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# Survival and dynamics of rings of co-orbital planets under perturbations

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## ABSTRACT

In co-orbital planetary systems, two or more planets share the same orbit around their star. Here we test the dynamical stability of co-orbital rings of planets perturbed by outside forces. We test two setups: (i) ‘stationary’ rings of planets that, when unperturbed, remain equally spaced along their orbit and (ii) horseshoe constellation systems, in which planets are continually undergoing horseshoe librations with their immediate neighbours. We show that a single rogue planet crossing the planets’ orbit more massive than a few lunar masses ( $0.01\text{--}0.04 M_{\oplus}$ ) systematically disrupts a co-orbital ring of 6, 9, 18, or 42 Earth-mass planets located at 1 au. Stationary rings are more resistant to perturbations than horseshoe constellations, yet when perturbed they can transform into stable horseshoe constellation systems. Given sufficient time, any co-orbital ring system will be perturbed into either becoming a horseshoe constellation or complete destabilization.

**Key words:** extraterrestrial intelligence – astrobiology – planets and satellites: dynamical evolution and stability.

## 1 INTRODUCTION

In a co-orbital system, two or more planets share the same orbit. The best-known cases of co-orbital systems are Jupiter’s Trojan asteroids, whose orbits oscillate (‘librate’) about Jupiter’s L4 and L5 Lagrange points,  $60^\circ$  ahead of and behind its orbit, and Saturn’s moons Janus and Epimetheus, which follow horseshoe orbits (Smith et al. 1980; Dermott & Murray 1981b). It is possible that co-orbital planetary systems may be common, although they remain to be discovered (e.g. Rowe et al. 2006). Simulations that include gas-driven migration frequently produce co-orbital planets in Trojan configurations, with two planets in each other’s mutual L4/L5 points (Cresswell & Nelson 2009; Izidoro et al. 2017; Raymond et al. 2018), and sometimes in horseshoe configurations as well (Rodríguez, Correa-Otto & Michtchenko 2019).

Salo & Yoder (1988) showed that co-orbital rings of planets could remain stable and maintain a fixed orbital spacing. The conditions for stability were that the ring must contain at least six planets, which must have equal masses. The planets must be evenly spaced around the star with a separation between planets of at least 12 mutual Hill radii  $R_{H,m}$ . The mutual Hill radius is defined as  $R_{H,m} = a(2m_p/3M_*)^{1/3}$ , where  $a$  is the orbital radius,  $m_p$  is the planet mass,

and  $M_*$  is the stellar mass. For Earth-mass planets at 1 au around a Solar-mass star, up to 42 planets can remain on stable orbits for Gyr time-scales (Smith & Lissauer 2010). We refer to these as ‘stationary’ rings of planets because their relative positions – when viewed in a co-rotating frame – remain fixed.

In a recent paper, we demonstrated the existence of *horseshoe constellations*, which represent a different type of co-orbital ring of planets (Raymond et al. 2023). In these systems, co-orbital planets do not remain stationary relative to one another, but rather undergo horseshoe oscillations with their immediate neighbours. A horseshoe constellation system at 1 au can contain at least 24 Earth-mass planets and remain stable for billions of years.

How easily do co-orbital rings of planets disrupt in the face of perturbations? This question is of interest, because the discovery of such systems is within the reach of current exoplanet observations (see, for instance, the TROY project: Lillo-Box et al. 2018a, b). In a previous paper (Raymond et al. 2023), we discussed possible formation pathways of systems containing many co-orbital planets. While natural pathways may exist (for instance, via fragmentation or coagulation within a ring of material around a young star), the natural formation of a ring of co-orbital planets is a very low-probability event. Rather, one might imagine that any such

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system may have instead been engineered by a highly advanced civilization.<sup>1</sup>

In this paper, we numerically assess the dynamical fragility of rings of co-orbital planets. We invoke the existence of a single, rogue protoplanet that dynamically perturbs the systems. We treat the mass of the rogue protoplanet as a free parameter, and run simulations with different numbers of co-orbital planets (6, 9, 18, or 42) in both stationary and horseshoe configurations.

## 2 NUMERICAL EXPERIMENT

Our general setup is analogous to that of Raymond et al. (2022), which assessed the perturbations that would disrupt the multi-resonant orbital structure of the TRAPPIST-1 exoplanet system (Gillon et al. 2017; Luger et al. 2017; Agol et al. 2021). The main difference is that in this study we only consider the perturbations from a single rogue protoplanet and neglect the case of a swarm of rogue planetesimals.

### 2.1 Simulations

Our simulations started with two components: a ring of co-orbital planets and a single rogue protoplanet. We tested seven different co-orbital rings: three horseshoe constellations and four stationary rings. The horseshoe rings contained 6, 9, and 18 Earth-mass planets each. We chose rings that were long-term stable from our previous study of such systems, by simply starting the planets spread far enough apart, with initial separations of 25 mutual Hill radii (see fig. 1 in Raymond et al. 2023). The stationary planet rings contained 6, 9, 18, and 42 planets of  $1 M_{\oplus}$  each, and were constructed by simply spreading the planets evenly in mean anomaly along a near-circular (eccentricity of  $10^{-5}$ ), co-planar orbit at 1 au. We first verified that each of the seven co-orbital rings was long-term stable by running 10 Gyr simulations using the hybrid algorithm in the Mercury integration package (Chambers 1999).

The orbit of the rogue protoplanet was chosen to cross the ring's orbital radius. The protoplanet's perihelion distance was randomly chosen between 0.5 and 1 au, its semimajor axis between 1.5 and 2.5 au, and its inclination between zero and  $10^{\circ}$ . While the exact orbital distribution of the rogue protoplanets does affect their angular momentum, it has only a small effect on the stability of the perturbed system (Raymond et al. 2022). We therefore did not systematically vary the rogue protoplanet's orbit.

For each ring of co-orbital planets we ran 150 simulations varying the mass of the rogue protoplanet in the range of  $0.001$ – $1 M_{\oplus}$ , sampled logarithmically. This mass range was chosen after a few test simulations. Each simulation was integrated for 100 Myr or until a collision or ejection occurred, again using the hybrid integrator in the Mercury integration package (Chambers 1999), which we showed in Raymond et al. (2023) to be as reliable as the Bulirsch-Stoer method (with accuracy parameter of  $10^{-15}$ ; see appendix A in that paper). For each system that remained stable for 100 Myr, we then ran a short simulation (in most cases of 1000 yr) with high-frequency outputs to assess the dynamical configuration of the final, perturbed system.

<sup>1</sup>Indeed, this scientific blog post dedicated to such systems invokes that such systems must be ‘Engineered’: see <https://planetplanet.net/2017/05/03/the-ultimate-engineered-solar-system/>.

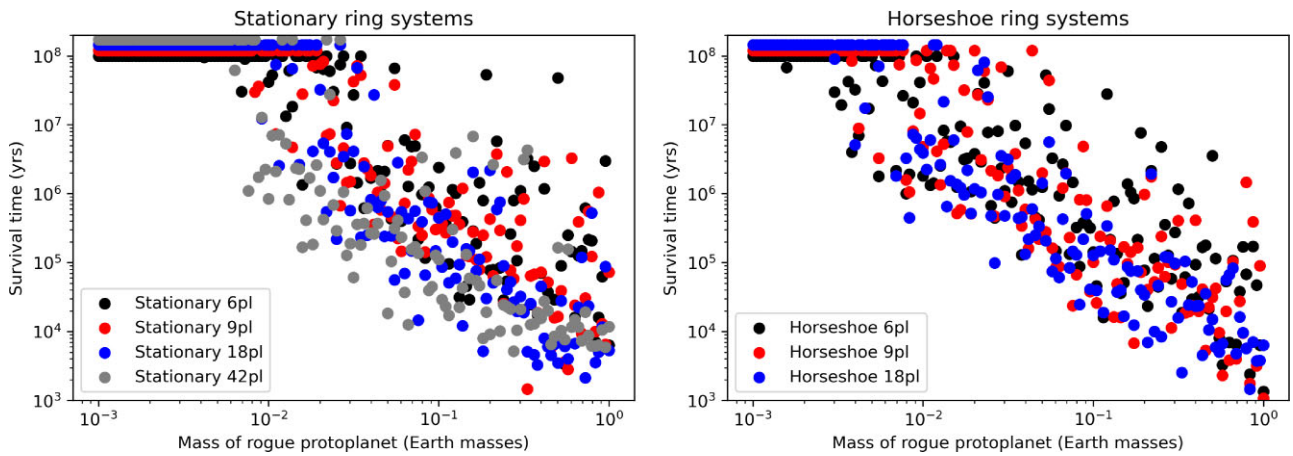
### 2.2 Results

Fig. 1 shows how long each system survived as a function of the mass of the rogue protoplanet, for both stationary (left) and horseshoe (right) systems. This figure makes three points. First, the critical mass above which a rogue protoplanet disrupts rings of co-orbital planets is close to a lunar mass ( $\sim 0.01 M_{\oplus}$ ). Second, the number of planets in a given co-orbital ring has little to no effect on the ring's stability. Stationary ring systems with 6, 9, 18, and 42 planets remained stable with rogue protoplanet masses of 0.0347, 0.0191, 0.0263, and  $0.0263 M_{\oplus}$ , respectively. Among horseshoe constellation systems with 6, 9, and 18 planets the maximum stable rogue protoplanet masses were 0.0151, 0.0437, and  $0.0120 M_{\oplus}$ , respectively. Finally, stationary ring systems are more resistant to perturbations than horseshoe systems. While the maximum rogue protoplanet mass that allowed for stability was similar between the horseshoe and stationary ring systems, more than three times more simulations with  $M_{\text{Rogue}} > 0.01 M_{\oplus}$  remained stable in stationary systems than in horseshoe systems, even if only considering the cases with 6, 9, or 18 planets.

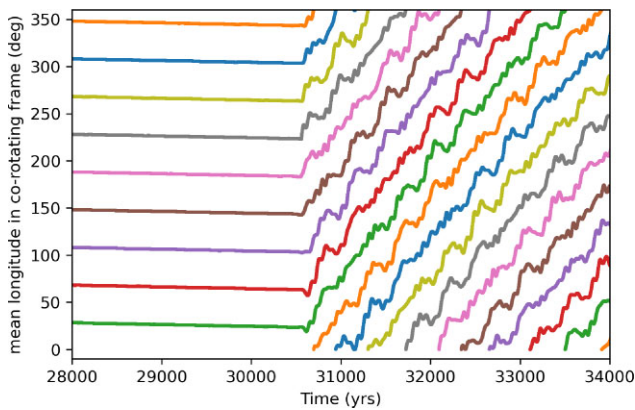
After being perturbed, the dynamics of horseshoe constellations does not change appreciably (at least, among those that remain stable). However, the dynamics of stationary rings can change significantly. Fig. 2 shows a 9-planet stationary ring being perturbed by a lunar-mass ( $0.12 M_{\oplus}$ ) protoplanet. Before any significant close encounters take place, the planets remain evenly spread out in longitude. However, after a strong close encounter between the rogue protoplanet and one planet in the ring after 30 500 yr, the system transitions into a horseshoe constellation state. The planets all precess much faster than in the stationary configuration, and undergo periodic mutual encounters that cause horseshoe librations (visible as ‘wiggles’ in Fig. 2). This change in orbital configuration can also be seen in the planets' orbital semimajor axes. The semimajor axes of planets in stationary rings remain perfectly constant in time. However, each horseshoe oscillation causes an exchange in orbital angular momentum such that the semimajor axes of planets in the horseshoe regime are not fixed in time but rather oscillate. The dynamics of horseshoe oscillations, and the transition between the tadpole and horseshoe regimes in the restricted case with two planets, depend largely on the Jacobi constant (see Dermott & Murray 1981a; Murray & Dermott 1999). In our simulations we find a gradient of outcomes between stationary rings and horseshoe systems (see below), implying that in the case of many planets there is no sharp transition between the two regimes.

Fig. 3 shows the short-term (100-yr) relative movement of co-orbital rings in two systems that both started off as stationary 9-planet co-orbital rings. The mass of the rogue protoplanet varied by an order of magnitude between the two systems, with  $M_{\text{Rogue}} = 0.001 M_{\oplus}$  (left-hand panel) and  $0.0105 M_{\oplus}$  (right-hand panel). The system with  $M_{\text{Rogue}} = 0.001 M_{\oplus}$  was only weakly perturbed: the planets' radial excursions (and semimajor axis oscillations) remained minimal and the system maintained its stationary configuration. This is the same behaviour seen in the system from Fig. 2 before the rogue protoplanet close encounter at 30 500 yr. In contrast, the system with  $M_{\text{Rogue}} = 0.0105 M_{\oplus}$  was perturbed into a horseshoe constellation system, with significant semimajor axis oscillations and frequent encounters between planets. This is similar to the behaviour of the system from Fig. 2 after the rogue protoplanet close encounter.

Fig. 4 shows the oscillation amplitudes of the semimajor axes of planets in each surviving co-orbital system as a function of the rogue protoplanet mass. The horseshoe systems all have similar amplitudes of oscillation of a little less than 0.01 au, regardless of  $M_{\text{Rogue}}$ . In



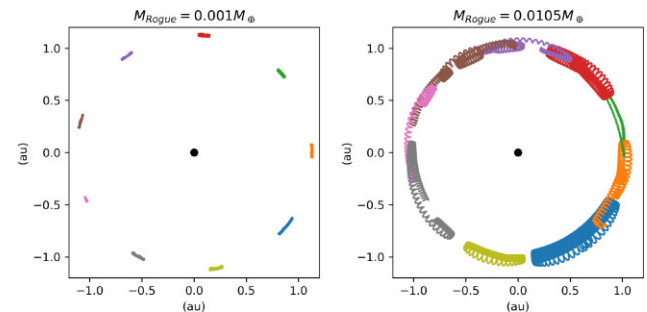
**Figure 1.** Survival of different co-orbital systems as a function of the mass of the rogue planetary embryo that was introduced into the system. The colours correspond to co-orbital systems with different number of planets. Among simulations that survived for the full 100 Myr integration, we introduced small vertical shifts for visibility.



**Figure 2.** Transformation of a stationary co-orbital ring into a horseshoe constellation system. The mean longitude of each planets is shown relative to the mean motion of a planet at 1 au (moving at  $360^\circ \text{yr}^{-1}$ ). The change in dynamical state was triggered by a close encounter between the rogue protoplanet and a planet at  $t \sim 30500$  yr.

contrast, for stationary ring systems the semimajor axis oscillation amplitude is a strong function of  $M_{\text{Rogue}}$ . This is because the strength of encounters with the rogue protoplanet determines the system’s orbital state. There is a continuum of semimajor axis oscillation amplitudes (and frequencies) governed by the closest approaches between planets (and the Jacobi constant; see Dermott & Murray 1981a). This can be explained naturally if the amplitude of semimajor axis oscillation increases for more massive rogue protoplanets.

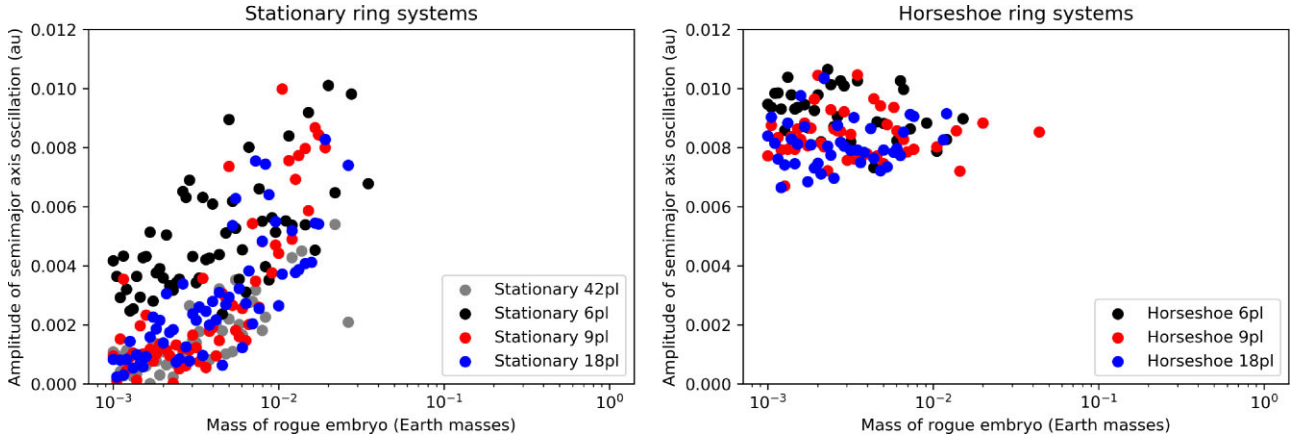
The fact that no surviving systems have semimajor axis oscillation amplitudes larger than  $\sim 0.01$  au is a clue that the patterns from Fig. 4 are the result of survivorship bias: any system that was excited to a higher level of oscillation was destabilized. This makes sense from a dynamical point of view, as higher oscillation amplitudes correlate with closer horseshoe encounters (Dermott & Murray 1981a), yet encounters closer than  $\sim 5$  Hill radii (or  $\sim 4$  mutual Hill radii for near-equal planet masses) result in disruption of the horseshoe system (Ćuk, Hamilton & Holman 2012). Rogue protoplanets with masses larger than a few lunar masses excite unstable horseshoe librations, leading to destabilization on a time-scale given by the



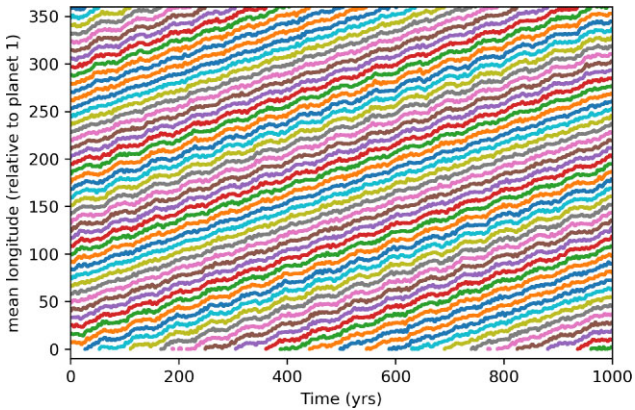
**Figure 3.** Two 9-planet ring systems that survived for 100 Myr but ended up in different dynamical states. Each panel shows the positions of each of the planets in the  $x$ - $y$  plane of their orbit over a  $\sim 100$ -yr time span as viewed in a frame that is co-moving at the mean motion of an isolated planet at 1 au. The radial excursion of the planets has been enhanced by a factor of ten. The simulation on the left was barely perturbed by its  $0.001 M_{\oplus}$  rogue embryo and remains in a near-stationary state. In contrast, the simulation on the right was perturbed by its roughly Moon-mass ( $0.0105 M_{\oplus}$ ) embryo into a horseshoe constellation.

protoplanet mass (see Fig. 1). Horseshoe systems are more fragile than stationary ring systems simply because stationary ring systems start with zero semimajor axis oscillation amplitudes whereas horseshoes start with significant amplitudes. In both types of systems, these oscillation amplitudes are amplified by perturbations from the rogue protoplanet. Even though individual perturbation events do not strictly always increase the oscillation amplitude (because this depends on the exact geometry of close encounters), horseshoe systems start out closer to the stability limit than stationary ring systems. Indeed, these inherent semimajor axis oscillations – which are a result of the construction of horseshoe constellation systems – are close enough to the stability limit that, although the probability of stability is a strong function of  $M_{\text{Rogue}}$ , the oscillation amplitude is not.

The 42-Earth system is maximally packed for long-term stable stationary rings (Salo & Yoder 1988; Smith & Lissauer 2010). In the co-orbital ring systems presented in Section 2 – with 6, 9, or 18 planets – there exist stable configurations in both the realm of stationary and horseshoe systems. In the 42-planet case, there is no corresponding stable horseshoe constellation system, as those



**Figure 4.** Amplitude of semimajor axis oscillations among systems that were stable for 100 Myr.



**Figure 5.** Evolution of the most strongly perturbed surviving 42-planet ring from our simulations. In this case, the rogue protoplanet mass was  $M_{\text{Rogue}} = 0.0219 M_{\oplus}$ . As in Fig. 2, the mean longitude of each planet is shown relative to the mean motion of a planet at 1 au (moving at  $360^{\circ} \text{ yr}^{-1}$ ). Despite the repeating colours, each continuous curve corresponds to a single planet.

have only been shown to exist with up to 24 planets (Raymond et al. 2023).

The 42-Earth stationary ring systems behave in a very similar way to stationary rings with fewer planets. The 42-planet systems are modestly less stable, with instability times that tend to be shorter for a given rogue protoplanet mass (Fig. 1), although the maximum stable rogue protoplanet mass for stability is roughly the same as in other systems.

There are no surviving 42-Earth stationary ring systems with large-amplitude semimajor axis oscillations. The maximum oscillation amplitude for 42-Earth rings is less than 0.006 au, significantly lower than for 6-, 9-, or 18-Earth rings. This is likely a consequence of the instability of horseshoe constellation systems above a given number of planets. Fig. 5 shows the evolution of the most strongly perturbed 42-Earth stationary ring that remained stable. While the mean longitudes of the planets do not evolve in a completely smooth way, each planet moves in concert such that there are no between planets closer than 10 mutual Hill radii. While the planets’ semimajor axes do oscillate, this appears to be due to cumulative relatively distant perturbations rather than horseshoe-type encounters.

### 3 DISCUSSION

#### 3.1 Implications for the detectability of co-orbital rings

In previous work we have put forth the idea that a highly advanced civilization could construct a planetary system to act as a cosmic signpost of its presence. We called such systems ‘SETI beacons’. In Clement et al. (2022) we showed how SETI beacons could be made from multiresonant planetary systems in which the period ratios of neighbouring planets produced a sequence of integers that would be recognizable as being non-natural (such as consecutive prime numbers). In Raymond et al. (2023) we proposed that horseshoe constellation systems with many planets may represent potential SETI beacons.

Stationary co-orbital rings of planets represent another flavor of SETI beacon. Yet, given the results of this study, we might expect horseshoe constellation systems to be more common than stationary rings. The reason is simply that there is nowhere in the Universe that is immune from gravitational perturbations. If one could construct a finely tuned stationary co-orbital system in isolation, it would invariably be affected by other nearby objects. It may be advantageous to build such a system late in a star’s lifetime, as the likelihood of perturbation would decrease in time as leftover planetesimals were dynamically and collisionally removed. Note that in Raymond et al. (2023) we simulated the evolution of co-orbital rings during post-main-sequence evolution of their (Sun-like) host stars, and showed that they survive and maintain their dynamical configurations (for more on the effects of post-main-sequence evolution on planetary systems, see Veras et al. 2011). Perhaps the most protected cosmic environment within reach of detection would be relatively close to a star on the outskirts of a galaxy (not too close or star–planet tidal effects might compromise dynamical stability; Rodríguez, Giuppone & Michtchenko 2013; Leleu, Robutel & Correia 2015). Yet the whole point of a SETI beacon is to be long-lasting, and even exceedingly rare perturbations will occur given sufficient time (and this applies regardless of the exact orbital configuration of the system). For instance, our Solar System will likely undergo a strong dynamical instability triggered by the close flyby of a star after the Sun has become a white dwarf (Zink, Batygin & Adams 2020). A carefully selected galactic environment could reduce the chances or the magnitude of external perturbations (although the Galaxy’s stellar environment will change drastically after the merger between the Milky Way and M31; e.g. Cox & Loeb 2008). Yet even relatively modest

perturbations (akin to the gravitational influence of a Moon-sized rogue protoplanet) would transform a stationary ring system into a horseshoe constellation.

### 3.2 Limitations

This study was admittedly simplified and limited. The parameter space of co-orbital planets is vast (see, for example, Laughlin & Chambers 2002; Giuppone, Benítez-Llambay & Beaugé 2012, for a discussion), and our study only covers a narrow region. Given the relatively simple, proof-of-concept nature of this work we did not think that there was much to gain by testing other system parameters such as the planet mass and orbital radius or the rogue protoplanet’s mass and orbital characteristics. This choice means that we limited ourselves to relatively short, intense perturbations and did not consider slower, more continuous ones such as those generated by a population of rogue planetesimals (see Raymond et al. 2022), other planets orbiting the same star, or even Galactic tides (for wide-orbit co-orbital rings). Tidal dissipation may also be important in co-orbital systems, especially on long time-scales or in systems that are relatively close to their host stars (Rodríguez et al. 2013; Leleu et al. 2015). In addition, cross-tides’, whereby one planet is torqued by the tidal bulge generated on the star by another planet (Touma & Wisdom 1994; Neron de Surgy & Laskar 1997; Lainey et al. 2017), may also be worth taking into account in future studies, as well as planet–planet tides (Wright 2018).

Finally, we emphasize that even though co-orbital rings of planets may be extremely rare, it would be a monumental event to discover such a system. They may be within the reach of current and upcoming exoplanet surveys such as TESS and PLATO (e.g. Barclay, Pepper & Quintana 2018), and we strongly encourage observers to keep them in mind when analysing unexpected signals.

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### DATA AVAILABILITY

All simulations and analysis code in this paper will be made available upon reasonable request.

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