

On the use of nonlinear normal modes and bifurcation tracking to design resonant MEMS sensors

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Due to their small size and high sensitivity, MEMS resonators are very good candidates for mass sensing devices. Classical MEMS-based sensing techniques rely on detecting a shift in frequency induced by an external perturbation (acceleration, addition of mass, ...) in a single MEMS resonator. The need for parallel mass sensing and highly sensitive devices led to the development of MEMS arrays [1] and alternative mass sensing techniques based on nonlinear phenomena [2].

In this paper, an array of two coupled electrostatically-actuated MEMS resonators is considered, as sketched in Figure 2a). All beams are identical and serve as actuation electrodes for the adjacent beams. Each beam i is subjected to lateral vibrations w_i due to nonlinear harmonic electrostatic forces generated by the adjacent beams. The two-beam model is detailed in [3]. The spatial dependence is removed with a Galerkin method using the linear undamped eigenmodes. The NNMs and their stability are computed with the harmonic balance method using 5 harmonics coupled with an arc-length continuation method [4].

In the case of a perfectly symmetric configuration (symmetric voltages, no additional mass), the system exhibits two main (pure) NNM branches corresponding to the out-of-phase and in-phase motions of the beams respectively, see Figure 2. A branch point (BP) bifurcation is found on the in-phase NNM. Beyond this BP, the in-phase NNM becomes unstable and a stable mixed NNM appears, whose modal shape is an asymmetric mix of the two pure modal shapes.

When a small mass δm is added on beam 1, a symmetry breaking occurs which turns the BP of Figure 2(a) into an imperfect bifurcation, as shown in Figure 3. As a result, the in-phase and mixed NNMs are transformed into an asymmetric in-phase NNM and an isolated NNM (INNMs) which is detached from the in-phase NNM. It is worth noting that the two plots of Figure 3 are switched if δm is added on beam 2 instead of beam 1.

The nonlinear forced response curves (NFRCs) of this asymmetric array are composed of a main NFRC supported by the pure NNMs and of isolated solutions detached from the main NFRC and supported by the INNMs. As a result, the main NFRC of beam 1 has a much lower amplitude than beam 2. This localization of motion resulting from symmetry breaking can be exploited for

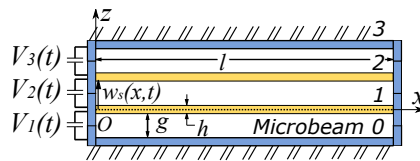


Figure 1: Two-beam MEMS array.

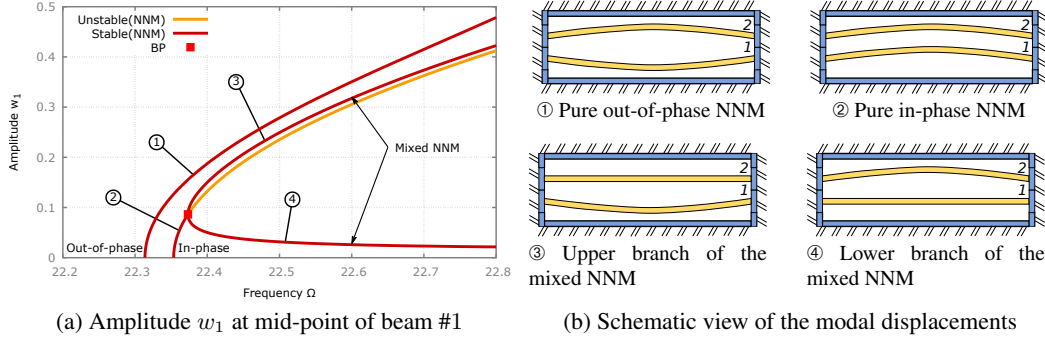


Figure 2: NNMs of the MEMS array in the perfectly symmetric case (no added mass). ①: out-of-phase NNM, ②: in-phase NNM, ③ and ④: mixed NNM.

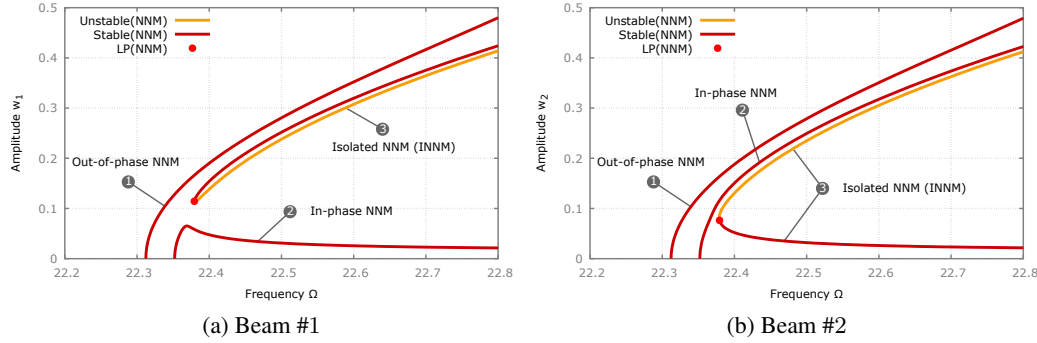


Figure 3: NNMs with their isolated branches (INNMs) after symmetry breaking due to added mass on beam #1. ①: out-of-phase NNM, ②: in-phase NNM, ③: isolated NNM.

mass detection. Real devices are likely to be inherently asymmetric due to manufacturing defects for instance. In this case, the mass detection is based on the reversal of localization of motion that occurs when the added mass exceeds the level of asymmetry.

References

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