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# Towards Retail Stores Automation: 6-DOF Pose Estimation combining Deep Learning Object Detection and Dense Depth Alignment Addendum 

Virgile Foussereau ${ }^{1,2}$, Iori Kumagai ${ }^{1}$, Guillaume Caron ${ }^{1,3}$

## I. INTRODUCTION

This document is meant to serve as an addendum to [1]. The main purpose of this addendum is to provide a mathematical demonstration to show that given a convex shape $S$ and its bounding box $B$, the center point is the only point guaranteed to be part of S. This justifies the choice made in [1] to use the bounding box center point as pose initialization for the detected object.

## II. ADDENDUM

Proposition 1. We consider the bounding box $B$ of a convex shape $S$ in the plane. The center point of $B$ is the only point guaranteed to be inside $S$.


Proof: We call $G$ the set of points that are guaranteed to be inside a convex shape $S$ given its bounding box B. We want to show that $G=\left(x_{0}, y_{0}\right)$ with $\left(x_{0}, y_{0}\right)$ the center point of B.

- Uniqueness: First, we show that $G \subseteq\left(x_{0}, y_{0}\right)$.

We consider the four corners of B : $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{1}\right),\left(x_{1}, y_{2}\right),\left(x_{2}, y_{2}\right)$. We consider only convex shapes with non-zero area, so we have $x_{1}<x_{2}$ and $y_{1}<y_{2}$.

[^0]- The segment $S_{1}$ between $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is a convex shape corresponding to the bounding box B , so $G \subseteq S_{1}$.
- Likewise, the segment $S_{2}$ between $\left(x_{2}, y_{1}\right)$ and ( $x_{1}, y_{2}$ ) is a convex shape corresponding to the bounding box B , so $G \subseteq S_{2}$.

Therefore $G \subseteq S_{1} \cap S_{2}$ and, as $S_{1} \cap S_{2}=\left(x_{0}, y_{0}\right)$, we have $G \subseteq\left(x_{0}, y_{0}\right)$.

Existence: We want to show that $\left(x_{0}, y_{0}\right) \in G$.
Let $S$ be a convex shape with bounding box $B$. We have at least one point from $S$ on each edge of $B$ :

$$
\begin{array}{lr}
\mathbf{p}_{\mathbf{A}}=\left(x_{1}, y_{A}\right) & y_{1} \leq y_{A} \leq y_{2} \\
\mathbf{p}_{\mathbf{B}}=\left(x_{2}, y_{B}\right) & x_{1} \leq x_{B} \leq x_{2} \\
\mathbf{p}_{\mathbf{C}}=\left(x_{C}, y_{2}\right) & y_{1} \leq y_{C} \leq y_{2} \\
\mathbf{p}_{\mathbf{D}}=\left(x_{D}, y_{1}\right) & x_{1} \leq x_{D} \leq x_{2}
\end{array}
$$

$\mathbf{p}_{\mathbf{A}}, \mathbf{p}_{\mathbf{B}}, \mathbf{p}_{\mathbf{C}}, \mathbf{p}_{\mathbf{D}} \in S$.
Note that these points are not necessarily distinct.
As $S$ is convex, we have segment $\left[p_{A}, p_{D}\right] \subseteq S$.
If $y_{A}=y_{1}$ or $x_{D}=x_{1}$, we directly have a point from S on line $L_{1}$ between $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$, namely $\left(x_{1}, y_{1}\right)$.

- If $y_{A}>y_{1}$ and $x_{D}>x_{1}$, then the segment $\left[p_{A}, p_{D}\right]$ is not parallel to the line $L_{1}$ between $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$, as neither of the two points are on the line. We can write the equation of line $L_{1}$ as: $L_{1}(x)=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\left(x-x_{1}\right)+y_{1}$. As $L_{1}\left(x_{1}\right)=y_{1}$ and $L_{1}$ is strictly increasing we have:

$$
y_{A}>L_{1}\left(x_{1}\right) \text { and } y_{1}<L_{1}\left(x_{D}\right)
$$

meaning that $\mathbf{p}_{\mathbf{A}}$ and $\mathbf{p}_{\mathbf{D}}$ are not on the same side of $L_{1}$. Therefore we obtain an intersection point $\left(u_{1}, v_{1}\right) \in$ $\left[p_{A}, p_{D}\right] \cap L_{1}$. As S is convex, we also have $\left(u_{1}, v_{1}\right) \in S$. Let's write the equations defining $u_{1}$ and $v_{1}$ :
$\exists \alpha, \beta \in[0,1],\left\{\begin{array}{l}u_{1}=\left(x_{D}-x_{1}\right) \alpha+x_{1}=\left(x_{2}-x_{1}\right) \beta+x_{1} \\ v_{1}=\left(y_{1}-y_{A}\right) \alpha+y_{A}=\left(y_{2}-y_{1}\right) \beta+y_{1} .\end{array}\right.$
We can solve these equations to find $\alpha$ and $\beta$ :
First we express $\beta=\frac{x_{D}-x_{1}}{x_{2}-x_{1}} \alpha$. Then by substitution we have:

$$
\begin{gathered}
\left(\left(y_{2}-y_{1}\right) \frac{x_{D}-x_{1}}{x_{2}-x_{1}}+y_{A}-y_{1}\right) \alpha=y_{A}-y_{1} \\
\Leftrightarrow \alpha=\frac{1}{1+\frac{y_{2}-y_{1}}{y_{A}-y_{1}} \frac{x_{D}-x_{1}}{x_{2}-x_{1}}} \\
\Leftrightarrow \alpha=\frac{1}{1+\frac{y_{2}-y_{1}}{y_{A}-y_{1}} \frac{x_{D}-x_{1}}{x_{2}-x_{1}}} .
\end{gathered}
$$

Therefore we can express the value of $v_{1}$ :

$$
v_{1}=y_{A}-\frac{y_{A}-y_{1}}{1+\frac{y_{2}-y_{1}}{y_{A}-y_{1}} \frac{x_{D}-x_{1}}{x_{2}-x_{1}}},
$$

and find an upper bound:

$$
\begin{array}{rlr}
v_{1} & \leq y_{A}-\frac{y_{A}-y_{1}}{1+\frac{y_{2}-y_{1}}{y_{A}-y_{1}}} \quad \text { as } x_{D} \leq x_{2} \\
& \leq y_{A}-\frac{\left(y_{A}-y_{1}\right)^{2}}{y_{A}+y_{2}-2 y_{1}} \\
& \leq \frac{y_{A}^{2}+y_{A} y_{2}-2 y_{A} y_{1}-y_{A}^{2}+2 y_{A} y_{1}-y_{1}^{2}}{y_{A}+y_{2}-2 y_{1}} \\
& \leq \frac{y_{A} y_{2}-y_{1}^{2}}{y_{A}+y_{2}-2 y_{1}} .
\end{array}
$$

Let's define the function $f: y \longmapsto \frac{y y_{2}-y_{1}^{2}}{y+y_{2}-2 y_{1}}$ for $y \in\left[y_{1}, y_{2}\right]$.
Then $f^{\prime}(y)=\frac{y_{2}\left(y+y_{2}-2 y_{1}\right)-y y_{2}+y_{1}^{2}}{\left(y+y_{2}-2 y_{1}\right)^{2}}$ and:

$$
\begin{aligned}
& f^{\prime}(y) \geq 0 \\
& \Leftrightarrow \quad y_{2}\left(y+y_{2}-2 y_{1}\right) \geq y y_{2}-y_{1}^{2} \\
& \Leftrightarrow \quad y_{2}^{2}-2 y_{1} y_{2} \geq-y_{1}^{2} \\
& \Leftrightarrow \quad\left(y_{2}-y_{1}\right)^{2} \geq 0
\end{aligned}
$$

Therefore f is increasing on $\left[y_{1}, y_{2}\right]$. As $f\left(y_{2}\right)=\frac{y_{2}^{2}-y_{1}^{2}}{2\left(y_{2}-y_{1}\right)}=\frac{y_{2}+y_{1}}{2}=y_{0}$, we have that $v_{1} \leq y_{0}$.

Likewise, we can find a point $\left(u_{2}, v_{2}\right) \in\left[\mathbf{p}_{\mathbf{C}}, \mathbf{p}_{\mathbf{B}}\right] \cap L_{1}$ with $\left(u_{2}, v_{2}\right) \in S$ and $v_{2} \geq y_{0}$.
$S$ is convex, so the segment $\left[\left(u_{1}, v_{1}\right),\left(u_{2}, v_{2}\right)\right]$ is a subset of $S$. As $\left(x_{0}, y_{0}\right),\left(u_{1}, v_{1}\right),\left(u_{2}, v_{2}\right)$ are three points of the line $L_{1}$ with $v_{1} \leq y_{0} \leq v_{2}$, we have $\left(x_{0}, y_{0}\right) \in\left[\left(u_{1}, v_{1}\right),\left(u_{2}, v_{2}\right)\right]$.

Therefore, $\left(x_{0}, y_{0}\right) \in G$.
$\therefore$ We have shown that $G=\left(x_{0}, y_{0}\right)$.

## References

[1] V. Foussereau, I. Kumagai, and G. Caron, "Towards retail stores automation: 6-dof pose estimation combining deep learning object detection and dense depth alignment," in IEEE/SICE SII, 2024, pp. 1-7.


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