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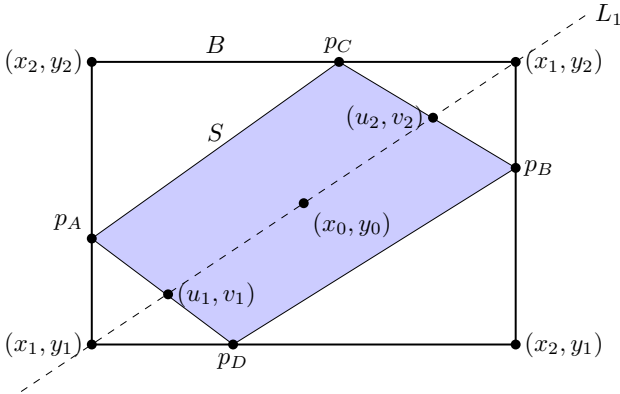
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I. INTRODUCTION

This document is meant to serve as an addendum to [1]. The main purpose of this addendum is to provide a mathematical demonstration to show that given a convex shape S and its bounding box B , the center point is the only point guaranteed to be part of S . This justifies the choice made in [1] to use the bounding box center point as pose initialization for the detected object.

II. ADDENDUM

Proposition 1. We consider the bounding box B of a convex shape S in the plane. The center point of B is the only point guaranteed to be inside S .



Proof: We call G the set of points that are guaranteed to be inside a convex shape S given its bounding box B . We want to show that $G = (x_0, y_0)$ with (x_0, y_0) the center point of B .

◆ **Uniqueness:** First, we show that $G \subseteq (x_0, y_0)$.

We consider the four corners of B : (x_1, y_1) , (x_2, y_1) , (x_1, y_2) , (x_2, y_2) . We consider only convex shapes with non-zero area, so we have $x_1 < x_2$ and $y_1 < y_2$.

· The segment S_1 between (x_1, y_1) and (x_2, y_2) is a convex shape corresponding to the bounding box B , so $G \subseteq S_1$.

· Likewise, the segment S_2 between (x_2, y_1) and (x_1, y_2) is a convex shape corresponding to the bounding box B , so $G \subseteq S_2$.

Therefore $G \subseteq S_1 \cap S_2$ and, as $S_1 \cap S_2 = (x_0, y_0)$, we have $G \subseteq (x_0, y_0)$.

◆ **Existence:** We want to show that $(x_0, y_0) \in G$.

Let S be a convex shape with bounding box B . We have at least one point from S on each edge of B :

$$\begin{aligned} \mathbf{p}_A &= (x_1, y_A) & y_1 &\leq y_A \leq y_2 \\ \mathbf{p}_B &= (x_2, y_B) & x_1 &\leq x_B \leq x_2 \\ \mathbf{p}_C &= (x_C, y_2) & y_1 &\leq y_C \leq y_2 \\ \mathbf{p}_D &= (x_D, y_1) & x_1 &\leq x_D \leq x_2 \end{aligned}$$

$\mathbf{p}_A, \mathbf{p}_B, \mathbf{p}_C, \mathbf{p}_D \in S$.

Note that these points are not necessarily distinct.

As S is convex, we have segment $[p_A, p_D] \subseteq S$.

· If $y_A = y_1$ or $x_D = x_1$, we directly have a point from S on line L_1 between (x_1, y_1) and (x_2, y_2) , namely (x_1, y_1) .

· If $y_A > y_1$ and $x_D > x_1$, then the segment $[p_A, p_D]$ is not parallel to the line L_1 between (x_1, y_1) and (x_2, y_2) , as neither of the two points are on the line. We can write the equation of line L_1 as: $L_1(x) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$. As $L_1(x_1) = y_1$ and L_1 is strictly increasing we have:

$$y_A > L_1(x_1) \text{ and } y_1 < L_1(x_D),$$

meaning that \mathbf{p}_A and \mathbf{p}_D are not on the same side of L_1 . Therefore we obtain an intersection point $(u_1, v_1) \in [p_A, p_D] \cap L_1$. As S is convex, we also have $(u_1, v_1) \in S$. Let's write the equations defining u_1 and v_1 :

$$\exists \alpha, \beta \in [0, 1], \begin{cases} u_1 = (x_D - x_1)\alpha + x_1 = (x_2 - x_1)\beta + x_1 \\ v_1 = (y_1 - y_A)\alpha + y_A = (y_2 - y_1)\beta + y_1. \end{cases}$$

We can solve these equations to find α and β :

First we express $\beta = \frac{x_D - x_1}{x_2 - x_1}\alpha$. Then by substitution we have:

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$$\begin{aligned}
((y_2 - y_1) \frac{x_D - x_1}{x_2 - x_1} + y_A - y_1) \alpha &= y_A - y_1 \\
\Leftrightarrow \alpha &= \frac{1}{1 + \frac{y_2 - y_1}{y_A - y_1} \frac{x_D - x_1}{x_2 - x_1}} \\
\Leftrightarrow \alpha &= \frac{1}{1 + \frac{y_2 - y_1}{y_A - y_1} \frac{x_D - x_1}{x_2 - x_1}}.
\end{aligned}$$

Therefore we can express the value of v_1 :

$$v_1 = y_A - \frac{y_A - y_1}{1 + \frac{y_2 - y_1}{y_A - y_1} \frac{x_D - x_1}{x_2 - x_1}},$$

and find an upper bound:

$$\begin{aligned}
v_1 &\leq y_A - \frac{y_A - y_1}{1 + \frac{y_2 - y_1}{y_A - y_1}} && \text{as } x_D \leq x_2 \\
&\leq y_A - \frac{(y_A - y_1)^2}{y_A + y_2 - 2y_1} \\
&\leq \frac{y_A^2 + y_A y_2 - 2y_A y_1 - y_A^2 + 2y_A y_1 - y_1^2}{y_A + y_2 - 2y_1} \\
&\leq \frac{y_A y_2 - y_1^2}{y_A + y_2 - 2y_1}.
\end{aligned}$$

Let's define the function $f : y \mapsto \frac{y y_2 - y_1^2}{y + y_2 - 2y_1}$ for $y \in [y_1, y_2]$.

Then $f'(y) = \frac{y_2(y + y_2 - 2y_1) - y y_2 + y_1^2}{(y + y_2 - 2y_1)^2}$ and:

$$\begin{aligned}
f'(y) &\geq 0 \\
\Leftrightarrow y_2(y + y_2 - 2y_1) &\geq y y_2 - y_1^2 \\
\Leftrightarrow y_2^2 - 2y_1 y_2 &\geq -y_1^2 \\
\Leftrightarrow (y_2 - y_1)^2 &\geq 0
\end{aligned}$$

Therefore f is increasing on $[y_1, y_2]$. As $f(y_2) = \frac{y_2^2 - y_1^2}{2(y_2 - y_1)} = \frac{y_2 + y_1}{2} = y_0$, we have that $v_1 \leq y_0$.

Likewise, we can find a point $(u_2, v_2) \in [\mathbf{p_C}, \mathbf{p_B}] \cap L_1$ with $(u_2, v_2) \in S$ and $v_2 \geq y_0$.

S is convex, so the segment $[(u_1, v_1), (u_2, v_2)]$ is a subset of S . As $(x_0, y_0), (u_1, v_1), (u_2, v_2)$ are three points of the line L_1 with $v_1 \leq y_0 \leq v_2$, we have $(x_0, y_0) \in [(u_1, v_1), (u_2, v_2)]$.

Therefore, $(x_0, y_0) \in G$.

\therefore We have shown that $G = (x_0, y_0)$. ■

REFERENCES

- [1] V. Foussereau, I. Kumagai, and G. Caron, "Towards retail stores automation: 6-dof pose estimation combining deep learning object detection and dense depth alignment," in *IEEE/SICE III*, 2024, pp. 1–7.