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Non-Conservative Nonlinear Modes Through Energy Resonance

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Summary. In this contribution, a method for the numerical computation of nonlinear modes for nonconservative systems is presented. The main idea is to introduce a conservative force term to compensate the non-conservative effects, in such a way that the condition of energy resonance is verified. As this formulation depends only on the system's characteristics and does not require the use of artificial negative damping parameters, it can be applied to generic systems with any type of non-conservative terms.

Introduction

Nonlinear Normal Modes (NNM), which extend the familiar modal analysis to the case of nonlinear mechanical systems, have been a central research topic in dynamics over the past decades. Besides being an interesting theoretical concept, they have been shown to lead to numerous practical applications thanks to their relation to forced resonances, localization, energy transfers and model reduction (see [1] for a review, as well as references therein). Simply stated, the idea behind all methods for the computation of NNMs is to construct an invariant manifold of the conservative equations of motion, either directly or through the continuation of periodic solutions on the manifold [2]. However, modes computed in this fashion do not take into account the presence of non-conservative effects. As a consequence, deviations from the actual backbone curves of the system are to be expected: while they remain small in the case of small proportional damping, they can be considerable when damping is large or when the nonlinear forces involved are fundamentally non-conservative. Hence, just as for the conservative case, efforts have been made to extend the concept of non-conservative modes to nonlinear systems, yielding Non-Conservative Nonlinear Modes (NCNM). Different methodologies for the computation of NCNMs exist, relying on generalizations of the manifold-construction, complex-mode and periodic-motion-continuation techniques. The latter of these, introduced by Krack [3], consists in adding an artificial damping term to compensate for all non-conservative effects and thus achieve periodic motions. This method was applied by Jahn and co-workers [4] to study self-excited vibrations, and in particular to detect Limit Cycle Oscillations (identified as special solutions with zero artificial damping along NCNM branches). Nevertheless, one of the assumptions of this method is that the non-conservative terms are frequency-independent, which is not the case of, e.g., systems with memory terms. The aim of the present contribution is to introduce a new method for the computation of NCNMs. To this end, the concept of Energy Resonance from linear modal analysis is generalized to the nonlinear case. Upon application of the Harmonic Balance Method (HBM), this leads to the formulation of a conservative equation where the effect of non-conservative terms of any form can be accurately taken into account, and whose (periodic) solutions correspond to NCNMs. As only a knowledge of the system's characteristics is needed in this formulation, this method is quite general and avoids the use of any additional, artificial parameters.

Energy resonance

Consider the equations of motion for a system of damped, unforced, nonlinear oscillators. By applying the HBM, they are expressed in the frequency domain with the Fourier coefficients of displacements \mathbf{X} and the fundamental frequency ω as unknowns, giving:

$$\begin{cases} \mathbf{R}(\mathbf{X}, \omega) = \mathbf{Z}(\omega)\mathbf{X} + \mathbf{F}_{\text{NL}}(\mathbf{X}, \omega) = \mathbf{0} \\ \mathbf{Z}(\omega) = \omega^2 \nabla^2 \otimes \mathbf{M} + \omega \nabla \otimes \mathbf{C} + \mathbf{I} \otimes \mathbf{K} \end{cases} \quad (1)$$

where $\mathbf{Z}(\omega)$ is the dynamical stiffness matrix containing structural inertial, damping and stiffness terms, and $\mathbf{F}_{\text{NL}}(\mathbf{X}, \omega)$ represents the Fourier coefficients of all nonlinear forces. Excluding the case of self-excited oscillations with limit cycles, this equation has no non-trivial solution since it contains both conservative and non-conservative terms. By simply ignoring all of the latter, a purely conservative system is obtained, whose solution yields the traditional, undamped nonlinear modes:

$$\mathbf{R}_P(\mathbf{X}, \omega) = (\omega^2 \nabla^2 \otimes \mathbf{M} + \mathbf{I} \otimes \mathbf{K})\mathbf{X} + \mathbf{F}_{\text{NL},c} = \mathbf{0} \quad (2)$$

Drawing a parallel with linear modal analysis, these solutions describe phase resonance. Another possibility, as proposed by Grenat [5], is to compensate the non-conservative terms through the addition of a fictitious stiffness (conservative) force in such a way that the underlying invariant manifold is kept unchanged. This is achieved by considering the condition for energy resonance rather than phase resonance, which can be expressed generically as:

$$\frac{\partial (\mathbf{X}^T \mathbf{X})}{\partial \omega} = 0 \quad (3)$$

The dynamical equilibrium is then given as a solution to the following conservative system:

$$\mathbf{R}_E(\mathbf{X}, \omega) = (\omega^2 \nabla^2 \otimes \mathbf{M} + \mathbf{I} \otimes \mathbf{K})\mathbf{X} + \mathbf{F}_{\text{NL},c} - \mathbf{G}_C(\mathbf{F}_{\text{NL},nc}) = \mathbf{0} \quad (4)$$

where the non-linear terms have been split into a conservative ($\mathbf{F}_{NL,c}$) and a non-conservative part ($\mathbf{F}_{NL,nc}$). The term \mathbf{G}_C is a function of damping and of the non-conservative part of the nonlinear terms. Eq.(4) can be used to compute normal modes regardless of the specific form of the non-conservative terms. Furthermore, as it is an autonomous equation, the usual continuation methods can be directly applied. Two examples are presented below:

1. Strongly-damped Duffing oscillator

$$\ddot{x}(t) + 2\zeta\dot{x}(t) + x(t) + \alpha x^3(t) = 0 \quad (5)$$

For viscous damping, it can be shown that the methodology described herein yields the following result, strictly equivalent to the natural frequency correction in the linear case:

$$\mathbf{G}_C(\mathbf{X}) = \frac{1}{2}\mathbf{I} \otimes (\mathbf{M}^{-1}\mathbf{C}\mathbf{C})\mathbf{X} \quad (6)$$

Fig. 1 shows the associated backbone curve obtained by continuation of NCNMs, and some nonlinear forced response curves (NFRC) computed for varying amplitudes of a sinusoidal excitation signal.

2. Cross-Flow Excited Tube in Loose Support

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}_{fs}\dot{\mathbf{q}}(t) + \mathbf{K}_{fs}\mathbf{q}(t) + \sum_{i=1}^N \left[\Lambda_i \int_0^t e^{-\beta_i\tau} \mathbf{q}(t-\tau) d\tau \right] + \mathbf{f}_{NL}(\mathbf{q}(t)) = \mathbf{0} \quad (7)$$

Details on this system can be found in [6]. The nonlinear terms represent impact forces and are thus conservative. However, the memory terms introduce frequency-dependent damping. By applying the proposed method, the fictitious force is found to be:

$$\mathbf{G}_C(\mathbf{Q}, \omega) = \widetilde{\mathbf{M}}_{\text{corr}}^{-1} \left[\frac{1}{2} ((\nabla \otimes \mathbf{C})^2 + (\mathbf{D}_{t\lambda}(\omega))^2) - (\nabla \otimes \mathbf{C})\mathbf{D}_{t\lambda} - \omega^2 \frac{\partial \mathbf{D}_{t\lambda}(\omega)}{\partial (\omega^2)} [(\nabla \otimes \mathbf{C}) - \mathbf{D}_{t\lambda}(\omega)] \right] \mathbf{Q} \quad (8)$$

where $\mathbf{D}_{t\lambda}$ contains the total non-conservative part of the memory terms in the frequency domain.

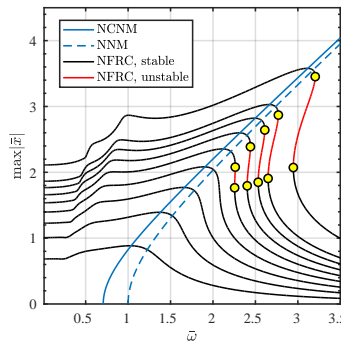


Figure 1: Backbones curves and associated NFRCs for a strongly damped Duffing oscillator.

Conclusions

By extending the concept of energy resonance to the nonlinear case, conservative systems which properly take into account the action of non-conservative terms can be formulated, which leads to the computation of accurate backbone curves for a wide class of systems. This method has been tested on various problems, yielding promising results. Some perspectives include the bifurcation analysis of NCNM curves, as well as the development of experimental methods to obtain them.

References

- [1] Avramov K.V. and Mikhlin Y.V. (2013) Review of Applications of Nonlinear Normal Modes for Vibrating Mechanical Systems. *Appl. Mech. Rev.* **65**(2).
- [2] Renson L., Kerschen G. and Cochelin B. (2016) Numerical computation of nonlinear normal modes in mechanical engineering. *J. Sound Vib.* **364**:177–206.
- [3] Krack M. (2015) Nonlinear modal analysis of nonconservative systems: Extension of the periodic motion concept. *Comput. Struct.* **154**:59-71.
- [4] Jahn M., Stender M., Tatzko S., Hoffmann N., Grolet A. and Wallaschek J. (2019) The extended periodic motion concept for fast limit cycle detection of self-excited systems. *Comput. Struct.* **227**:106-139.
- [5] Grenat C., Baguet S., Dufour R. and Lamarque C-H. (2018) Generalization of conservative nonlinear normal modes to the non-conservative equation of motion. *Proceedings of ASME IDETC/CIE 2018 Conference*.
- [6] Piteau P., Delaune X., Antunes J. and Borsoi L. (2012) Experiments and computations of a loosely supported tube in a rigid bundle subjected to single-phase flow. *J. Fluids Struct.* **28**:56-71.