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# Towards Retail Stores Automation: 6-DOF Pose Estimation combining Deep Learning Object Detection and Dense Depth Alignment - Addendum

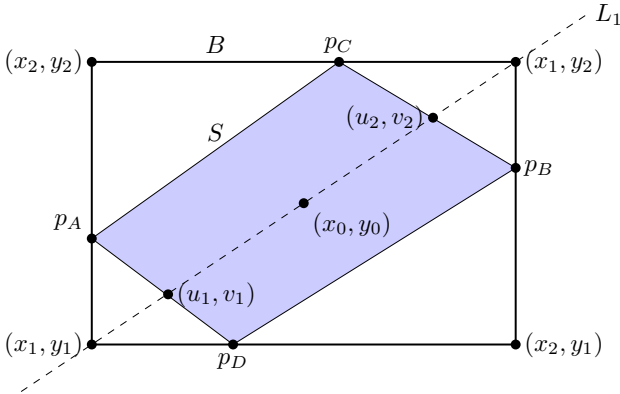
Virgile Foussereau<sup>1,2</sup>, Iori Kumagai<sup>1</sup>, Guillaume Caron<sup>1,3</sup>

## I. INTRODUCTION

This document is meant to serve as an addendum to [1]. The main purpose of this addendum is to provide a mathematical demonstration to show that given a convex shape  $S$  and its bounding box  $B$ , the center point is the only point guaranteed to be part of  $S$ . This justifies the choice made in [1] to use the bounding box center point as pose initialization for the detected object.

## II. ADDENDUM

**Proposition 1.** We consider the bounding box  $B$  of a convex shape  $S$  in the plane. The center point of  $B$  is the only point guaranteed to be inside  $S$ .



*Proof:* We call  $G$  the set of points that are guaranteed to be inside a convex shape  $S$  given its bounding box  $B$ . We want to show that  $G = (x_0, y_0)$  with  $(x_0, y_0)$  the center point of  $B$ .

◆ **Uniqueness:** First, we show that  $G \subseteq (x_0, y_0)$ .

We consider the four corners of  $B$ :  $(x_1, y_1)$ ,  $(x_2, y_1)$ ,  $(x_1, y_2)$ ,  $(x_2, y_2)$ . We consider only convex shapes with non-zero area, so we have  $x_1 < x_2$  and  $y_1 < y_2$ .

· The segment  $S_1$  between  $(x_1, y_1)$  and  $(x_2, y_2)$  is a convex shape corresponding to the bounding box  $B$ , so  $G \subseteq S_1$ .

· Likewise, the segment  $S_2$  between  $(x_2, y_1)$  and  $(x_1, y_2)$  is a convex shape corresponding to the bounding box  $B$ , so  $G \subseteq S_2$ .

Therefore  $G \subseteq S_1 \cap S_2$  and, as  $S_1 \cap S_2 = (x_0, y_0)$ , we have  $G \subseteq (x_0, y_0)$ .

◆ **Existence:** We want to show that  $(x_0, y_0) \in G$ .

Let  $S$  be a convex shape with bounding box  $B$ . We have at least one point from  $S$  on each edge of  $B$ :

$$\begin{aligned} \mathbf{p}_A &= (x_1, y_A) & y_1 &\leq y_A \leq y_2 \\ \mathbf{p}_B &= (x_2, y_B) & x_1 &\leq x_B \leq x_2 \\ \mathbf{p}_C &= (x_C, y_2) & y_1 &\leq y_C \leq y_2 \\ \mathbf{p}_D &= (x_D, y_1) & x_1 &\leq x_D \leq x_2 \end{aligned}$$

$\mathbf{p}_A, \mathbf{p}_B, \mathbf{p}_C, \mathbf{p}_D \in S$ .

Note that these points are not necessarily distinct.

As  $S$  is convex, we have segment  $[p_A, p_D] \subseteq S$ .

· If  $y_A = y_1$  or  $x_D = x_1$ , we directly have a point from  $S$  on line  $L_1$  between  $(x_1, y_1)$  and  $(x_2, y_2)$ , namely  $(x_1, y_1)$ .

· If  $y_A > y_1$  and  $x_D > x_1$ , then the segment  $[p_A, p_D]$  is not parallel to the line  $L_1$  between  $(x_1, y_1)$  and  $(x_2, y_2)$ , as neither of the two points are on the line. We can write the equation of line  $L_1$  as:  $L_1(x) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$ . As  $L_1(x_1) = y_1$  and  $L_1$  is strictly increasing we have:

$$y_A > L_1(x_1) \text{ and } y_1 < L_1(x_D),$$

meaning that  $\mathbf{p}_A$  and  $\mathbf{p}_D$  are not on the same side of  $L_1$ . Therefore we obtain an intersection point  $(u_1, v_1) \in [p_A, p_D] \cap L_1$ . As  $S$  is convex, we also have  $(u_1, v_1) \in S$ . Let's write the equations defining  $u_1$  and  $v_1$ :

$$\exists \alpha, \beta \in [0, 1], \begin{cases} u_1 = (x_D - x_1)\alpha + x_1 = (x_2 - x_1)\beta + x_1 \\ v_1 = (y_1 - y_A)\alpha + y_A = (y_2 - y_1)\beta + y_1. \end{cases}$$

We can solve these equations to find  $\alpha$  and  $\beta$ :

First we express  $\beta = \frac{x_D - x_1}{x_2 - x_1}\alpha$ . Then by substitution we have:

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$$\begin{aligned}
((y_2 - y_1) \frac{x_D - x_1}{x_2 - x_1} + y_A - y_1) \alpha &= y_A - y_1 \\
\Leftrightarrow \alpha &= \frac{1}{1 + \frac{y_2 - y_1}{y_A - y_1} \frac{x_D - x_1}{x_2 - x_1}} \\
\Leftrightarrow \alpha &= \frac{1}{1 + \frac{y_2 - y_1}{y_A - y_1} \frac{x_D - x_1}{x_2 - x_1}}.
\end{aligned}$$

Therefore we can express the value of  $v_1$ :

$$v_1 = y_A - \frac{y_A - y_1}{1 + \frac{y_2 - y_1}{y_A - y_1} \frac{x_D - x_1}{x_2 - x_1}},$$

and find an upper bound:

$$\begin{aligned}
v_1 &\leq y_A - \frac{y_A - y_1}{1 + \frac{y_2 - y_1}{y_A - y_1}} && \text{as } x_D \leq x_2 \\
&\leq y_A - \frac{(y_A - y_1)^2}{y_A + y_2 - 2y_1} \\
&\leq \frac{y_A^2 + y_A y_2 - 2y_A y_1 - y_A^2 + 2y_A y_1 - y_1^2}{y_A + y_2 - 2y_1} \\
&\leq \frac{y_A y_2 - y_1^2}{y_A + y_2 - 2y_1}.
\end{aligned}$$

Let's define the function  $f : y \mapsto \frac{y y_2 - y_1^2}{y + y_2 - 2y_1}$  for  $y \in [y_1, y_2]$ .

Then  $f'(y) = \frac{y_2(y + y_2 - 2y_1) - y y_2 + y_1^2}{(y + y_2 - 2y_1)^2}$  and:

$$\begin{aligned}
f'(y) &\geq 0 \\
\Leftrightarrow y_2(y + y_2 - 2y_1) &\geq y y_2 - y_1^2 \\
\Leftrightarrow y_2^2 - 2y_1 y_2 &\geq -y_1^2 \\
\Leftrightarrow (y_2 - y_1)^2 &\geq 0
\end{aligned}$$

Therefore  $f$  is increasing on  $[y_1, y_2]$ . As  $f(y_2) = \frac{y_2^2 - y_1^2}{2(y_2 - y_1)} = \frac{y_2 + y_1}{2} = y_0$ , we have that  $v_1 \leq y_0$ .

Likewise, we can find a point  $(u_2, v_2) \in [\mathbf{p_C}, \mathbf{p_B}] \cap L_1$  with  $(u_2, v_2) \in S$  and  $v_2 \geq y_0$ .

$S$  is convex, so the segment  $[(u_1, v_1), (u_2, v_2)]$  is a subset of  $S$ . As  $(x_0, y_0), (u_1, v_1), (u_2, v_2)$  are three points of the line  $L_1$  with  $v_1 \leq y_0 \leq v_2$ , we have  $(x_0, y_0) \in [(u_1, v_1), (u_2, v_2)]$ .

Therefore,  $(x_0, y_0) \in G$ .

$\therefore$  We have shown that  $G = (x_0, y_0)$ . ■

## REFERENCES

- [1] V. Foussereau, I. Kumagai, and G. Caron, "Towards retail stores automation: 6-dof pose estimation combining deep learning object detection and dense depth alignment," in *IEEE/SICE III*, 2024, pp. 1–7.