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► **To cite this version:**

P. Grison. Automatic search for periodic light curves with any kind of shape in unevenly spaced data.. Astronomy and Astrophysics - A&A, 1994, 289, pp.404-410. hal-04233538

HAL Id: hal-04233538

<https://hal.science/hal-04233538>

Submitted on 9 Oct 2023

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Automatic search for periodic light curves with any kind of shape in unevenly spaced data

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Received 26 October 1993 / Accepted 17 March 1994

Abstract. It is shown that the "periodogram analysis" commonly used to find sinusoid-like periodicities can be improved to search for variable star light curves of any kind of shape. The use of a particular set of orthogonal functions for the Least Square Model fitted at different test frequencies permits the derivation of a generalization of the periodogram and a probability of detection better adapted to the detection of any sort of periodic light curve. The results of Monte Carlo simulations are presented to illustrate the advantage of the new method.

Key words: stars: variable – stars: period – methods: analytical – methods: data analysis

1. Introduction

The search for periodic phenomena is a key problem in various branches of physics. With the development of modern observational and storage techniques, the quantity of data that can be collected has increased enormously and it has become necessary to develop numerical methods which are able to identify quickly and with confidence whether a periodicity is present in a sequence of unevenly spaced observations. EROS (Experience de Recherche d'Objets Sombres; see Aubourg et al. 1993) is an example of a huge database comprising a sample of ~ 5 million stars in the Large Magellanic Cloud with ~ 130 photometric measurements in two colours (blue and red) per star. The problem is to identify systematically and as well as possible all the variable stars in the EROS database, irrespective of the form of their light curves. Very recently Gregory & Loredó (1992) have described a purely statistical solution to this problem.

This paper presents a different approach based on Fourier transform analysis. Fourier transforms are a well known family of algorithm commonly used in astronomy which can be computed quickly and which detect a periodic signal efficiently; a good overview of the periodogram theoretical background can be found in Scargle (1982) who derived a new expression for the periodogram in the case of an unevenly spaced data set and who pointed out a relevant statistical test for detection of the signal.

Additionally, different kinds of fast algorithm have been developed and can be useful in the case of a large sample (Press et al. 1989; Kurtz 1985; Ponman 1981). It should also be mentioned that if there are significant gaps in the data, some improved methods are available: use of windows is detailed by Harris (1978); and techniques which fill the gaps to produce evenly sampled data prior to calculation of the power spectrum are described by Carbone et al. (1992). However, numerical simulations made to compare different kinds of period-determination techniques have shown that the more the shape of the periodic signal differs from a sinusoid the less the periodogram is efficient (see Heck et al. 1985; Pérez de la Blanca et al. 1981). This result is not surprising since Scargle (1982) has shown the equivalence between periodogram analysis and the fitting of a sinusoid at different frequencies (see also Lomb 1975). It would be better to use a more general model taking into account several harmonics in a Fourier series to fit exactly the real shape of the periodic function (see Vanicek 1969, 1970), but unfortunately this becomes a heavy numerical problem and the useful statistical properties are lost. The purpose of this paper is to show that an alternative way is possible which overcomes such difficulties.

Section 2 presents a review of periodogram fundamentals.

In Sect. 3 we describe the analytical expression of an appropriate set of orthogonal functions used to define a generalized model to fit the data. A new expression for the "Generalized Periodogram" (hereafter GP) applied to find periodicities for any kind of variable star is derived in Sect. 4. A new expression for the probability of detection as a simple function of the GP is presented in Sect. 5. Section 6 gives the results from a Monte Carlo simulation to compare the performance of the new approach with the old, and justify its utility. Some additional points are discussed in Sect. 7 and a conclusion is given in Sect. 8.

2. Periodogram analysis

Scargle (1982) first derived a new expression for the periodogram (or power spectrum) in the case of an unevenly spaced data set. He then pointed out a statistical test on this Modified Periodogram (hereafter MP) which estimates the significance

of the period. The MP defined below (Eq. 2) is also equivalent to the reduction of the sum of squares $(1/2\sigma^2) \sum_{j=1}^N [X^2(t_j) - (\sum_{j=1}^N (X(t_j) - M_1(t_j))^2)]$ where $X(t_j) = x(t_j) - \bar{x}$ is the time series of N measures with zero mean and variance σ^2 ; the mean \bar{x} of the observational data is given by $(1/N) \sum_{j=1}^N x(t_j)$; the total variance σ^2 is estimated by $1/(N-1) \sum_{j=1}^N X(t_j)^2$; and M_1 is the least square model of a sinusoid which can be expressed in the form:

$$M_1(t) = A \cos(\omega t - \tau) + B \sin(\omega t - \tau) \quad (1)$$

The analytical expression for the MP is given by:

$$P_1(\omega) = \frac{1}{2\sigma^2} (A^2(\omega)C(\omega) + B^2(\omega)S(\omega)) \quad (2a)$$

where:

$$A(\omega) = \frac{\sum_{j=1, N} X(t_j) \cos(\omega t_j - \tau)}{C(\omega)}$$

$$B(\omega) = \frac{\sum_{j=1, N} X(t_j) \sin(\omega t_j - \tau)}{S(\omega)}$$

$$C(\omega) = \sum_{j=1, N} \cos^2(\omega t_j - \tau)$$

$$S(\omega) = \sum_{j=1, N} \sin^2(\omega t_j - \tau)$$

$$\tau = \frac{1}{2} \arctan \left(\frac{\sum_{j=1, N} \sin(2\omega t_j)}{\sum_{j=1, N} \cos(2\omega t_j)} \right)$$

Here $A(\omega)$ and $B(\omega)$ are the parameters estimated by the Least Square Method (hereafter LSM) of the model M_1 (Eq.1), and τ is an additional parameter which results in the LSM correlation terms being zero (see Lomb 1976) or equivalently makes the periodogram time independent.

The main interest of this method is the statistical behaviour of the MP. Scargle (1982) has shown that when X is pure gaussian random noise, $P_1(\omega)$ presents an exponential probability distribution in the case of arbitrary sampling. A direct consequence of this statistical property is the existence of a confidence level for the detection of periodicities. The basic test to decide whether a period is not a random noise fluctuation is to search for $P_1 \max$ (the maximum of P_1 over the set of test frequencies) and to compute the probability of finding such a maximum in a pure noise case: $Pr(1) = Pr(P_1(\omega_i) > P_1 \max, i = 1, N_f) = 1 - (1 - \exp(-P_1 \max))^{N_i}$, where N_i is the number of independent frequencies and N_f the number of test frequencies calculated in the periodogram. The natural set of independent frequencies (even sampling) is $\omega_n = 2\pi n/T$, $n = 0, 1, 2, \dots, N/2$ with T the total observational time interval. In

the unevenly-sampled case it is desirable to adopt an over sampling *OFAC* and additionally a hypersampling *HIFAC*, such that $N_f = OFAC * HIFAC * N/2$ (this point is discussed further in Sect. 7). Koen (1990) has criticized the validity of Eq. 2a because σ^2 is also a random variable estimated through $s^2 = 1/(N-1) \sum_{j=1}^N X(t_j)^2$ (hereafter σ^2 represents the true value of the variance and s^2 its estimation). Consequently the periodogram P_1 should be rewritten:

$$P_1(\omega) = \frac{1}{2s^2} (A^2(\omega)C(\omega) + B^2(\omega)S(\omega)) \quad (2b)$$

and the probability of false detection $Pr(1)$ associated with the random variable P_1 is thus proportional to the ratio of two random variables with χ^2 distributions of 2 and $N-1$ degrees of freedom, which is the Snédécór distribution $F_{2, N-1}$ with the same degrees of freedom, whence:

$$Pr(1) = 1 - \left(\int_0^{P_1 \max} f_{F_{2, N-1}}(x) dx \right)^{N_i} \quad (3)$$

where f_F is the probability density function ($f_F(x) = dF(x)/dx$) of the cumulative distribution function F defined as the probability of finding a random variable ξ in the range $[-\infty, x]$ i.e. $F(x) = Pr(\xi \leq x) = \int_{-\infty}^x f(t) dt$. In this case and for a large data set the $F_{2, N-1}$ distribution approaches the exponential distribution.

3. The model and the orthogonal functions

The first step to improving the above periodogram analysis is to take a model such as $\sum_{i=1}^k A_i \cos(i\omega t - \tau_i) + B_i \sin(i\omega t - \tau_i)$, where k is the number of harmonics needed to fit the shape of the curve (see e.g. Vaniceck 1969, 1970). In the general case of unevenly spaced data it quickly becomes difficult to compute the coefficients A_i and B_i ; furthermore, the significance test cannot be applied because of the non-orthogonality of the functions $\cos(i\omega t - \tau_i)$, $\sin(i\omega t - \tau_i)$. We show in this section that we can always find an appropriate set of orthogonal function (Φ_i^c, Φ_i^s) which are linear combinations of the preceding ones. The resulting general least square model is a linear combination of k pairs of functions (Φ_i^c, Φ_i^s) :

$$M_k = \sum_{i=1, k} A_i \Phi_i^c + B_i \Phi_i^s \quad (4)$$

The functions Φ are:

$$\begin{aligned} \Phi_1^c &= \cos(\omega t - \tau_1) \\ \Phi_1^s &= \sin(\omega t - \tau_1) \\ \Phi_2^c &= \cos(2\omega t - \tau_2) - \alpha_{12}^c \Phi_1^c - \beta_{12}^c \Phi_1^s \\ \Phi_2^s &= \sin(2\omega t - \tau_2) - \alpha_{12}^s \Phi_1^c - \beta_{12}^s \Phi_1^s \\ \Phi_k^c &= \cos(k\omega t - \tau_k) - \sum_{i=1, k-1} (\alpha_{ik}^c \Phi_i^c + \beta_{ik}^c \Phi_i^s) \\ \Phi_k^s &= \sin(k\omega t - \tau_k) - \sum_{i=1, k-1} (\alpha_{ik}^s \Phi_i^c + \beta_{ik}^s \Phi_i^s) \end{aligned} \quad (5)$$

in which the coefficients $\alpha_{ik}^c, \beta_{ik}^c, \alpha_{ik}^s, \beta_{ik}^s, \tau_k$ are determined by the orthogonality conditions:

$$\begin{aligned} \sum_{j=1, N} \Phi_i^c(t_j) \Phi_k^c(t_j) &= 0 \quad i = 1, k-1 \quad k > 1 \\ \sum_{j=1, N} \Phi_i^c(t_j) \Phi_k^s(t_j) &= 0 \quad i = 1, k-1 \quad k > 1 \\ \sum_{j=1, N} \Phi_i^s(t_j) \Phi_k^c(t_j) &= 0 \quad i = 1, k-1 \quad k > 1 \\ \sum_{j=1, N} \Phi_i^s(t_j) \Phi_k^s(t_j) &= 0 \quad i = 1, k-1 \quad k > 1 \\ \sum_{j=1, N} \Phi_k^c(t_j) \Phi_k^s(t_j) &= 0 \end{aligned} \quad (6)$$

To solve these equations, one can proceed step by step. The first parameter to find is τ_1 , for which we have $\sum \Phi_1^c \Phi_1^s = 0$. It is given by the expression for τ following Eq. 2a ($\tau_1 = \tau$). Then, using this result, it is possible to solve the five equations involving the second harmonic for the five unknown parameters $\alpha_{12}^c, \alpha_{12}^s, \beta_{12}^c, \beta_{12}^s, \tau_2$, and so on for the following harmonics up to the desired final harmonic k . The general expressions for the coefficients are:

$$\begin{aligned} \alpha_{ik}^c &= \cos \tau_k A_{ik}^c + \sin \tau_k A_{ik}^s \quad i = 1, k-1 \quad k > 1 \\ \alpha_{ik}^s &= \cos \tau_k A_{ik}^s - \sin \tau_k A_{ik}^c \quad i = 1, k-1 \quad k > 1 \\ \beta_{ik}^c &= \cos \tau_k B_{ik}^c + \sin \tau_k B_{ik}^s \quad i = 1, k-1 \quad k > 1 \\ \beta_{ik}^s &= \cos \tau_k B_{ik}^s - \sin \tau_k B_{ik}^c \quad i = 1, k-1 \quad k > 1 \end{aligned} \quad (7)$$

$$\tau_k = \frac{1}{2} \arctan \left(\frac{\sum_{j=1, N} \sin(2k\omega t_j) - 2N_k}{\sum_{j=1, N} \cos(2k\omega t_j) + 2D_k} \right)$$

where

$$N_k = \sum_{i=1, k-1} \left(N_{ik}^A \sum_{j=1, N} \Phi_i^c(t_j)^2 + N_{ik}^B \sum_{j=1, N} \Phi_i^s(t_j)^2 \right)$$

$$D_k = \sum_{i=1, k-1} \left(D_{ik}^A \sum_{j=1, N} \Phi_i^c(t_j)^2 + D_{ik}^B \sum_{j=1, N} \Phi_i^s(t_j)^2 \right)$$

and

$$N_{ik}^A = A_{ik}^c A_{ik}^s$$

$$N_{ik}^B = B_{ik}^c B_{ik}^s$$

$$D_{ik}^A = \left(\frac{(A_{ik}^s)^2 - (A_{ik}^c)^2}{2} \right)$$

$$D_{ik}^B = \left(\frac{(B_{ik}^s)^2 - (B_{ik}^c)^2}{2} \right)$$

$$A_{ik}^c = \frac{\sum_{j=1, N} \Phi_i^c(t_j) \cos(k\omega t_j)}{\sum_{j=1, N} \Phi_i^c(t_j)^2}$$

$$A_{ik}^s = \frac{\sum_{j=1, N} \Phi_i^c(t_j) \sin(k\omega t_j)}{\sum_{j=1, N} \Phi_i^c(t_j)^2}$$

$$B_{ik}^c = \frac{\sum_{j=1, N} \Phi_i^s(t_j) \cos(k\omega t_j)}{\sum_{j=1, N} \Phi_i^s(t_j)^2}$$

$$B_{ik}^s = \frac{\sum_{j=1, N} \Phi_i^s(t_j) \sin(k\omega t_j)}{\sum_{j=1, N} \Phi_i^s(t_j)^2}$$

These equations define the analytical expression for the set of orthogonal functions used to describe the LSM model M_k (Eq. 1). If desired these functions can be rewritten in term of a classical Fourier series $\sum \mathcal{A}_i \cos(i\omega t - \tau_i) + \mathcal{B}_i \sin(i\omega t - \tau_i)$, where $\mathcal{A}_i = A_i - \sum \alpha_{ij}^c A_j - \sum \alpha_{ij}^s B_j$, $\mathcal{B}_i = B_i - \sum \beta_{ij}^c A_j - \sum \beta_{ij}^s B_j$.

4. The generalized periodogram

The LSM gives estimates for the A_k, B_k coefficients:

$$A_k = \frac{\sum_{j=1, N} X_j \Phi_k^c(t_j)}{\sum_{j=1, N} \Phi_k^c(t_j)^2}$$

$$B_k = \frac{\sum_{j=1, N} X_j \Phi_k^s(t_j)}{\sum_{j=1, N} \Phi_k^s(t_j)^2} \quad (8)$$

We can derive a new expression for the periodogram defined by $P_k = (1/2ks^2)(\sum X_i^2 - \sum (X_i - M_k)^2)$, where M_k (Eq. 4) is the best fit to the data. It is easy to show that P_k can be expressed as:

$$P_k = \frac{1}{2ks^2} \sum_{i=1, k} p_i \quad (9)$$

$$p_i = A_i^2 \sum_{j=1, N} \Phi_i^c(t_j)^2 + B_i^2 \sum_{j=1, N} \Phi_i^s(t_j)^2$$

Note that the case $k=1$ corresponds to the MP (Eq. 2b). Figure 1 shows the variation of P over a set of test frequencies, up to the fourth harmonic in the Fourier expansion for simulated data of a semi detached eclipsing binary of 4.5 days period. Note the best contrast between the true frequency and the others is obtained for the third order.

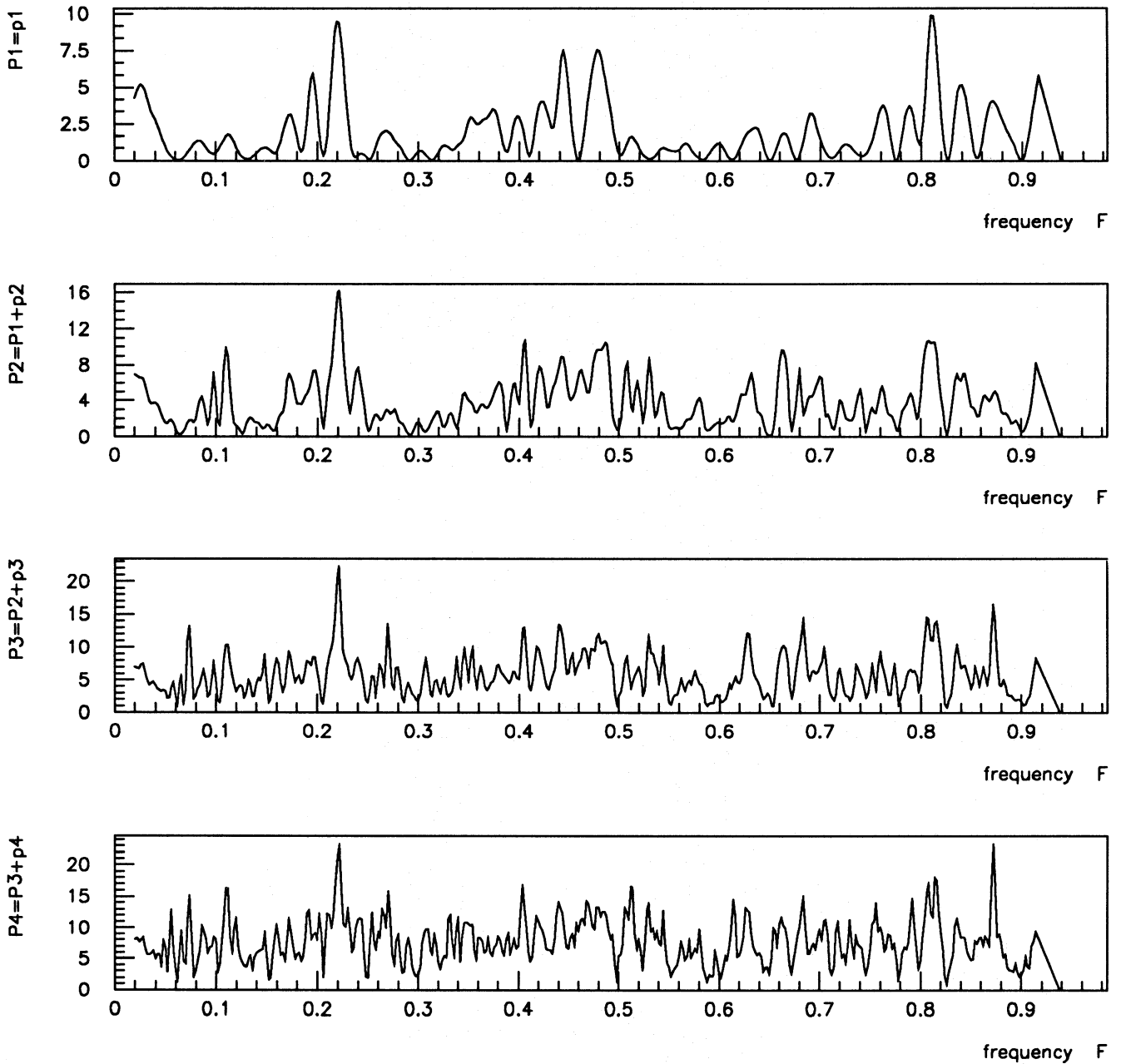


Fig. 1. The generalized periodogram (Eq. 9) versus the frequency for the first four Fourier harmonics of a simulated semi-detached eclipsing binary of 4.5day period ($F=0.22 \text{ day}^{-1}$)

5. Probability of false detection

The main advantage of the GP method is the existence of a simple statistical test which allows us to decide whether or not the period found is a pure noise effect. The fact of having chosen an orthogonal base instead of the classical Fourier series decomposition transforms the periodogram P_k proportional to the sum of squares of $2k$ independent terms $(A_i(\sum(\Phi_i^c)^2)^{\frac{1}{2}}, B_i(\sum(\Phi_i^s)^2)^{\frac{1}{2}})$. Under the assumption that X is a pure normal noise $N(0, \sigma^2)$ with zero mean and variance σ^2 , the $2k$ terms are also normal (Scargle 1982) with zero mean and

variance $\sigma_c^2 = \sigma^2 \sum \Phi_i^{c2}$, $\sigma_s^2 = \sigma^2 \sum \Phi_i^{s2}$. Furthermore, the sum of the squares of $2k$ random normal variables of unit variance is a random variable with a χ^2 distribution of $2k$ degrees of freedom. From this property and rewriting $P_k = (\sum_{i=1}^k p_i / 2k\sigma^2) / (s^2/\sigma^2)$ we can deduce a probability of false detection $Pr(k)$ as the generalization of $Pr(1)$ (Eq. 3) in the following form (see Eq. 3 for the definition of f_F):

$$Pr(k) = 1 - \left(\int_0^{P_k^{max}} f_{F_{2k, N-1}}(x) dx \right)^{N_i} \quad (10)$$

$$\int_0^{P_k^{max}} f_{F_{2k, N-1}}(x) dx = 1 - I_{(N-1)/(N-1+2kP_k^{max})} \left(\frac{N-1}{2}, k \right)$$

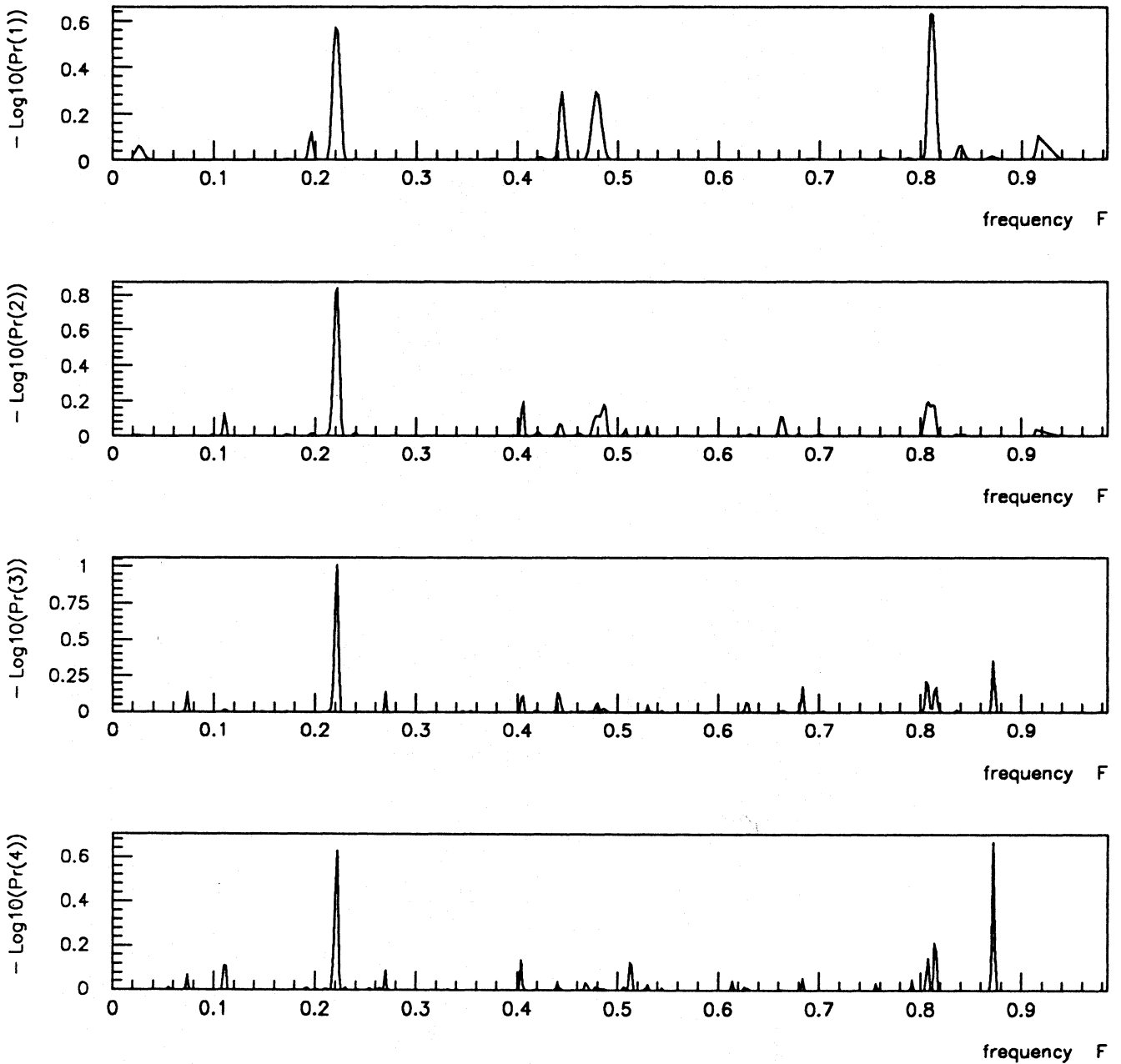


Fig. 2. The generalized probability of detection $Pr(i)$, $i = 1, 4$ given by Eq. 10 for the same semi-detached eclipsing binary simulated in Fig. 1

where I is the incomplete beta function (see Press et al. 1989), $P_{k,max}$ is the maximum of P_k over the set of test frequencies, and N_i is the number of independent frequencies (see Sect. 7 for remarks about N_i). For the same example given in Fig. 1, Fig. 2 shows the false alarm probability for the first to fourth order development $Pr(1) \dots Pr(4)$, or equivalently the confidence level of the tested periods $CL = 1 - Pr(i)$. The minimum of the probability $Pr(i)$ is obtained for $i = 3$ which corresponds also to the best contrast of the P_i drawn in Fig. 1. The fact in this example that $Pr(4) > Pr(3)$ shows that the additional term p_4 in $P_4 = P_3 + p_4$ is not statistically meaningful.

6. Monte Carlo simulation

Monte Carlo simulation is a good tool for quantifying the efficiency of the GP compared to the MP in the case of a periodic non-sinusoidal light curve. Light curve simulations corresponding to the model of a semi-detached eclipsing binary already considered were made of the form $a[\cos(\omega t/t_0) - 0.75 \cos(2\omega t/t_0) + 0.5 \cos(3\omega t/t_0) - 0.25 \cos(4\omega t/t_0)] + b$, where the parameter a is proportional to the signal-to-noise ratio, and b is normal noise $N(0,1)$. The value of a was varied from 0 to 5 in steps of 0.05; the number of measurements was $N = 50$, randomly spaced. In accordance with Scargle (1982, Appendix D) the number of test frequencies N_f was taken equal

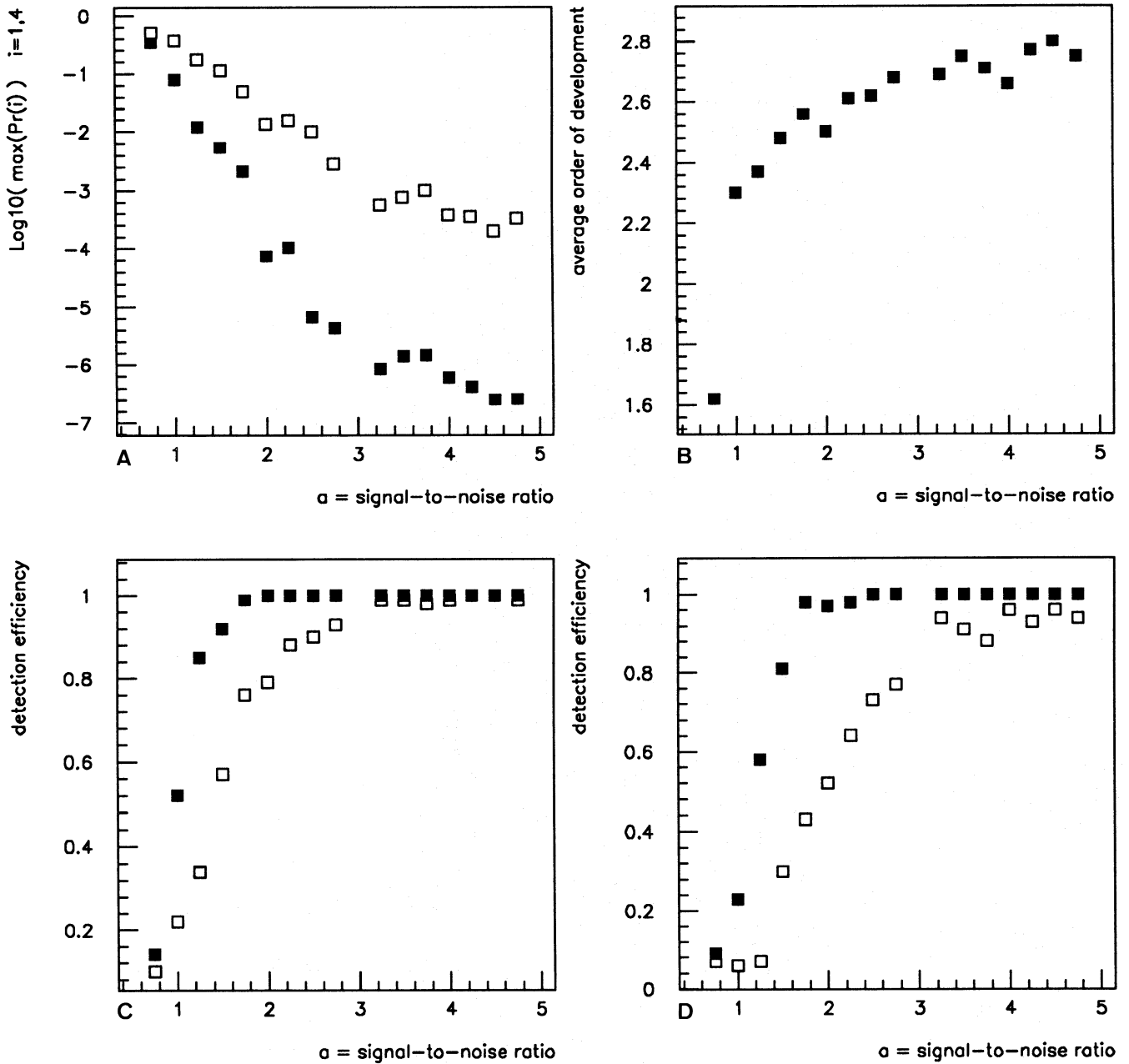


Fig. 3. **a** The minimum probability of false detection versus the signal-to-noise ratio of the signal a for the MP (open squares) and the GP (filled squares). **b** Average order of development used to compute the GP versus the signal to noise ratio (to compare with 1 for the case of the MP). **c,d** detection efficiencies versus the signal to noise ratio for a confidence level better than 99% and 99.9%

to the number of Nyquist frequencies N_N without oversampling $N_f = N_N = N/2 = N_i$ where N_i is the number of independent frequencies used to calculate the probability $\text{Pr}(i)$ ($i = 1, 4$). For each value of a , $N_t = 100$ independent trials were computed and the probability of false detection $\text{Pr}(i)$ defined in Eq. 10 was estimated, with the true number of independent frequencies N_i taken equal to the number of independent frequencies for each trial N_i multiplied by the number of trials N_t . The computation of the GP for each test frequency is done in the following order:

1) $P_1, \text{Pr}(1)$,

Only higher peaks of P_1 are studied to the next order if $\text{Pr}(1) < \text{Pr}_{max}$ where Pr_{max} is an appropriate limit on the probability (for example $\text{Pr}_{max} = 0.2$), then:

2) $P_2, \text{Pr}(2)$,

then if $\text{Pr}(2) < \text{Pr}_{max}$ and $\text{Pr}(2) < \text{Pr}(1)$,

3) $P_3, \text{Pr}(3)$...and so on.

Figure 3 gives the Monte Carlo results for the GP and MP methods. The significant difference between GP (filled squares) and MP (open squares) proves that GP performs better than MP in

the case of periodic light curves with shapes different from a sinusoid such as those of semi-detached eclipsing binaries.

7. Discussion

- a) The choice of test and independent frequencies:

The first step in searching for periodicities is to define a set of test frequencies for which the GP will be computed. In the case of evenly sampled data and for a sinusoidal light curve the set of Nyquist frequencies ($\omega_n = 2\pi n/T, n = 0, 1, 2, \dots, N/2$, where T is the total observational time interval) is the best choice, because it is the lowest number of frequencies required to recover the original data, and it is also by definition a set of independent frequencies. In the general case of unevenly spaced data and for a non-sinusoidal light curve, it seems useful to increase the natural Nyquist number of test frequencies by an oversampling (*OFAC*) and a hypersampling (*HIFAC*) factor which define a larger set of test frequencies ($\omega_n = 2\pi n/(T * OFAC)$, $n = 0, 1, 2, \dots, N * OFAC * HIFAC/2$). Basically, we can choose $OFAC \simeq k$ (k being the highest harmonic we hope to detect) because the higher the harmonics in the light curve, the more frequencies we need to find them. The hypersampling defines the largest frequency we can hope to find, and its value is mainly dependent upon the data sampling. In cases of sampling where there are no large gaps $HIFAC = 2$ is commonly used. This sets the smallest period searched for equal to the average time between observations. In the cases where significant gaps are present in the data, *HIFAC* can be increased in order to find periodicities comparable of the average time between observations. The appropriate value for the number of independent frequencies N_i is unclear and changes with the sample (see Horne et al. 1986). Until there is a better understanding of this parameter, it is safer to adopt $N_i = N_t = N_N * OFAC * HIFAC$ so as not to underestimate the probability of false detection $Pr(k)$ (Eq. 10) or equivalently overestimate the confidence level $(1 - Pr(k))$.

- b) Pseudo-periodic light variation:

The GP method is strictly speaking only applicable for periodic variable stars. We could think of generalizing this approach to pseudo-periodic light curves ($f(t) = f_1(t/T_1) + f_2(t/T_2) + \dots + f_M(t/T_M)$, $|f_1| > |f_2| > \dots > |f_M|$) with M different periods not in rational ratios. However this means that one should analyse $N_1 * N_2 * \dots * N_M$ test frequencies, increasing by the same factor the probability of false detection (see Eq. 10 with $N_i = N_1 * N_2 * \dots * N_M$), and the computing time. However, if the change $|\Delta f_2| \ll |\Delta f_1|$ over the time interval studied, we could hope to find (T_1, f_1) in a first step and in a second step, after subtraction of f_1 from the data, try to detect (T_2, f_2) , and so on.

- c) Light curves with nearly equal eclipses:

Some eclipsing binaries have similar primary and secondary minima. The GP formulation obviously detects these objects with half their period. Such cases are easily recognized during inspection of phased light curves.

8. Conclusion

In this paper we have derived a new expression of the periodogram in order to search for non simple sinusoidal periodic light curves with maximum efficiency, independent of the shape of the variations. The expression of the GP (Eq. 5) with its associated probability $Pr(k)$ (Eq. 6) performs better than the common MP without any significant increase of computing time. A direct application of such analysis is the detection of all periodic variable stars in a large data sample. A paper is in preparation concerning new eclipsing binary stars in the LMC found in this way in data from the EROS survey.

Acknowledgements. I thank J.P. Beaulieu, R. Ferlet, W. Tobin and A. Vidal Madjar for their thoughtful comments, and the referee for useful suggestions.

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