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► To cite this version:

Domenico Gattuso, Rochdi Merzouki, Domenica Savia Pellicanò, Benoît Trouillet. Performance of a freight platform equipped with automated handling units. Analysis thought an optimization model. 25th Euro Working Group on Transportation Meeting (EWGT 2023), Sep 2023, Santander, Spain. hal-04230533v2

HAL Id: hal-04230533

<https://hal.science/hal-04230533v2>

Submitted on 1 Feb 2024

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25th Euro Working Group on Transportation Meeting (EWGT 2023)

Performance of a freight platform equipped with automated handling units. Analysis through an optimization model

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Abstract

Automation should help the freight platforms to be more competitive and sustainable. The automated handling units (AHUs) allow to speed up the freight handling operations by eliminating time wasters, increasing safety, and also allowing for a reduction in traditional labour; but they require continuous monitoring and intelligent management entrusted to specialized personnel. The use of AHUs is also seen positively from an energy and environmental sustainability point of view.

The paper focuses on a smart organizational and functional structure of a freight platform equipped with automated handling vehicles. The study proposes the mathematical formulation of an optimization model that aims to minimize the operating times of handling goods in a logistics platform through the introduction of fully automated vehicles that replace traditional systems.

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Peer-review under responsibility of the scientific committee of the 25th Euro Working Group on Transportation Meeting (EWGT 2023)

Keywords: Freight platform; automated handling units; optimisation model

1. Introduction

The Handling Units (HUs) are fundamental components in the transport and logistics sector; perform functions such as freight handling, moving and storing (Amjath et al., 2022); they are responsible for short-distance goods transfer within a manufacturing facility, warehouse, or plant as per pre-established requirements (Sahu et al., 2017).

HU management plays a fundamental role in the entire logistics process because it affects production times and costs; some research has shown that material handling activities represent a share that can vary from 15% to 70% of the total production cost, depending on the type of product (Soufi et al. 2021).

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Management of freight services is a very complex process that includes several phases and has the main objective of improving the performance of the whole supply chain. In recent years, much attention has been paid to Fleet Management (FM), a tactical-operational activity, developed by private and public companies, for the management of passenger and freight transport services in order to optimize the operation of transport fleets, satisfy customers demand and reduce costs. FM is a broad concept that covers both fleet sizing and configuration as well as vehicle allocation and routing (Monnerat et al., 2019).

There are several mathematical models and quantitative methods developed to optimize and simulate the operation of transport fleets able to support managers' decisions. These models are sometimes simplified and do not consider important real aspects.

In the literature of the sector, there are some researches that have outlined a state of the art in the field of FM. Baldacci et al. (2008) presented an overview of the proposed approaches on the Vehicle Routing Problem (VRP), one of the most studied optimization problems which belongs to a general class referred to as fleet composition and routing problems. Among the first researchers in the sector, Dantzig and Ramser (1959) investigated the different variants of the problem, for example the Capacitated VRP (CVRP) in which the only constraint considered is the capacity, or the VRP with Time Windows (VRPTW) in which customers can be served in a specific time interval (Liu and Shen, 1999). VRP variants consider more deposits, more trips to be made, more types of vehicles, etc. (Cordeau et al., 2006; Bräysy et al., 2002; Toth and Vigo, 2002).

For each problem class there are sub-categories that consider heterogeneous fleets; there is the fixed heterogeneous fleet (HFF) problem and the fleet size and composition problem (FSM). The FSM includes vehicle acquisition costs and depreciation costs in the objective function and differs from the CVRP because it includes the composition of the vehicle fleet in the definition of the problem. The goal is therefore to minimize a total cost function that includes fixed vehicle operating costs and variable routing costs. Baldacci et al. (2008) collected the literature studies in five categories: the first two consider the problems related to a heterogeneous fleet of vehicles (Heterogeneous Fleet VRP-HVRP); the other three refer to problems of mixed VRP (Mixed Fleet VRP-FSM) in which the problem of fleet sizing is also addressed. The study underlying this research was first investigated by Golden et al. (1984). Another collection of scientific models has been proposed by Hoff et al. (2010) who considered the problem of fleet size and mixed vehicle routing (FSMVRP) in the industrial sector and, specifically, he referred to freight transport by road and sea. Hoff et al. (2010) classified the works on the VRP considering three criteria: solution method, type of problem and mode of transport.

A particular state of the art was proposed by Bielli et al. (2011) who analyzed the literature studies relating to FM organizing them both in relation to the type of problem and according to the different modes of transport. The authors referred to problems related to: vehicle routing and scheduling; dynamic fleet management; city logistics; urban public transport; dial-a-ride transport; air transport; maritime transport; rail and intermodal transport.

Pillac et al. (2013) have proposed a general description of VRP focusing on and solution methods for dynamic vehicle routing problems that have been formulated to accommodate real-world applications where information is constantly evolving and sometimes reflecting the possible uncertainty about available data.

Srivatsa Srinivas and Gajanand (2017) have introduced a review of existing models of VRP, planner behaviour models in the VRP context and driver behaviour models and provides a motivation to integrate these models in a stochastic traffic environment to produce practical, economic and driver-friendly logistics solutions. The research provides valuable insights on the relevance of behavioural issues in logistics and highlights the modelling implications of incorporating planner and driver behaviour in the framework of routing problems.

Moghdani et al. (2021) have presented a literature review on green VRP classifying the works in relation to the different aspects of VRP addressed. Sar and Ghadimi (2023) have investigated the state of the-art of VRP in Reverse Logistics operations by reviewing published peer-reviewed articles published between 2000 and 2022.

The paper focuses attention on FM in a freight interchange platform equipped with automated vehicles (AGV) which represent new organizational solutions and allow activities to be developed autonomously and efficiently, eliminating wasted time, increasing safety, guaranteeing a certain level of sustainability.

The authors have already addressed the topic of logistics platform automation (*Gattuso et al., 2022; Gattuso and Pellicanò, 2022*); in this study, the mathematical formulation of an optimization model is proposed that seeks to minimize the operating times of freight handling in a platform through the introduction of automated handling units.

The paper is organized in three sections; the first part describes the methodological approach used in the study; in the second one, the formulation of the optimization problem is described; in the last section the results of an application are presented.

2. Methodological Approach

A freight platform, like any logistics system, has to deal with several organizational problems every day (economic, financial, technological, production and, last but not least, logistics). To solve problems, it is necessary to adopt often complex and moreover timely decisions, since every minute of delay can mean loss of capital or, more generally, damage to the system. It is rarely possible, convenient or lawful to carry out experiments on the real system, even if this is the only way to have sure answers; in order to make choices that are as rational and correct as possible, it is therefore necessary to conduct experiments using a model which, in a more or less simplified way, represents the reality to be investigated

The models and therefore the techniques that can be used in such circumstances can essentially be divided into three categories, as shown in Figure 1: analytical models; simulation models; optimization models.

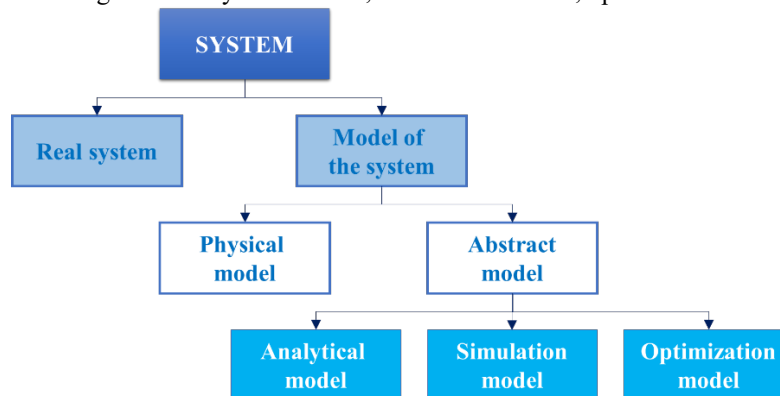


Fig. 1. Classification of survey methods

The attention is placed on the first class of methods and involves the formalization of the problem in a mathematical model, from the resolution of which it is possible to obtain the solution to the problem.

A mathematical model is a model built using the language, tools and principles of mathematical analysis, probability theory and statistics. The concept of analytical models is referred to operations research, which provides mathematical tools to support decision-making activities when it comes to managing and coordinating limited activities and resources, in order to maximize or minimize an objective function.

A mathematical model is often built with the aim of faithfully representing a certain real object or phenomenon to provide predictions about the future state. Generally, the model describes the probable evolution of a phenomenon or of a physical system on the basis of initial data (basic conditions) provided by the user (input) returning final data (output). The effectiveness of the model can therefore be measured by comparing the final data with the actual observed result of the evolution of the phenomenon or system. The drafting of a mathematical model embraces many fields of pure and applied science precisely because it allows the overcoming of the simple qualitative treatment to instead obtain a quantitative treatment of the phenomenon or of the system of interest.

Operations research plays an important role in decision-making because it allows to make the best choices to achieve a certain goal by respecting constraints that are imposed from the outside and are not under the control of those who have to make the choices; it takes care of formalizing a problem in a mathematical model and calculating

an optimal, when possible, or an approximate (also called sub-optimal) solution for it. The aim of the operations research is the study and development of methodologies for solving decision-making problems. The problems faced in the field of operations research are typically those in which decisions have to be made on the use of resources available in limited quantities in order to respect an assigned set of constraints, maximizing the benefit obtainable from the use of the resources.

The steps for building an analytical model can be summarized as follows:

- 1. identification of the problem; this phase involves the identification of the controllable variables (decision variables that can be continuous or discrete) and of the non-controllable ones, together with the choice of the economic function to maximize (e.g. the overall gain) or to minimize (e.g. the overall cost); this function is called objective function;
- 2. analysis of reality and collection of useful data;
- 3. construction of the model, which has to be a good (simplified) representation of the problem; it is not something static and definitive, but it can possibly be modified in a subsequent revision to make it more coherent to the problem. The mathematical model normally consists of an economic function to be optimized and a system of constraints expressed by equations and/or inequalities (these constraints can be of a physical, economic or regulatory type);
- 4. determination of one or more solutions using an algorithm. An algorithm is a sequence of operations aimed at determining the decision variables contained in the model; algorithms can be exact or heuristic. It is a question of identifying the value of the decision variables that maximize/minimize the objective function in compliance with the constraints;
- 5. analysis of the results obtained. It is checked whether the theoretical function offers the expected advantages and the representativeness of the model is verified; if necessary, based on the results of this phase, a modification is made to the model itself, and the procedure is repeated.

These steps are not to be seen as strictly sequential; in a real decision-making process it is frequent the case that one of the phases requires modifications of the results obtained in a previous phase. For example, in the construction of the model the need for new data in addition to those already collected may emerge. Data collection presupposes an idea of the type of model that will be built.

3. Formulation of the optimization problem

Optimization is a well-known problem in computer science, economics and mathematics. The main interest is to obtain the best solution among all feasible solutions to a problem; the main difficulty is to design a robust and executable model in order to propose a feasible solution with all the properties needed for a good solution. In the literature of sector there are some references like *Erdoğan and Miller-Hooks (2012)* who proposed MILP formulation for Green Vehicle Routing Problems (GVRP) or *Schneider et al. (2014)* who studied the customer time windows, demand and capacity constraints using Electric VRP. Based on *Erdoğan and Miller-Hooks (2012)* approach, *Masmoudi et al. (2022)* developed a simulated annealing heuristics to solve the GVRP.

Optimization could provide a better overview of resources used by an operation to maximize or minimize the profit or cost. Optimization problems can be divided into categories based on the variables, and variables can be classified as continuous variables or discrete variables, called continuous optimization or discrete optimizations, respectively. The optimal value should come from a continuous function in continuous optimization. In discrete optimization objects, for example, integer, graph or permutation objects should be found from a countable set.

The primary goal of the work is to build a model and use that model to optimize the operation for the targeted missions. A second objective is to show the feasibility of this approach in an industrial context. To achieve the primary goal (Fig. 2), the main goal has subdivided into smaller classic tasks as follows.

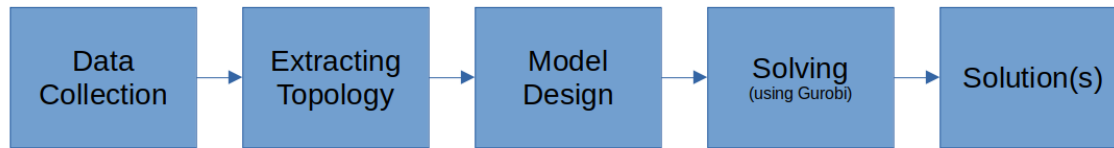


Fig. 2. Operations design

First of all, the *data collection* has to be done from the studied platform. The studied platform is represented in Fig. 3. Data extraction from a freight platform involves retrieving relevant information from the platform's topology (*extracting topology*); this process typically includes identifying the required data fields which represents the virtual grid (in black on crossdock "A" in Fig. 3). The extracted data is then transformed into a suitable format for analysis or integration into other systems, often using techniques like data cleaning, normalization, and structuring (.json format). Finally, the extracted and transformed data can be used for feeding into mathematical modeling algorithms for predictive analysis. The goal is to find routes for Automated HUs (AHUs) in a freight platform. Routes are evaluate based on the total time spent to complete all missions. To achieve this goal, a virtual grid is created on the platform representing all possible roads. The used approach involves structuring the routes in terms of blocks and crossroads. A block is a portion of the road that can be occupied by at most one AHU, thus avoiding collisions. The AHU moves from block to block via crossroads. It is assumed that these nodes are static.

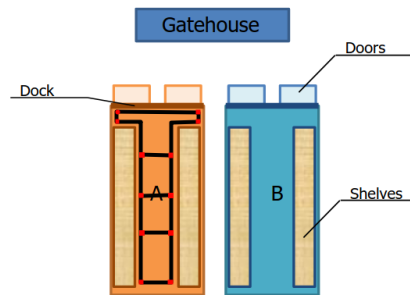


Fig. 3: Logistic platform and virtual grid

The next step is related to the mathematical model design. The decision variables have to be defined that represent the transportation routes and schedules for the shipment. There are two kinds of constraints to consider; there are variables that denote the logical behavior of the AHU, including:

- ↯ the departure and arrival blocks assigned to an AHU;
- ↯ when an AHU leaves canton, the canton became free and, the opposite, if the AHU enters in a canton, the canton is occupied;
- ↯ it is necessary to ensure that the crossroad between two cantons is either free or busy;
- ↯ it is necessary to ensure that the topology is respected, as a virtual grid has been defined;
- ↯ the connectivity of the AHUs routes must be maintained;
- ↯ U-turns are prohibited.

This representation involves fifteen equations or inequalities. Some constraints are introduced to define the hourly intervals of the AHU's presence in each of these locations. Once these time intervals are known, security constraints can be imposed to prevent two AHUs from being in the same canton or crossroads at the same time. The equations are thus obtained:

- 7.8 Hourly interval of occupation of a block: the hourly interval of occupation of a block by an AHU is an interval, where the time bounds are expressed in units of time starting from time zero, which corresponds to the date reception of missions by the “macro” layer. These are constrained by the nominal crossing time and the block exit time, which are considered according to the canton crossed previously.
- 7.8 Origin: the AHUs occupy their original canton from time zero. The beginning of the occupation interval of these blocks must, therefore, be initialized to zero.
- 7.8 Succession of blocks: When an AHU passes from one block to another, the time intervals of occupation of the two blocks must follow one another, taking into account the duration of crossing the crossroads between the two cantons.
- 7.8 Time slots of unused blocks: the AHU that do not pass through a canton see their time intervals be set to time zero, in order to reduce the number of unassigned variables of the mathematical model to solve. Thus, there is a relationship between some decisions variables and the hourly intervals of occupation of the blocks.
- 7.8 Hourly interval of intersection occupation: the hourly interval of occupation of an intersection by an AHU is an interval, where the time bounds are expressed in seconds, starting from time zero corresponding to the date reception of missions by the “macro” layer. these are constrained by the nominal crossing time and by the exit time from a considered intersection according to the canton previously crossed and the canton of destination.
- 7.8 Crossing crossroads: when crossing a block to another block, the AHU crosses the crossroads between the two blocks. The time interval corresponding to the crossing of this crossroads depends on its date of exit from initial block.
- 7.8 Time interval of unused intersections: the AHU not passing through a crossroads see their time intervals be set to time zero, in order to reduce the number of unassigned variables of the mathematical model to solve.

The proposed mathematical model has expanded to include eight equations. The construction of the mathematical model is completed by incorporating the security constraints.

- 7.8 Order of occupation of the cantons: the order of occupation of the block by two AHUs is characterized by additional binary variables. The concept is based on the requirement that one AHU must be scheduled before the other.
- 7.8 Township occupancy: if the two considered AHU pass through the same canton, they use this resource at different times.
- 7.8 Order of intersection occupancy: the order of occupation of an intersection by two is characterized by one more binary variable. One of AHU must be scheduled before the other.
- 7.8 Occupation of crossroads: The same considerations as before are used to prohibit the presence of two different AHU at the same crossroads.

For problem *solving*, a robust and executable model has to be defined. It is very important to define the model accurately to address the problem. Here there are forty equations, each characterized a property in the solution. There is a lot of variables: about $B2A+(B+C)A2+5A(B+C)$ binary variables where A is the number of AHU, B the number of cantons and C the number of crossroads. An objective function has to be defined to optimize the transportation process based on some criteria, such as, in this paper, minimizing transportation time; the minimizing cost, or maximizing vehicle utilization, is also considered. Finally, according to the objective, a set of routes has to be found for all the vehicles. It would be essential to see how the parameters are working. Knowing the computation time and CPU consumption will also be necessary to solve the problem. It is necessary to know what is a good solution through the definition of a performance evaluation. The solutions can be differentiated by the value of the objective function; scenarios development conclude the resolution.

Finally, *solutions* are obtained. Mixed Integer Linear Programming (MILP) and Constrained Programming (CP) along with commercial Gurobi solver used to conduct this experimental research. A timeout parameter is used to ensure a prompt response. The consequence is two kinds of results:

- ↯ No solution: the system is too constrained. We use a heuristic (a greedy algorithm) to find a good solution quickly;
- ↯ Some solutions: GUROBI proposes the best solution found in the time granted. The best solution is evaluated by an objective function (the total time required for all missions).

4. Results and discussion

The problem is solved in a standard platform. The solution is computed for a problem with a number of cantons between 10 or 200. The number of AHU varies from 2 to 16. The results of the heuristics are compared with optimal solutions found by Gurobi. The heuristics are run on a PC with a 2.7 GHz CPU and 32 GB of RAM. These results are shown in Table 1. The column “C” represents the cost of the solution found. The column “T” represents the computing time in seconds. The column “G” represents the difference between the value of the heuristic solution and the Gurobi solution.

Table 1. Results.

	Heuristic			Gurobi	
	C	T(s)	G(%)	C	T(s)
C_10_2	20,2	<1	0	20.2	<1
C_50_4	81,7	<1	0	81.7	<1
C_100_8	172,5	<1	0	172.5	<1
C_200_8	1078,5	10	9,8	982.5	16629
C_200_16	N/A	N/A	N/A	N/A	N/A

With up to 4 AHUs, the optimal solution can be found in a reasonable time. The greedy algorithm finds the optimal solution for 5 sets on 7. For the last set with 16 AGV, Gurobi is unable to find a solution in a predefined time. The heuristic always finds a solution in less than one minute, with a gap with the optimal solution varying between 0% and 9.8%. The naming is done as follow: C_X_Y where X is the number of cantons and Y the number of AHU. The following results are obtained in the next array.

Gurobi is a powerful mathematical optimization solver that can be used to solve complex optimization problems, including those in the transportation industry. By using Gurobi, freight platform operators can optimize the allocation of resources such as AHU, warehouses, and staff, to ensure that goods are moved quickly and efficiently.

For instance, the problem is solved with eight AHUs in a small platform (about 100 cantons). The results obtained showed that it is possible to find the best solution for all missions (eighty missions) in less than one second. However, with the solving scale increasing, it’s hard to handle by GUROBI, and the memory occupied increases sharply. Thus, we use some greedy heuristics when the size grows. The opportunity to find some best ways in a few times if the exact method can’t propose something in a defined duration.

5. Conclusions

In this paper, the AHU routing problem in a freight platform is studied. An integer linear arc-based model and a path-based model are formulated. The problem of performance optimization of a freight platform using Gurobi can be formulated as a mathematical optimization problem. The objective is to minimize the total cost of moving goods from the origin to the destination while meeting a set of constraints. The constraints may include the capacity of the warehouses, the availability of AHU, and the delivery time windows. The arc-based model can be solved by GUROBI. Indeed, for a small example, it works well. This methodology suffers with scalability but it can be avoid using greedy heuristics. Thus, we don't find the best solution but a good solution. However, by using optimization software such as Gurobi, freight platform operators can optimize the allocation of resources and improve the

efficiency of the platform. This can result in significant cost savings, reduced delays, and a more sustainable transportation system.

By incorporating time windows in optimization models using Gurobi, freight platforms can optimize the transportation process to ensure that shipments are picked up and delivered within their time windows while minimizing transportation time and cost. This can lead to significant efficiency gains and cost savings for the platform and its customers.

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