



HAL
open science

Fraction Knowledge in Adults With Persistent Mathematics Difficulties

Parnika Bhatia, Jessica Léone, Marie-Line Gardes, Jérôme Prado

► **To cite this version:**

Parnika Bhatia, Jessica Léone, Marie-Line Gardes, Jérôme Prado. Fraction Knowledge in Adults With Persistent Mathematics Difficulties. *Learning Disability Quarterly*, In press, pp.073194872311713. 10.1177/07319487231171380 . hal-04236417

HAL Id: hal-04236417

<https://hal.science/hal-04236417>

Submitted on 11 Oct 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Fraction knowledge in adults with persistent mathematics difficulties

Parnika Bhatia¹, Jessica Léone¹, Marie-Line Gardes^{2*}, & Jérôme Prado^{1*}

¹ Lyon Neuroscience Research Center (CRNL), INSERM U1028 - CNRS UMR5292, University of Lyon, France

² Haute école pédagogique (HEP) Vaud, UER MS, Avenue de Cour 33 – 1014, Lausanne, Switzerland

* These authors shared senior authorship

Author Contact Information

Parnika Bhatia

Lyon Neuroscience Research Center, CH Le Vinatier, 95 bd Pinel, 69675 Bron Cedex, France

parnika.bhatia@etu.univ-lyon1.fr

Abstract

Fractions are challenging for both typically achieving children and adults. Although some prior research has focused on fraction difficulties of children with mathematics difficulties (MD), persistent difficulties encountered by adults with MD remain unknown. It is possible that these adults may be able to compensate for some deficits. Here, we administered an un-timed, paper-based fraction achievement test to adults with and without MD to compare their knowledge of fractions. Compared to controls, adults with MD performed worse in fraction number lines, fraction concepts, fraction arithmetic, and word problems. However, no difference in performance between the two groups was observed on symbolic representations. This suggests that adults with MD might be able to perform rote procedures such as transcoding from a verbal to a symbolic representation but are severely impaired for fraction number line, fraction concept, and fraction arithmetic. Exploratory error pattern analyses for fraction number line and fraction arithmetic further revealed mistakes similar to those observed in prior studies on children with MD, indicating core deficits in fraction understanding in individuals with MD.

Keywords: adults, mathematics difficulties, fraction knowledge

Despite their importance and widespread usage, fractions are typically hard to grasp for adults and children across the world (Chan, Leu, & Chen, 2007; Ni, 2001; Yoshida & Sawano, 2002). First, fractions are difficult because of the complexity of their sub-constructs (ratio, measure, part-whole, and operator as conceived by Kieren, 1980) and the limited understanding of the part-whole model (Pitkethly & Hunting, 1996). For instance, frequently used part-whole relationships in school may hinder the acquisition of other concepts such as improper fractions, fair shares, and, the property of infinite divisibility of fractions (Behr et al., 1983; Pitkethly & Hunting, 1996 as in Misquitta, 2011). Second, because the bipartite structure of fractions (two natural numbers separated by a horizontal line) is different from that of whole numbers, learners may struggle to understand the overall magnitude of fractions (Hiebert, 1985). Instead, they might overly rely on fraction components (numerator and denominator) to estimate fractional magnitude. This phenomenon, known as the “whole-number bias”, often leads to errors in problems that require holistic processing of fractions (Ni & Zhou, 2005; Vamvakoussi et al., 2012; Van Hoof et al., 2013). Third, the procedures required to solve fraction arithmetic are varied and complex (Lortie-Forgues et al., 2015). For instance, adding fractions with common denominators (e.g., $2/6 + 3/6$) requires one to maintain the denominator constant and add the numerator, whereas multiplying the same fractions (e.g., $2/6 \times 3/6$) requires one to multiply both the numerator and denominator. Thus, arithmetic procedures may be an important source of difficulty for individuals when manipulating fractions.

Mathematics difficulties and fractions

As should be clear from the review above, learning and understanding fractions are difficult for most typically achieving (TA) children and adults. Yet, fractions might be even more difficult for individuals with mathematics difficulties (MD), which represents both students with

low performance as well as students with a diagnosed math disability (Nelson & Powell, 2017). Indeed, a growing number of studies suggest that children who experience MD appear to struggle with many aspects of fraction skills, including fraction concepts, arithmetic, estimation of fraction magnitudes, and word problems (Bailey et al., 2015; Hansen et al., 2017; Hecht & Vagi, 2010; Hunt, 2015; Ikhwanudin et al., 2019; Mazzocco & Devlin, 2008; Mazzocco et al., 2013; Malone & Fuchs, 2017; Morano et al., 2019; Newton et al., 2014). These studies provide important insights into the general relationship between fraction knowledge and low mathematics achievement.

For instance, Mazzocco and colleagues (2013) noted that students with MD struggled in a fraction magnitude comparison task until grade 7 as compared to TA and low achieving students who had achieved ceiling effects for accuracy by grades 5 and 4, respectively. Additionally, children with MD find it difficult to identify equivalent ratios, name decimals, rank order fractions, and decimals, and consistently prefer visual models to symbolic notations to represent fractions (Mazzocco & Devlin, 2008). In another study, children with MD were found to have poorer performance than TA peers in tasks involving fraction arithmetic, fraction estimation, and fraction word problems (Hecht & Vagi, 2010). At-risk fourth graders also applied whole number logic to fractions leading to errors on fraction ordering tasks (Malone & Fuchs., 2016). Finally, not only do students with MD perform poorly on fraction-based tasks, but they also start and stay far behind their TA peers in school. For example, using growth trajectory analyses, Hansen et al. (2017) showed slow growth for students with MD on fraction concepts. This slow growth further adds to the achievement gap by the time students reach high school. Therefore, there is emerging evidence that children with MD significantly struggle with fraction knowledge.

To our knowledge, there is very limited quantitative research on fraction skills in adults with MD. One recent study noted that adults with persistent MD perform poorly on a timed-match to sample fraction task as compared to controls (Bhatia et al., 2020). In another study on two adults with MD, the error patterns on fraction tasks were shown to be similar to those obtained for students in the study by Mazzocco et al. (2013) (Lewis, 2016). However, there are two main limitations of the abovementioned studies; (i) in the first study, the task employed was timed and the focus was on the property of equivalence/matching which could have been achieved using estimation strategies, (ii) the second study had a very limited sample size ($n=2$) and focused on specific misunderstandings of adults with MD. Thus, both these studies fail to provide a detailed exploration of the struggles faced by adults with persistent MD on different types of fraction competencies. Given the persistence of mathematics difficulty in adult life and the importance of fraction knowledge for academic and professional success, it is imperative to improve our understanding of the fraction difficulties faced by adults with MD.

A deficit in fraction magnitude knowledge as the main cause for fraction impairments in individuals with mathematics difficulties?

The integrated theory of numerical development (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014) suggests that a major source of fraction difficulties in those individuals might be a poor grasp of fraction magnitudes. Indeed, fraction magnitude knowledge may play a critical role in learning fraction concepts and arithmetic (Tian & Siegler, 2017). As emphasized by Tian and Siegler (2017), “numerical development involves learning about the characteristics that unite all types of real numbers as well as the characteristics that differentiate them” (p. 615, Tian & Siegler, 2017). Notably, the main feature uniting whole numbers and rational numbers is that both represent numerical magnitudes and, as a result, can be mapped onto number lines. Thus,

the theory posits that difficulties with many fraction concepts and procedures may be traced back to a poor understanding of fraction magnitudes.

Studies typically measure fraction magnitude knowledge using number line estimation tasks, in which fractions have to be mapped onto lines (usually marked from 0-1 or 0-5) (Jordan et al., 2017). Consistent with the integrated theory of numerical development, these studies have found that children with mathematics difficulties tend to exhibit poorer performance than their TA peers on fraction number line tasks. For example, students with MD (who score below 35% on state assessment tests) were less accurate than TA students on fraction number line estimation tasks (0-1 and 0-5) (Siegler & Pyke, 2013). In a recent study, students with diagnosed learning disabilities also showed poor performance on number line estimation tasks as compared to circular models (measure part-whole understanding) (Morano et al., 2019). The number line estimation task also showed stronger relation with mathematics achievement and fraction magnitude comparison tasks than circular models, indicating that the part-whole knowledge usually acquired through circle models might be easier to develop but magnitude knowledge (number line tasks) is more important for learning of fraction concepts (Morano et al., 2019). Intervention studies also show that practicing number line tasks may lead to improved fraction knowledge in individuals with MD (Barbieri et al., 2019; Fuchs et al., 2013; Fuchs et al., 2016; Saxe et al., 2013). Therefore, consistent with the integrated theory of numerical development, several lines of research suggest that limited understanding of fraction magnitude may be at the core of struggles with fractions in individuals with MD (Amalina, 2019; Morano & Riccomini, 2019; Tian & Siegler, 2017).

Rationale of the Current Study

Previous studies that examined the relationship between fraction skills and MD, however, have at least two main limitations. First, prior studies almost exclusively focused on children who are in the process of learning about fractions in school. This raises the possibility that at least some identified fraction difficulties might not necessarily be persistent. For instance, adults might be able to develop compensatory strategies that might mask some of the difficulties identified in children (Lewis & Lynn, 2018a). This is suggested by both neuroimaging and behavioral studies. For instance, Cappelletti and Price (2014) found adults with MD (as compared to TA adults) to exhibit stronger frontal activations associated with faster response latencies, suggesting compensatory mechanisms to make up for the inefficient activation in the number-related parietal regions (Cappelletti & Price, 2014). This echoes with compensation mechanisms that may be used in adults with reading difficulties (Hancock et al., 2017). Therefore, it is important to perform research on skills of adults (not only children) with MD, as they may have different profiles of difficulties (Lewis & Lynn, 2018b). Second, previous studies on adults with MD largely focused on specific fraction competencies and a trial-by-trial analysis of their difficulties (Lewis, 2016; Lewis & Lynn, 2018). These studies did not necessarily provide a broad assessment of the difficulties that adults with MD exhibit on different fraction competencies taught at school and required for understanding higher mathematical concepts. In other words, a quantitative analysis of the performance of adults with persistent MD on a variety of fraction skills remains necessary to determine which skills remain impaired, and which skills can be compensated for.

To fill this gap in the literature, we designed a paper-based fraction achievement test that assessed fraction knowledge. The test was based on the French curriculum standards for fraction learning. In France, fractions are introduced to students in 4th grade as one of the first rational

numbers to aid in the holistic development of mathematical knowledge. During elementary school (4th and 5th grade), students are introduced to the part-whole and measure concepts of fractions. The fractions studied during this phase are relatively simple (e.g. half, third, quarter, fifth, sixth, tenth, hundredths). Fourth and fifth graders notably work on transcoding different representations of fractions, understanding equivalence, comparing fractions with the same denominator, and adding fractions with the same denominators. Starting in 6th grade, students are taught to use fractions as quotients and work on increasingly complex problems involving how to reduce a fraction, compare them, and manipulate them in arithmetic problems. Other aspects of fractions such as ratios and use as operators on other numbers are introduced in 7th grade. The fraction achievement test designed in the lab encompassed almost all fraction skills that are learned in the French school system (and are required for proficiency in fractions). It notably assessed four major competencies that have also been evaluated in Rodrigues, Jordan, and Hansen, (2019): fraction concepts, fraction arithmetic, fraction number line, and word problems. In addition to these competencies, we also assessed the symbolic representation of fractions (i.e., transcoding between Arabic numbers and numeral). This was because transcoding skills are important to the development of mathematical skills (Geary et al., 2000; Moeller et al., 2011; Moura et al., 2015) and emphasized in the French national curriculum standards (see above). The correspondence between the French national curriculum standards for fractions and each question in the fraction achievement test is shown on Supplementary Table 1.

Here we used this assessment of fraction skills to compare the performance of adults with MD to TA adults who were matched for age, reading skill, and verbal IQ. We specifically aimed to test which fraction competencies remained the most impaired in adults with MD, despite years spent following the same standardized fraction curriculum in school. Consistent with prior

studies in children with MD, we expected that adults with MD would be impaired in many of these competencies, though adults might be able to mask some impairments with compensatory strategies (as compared to children). According to the integrated theory of numerical development, one would notably expect that adults with MD might still show important deficits in fraction number line estimation, as difficulties with understanding fraction magnitude have been posited to be the main source of fraction impairments in children with mathematics difficulties.

Method

Participants

Twenty typically achieving participants ($M = 22.4$, range = 19-29, 10 females) were recruited through generic advertisements on social media, a mailing list sent to several universities in Lyon, France, and by word of mouth. Twenty-two participants with MD were recruited in 3 different ways: (i) through the “mission handicap” program at the University of Lyon (a program aimed at helping college students with disabilities), (ii) by sending a targeted email to neuropsychologists and clinicians in the Lyon area, and (iii) by advertisements on social media targeting adults who either had been formally diagnosed with developmental dyscalculia or had experienced MD. However, only 18 out of these 22 participants met our criteria for MD (see below) ($M = 22.9$, range = 19-28, 16 females). The performance of a subsample of these MD participants ($n=13$) on a more restricted fraction match-to-sample task was already analyzed in Author (year). All participants were native French speakers and did not report a prior history of neurological disease or psychiatric disorders. Written informed consent was given by each participant before the experiment. Participants were given 30 euros for participating in the study.

The experiment was performed in accordance with the ethical standards established by the Declaration of Helsinki (see Table 1).

Procedure

Data collection (approximately 3 hours) was done in the laboratory. Participants were administered various tests to assess their cognitive, mathematical, and reading skills. Verbal IQ and spatial IQ were estimated using the verbal reasoning and matrix reasoning subtest of the WAIS-IV (Wechsler, 2008). Mathematics skills were assessed using 2 subtests of the 3rd version of the Woodcock-Johnson Test of Achievement (WJ-III) (Woodcock, Mather, McGrew, & Wendling, 2001): Mathematics Fluency and Applied Problems. The Mathematics Fluency subtest is a timed test in which participants have to solve as many single-digit additions, subtraction, multiplication, and division problems as they can within 3 min. The Applied Problems subtest is un-timed and measures the ability to analyze basic numerical concepts and oral word problems. This test stops after 6 consecutive errors or when the last item is reached. We chose to use the 3rd version of the WJ rather than the more recent 4th version because only the former has been translated in French (Schwartz et al., 2018, 2020). Reading fluency was assessed with the Alouette-R test (Lefavrais, 1967). This test requires participants to read a 265-word text aloud in 3 minutes and measures the number of words read, the time required, and the number of pronunciation errors to evaluate the reading speed and efficiency, respectively. Finally, participants were administered a paper-pencil based, un-timed, Fraction Achievement Test developed in the lab (see below). A 10-minute break was given between the tests such that the WAIS-IV and WJ-III were carried out before the break and Alouette and Fraction Achievement tests were carried out after the break.

Criteria for defining MD

To be classified as having MD, participants had to have been diagnosed by a clinical specialist and/or complained of mathematics difficulties since school. They also had to perform (i) above the 25th percentile on the verbal reasoning subtest of the WAIS-IV (therefore indicating normal verbal IQ), (ii) at or below the 10th percentile on at least one of the mathematics sub-tests of fluency and applied problems and (iii) below the 25th percentile on the average of the mathematics sub-tests of fluency and applied problems. Using these two mathematics subtests allowed us to assess a range of mathematical skills, such as single-digit arithmetic, calculating money, time, applying basic formulas (Pythagoras theorems), and equation solving. Further, the use of both a stringent (i.e., 10th percentile on at least one subtest) and more lenient (25th percentile on both subtests) cutoff criteria allowed us to adopt a relatively strict definition of MD while maximizing sample size (Geary, 2004; Murphy et al., 2007). Four participants diagnosed by clinical specialists did not have a score below the 10th percentile on either of the sub-tests and were excluded from the analysis. Of the 18 MD participants, 7 participants scored below the 10th percentile on both the mathematics subtests and 9 participants scored below the 10th percentile on only the mathematics fluency subtest. All 18 participants scored below the 25th percentile on the average of both the subtests. Additionally, the 18 MD participants also had average to superior verbal IQ, as reflected by standardized scores ranging from the 25th percentile (i.e., 90) to the 98th percentile (i.e., 130) on the verbal reasoning subtest of the WAIS-IV.

Of the 18 participants included in the MD group, 9 had been diagnosed by a clinical specialist (speech therapist, occupational therapist, and/or neuropsychologist) using a variety of behavioral assessments at one or more time points before the age of 18. The remaining 9 participants reported that either they or their teachers (or professors) suspected they had MD. They were interviewed by a clinical specialist (i.e., neuropsychologist), who notably asked these

participants about their onset of mathematics difficulties and the different areas in which they were impacted in their daily lives. The participants complained of mathematics difficulties since grade 4th/ 5th in the school and reported difficulties in performing one or more of the basic daily activities such as grocery shopping, banking, reading analog time, and filing taxes.

All participants had reading scores above the cutoff criterion for dyslexia (i.e., reading speed score above 8.7 and/or reading efficiency score above 402.26) (Cavalli et al., 2018).

Fraction Achievement Test

The Fraction Achievement Test included a total of 24 questions with different items/sub-parts assessing 5 main competencies. Example questions are shown in Figure 1 and the entire translated version of the test is included in the Appendix.

First, fraction concepts (Figure 1A) were measured using a total of 13 questions (question no. 1, 2, 3, 4a, 4b, 5, 6, 11, 12, 13, 16, 17, 18). The items assessed part-whole understanding of area models, set models, equivalence, comparing fractions, ordering fractions, and, mixed fractions. For example, the part-whole understanding task included items where participants had to construct and color fractions like $\frac{1}{2}$ and $\frac{5}{4}$ (Q4a, b), circle $\frac{2}{5}$ of the figure (Q.5). The equivalence task included seven items (e.g. $\frac{3}{4}=\frac{4}{5}$, $\frac{3}{5}=\frac{35}{10}$, $\frac{2}{1}=1$, $\frac{3}{2}=\frac{6}{4}$, $\frac{4}{4}=1$, $\frac{5}{3}=\frac{3}{5}$, $\frac{2}{3}=\frac{12}{13}$; Q.12) where participants had to choose the correct pair. Second, fraction arithmetic skills (Figure 1B) were measured using 4 questions (Q.14, 15, 19, 20). Each question had 3 to 5 items and participants were presented with an addition problem written in symbolic form. Problems consisted of proper, improper fractions, and whole numbers (e.g. $4 + \frac{1}{2}$, $1 + \frac{7}{2}$, $\frac{3}{4} + \frac{5}{4}$, $2 + \frac{2}{3}$). Third, the symbolic representation of fractions (Figure 1C) was tested using 2 types of questions (Q. 7, 8) consisting of 4 items each. The first type was identifying the verbal

representations of fractions (e.g. three halves) and writing the symbolic form ($\frac{3}{2}$). The second type was identifying the symbolic form and writing verbal representation. Fourth, the fraction number line (Figure 1D) was assessed using two questions (Q. 9, 10) with three items each. Three items involved placing three fractions on the number line (e.g. $\frac{8}{5}$, $\frac{6}{10}$, $\frac{16}{5}$; Q.9), and the other three involved finding the three fractions marked on the number line. Fifth, word problem-solving skills (Figure 1E) were measured using four-word problems (Q. 21, 22, 23, 24). For example, one of the problems read, ‘Assia read 24 pages of an adventure book. Tom read one-third of it. How many pages did Tom read?’. Cronbach’s α ranged from 0.771 to 0.965 across all 5 measures (fraction concepts: 0.860, arithmetic skills: 0.965, symbolic representation: 0.944, number line: 0.873, word problems: 0.771), indicating acceptable to excellent internal consistency. The inter-rater reliability for categorization of questions by three independent researchers for all the above measures was very good (Cohen’s kappa = 0.84). All fractions were presented vertically with a horizontal bar between the numerator and denominator (a notation used in the French curriculum). **[insert Figure 1 here]**

Scoring test items

The fraction achievement test was scored using a template with correct answers by a neuropsychologist and rechecked by two other researchers. The data entry was also checked independently by these two researchers to identify incorrect entries as well as correction errors on the paper-based tests. Any discrepancy in scoring or data entry was discussed among the three coders and if one of the coders was not convinced the item was marked for rechecking by a researcher in mathematics education in the lab. The inter-rater reliability between the final two researchers was very strong (Cohen’s kappa = 1). For each item, the correct response was scored

1 and the incorrect/ no response (marked as ‘do not know/?’ by the participant) was scored 0. The percentage correct was calculated for each of the five competencies.

Error Pattern Analysis

An exploratory follow-up analysis was carried out to examine the errors made by adults with MD for the competencies that differed significantly between the two groups (i.e., fraction arithmetic, fraction number line, and word problems). For each of the items attempted, the errors were recorded in detail for each participant. Identifiable error patterns with maximum frequency were then examined and compared to prior literature on fraction knowledge in children with MD. This was done to identify the most common and persistent difficulties encountered by individuals with MD. For instance, prior literature on children with MD notes difficulties in ordering fractions, using whole number strategies for fraction arithmetic, and the inability to represent fractions on the number line (Mazzocco & Devlin, 2008; Siegler & Pyke, 2013; Morano et al., 2019). While prior literature informs some common misconceptions in both children and adults with MD, this analysis is posteriori and the findings should be interpreted with caution. Additionally, some items could not be examined for errors as either they were not attempted (marked as “do not know/?”) or the steps for the computation were not shown by the participants. The number of adults with MD that exhibited the error pattern, did not attempt the questions, did not list the steps of computation, and/or, solved the questions correctly have been mentioned in the results.

Analyses

Statistical analyses were conducted using the Jamovi software (www.jamovi.org/). Shapiro Wilk W tests indicated that none of the fraction measures except fraction concepts (i.e.,

fraction arithmetic, symbolic representation, fraction number line, and, word problems) were normally distributed (all $ps < 0.001$). Nonetheless, because ANOVAs are known to be robust to violations of normality given our sample sizes (Schmider et al., 2010), percentage scores associated with all fraction competencies were analyzed using ANOVAs with the between-subject factor Group (MD and TA).

Data availability

The test (in the original French and a translated version in English), as well as the scored data and the anonymized scan of the questionnaire for each participant, are available via the Open Science Foundation (OSF) at https://osf.io/dhkzc/?view_only=4656f9c57a9144699ebe1cf1e3646805.

Results

Demographics and psychometric measures

A preliminary analysis was conducted to check for between-group differences in background measures (see Table 2). There was no significant age difference between the two groups ($F(1, 36) = 0.17, p = 0.67$). Although the groups differed in terms of matrix reasoning ($F(1, 36) = 19.33, p < 0.001$), there was no significant difference in terms of verbal reasoning ($F(1, 36) = 2.11, p = 0.15$). We also found no significant differences for reading efficiency ($F(1, 36) = 2.72, p = 0.108$) and reading speed ($F(1, 36) = 2.75, p = 0.106$) between the two groups. Thus, the groups were matched in terms of age, verbal IQ, and reading skills. Additionally, there were significant differences between the Woodcock-Johnson III subtests of mathematics fluency ($F(1, 36) = 98.40, p < 0.001$), and applied problems ($F(1, 36) = 85.81, p < 0.001$) between

adults with and without MD. Thus, individuals with MD had lower mathematics skills than TA participants.

Fraction knowledge

Scores for each fraction competency as a function of the group are displayed in Table 3. Scores associated with each fraction competency were analyzed using between-subjects ANOVA. A statistically significant effect of Group was observed for fraction concept, ($F(1, 36) = 40.84, p < 0.001, \eta^2p = 0.53$); fraction arithmetic, ($F(1, 36) = 89.29, p < 0.001, \eta^2p = 0.71$); fraction number lines, ($F(1, 36) = 29.97, p < 0.001, \eta^2p = 0.45$); and word problems, ($F(1, 36) = 43.14, p < 0.001, \eta^2p = 0.54$). This indicated lower scores for the MD than for the TA group. However, there was no significant difference between TA and MD participants for symbolic representation ($F(1, 36) = 0.72, p = 0.402, \eta^2p = 0.02$). Thus, differences between MD and TA individuals were specific to some competencies. **[insert Figure 2 here]**

Exploratory analysis of error patterns of adults with MD

An in-depth exploration of error patterns of adults with MD was also conducted for items related to the fraction number line, arithmetic, and word problems. The distributions of the most consistent error patterns for the MD group are shown in Figure 3. **[insert Figure 3 here]**.

Fraction Number Line

Fraction number line knowledge was tested on Q. 9 and Q. 10. In Q.9, participants were asked to place the fractions $\frac{8}{5}, \frac{6}{10}, \frac{16}{5}$ on a graduated 0-4 number line consisting of four units (0-1, 1-2, 2-3, 3-4) segmented into 5 parts each. Six adults with MD “did not know” how to solve all three items on Q.9 and hence, did not attempt the question. Of the remaining 12 adults, six adults

struggled to place the fraction $\frac{6}{10}$ on the number line. The most common errors that could be interpreted pertained to finding the equivalent fraction $\frac{3}{5}$. This might have led to the participants placing the fraction at either less than or more than 1 mark, revealing an inconsistency in understanding the magnitude $\frac{6}{10}$. Four participants out of these six were able to place the other two fractions correctly. Thus, revealing an understanding of magnitude when the fraction and the parts on the number line were the same or equal. Because the participants were not asked about their reasoning behind placing the fractions these errors are mostly interpreted by comparing participants' responses on other items. Therefore, these exploratory findings should be taken with caution. In Q.10, participants had to find the fraction corresponding to the points $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{9}{4}$ on a 0-3 number line with sub-units segmented into 4 parts. Six adults with MD did not attempt the question and two adults solved all the items correctly. Of the remaining 10 participants, three mistakenly counted the tick mark as a unit on the number line similar to the unit of a ruler. This faulty strategy led adults with MD to count the tick marks starting with 0 (instead of counting the number of parts divided by the tick marks) which resulted in fractions with a denominator of 5 instead of 4 on the 0-3 number line (segmented in 4 parts) (see Zhang, Stecker, & Beqiri, 2017 for details on ruler-tick-mark-counting strategy). Others (n = 4) counted the total number of parts as twelve on the 0-3 number line (sub-units segmented in 4 parts) and marked the points as multiples of one-twelfth indicating that the participants might have erred at making the distinction between 0-3 number line made up of three 0-1 units leading them to count the total number of parts as twelve instead of four. This finding is in line with prior research showing students struggling with multiple levels of units (Boyce & Norton, 2016; Hackenberg, 2013; Hackenberg & Lee, 2015 as in Zhang et al., 2017). For the rest of the participants (n=3), the errors could not be identified.

Fraction Arithmetic

Fraction arithmetic problems were of four types: (1) Converting improper fractions to whole numbers and fractions less than 1 (2) Addition of whole numbers to fractions (3) Addition of fractions with like denominators and (4) Addition of fractions with unlike denominators. For type 1, only two adults were able to solve all the items correctly and six adults did not attempt any of the items. Of the remaining 10 participants, six adults with MD showed a classical error (whole number bias) by working separately with the components of the fraction. For example, to convert $\frac{10}{3}$ the participant decomposed the numerator 10 to the whole number 8 and fraction $\frac{2}{3}$, without considering the role of the denominator ($\frac{10}{3} = 8 + \frac{2}{3}$). One adult with MD added all the numbers together ($\frac{10}{3} = 13$) indicating the misunderstanding about fractions as a distinct number form. The remaining three adults either solved the problems incompletely or made errors that could not be deciphered. For type 2, six adults did not attempt the question and four adults were able to solve one or more items correctly revealing an understanding of the addition of whole numbers to fractions. Of the remaining eight adults with MD, six adults added the whole number to the numerator (e.g. $4 + \frac{1}{2} = \frac{5}{2}$). One adult added the whole number arbitrarily to either the numerator or the denominator or both (e.g. $4 + \frac{1}{2} = \frac{5}{3}$) showing the classical whole number bias by operating on the components of fractions separately, revealing a lack of understanding of holistic fraction magnitudes. The addition of fractions with like and unlike denominators (type 3 and 4) was also challenging for adults with MD. Two adults with MD did not attempt both types of problems. 10 adults with MD were able to add the fractions with like denominators and four were able to add with unlike denominators correctly. Six adults with MD were not able to solve the addition of like denominators, five adults added the numerators and denominators across the

two fractions (e.g. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{2b}$) and one showed a non-classical error. For the unlike denominator items, eight adults with MD added the numerators and denominators separately (e.g. $\frac{2}{2} + \frac{1}{6} = \frac{3}{9}$). Interestingly, only one adult with MD was able to correctly solve all types of fraction arithmetic problems.

Word Problems

For word problems, the most common errors were found in calculations and the use of wrong operations. Nine adults with MD made a calculation error and two among these also showed difficulty in the choice of operation utilized. Five adults with MD were unable to understand “one-third of 24”. The rest of the participants did not show their computation steps and so, their error patterns were unidentifiable. Except for three adults with MD, all the others were able to solve one or more than one-word problems correctly.

Discussion

In the present study, we used an un-timed paper-based fraction test to compare the fraction skills of adults with and without MD. We found that adults with MD showed large deficits in fraction magnitude knowledge (as measured by the number line task), fraction concepts, and fraction arithmetic as compared to TA adults. Additionally, adults with MD showed poor performance on word problems than TA adults. Interestingly, no difference was observed in symbolic representations, suggesting that adults with MD may be able to compensate for some difficulties with fractions.

Adults with MD show deficits in fraction magnitude knowledge, fraction concepts, fraction arithmetic, and, word problems

Our finding that fraction number lines are one of the most challenging tasks for adults with MD could partially be accounted for by the integrated theory of numerical development. This theory posits that fraction magnitude knowledge is (i) central to the development of fraction learning and (ii) impaired in children with mathematics difficulties (Tian & Siegler, 2017; Siegler et al., 2011). As such, the theory places great emphasis on fraction number lines because number lines are an important tool for measuring fraction magnitude knowledge (Hansen et al., 2017; Jordan et al., 2017). Overall, the theory is supported by studies showing that children with mathematics difficulties often exhibit poor performance on number line tasks (Bailey et al., 2015; Siegler & Pyke, 2013). It is also supported by intervention studies where number lines were used to develop fraction magnitude knowledge in children with mathematics difficulties (Barbieri et al., 2019; Fuchs et al., 2016; Fuchs et al., 2013; Saxe et al., 2013). These studies revealed an improvement in fraction magnitude knowledge (as well as enhanced learning of other fraction competencies) for children with mathematics difficulties.

To our knowledge, however, only one previous quantitative study to date had examined the fraction magnitude knowledge of adults with MD. In Bhatia et al. (2020), adults with dyscalculia were shown to perform poorly on a fraction matching/equivalence task compared to controls, indicating a possible impaired fraction magnitude knowledge. However, the task used in Bhatia et al. (2020) was timed and was limited in that it only assessed fraction or ratio equivalence. Thus, no specific conclusions could be drawn about the type of competencies that were challenging for adults with MD from that study. The present findings significantly extend these previous results by providing a comprehensive survey of fraction impairments in adults with MD.

Additionally, it might have been argued that adults with mathematics difficulties may be able to compensate for their difficulties in fraction magnitude knowledge. Indeed, we found that impaired fraction magnitude knowledge as measured by the number line task in individuals with MD is pervasive and persists through adulthood. Interestingly, the most common errors observed in most of the adults with MD on the fraction number line task were similar to those observed in children in prior studies. For instance, most adults with MD counted either the tick marks (not the segmented parts) or the total number of parts on the entire number line (not the sub-units), which is a common error pattern observed in prior studies on children with MD (Zhang et al., 2017). These overlapping findings might suggest the pervasive challenges on number line tasks where identifying the number of parts and understanding the multiple levels of units on a number line is difficult for both adults and children with MD. Interestingly, a majority of TA adults did not show these error patterns indicating a sound magnitude understanding. This finding adds to the emerging evidence that fraction number line tasks may be an interesting tool to diagnose fraction impairments in both children and adults. For example, a recent study aimed at identifying middle school students who might be at a risk for developing mathematics difficulties found fraction number lines to be an accurate screener for later risk status (Rodrigues et al., 2019).

Individuals with MD also showed lower performance on other fraction magnitude knowledge items such as ordering and comparing fractions as measured by the fraction concept measure. As compared to the TA group, the MD group showed a much larger variability in their performance on fraction concepts as compared to fraction number lines, indicating differential challenge to access fraction magnitudes depending on the type of task. Indeed, a prior study on children with learning disabilities shows persistent difficulties on the fraction number line task as compared to tasks that are comparing fractions (Morano et al., 2019). These findings seem to

suggest that difficulties in fraction knowledge could also potentially be explained by the type of task measuring fraction magnitudes. Past studies on fraction concepts have also shown a weak understanding of similar tasks in students with MD (Hecht & Vagi, 2010; Mazzocco & Devlin, 2008). Therefore, it is not surprising that adults with MD in our study find fraction concepts challenging especially complex concepts like mixed-fractions.

Fraction arithmetic is another domain in which MD participants performed worse than TA participants in our study. Studies on middle school children with mathematics difficulties have previously shown poor performance on fraction arithmetic tasks as compared to TA students (Bailey et al., 2015; Hansen et al., 2017; Ikhwanudin et al., 2019; Morano et al., 2019; Newton et al., 2014). Errors with fraction arithmetic may have different sources. For example, students with mathematics difficulties may select incorrect procedures, such as applying the denominator procedure for the same denominator problems (e.g. $1/3 + 1/3 = 2/6$) (Ikhwanudin et al., 2019; Newton et al., 2014). Adults with mathematics difficulties may also add the numerators and denominators separately (e.g. $1/3 + 2/5 = 3/8$) similar to children with mathematics difficulties (Lewis, 2014). A closer look at the error patterns of MD adults in our study indicates that 44% of the adults added denominators and numerators separately for both common and different denominator problems (e.g. $\frac{a}{b} + \frac{c}{b} = \frac{a+b}{2b}$ or $\frac{a}{b} + \frac{c}{d} = \frac{a+b}{c+d}$). This classical error has also been seen in several studies on children with MD (Ikhwanudin et al., 2019; Newton et al., 2014) and highlights the persistent lack of understanding of fraction procedures in MD populations irrespective of age and experience. Conversely, TA adults in the current study did not commit any classical errors showing a good understanding of fraction procedures.

Lastly, adults with MD showed poor performance in word problems than TA adults. To our knowledge, no prior studies have examined fraction word problem skills in adults with MD.

Our findings are in line with studies on students with MD. For instance, a study in 4th graders found poor performance on fraction word problems for children with MD as compared to TA children (Hecht & Vagi, 2010). Additionally, the study also showed less improvement in word problem skills from 4th to 5th grade in children with MD as compared to TA children. Recently, Hughes & colleagues, 2020 noted difficulties in fraction word problems in fifty-one 4th and 5th graders with MD. About 45% of the students could not give the correct answer to the problem and of those who were accurate, only 1 student was able to explain his/her reasoning. Thus, difficulties with fraction word problems in individuals with MD are likely to originate early in school and persist through adulthood.

Adults with MD do not show severe deficits in symbolic representation

Interestingly, both adults with MD and TA adults show near ceiling performance on symbolic representations of fractions. In other words, adults with MD had no difficulty naming fractions represented in symbolic form and writing the symbolic fractions that were represented in numeral form. To our knowledge, this is the first time that this fraction transcoding competency has been measured in adults with MD. One plausible explanation for the level of performance adults exhibit on this task could be the rote memorization of procedures involved in naming fractions. Thus, indicating that adults with MD are not impaired when it comes to naming fractions and may do so with rote procedures that are not based on an understanding of integrated fraction magnitudes.

Limitations and future research

The current study has limitations that might be addressed in future research. Although the fraction achievement test was matched on competencies to the French national curriculum

standards and categorized similarly to the screener in Rodrigues et al., 2019, it had two drawbacks. First, we initially developed the test for assessing children in late elementary and early middle school. As a result, it did not include problems involving fraction subtraction, multiplication, and division (which are learned in later grades). A limitation is thus that we do not have a clear picture of all fraction arithmetic skills as the findings are limited to fraction addition problems. Second, the high performance of adults with MD on symbolic representations clearly indicates that they did not have issues transcoding the symbolic fractions we presented. Importantly, task difficulty in the task was varied by including an equal proportion of proper and improper fractions (improper fractions are more complex than proper fractions). Nonetheless, these problems were mostly limited to simple one-digit fractions with relatively easy denominators (e.g., thirds and halves). Thus, it is likely that adults with MD might have experienced more difficulty with larger and more difficult fractions.

The sample might also have two limitations. First, our sample size remains limited. Therefore, future studies would need to replicate the present findings with independent samples. Second, we ensured that adults with and without MD were matched for age, reading skills, as well as for the verbal reasoning sub-test on WAIS-IV. Nonetheless, the groups differed significantly in the matrix reasoning subtest. Therefore, in addition to mathematical difficulties, adults with MD in our sample might also have weak non-verbal abstract and spatial reasoning skills (measured by matrix reasoning sub-test).

Lastly, due to our fraction test being quantitative, it was difficult to understand the exact reasoning used to solve different fraction questions by adults with MD. Thus, we were limited in identifying their misunderstandings and compensatory strategies. More generally, although the fraction achievement test gives us an exhaustive picture of the different types of competencies

practiced at school and reveals specific persistent difficulties for adults with MD, it is not a tool designed for understanding the reasoning behind these difficulties. Therefore, future studies might need to systematically examine (i) the relation between nonverbal reasoning abilities and fraction understanding of adults with MD and (ii) an item-wise qualitative assessment of reasoning strategies underlying different fraction competencies.

Implications for practice

The current study highlights the difficulties adults with MD encounter when solving fraction problems. A closer look at the error patterns reveals that the faulty strategies used by adults with MD when solving fraction number lines and fraction arithmetic are similar to prior studies on children with MD (Ikhwanudin et al., 2019; Newton et al., 2014; Zhang et al., 2017). These persistent challenges might be indicative of an underlying deficit in fractions in individuals with developmental dyscalculia.

Finally, our study also indicates that specific fraction competencies could be used as potential diagnostic indicators for identifying MD in middle/ high school students and adult populations. Indeed, one of the reasons for the dearth of research in these populations is the absence of a common or standard instrument to aid in the identification and categorization of MD (Murphy et al., 2007). Interestingly, our results on impairments in fraction number line performance are in line with a study on middle school students that show fraction number lines to be an accurate screener for identifying individuals at risk for dyscalculia (Rodrigues et al., 2019).

Conclusion

In conclusion, this study contributes to the literature on mathematics difficulties by identifying specific, persistent difficulties that adults with MD encounter when solving a variety

of fraction competencies. To our knowledge, this is the first study to compare the performance on fraction competencies between adults with and without MD. Findings suggest that adults with MD face difficulties when solving fraction number lines, fraction concepts, fraction arithmetic, and word problems as compared to TA adults. The study also emphasizes the significance of fraction magnitude knowledge as an intervention and diagnostic tool for MD and also adds to growing literature on the importance of number lines in building a holistic fraction understanding.

References

- Amalina, I., Amalina, I. K., & Fuad, Y. (2019, December). The fraction magnitude knowledge through representations at students with mathematics difficulties. In *Mathematics, Informatics, Science, and Education International Conference (MISEIC 2019)*. Atlantis Press. <https://dx.doi.org/10.2991/miseic-19.2019.21>
- Bailey, D. H., Zhou, X., Zhang, Y., Cui, J., Fuchs, L. S., Jordan, N. C., Gersten, R., & Siegler, R. S. (2015). Development of fraction concepts and procedures in U.S. and Chinese children. *Journal of Experimental Child Psychology, 129*, 68–83. <https://doi.org/10.1016/j.jecp.2014.08.006>
- Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2019). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. *Journal of Educational Psychology*. <https://doi.org/10.1037/edu0000384>

- Behr, M. J., Lesh, R., Post, T., & Silver, E. A. (1983). Rational number concepts. *Acquisition of mathematics concepts and processes*, 91-126.
- Bhatia, P., Delem, M., Léone, J., Boisin, E., Cheylus, A., Gardes, M.-L., & Prado, J. (2020). The ratio processing system and its role in fraction understanding: Evidence from a match-to-sample task in children and adults with and without dyscalculia. *Quarterly Journal of Experimental Psychology*, 174702182094063. <https://doi.org/10.1177/1747021820940631>
- Boyce, S., & Norton, A. (2016). Co-construction of fractions schemes and units coordinating structures. *The Journal of Mathematics Behavior*, 41, 10–25.
doi:10.1016/j.jmathematicsb.2015.11.003
- Cappelletti, M., & Price, C. J. (2014). Residual number processing in dyscalculia. *NeuroImage: Clinical*, 4, 18-28.
- Cavalli, E., Colé, P., Leloup, G., Poracchia-George, F., Sprenger-Charolles, L., & El Ahmadi, A. (2018). Screening for dyslexia in french-speaking university students: an evaluation of the detection accuracy of the alouette test. *Journal of learning disabilities*, 51(3), 268–282.
<https://doi.org/10.1177/0022219417704637>
- Chan, W.-H., Leu, Y.-C., & Chen, C.-M. (2007). Exploring group-wise conceptual deficiencies of fractions for fifth and sixth graders in Taiwan. *The Journal of Experimental Education*, 76(1), 26–57. <https://doi.org/10.3200/JEXE.76.1.26-58>
- Fuchs, L. S., Malone, A. S., Schumacher, R. F., Namkung, J., Hamlett, C. L., Jordan, N. C., Siegler, R. S., Gersten, R., & Changas, P. (2016). Supported self-explaining during fraction intervention. *Journal of Educational Psychology*, 108(4), 493–508.
<https://doi.org/10.1037/edu0000073>
- Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., Jordan, N. C., Siegler, R., Gersten, R., & Changas, P. (2013). Improving at-risk learners'

- understanding of fractions. *Journal of Educational Psychology*, 105(3), 683–700.
<https://doi.org/10.1037/a0032446>
- Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of experimental child psychology*, 77(3), 236-263.
<https://doi.org/10.1006/jecp.2000.2561>
- Geary, D. C. (2004). Mathematics and Learning Disabilities. *Journal of Learning Disabilities*, 37(1), 4–15. <https://doi.org/10.1177/00222194040370010201>
- Hackenberg, A. J. (2013). The fractional knowledge and algebraic reasoning of students with the first multiplicative concept. *The Journal of Mathematical Behavior*, 32, 538–563.
doi:10.1016/j.jmathematicsb.2013.06.007
- Hackenberg, A. J., & Lee, M. (2015). Relationships between students' fractional knowledge and equation writing. *Journal for Research in Mathematics Education*, 46, 196–243.
doi:10.5951/jresemathematicseduc.46.2.0196
- Hancock, R., Richlan, F., & Hoefft, F. (2017). Possible roles for fronto-striatal circuits in reading disorder. *Neuroscience & Biobehavioral Reviews*, 72, 243-260.
- Hansen, N., Jordan, N. C., & Rodrigues, J. (2017). Identifying learning difficulties with fractions: A longitudinal study of student growth from third through sixth grade. *Contemporary Educational Psychology*, 50, 45–59. <https://doi.org/10.1016/j.cedpsych.2015.11.002>
- Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*, 102(4), 843–859.
<https://doi.org/10.1037/a0019824>
- Hiebert, J. (1985). Children's knowledge of common and decimal fractions. *Education and Urban Society*, 17(4), 427-437.

- Hughes, E. M., Riccomini, P. J., & Lee, J.-Y. (2020). Investigating written expressions of mathematical reasoning for students with learning disabilities. *The Journal of Mathematical Behavior*, 58, 100775.
<https://doi.org/10.1016/j.jmathematicsb.2020.100775>
- Hunt, J. H. (2015). Notions of equivalence through ratios: Students with and without learning disabilities. *The Journal of Mathematical Behavior*, 37, 94–105.
<https://doi.org/10.1016/j.jmathematicsb.2014.12.002>
- Ikhwanudin, T., Prabawanto, S., & Wahyudin, W. (2019). The Error Pattern of Students with Mathematics Learning Disabilities in the Inclusive School on Fractions Learning. *International Journal of Learning, Teaching and Educational Research*, 18(3), 75–95.
<https://doi.org/10.26803/ijlter.18.3.5>
- Jordan, N. C., Rodrigues, J., Hansen, N., & Resnick, I. (2017). Fraction Development in Children: Importance of Building Numerical Magnitude Understanding. In *Acquisition of Complex Arithmetic Skills and Higher-Order Mathematics Concepts* (pp. 125–140). Elsevier. <https://doi.org/10.1016/B978-0-12-805086-6.00006-0>
- Kieren, T. E. (1980). The rational number construct: Its elements and mechanisms. *Recent research on number learning*, 125-149.
- Lewis, K. E. (2016). Beyond Error Patterns: A Sociocultural View of Fraction Comparison Errors in Students with Mathematic Learning Disabilities. *Learning Disability Quarterly*, 39(4), 199–212. <https://doi.org/10.1177/0731948716658063>
- Lewis, K. E., & Lynn, D. M. (2018). An insider's view of a mathematics learning disability: Compensating to gain access to fractions. *Investigations in Mathematics Learning*, 10(3), 159–172. <https://doi.org/10.1080/19477503.2018.1444927>

- Lewis, K. E., & Lynn, D. M. (2018). Access Through Compensation: Emancipatory View of a Mathematics Learning Disability. *Cognition and Instruction*, 36(4), 424–459.
<https://doi.org/10.1080/07370008.2018.1491581>
- Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review*, 38, 201–221.
<https://doi.org/10.1016/j.dr.2015.07.008>
- Malone, A. S., & Fuchs, L. S. (2016). Error Patterns in Ordering Fractions Among At-Risk Fourth-Grade Students. *Journal of Learning Disabilities*, 50(3), 337–352.
<https://doi.org/10.1177/0022219416629647>
- Mazzocco, M. M. M., & Devlin, K. T. (2008). Parts and ‘holes’: Gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Science*, 11(5), 681–691. <https://doi.org/10.1111/j.1467-7687.2008.00717.x>
- Mazzocco, M. M. M., Myers, G. F., Lewis, K. E., Hanich, L. B., & Murphy, M. M. (2013). Limited knowledge of fraction representations differentiates middle school students with mathematics learning disability (dyscalculia) versus low mathematics achievement. *Journal of Experimental Child Psychology*, 115(2), 371–387.
<https://doi.org/10.1016/j.jecp.2013.01.005>
- Misquitta, R. (2011). A Review of the Literature: Fraction Instruction for Struggling Learners in Mathematics. *Learning Disabilities Research & Practice*, 26(2), 109–119.
<https://doi.org/10.1111/j.1540-5826.2011.00330>
- Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H. C. (2011). Early place-value understanding as a precursor for later arithmetic performance—A longitudinal study on numerical development. *Research in developmental disabilities*, 32(5), 1837-1851.
<https://doi.org/10.1016/j.ridd.2011.03.012>

- Morano, S., & Riccomini, P. J. (2019). Is a Picture Worth 1,000 Words? Investigating Fraction Magnitude Knowledge Through Analysis of Student Representations. *Assessment for Effective Intervention*, 153450841882069. <https://doi.org/10.1177/1534508418820697>
- Morano, S., Riccomini, P. J., & Lee, J. (2019). Accuracy of Area Model and Number Line Representations of Fractions for Students with Learning Disabilities. *Learning Disabilities Research & Practice*, 34(3), 133–143. <https://doi.org/10.1111/ldrp.12197>
- Moura, R., Lopes-Silva, J. B., Vieira, L. R., Paiva, G. M., Prado, A. C. D. A., Wood, G., & Haase, V. G. (2015). From “five” to 5 for 5 minutes: Arabic number transcoding as a short, specific, and sensitive screening tool for mathematics learning difficulties. *Archives of Clinical Neuropsychology*, 30(1), 88-98. <https://doi.org/10.1093/arclin/acu071>
- Murphy, M. M., Mazzocco, M. M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive Characteristics of Children With Mathematics Learning Disability (MLD) Vary as a Function of the Cutoff Criterion Used to Define MLD. *Journal of Learning Disabilities*, 40(5), 458–478. <https://doi.org/10.1177/00222194070400050901>
- Newton, K. J., Willard, C., & Teufel, C. (2014). An Examination of the Ways That Students with Learning Disabilities Solve Fraction Computation Problems. *The Elementary School Journal*, 115(1), 1–21. <https://doi.org/10.1086/676949>
- Ni, Y. (2001). Semantic Domains of Rational Numbers and the Acquisition of Fraction Equivalence. *Contemporary Educational Psychology*, 26(3), 400–417. <https://doi.org/10.1006/ceps.2000.1072>
- Ni, Y., & Zhou, Y.-D. (2005). Teaching and Learning Fraction and Rational Numbers: The Origins and Implications of Whole Number Bias. *Educational Psychologist*, 40(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

- Pitkethly, A., & Hunting, R. (1996). A review of recent research in the area of initial fraction concepts. *Educational Studies in Mathematics*, 30(1), 5–38.
<https://doi.org/10.1007/BF00163751>
- Rodrigues, J., Jordan, N. C., & Hansen, N. (2019). Identifying Fraction Measures as Screeners of Mathematics Risk Status. *Journal of learning disabilities*, 52(6), 480-497.
<https://doi.org/10.1177%2F0022219419879684>
- Rubinsten, O., & Henik, A. (2005). Automatic activation of internal magnitudes: a study of developmental dyscalculia. *Neuropsychology*, 19(5), 641.
<https://psycnet.apa.org/doi/10.1037/0894-4105.19.5.641>
- Saxe, G. B., Diakow, R., & Gearhart, M. (2013). Towards curricular coherence in integers and fractions: A study of the efficacy of a lesson sequence that uses the number line as the principal representational context. *ZDM*, 45(3), 343–364. <https://doi.org/10.1007/s11858-012-0466-2>
- Schwartz, F., Epinat-Duclos, J., Léone, J., Poisson, A., & Prado, J. (2020). Neural representations of transitive relations predict current and future math calculation skills in children. *Neuropsychologia*, 141, 107410.
- Schwartz, F., Epinat-Duclos, J., Léone, J., Poisson, A., & Prado, J. (2018). Impaired neural processing of transitive relations in children with math learning difficulty. *NeuroImage: Clinical*, 20, 1255-1265.
- Schmider, E., Ziegler, M., Danay, E., Beyer, L., & Bühner, M. (2010). Is it really robust? Reinvestigating the robustness of ANOVA against violations of the normal distribution assumption. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*, 6(4), 147.

- Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology*, *49*(10), 1994–2004.
<https://doi.org/10.1037/a0031200>
- Siegler, R. S., & Lortie-Forgues, H. (2017). Hard Lessons: Why Rational Number Arithmetic Is So Difficult for So Many People. *Current Directions in Psychological Science*, *26*(4), 346–351. <https://doi.org/10.1177/0963721417700129>
- The jamovi project (2020). *jamovi* (Version 1.2) [Computer Software]. Retrieved from <https://www.jamovi.org>
- Tian, J., & Siegler, R. S. (2017). Fractions Learning in Children With Mathematics Difficulties. *Journal of Learning Disabilities*, *50*(6), 614–620.
<https://doi.org/10.1177/0022219416662032>
- Torbeyns, J., Schneider, M., Xin, Z., & Siegler, R. S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, *37*, 5–13.
<https://doi.org/10.1016/j.learninstruc.2014.03.002>
- Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior*, *31*(3), 344–355. <https://doi.org/10.1016/j.jmathematics.2012.02.001>
- Van Hoof, J., Lijnen, T., Verschaffel, L., & Van Dooren, W. (2013). Are secondary school students still hampered by the natural number bias? A reaction time study on fraction comparison tasks. *Research in Mathematics Education*, *15*(2), 154–164.
<https://doi.org/10.1080/14794802.2013.797747>
- Wechsler, D. (2008). Wechsler adult intelligence scale–Fourth Edition (WAIS–IV). San Antonio, TX: NCS Pearson, 22(498), 1.

Woodcock, R. W., Mather, N., McGrew, K. S., & Wendling, B. J. (2001). Woodcock-Johnson III tests of cognitive abilities. Itasca, IL: Riverside Publishing Company.

Yoshida, H., & Sawano, K. (2002). Overcoming cognitive obstacles in learning fractions: Equal-partitioning and equal-whole1. *Japanese Psychological Research*, 44(4), 183–195.

<https://doi.org/10.1111/1468-5884.00021>

Zhang, D., Stecker, P., & Beqiri, K. (2017). Strategies Students With and Without Mathematics Disabilities Use When Estimating Fractions on Number Lines. *Learning Disability Quarterly*, 40(4), 225–236. <https://doi.org/10.1177/0731948717704966>

Table 1*Demographic information for all included participants*

Characteristics	TA (n = 20)	MD (n =18)
	<i>Mean (range)</i>	
Age (years)	22.4 (19-29)	22.9 (19-28)
	<i>Number of participants</i>	
Gender		
Male/Female	10/10	2/16
Level of Education		
Middle - High School	0	1
Undergraduate	11	11
Graduate and above	9	6
Right Handedness	19	16

Table 2*Psychometric measures of TA and MD participants*

Measure	TA (n = 20)	MD (n= 18)
	<i>Mean (range)</i>	<i>Mean (range)</i>
<i>WAIS-IV</i>		
Verbal Reasoning ⁽¹⁾	116 (85 - 135)	110 (90-130)
Matrix Reasoning ^{(1)***}	106 (85 - 140)	89.2 (65-100)
<i>WJ-III</i>		
Mathematics Fluency ^{(1)***}	97.8 (82 – 111)	70.4 (54-82)
Applied Problems ^{(1)***}	108 (96 - 126)	84.8 (67-102)
<i>Alouette-R</i>		
Reading Efficiency ⁽²⁾	469 (332 - 584)	433 (282-532)
Reading Speed ⁽²⁾	9.96 (7.09 - 12.3)	9.20 (6.02-11.2)

Notes. (1) standardized score (Mean = 100, SD = 15), (2) raw score. Group differences

indicated by ***= p < 0.001.

Table 3*Percent score on each fraction competency as a function of group*

Fraction Competency	<i>TA</i>		<i>MD</i>		<i>(Cronbach's α)</i>
	<i>Mean (range)</i>	<i>SD</i>	<i>Mean (range)</i>	<i>SD</i>	
Fraction Concepts	94.2 (82.8-100)	5.32	67.0 (34.5 – 94.8)	18.19	
<i>Equivalence</i>	98.6 (85.7-100.0)	4.4	73.0 (0-100)	30.2	0.837
<i>Comparing Fractions</i>	94.7 (75.0-100)	7.7	65.3 (12.5-100)	28.0	0.913
<i>Ordering Fractions</i>	87.5 (0-100)	27.5	55.5 (0-100)	29.1	0.410
<i>Mixed Fractions</i>	100	0	40.3 (0-100)	43.0	0.943
<i>Part-whole understanding of area models and Set models</i>	92.6 (79.3-100)	6.7	71.1 (41.4-96.6)	15.0	0.872
Fraction Arithmetic	95 (68.8 – 100)	7.19	26.4 (0 – 93.8)	31.62	0.965
Symbolic Representation	99.4 (87.5 - 100)	2.79	96.5 (37.5 – 100)	14.73	0.944
Fraction Number Line	72.5 (0 - 100)	39.83	14.8 (0 – 66.7)	21.31	0.873
Word problems	99 (80- 100)	4.47	50 (0 -100)	33.07	0.771

Table 4.*Percentage of adults with MD with similar error patterns*

Fraction Competency		Type of Error				
	Item Type	Specific errors	Unidentified error	Don't Know	Solved all items correctly	
Fraction Number Line*	Q.9	<i>UNR</i>	16.66	33.33	0	
		50.00				
Fraction Arithmetic**	Q.10	<i>CTM</i>	16.66	33.33	11.11	
		16.66				
		<i>SBU</i>				
		22.22				
Fraction Arithmetic**	1	<i>DCM</i>	33.33	16.66	33.33	11.11
		<i>NTD</i>	5.55			
	2	<i>AWN</i>	38.88	5.55	33.33	22.22
	3	<i>AND</i>	27.77	5.55	11.11	55.55
	4	<i>AND</i>	44.44	22.22	11.11	22.22
Word problems***		<i>CE</i>	38.88	22.22	16.66	11.11
		<i>IO</i>	11.11			

Notes. *Fraction Number Line UNR: Unable to reduce the fraction and/or understand if fraction is greater or lesser than 1, CTM: Counted tick mark on the number line, SBU: Error in identifying sub-units of multiple unit on a number line. **Fraction Arithmetic items were of four types, 1) Converting improper fractions to whole numbers and fractions less than 1, 2) Addition of whole numbers to fractions, 3) Addition of fractions with like denominators and 4)

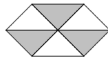
Addition of fractions with unlike denominators. DCM: Decomposed the fraction using only the numerator, NTD: Added the numerator to the denominator, AWN: Added the whole number either to the Numerator or the Denominator, AND: Added the numerator and denominator across both fractions. ***Word problem CE: calculation error, IC: Incorrect operation

Figure 1

Example questions from Fraction Achievement Test for A) Fraction Concepts B) Fraction Arithmetic C) Fraction Number Line D) Symbolic Representations E) Word Problems

A.

Q3. Circle the figures where the shaded part represents $\frac{1}{2}$ of the total area



Q12. Tick the correct box.

	True	False
$\frac{3}{4} = \frac{4}{5}$		

B.

Q15. Write each sum as a single fraction.

$$4 + \frac{1}{2} = \frac{\dots}{\dots}$$

Q19. Calculate the sum of the fractions given below :

$$\frac{3}{4} + \frac{5}{4} =$$

C.

Q7. Write the fractions in number form

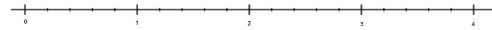
trois demis : $\frac{\dots}{\dots}$

Q8. Write the fractions presented below in words.

$\frac{5}{4}$:

D.

Q9. Place the three fractions on the graduated number line below: $\frac{8}{5}$; $\frac{6}{10}$; $\frac{16}{5}$



Q10. Find the fractions that correspond to the points A, B, and, C.



E.

Q21. Assia read 24 pages of an adventure book. Tom read one third of it. How many pages did Tom read?

Q23. Problem: A wagon has 60 seats. 20 seats are already occupied by passengers. What fraction of the total seats do occupied seats represent?

Figure 2

Percent correct score of adults with and without MD for A) Fraction Concepts B) Fraction Arithmetic C) Fraction Number Line D) Symbolic Representations E) Word Problems. Each dot represents the score of an individual. Error bars represent 95% confidence intervals.

