

Convergence study

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I. ADAPTIVE MESH REFINEMENT

The simulation goes in two steps. First we create the stagnation point flow. Once a stationary flow is reached we inject the bubble. We check the numerical convergence of these two steps separately.

Basilisk works with adaptive mesh refinement. In order to test numerical convergence, one should vary the minimum grid size $\Delta_{\min} = L/2^{Le}$ and the maximum grid size $\Delta_{\max} = L/2^{\ell e}$, where $L = 10$ is the box size, and Le and ℓe are the maximum and minimum levels of refinement respectively. Note that bubble radius is set to 1. The grid is refined based on the interface position and the local velocity (see [1] for a detailed description of the refinement procedure).

In our study, we use adaptive mesh refinement with a maximum level $Le = 9$ for precursors simulations and $Le = 10$ when the bubble is present. In both cases we use $\ell e = 5$. If the grid were of constant cell size, Δ_{\min} , the number of points would be 512^2 and 1024^2 cells respectively. The choice of these two levels is explained in what follows.

II. FLOW'S CONVERGENCE STUDY

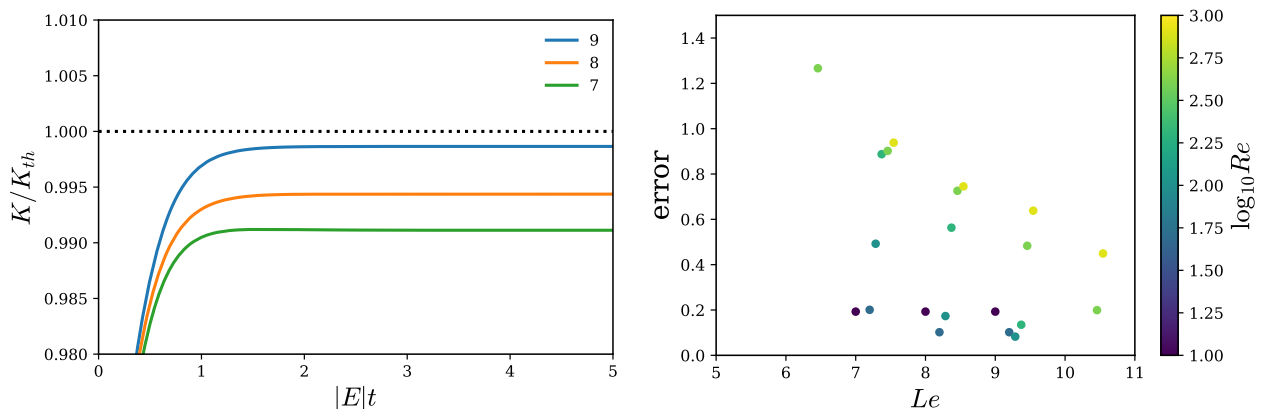


FIG. 1. a) Typical temporal evolution of the kinetic energy for different maximum levels of refinement at $Re = 200$. A stationary state is reached after approximately $1/|E|$, with an error smaller than 1% for all Le on the total kinetic energy. b) Error, defined in equation (2), as a function of the maximum level Le and for all the Reynolds numbers. Points have been horizontally shifted for readability reasons.

We test the numerical convergence for the precursor simulations by comparing the total kinetic energy in the box in the absence of a bubble to the predicted value. Given the theoretical velocity field in a stagnation point flow we get the theoretical value:

$$K_{th} = \frac{1}{\pi L^3} \int_0^L dx \int_0^L dy \frac{1}{2} \rho \mathbf{u}^2 = \frac{11}{48} E^2 L^2 \quad (1)$$

The measured kinetic energy in the simulations is written K . We define the error as:

$$\text{error} = 100. \frac{|K - K_{th}|}{K_{th}} \quad (2)$$

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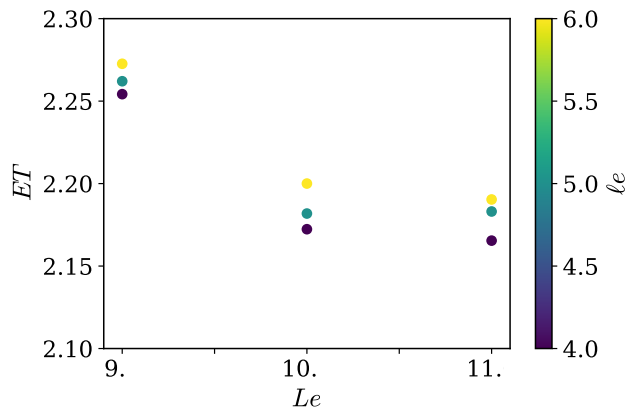


FIG. 2. Example of the variation of the dimensionless lifetime ET , for a bubble at $Re = 400$ and $We = 8$, with the numerical parameters Le and le .

We look at the evolution of the kinetic energy as a function of Le . Figure 1a) shows an example of the typical temporal evolution of the kinetic energy for three maximum levels of refinement, for a simulation at Reynolds 200. The kinetic energy starts from 0 and quickly reaches a plateau after $t \sim 1/|E|$. The corresponding error is less than 1% as soon as $Le > 7$. Figure 1b displays the error for the six different values of Re ranging from 10 to 800, as a function of Le . As soon as $Le \geq 8$, the error is less than 1%.

III. BUBBLE'S CONVERGENCE STUDY

We test the numerical convergence in the presence of a bubble by computing the dimensionless bubble lifetime, ET , as a function of the numerical parameters. An example is given in figure 2 for $Re = 400$ and $We = 8$, where we vary both the minimum and the maximum grid size. The maximum grid size does not affect significantly bubble lifetime. We thus keep the same value as for precursors simulation, $le = 5$. The relative difference between $Le = 10$ and $Le = 11$, $(T_{11} - T_{10})/T_{11}$, being less than a percent for all le , we fix $Le = 10$.

[1] J. A. Van Hooft, S. Popinet, C. C. Van Heerwaarden, S. J. Van der Linden, S. R. De Roode, and B. J. Van de Wiel, Towards adaptive grids for atmospheric boundary-layer simulations, *Boundary-layer meteorology* **167**, 421 (2018).