressed in the inertial reference frame (time domain), $\boldsymbol{B}_v$ is the viscous damping matrix, Ω is the rotational speed of the system and $\boldsymbol{J}$ is an operator deriving from the passage from rotating to inertial reference frame, defined as (for a 2 DOFs system) :

$$\boldsymbol{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{2}$$

Considering a harmonic excitation with angular frequency ω , the equation 1 becomes:

$$\{\boldsymbol{F}_v\}_{fr} = (-\omega + \Omega\,\boldsymbol{J})\,\boldsymbol{B}_v\,\{\boldsymbol{X}\}_{fr} \tag{3}$$

$\{\boldsymbol{X}\}_{fr}$ represents the oscillation amplitude in the frequency domain. The first term of the sum represents the contribution of the whirling motion on the damping forces with a stabilizing effect while the second term, known as *circulatory damping*, produces a destabilizing effect on the rotating system.

From this assumption and using a specific representation for the hysteretic damping matrix [Gen04] [GA10] , a model is developed to represent the hysteretic damping force acting on the rotor:

$$\{\boldsymbol{F}_h\}_{fr} = \begin{cases} \frac{i}{Q}\boldsymbol{K}\left\{\boldsymbol{X}\right\}_{fr} & if\ \omega > \Omega\ (sub-critical\ regimes) \\ \\ \frac{i}{Q}\boldsymbol{K}\,\boldsymbol{J}\left\{\boldsymbol{X}\right\}_{fr} & if\ \omega < \Omega\ (super-critical\ regimes) \end{cases} \tag{4}$$

Where $\boldsymbol{K}$ is the stiffness matrix, Q is the loss factor defined as the energy dissipated per cycle, ω is the whirling angular frequency and Ω is the rotating speed. The models of viscous and hysteretic damping described in equations 3 and 4 are applied to a Jeffcott rotor projecting the motion on the first vibration mode. The viscous damping factor related to the considered vibration mode is defined as:

$$\zeta_{viscous} = \zeta_M(\omega_0 \pm \Omega) + \zeta_F\,\omega_0 \tag{5}$$

Where the damping factors ζ_M (for moving parts) and ζ_F (for fixed parts) represent respectively the contribution of the moving and fixed parts on the modal damping of a specific vibration mode. On the other hand, using the hysteretic damping model, the damping factor of the mode is defined as :

$$\zeta_{hysteretic} = \frac{\omega_0}{Q}\,sign(\omega_0 \pm \Omega) + \zeta_F\,\omega_0 \tag{6}$$

The system is defined as "*stable*" when the modal damping associated to a specific vibration mode is positive in accord with the convention adopted by G.Genta [Gen04]. The results are summarized in the following table:

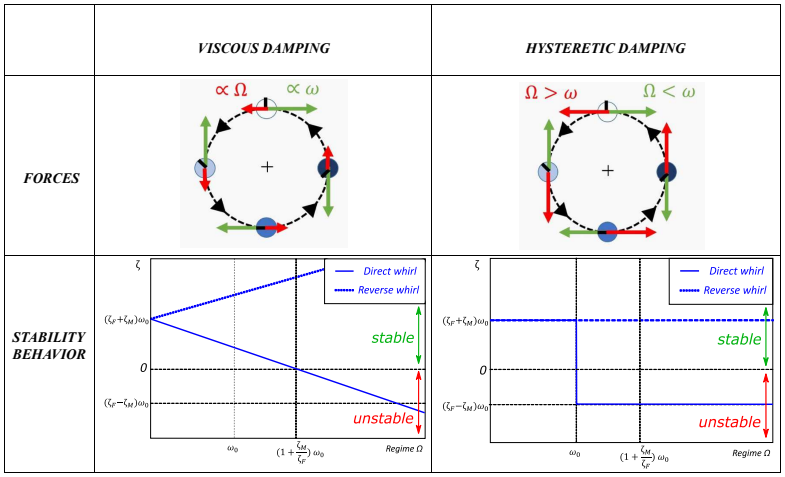

Figure 1: Damping forces and stability behavior

The first line represents the damping forces on the systems during a whirling movement while the second line represents the stability behavior of the system with respect to the regime.

3 Conclusion

The viscous damping model, presented in the equation 1, is composed by two terms : the first term of the sum is proportional to the excitation frequency and it is dominates the second term in subcritical regimes. The second term, known as circulation

term, is proportional to the rotational speed and dominates in supercritical regimes. In the case of a direct whirl motion, these two terms cancel each other out when the regime is equal to the excitation frequency.

Concerning the hysteretic damping force, presented in the equation 4, the force direction depends only on the sum $(\omega - \Omega)$. The stabilizing behavior depends on the regime. Indeed, as seen in figure 1 , it is stabilizing in subcritical regimes and destabilizing in supercritical regimes, but only if the sum $\zeta_F - \zeta_M < 0$.

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