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Characterization of porous materials with a rigid frame via reflected waves

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The inverse problem for waves reflected by porous material is solved at oblique incidence, and an inverse scattering calculation of porosity and tortuosity is given for air-saturated plastic foam samples. The interaction of the sound pulse with the fluid-saturated porous material is described by a time-domain equivalent fluid model. The sensitivity of the porosity and tortuosity is studied and it shows their effect on the reflection coefficient at the first interface. This study shows that porosity is much more sensitive than tortuosity to reflection, especially when the incident angle is less than its critical value, at which the reflection coefficient vanishes. Some advantages and perspectives of this method are discussed. © 2003 American Institute of Physics. [DOI: 10.1063/1.1629386]

I. INTRODUCTION

Porous media filled with air,^{1–3} such as fibrous mats, plastic foams, and various felts, are used extensively in the automobile, aeronautical, and building industries to attenuate sound waves. Lately, low frequency (i.e., in the 20–600 kHz range) ultrasonic techniques have amply proven their worth as powerful tools by which to probe the acoustic properties of these materials.

Determining the properties of a medium using waves reflected by or transmitted through the medium is an inverse scattering problem.^{4–8} Such problems are often approached by taking a physical model of the scattering process, generating a synthetic response for a number of assumed parameter values, and adjusting these parameters until a reasonable level of correspondence is attained between the synthetic response and the data observed.

Ultrasonic materials can often be characterized by measuring the attenuation coefficient and phase velocity in the frequency domain^{9–11} or by measuring transmitted and reflected waves in the time domain.^{4–8,12} In the frequency domain, measurement of the attenuation coefficient may be more robust than measurement of the phase velocity. In these situations, application of the Kramers–Kronig dispersion relations¹³ may allow the phase velocity to be determined from the measured attenuation coefficient.

To efficiently cope with the specific problems that occur in transient acoustic field propagation, new approaches are required.^{12,14} At present, most signal propagation analyses

are performed in the frequency domain using the Fourier transform, and the results are translated into the time domain, and vice versa. However this has several limitations. The first one is that transformation is difficult to compute numerically with sufficient accuracy for nonanalytical functions. For example, using the Fourier transform to obtain time-domain results for a lossy material is a more complicated approach than using a true time-domain analysis, and the numerical results are less accurate. The second disadvantage is that by working in the frequency domain, some numerical information is lost or hard to recover. For example, in the case of noisy data it may be difficult to reconstruct the chronological events of a signal by phase unwrapping. Consequently, it is difficult to obtain a deep understanding of transient signal propagation using the frequency-domain method.

A time-domain approach differs from frequency analysis in that the susceptibility functions that describe viscous and thermal effects are convolution operators that act on the velocity and pressure fields and therefore a different algebraic formalism must be applied to solve the wave equation. The time-domain response of the material is described by an instantaneous response and a “susceptibility” kernel responsible for memory effects. In the past, many authors have used fractional calculus as an empirical method to describe the properties of viscoelastic materials, e.g., Caputo¹⁵ and Bagley and Torvik.¹⁶ The observation that asymptotic expressions of stiffness and damping in porous materials are proportional to fractional powers of frequency suggests that time derivatives of fractional order might describe the behavior of sound waves in this kind of material, including relaxation and frequency dependence.

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In this article, we solve the inverse problem for a slab of porous material with a rigid frame via reflected waves for different incident angles. An optimization procedure based on the least-square method is used that gives an estimate of the porosity and tortuosity of air-saturated porous materials. This method is based on a temporal equivalent fluid description of the propagation of sound in fluid-saturated porous material. This model was initially introduced by the authors⁶ to describe the memory effect of the wave during its propagation in the time domain. The advantage of this method is its simplicity compared to classical methods (summarized in this article).

The article is outlined as follows. In Sec. II, the parameters for ultrasonic transport in air-saturated porous material are defined and the classical experimental techniques used to estimate them are given. Section III describes a time-domain model and the basic equations of wave propagation in porous material. Section IV is devoted to the direct problem and expression of the reflection and transmission kernels in the time domain at oblique incidence. The sensitivity of the porosity and tortuosity and of the incident angle is discussed, showing the effect of each parameter on the reflection coefficient at the first interface. Section VI contains the numerical solution of the inverse problem based on the least-square method. Experimental validation that relies on ultrasonic measurement is discussed for air-saturated industrial plastic foams.

II. ULTRASONIC TRANSPORT PARAMETERS IN AIR-SATURATED POROUS MATERIAL

The quantities involved in sound propagation in porous materials can be defined locally, on a microscopic scale. However, this type of study is generally difficult because of the complicated geometry of the frames. Only the mean values of the quantities involved are of practical interest. Averaging must be performed on a macroscopic scale using volumes with dimensions sufficiently large for the average to be significant. At the same time, these dimensions must be much smaller than the wavelength. Even on a macroscopic scale, a description of sound propagation in porous material can be very complicated, since sound also propagates within the frame of the material. If the frame is motionless, the porous material can be replaced, macroscopically, by an equivalent fluid. If the porosity is low, the equivalent fluid has the same properties as air in porous material on a macroscopic scale.

In 1987, a substantial contribution was made by Johnson *et al.*,¹⁷ who put forth a theory of dynamic fluid flow (i.e., as a function of frequency) in porous media, thereby introducing the concept of dynamic tortuosity and permeability as well as viscous characteristic length. Originally, their research mainly focused on geophysical and petroleum industry applications. The Johnson *et al.* model was used for different kinds of porous materials.^{1,4-13,17-24} For gas-saturated porous media, Allard¹ produced an analogous theory for thermal effects by introducing the concept of thermal characteristic length. Lafarge *et al.*^{22,23} extended this theory by adding the concept of thermal permeability which plays an important role in low frequency approximations.

A. Porosity

One important parameter which appears in theories of sound propagation in porous materials is porosity ϕ . Porosity is the relative volume fraction of the air contained in the material. Unlike other parameters included in the description of different various phenomena occurring in acoustic propagation of porous media at a high frequency range, such as tortuosity,¹⁷ viscous characteristic length,¹⁷ and thermal characteristic length,¹ or at a low frequency range, such as flow resistivity¹⁷ and thermal permeability,^{22,23} porosity is a key parameter that plays an important role in propagation at all frequencies. As such, in studies of the acoustic properties of porous materials, it is extremely useful to be able to measure this parameter.

Beranek²⁵ described an apparatus (a porosimeter) used to measure the porosity of porous materials. This device was based on the equation of state for ideal gases at constant temperature (i.e., Boyle's law). Porosity can be determined by measuring the change in air pressure that occurs with a known change in volume of the chamber containing the sample. In the Beranek apparatus, changes in both pressure and volume are monitored using a U-shaped fluid-filled manometer. An alternative technique for measuring porosity is a dynamic method proposed by Leonard.²⁶ Techniques that use water as the pore-filling fluid, rather than air, are common in geophysical studies.^{27,28} Mercury has been used as the pore-filling fluid in other applications.²⁹ However, for many materials, the introduction of liquids into the material is not appropriate. A similar device to that of Beranek's, that involves the use of an electronic pressure transducer, was introduced by Champoux *et al.*³⁰ This device can be used to measure very slight changes in pressure accurately, and the output can be recorded by a computer. Recently, a simple alternative method for measuring porosity via waves reflected at the first interface of a slab of air-saturated porous material was proposed by the authors, and it gave good results for plastic foams^{7,8,31} and random bead packing.³²

B. Tortuosity

Tortuosity α_∞ is an important geometrical parameter included in the description of inertial coupling between the fluid and the structure of porous material in the high frequency range. This parameter represents the refraction index for porous materials.⁶ The concept of tortuosity is not recent, and it is found with different notations and meanings in previous studies. It is the structure form factor k_s according to Zwikker and Kosten³³ or parameter q^2 according to Attenborough.³⁴ In the case of cylindrical pores at angle ϑ to the direction of propagation, $\alpha_\infty = 1/\cos^2 \vartheta$. In Carman's book,³⁵ tortuosity is related to $(\cos \vartheta)^{-1}$. An electrical method for measuring tortuosity was developed by Carman,³⁵ Brown,³⁶ and Johnson *et al.*,²⁸ which, however, can only be used if the frame does not conduct electricity. The porous material is saturated with a conducting fluid, and the resistivity of the saturated material is measured between two electrodes using the relation

$$\alpha_\infty = \phi \frac{r_c}{r_f}, \tag{1}$$

where r_c and r_f are the measured resistivities of the saturated material and the fluid, respectively. From relation (1), it can be seen that the tortuosity is independent of the shape of the pore cross sections. By introducing the concept of dynamic tortuosity for common porous materials with arbitrary pore shape, Johnson *et al.*¹⁷ gave another definition of tortuosity,

$$\alpha_\infty = \lim_{\omega \rightarrow \infty} \frac{\langle u_m^2 \rangle_V}{\langle \mathbf{u}_m \rangle_V^2}, \tag{2}$$

where ω is the angular frequency and u_m is the microscopic velocity for a steady flow of inviscid fluid. The symbol $\langle \rangle_V$ denotes the average of the fluid volume V and the term $\langle \mathbf{u}_m \rangle$ is interpreted as macroscopic velocity. Johnson *et al.*¹⁷ proposed a method for measuring tortuosity using superfluid ⁴He as the pore fluid.

Tortuosity takes a low value ($\alpha_\infty = 1$) for porous material with straight pores, and high values ($\alpha_\infty = 2,3$) for porous media with high resistivity. This parameter can also be evaluated using ultrasonics for measurement in the frequency domain,⁹⁻¹¹ from the attenuation or phase velocity, or in the time domain by solving the inverse problem in the transmitted⁴⁻⁶ or reflected mode^{7,8,31,32}

III. MODEL

In porous material acoustics, a distinction can be made between two situations depending on whether the frame is moving or not. In the first case, wave dynamics due to the solid frame–fluid coupling are clearly described by Biot theory.^{37,38} In air-saturated porous media, the structure is generally motionless and the waves propagate only in the fluid. This case is described by the equivalent fluid model which is a particular case of the Biot model, in which fluid–structure interactions are taken into account in two frequency response factors: dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al.*¹⁷ and dynamic compressibility of the air in the porous material $\beta(\omega)$ given by Allard.¹ In the frequency domain, these factors multiply the fluid’s density and compressibility, respectively, and represent the deviation of the fluid’s behavior in free space as the frequency increases. In the time domain, they act as operators and in the asymptotic domain (high frequency approximation) with their expressions given^{4-8,12} by

$$\tilde{\alpha}(t) = \alpha_\infty \left(\delta(t) + \frac{2}{\Lambda} \left(\frac{\eta}{\pi \rho_f} \right)^{1/2} t^{-1/2} \right), \tag{3}$$

$$\tilde{\beta}(t) = \left(\delta(t) + \frac{2(\gamma-1)}{\Lambda'} \left(\frac{\eta}{\pi Pr \rho_f} \right)^{1/2} t^{-1/2} \right). \tag{4}$$

In these equations, $\delta(t)$ is the Dirac function, Pr is the Prandtl number, η and ρ_f are fluid viscosity and fluid density, respectively, and γ is the adiabatic constant. The relevant physical parameters of the model are the medium’s tortuosity α_∞ initially introduced by Zwikker and Kosten,³³ and viscous and the thermal characteristic lengths Λ and Λ' intro-

duced by Johnson *et al.*¹⁷ and Allard.¹ In this model time convolution of $t^{-1/2}$ with a function is interpreted as a semiderivative operator according to the definition of the fractional derivative of order ν given in work by Samko and colleagues³⁹

$$D^\nu[x(t)] = \frac{1}{\Gamma(-\nu)} \int_0^t (t-u)^{-\nu-1} x(u) du, \tag{5}$$

where $\Gamma(x)$ is the gamma function.

In this framework, the basic equations of our model obtained from the conservation of momentum and conservation of mass can be written as

$$\rho_f \tilde{\alpha}(t) * \frac{\partial v_i}{\partial t} = -\nabla_i p$$

and

$$\frac{\tilde{\beta}(t)}{K_a} * \frac{\partial p}{\partial t} = -\nabla \cdot \mathbf{v}, \tag{6}$$

where the asterisk $*$ denotes the time convolution operation, p is the acoustic pressure, \mathbf{v} is the particle velocity, and K_a is the bulk modulus of air. The first equation is the Euler equation, the second one is the constitutive equation.

In the plane (xoz) constitutive Eqs. (6) can be written as

$$\begin{aligned} \rho_f \alpha_\infty \frac{\partial v_x(x,z,t)}{\partial t} + \frac{2\rho_f \alpha_\infty}{\Lambda} \left(\frac{\eta}{\pi \rho_f} \right)^{1/2} \\ \times \int_0^t \frac{\partial v_x(x,z,t')/\partial t'}{\sqrt{t-t'}} dt' = -\frac{\partial p(x,z,t)}{\partial x}, \\ \rho_f \alpha_\infty \frac{\partial v_z(x,z,t)}{\partial t} + \frac{2\rho_f \alpha_\infty}{\Lambda} \left(\frac{\eta}{\pi \rho_f} \right)^{1/2} \\ \times \int_0^t \frac{\partial v_z(x,z,t')/\partial t'}{\sqrt{t-t'}} dt' = -\frac{\partial p(x,z,t)}{\partial z}, \tag{7} \\ \frac{1}{K_a} \frac{\partial p(x,z,t)}{\partial t} + \frac{2(\gamma-1)}{K_a \Lambda'} \left(\frac{\eta}{\pi \rho_f Pr} \right)^{1/2} \\ \times \int_0^t \frac{\partial p(x,z,t')/\partial t'}{\sqrt{t-t'}} dt' = -\frac{\partial v_x(x,z,t)}{\partial x} - \frac{\partial v_z(x,z,t)}{\partial z}, \end{aligned}$$

where v_x and v_z are the components of particle velocity along axes x and z .

In these equations, the convolutions express the dispersive nature of the porous material. They take into account memory effects due to the fact that the medium’s response to wave excitation is not instantaneous but instead needs some time to take effect.

IV. DIRECT PROBLEM

The direct scattering problem involves determining the scattered field as well as the internal field that arises when a known incident field impinges on the porous material with known physical properties. To compute the solution of the direct problem one needs to know the Green’s function^{4,40} of

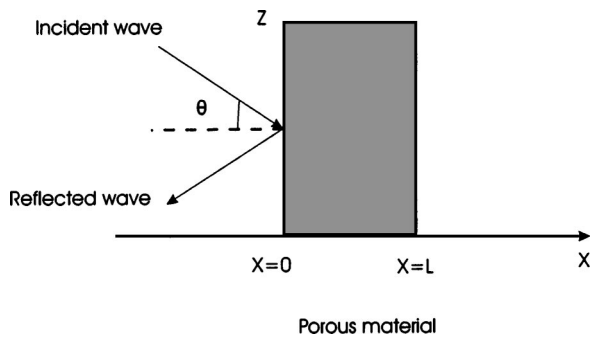


FIG. 1. Problem geometry.

the modified wave equation in a porous medium. In this case, the internal field is given by the time convolution of the Green's function with the incident wave, and the reflected and transmitted fields are deduced from the internal field and the boundary conditions.

Here in Sec. IV some notation is introduced. The problem geometry is shown in Fig. 1. A homogeneous porous material occupies region $0 \leq x \leq L$. This medium is assumed to be isotropic and to have a rigid frame. A short sound pulse impinges at oblique incidence on the medium from the left, giving rise to an acoustic pressure field $p(x, z, t)$ and an acoustic velocity field $\mathbf{v}(x, z, t)$ within the material, which satisfy the system of Eqs. (7) and can be written as

$$a \frac{\partial v_x(x, z, t)}{\partial t} + b D^{1/2} [v_x(x, z, t)] = - \frac{\partial p(x, z, t)}{\partial x}, \quad (8)$$

$$a \frac{\partial v_z(x, z, t)}{\partial t} + b D^{1/2} [v_z(x, z, t)] = - \frac{\partial p(x, z, t)}{\partial z}, \quad (9)$$

$$d \frac{\partial p(x, z, t)}{\partial t} + f D^{1/2} [p(x, z, t)] = - \frac{\partial v_x(x, z, t)}{\partial x} - \frac{\partial v_z(x, z, t)}{\partial z}, \quad (10)$$

with

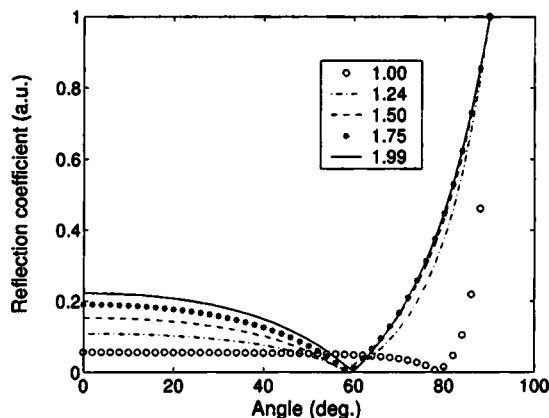


FIG. 2. Variation of the reflection coefficient at the first interface r with the incident angle θ for fixed porosity value $\phi=0.9$ and for different values of tortuosity $\alpha_\infty=1.99$ (solid line), 1.75 (star), 1.5 (dashed line), 1.24 (dash-dotted line), and 1 (circle).

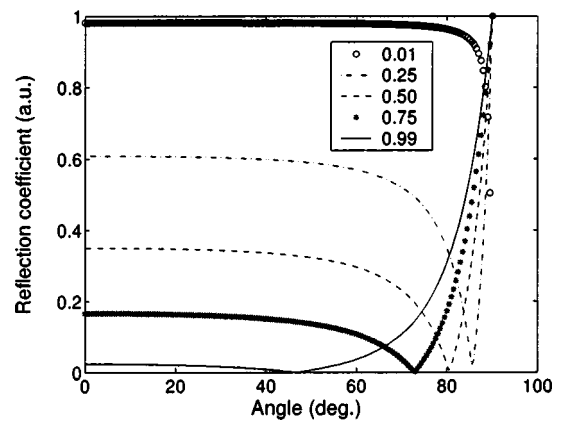


FIG. 3. Variation of the reflection coefficient at the first interface with the incident angle for fixed tortuosity value $\alpha_\infty=1.1$ and for different values of porosity $\phi=0.99$ (solid line), 0.75 (star), 0.50 (dashed line), 0.25 (dash-dotted line), and 0.01 (circle).

$$a = \rho_f \alpha_\infty, \quad b = \frac{2 \rho_f \alpha_\infty}{\Lambda} \left(\frac{\eta}{\rho_f} \right)^{1/2},$$

$$d = \frac{1}{K_a},$$

and

$$f = \frac{2(\gamma-1)}{K_a \Lambda'} \left(\frac{\eta}{\rho_f P_r} \right)^{1/2}.$$

In the region of $x \leq 0$, the incident pressure wave is given by

$$p^i(x, z, t) = p^i \left(t - \frac{x \cos \theta}{c_0} - \frac{z \sin \theta}{c_0} \right), \quad (11)$$

where c_0 is the velocity of the free fluid ($x \leq 0$); $c_0 = \sqrt{K_a / \rho_f}$.

In the region of $0 \leq x \leq L$, the pressure wave is given by

$$p(x, z, t) = p \left(t - \frac{x \cos \theta'}{c'} - \frac{z \sin \theta'}{c'} \right), \quad (12)$$

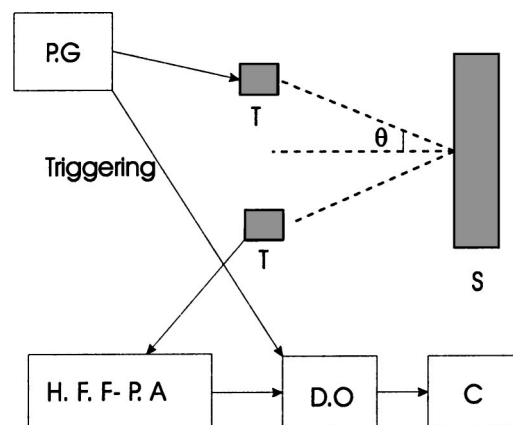


FIG. 4. Experimental setup of ultrasonic measurements in reflected mode. PG: Pulse generator, HFFPA: high frequency filtering preamplifier, DO: digital oscilloscope, C: computer, T: transducer, S: sample.

TABLE I. Measured values of porosity and tortuosity using classical methods (after Refs. 4–6 and 9–11).

Material	M1	M2	M3
Tortuosity	1.1	1.1	1.5
Porosity	0.95	0.99	0.86

where c' is the velocity in the porous material ($0 \leq x \leq L$), and the refraction angle θ' is given by Descartes–Snell law¹ as

$$\frac{\sin \theta}{c_0} = \frac{\sin \theta'}{c'} \tag{13}$$

To simplify the system of Eqs. (7), we can then use the following property:

$$\frac{\partial}{\partial z} = -\frac{\sin \theta}{c_0} \frac{\partial}{\partial t} \tag{14}$$

which implies

$$\frac{\partial v_z}{\partial z} = -\frac{\sin \theta}{c_0} \frac{\partial v_z}{\partial t}$$

and

$$\frac{\partial p}{\partial z} = -\frac{\sin \theta}{c_0} \frac{\partial p}{\partial t} \tag{15}$$

From Eqs. (9) and (15), we then obtain the relation,

$$\frac{\partial v_z}{\partial z} = -\frac{\sin^2 \theta}{c_0^2} (a + bD^{-1/2})^{-1} * \frac{\partial p}{\partial t} \tag{16}$$

Using Eqs. (8), (10), and (16), the equation system [Eqs. (8)–(10)] can thus be simplified to

$$a \frac{\partial v_x(x, z, t)}{\partial t} + bD^{1/2}[v_x(x, z, t)] = -\frac{\partial p(x, z, t)}{\partial x} \tag{17}$$

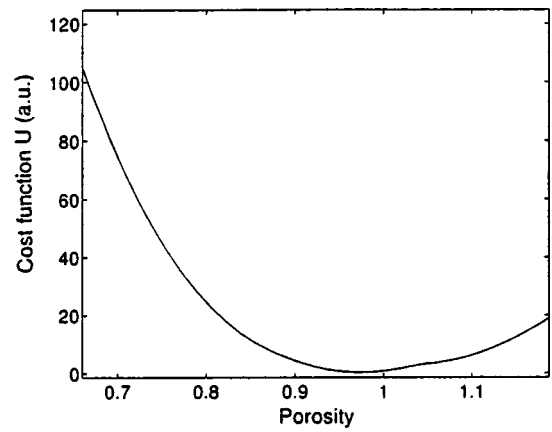


FIG. 6. Variation of cost function U with the porosity for plastic foam M1.

$$\begin{aligned} d \frac{\partial p(x, z, t)}{\partial t} + fD^{1/2}[p(x, z, t)] \\ = -\frac{\partial v_x(x, z, t)}{\partial x} + \frac{\sin^2 \theta}{c_0^2} (a + bD^{-1/2})^{-1} * \frac{\partial p(x, z, t)}{\partial t} \end{aligned} \tag{18}$$

From Eqs. (17) and (18), we derive the generalized lossy wave equation in the time domain along the x axis as

$$\begin{aligned} \frac{\partial^2 p(x, z, t)}{\partial x^2} - A \frac{\partial^2 p(x, z, t)}{\partial t^2} - BD^{3/2}[p(x, z, t)] \\ - C \frac{\partial p(x, z, t)}{\partial t} = 0, \end{aligned} \tag{19}$$

where coefficients A , B , and C are constants given, respectively, by

$$\begin{aligned} A &= \frac{1}{c_0^2} (\alpha_\infty - \sin^2 \theta), \\ B &= \frac{2\alpha_\infty}{K_a} \sqrt{\frac{\rho_f \eta}{\pi}} \left(\frac{1}{\Lambda} + \frac{\gamma - 1}{\sqrt{Pr\Lambda'}} \right), \end{aligned}$$

and

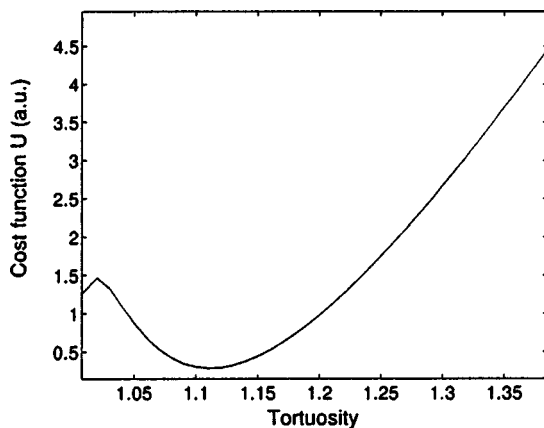


FIG. 5. Variation of cost function U with the tortuosity for plastic foam M1.

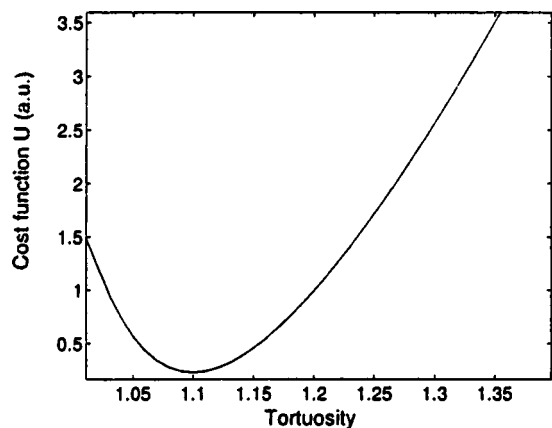


FIG. 7. Variation of cost function U with the tortuosity for plastic foam M2.

$$C = \frac{4\alpha_\infty(\gamma-1)\eta}{K_a\Lambda\Lambda'\sqrt{Pr}} \tag{20}$$

The first one is related to the velocity of the wave projected along the x axis, $c = c_0/\sqrt{\alpha_\infty - \sin^2\theta}$. The other coefficients are essentially dependent on the characteristic lengths, Λ and Λ' , and express viscous and thermal interactions between the fluid and structure. Constant B governs signal dispersion while C is responsible for wave attenuation. Obviously, these three coefficients can be used to determine parameters α_∞ , Λ , and Λ' . One way in which to solve Eq. (19) with suitable initial and boundary conditions is by using the Laplace transform. The approach is quite simple although the inverse Laplace transform involves tedious calculations.⁴⁰ A suitable setting for introducing a time-domain solution of the modified wave propagation, Eq. (19), is provided by the following model.

If the incident sound wave is launched in region $x \leq 0$, then the expression of the pressure field in the region on the left of the material is the sum of the incident and reflected fields,

$$p_1(x,t,\theta) = p^i\left(t - \frac{x \cos \theta}{c_0}\right) + p^r\left(t + \frac{x \cos \theta}{c_0}\right), \quad x < 0. \tag{21}$$

Here, $p_1(x,t,\theta)$ is the field in region $x < 0$, and p^i and p^r denote the incident and reflected fields, respectively.

The incident and reflected fields are related by the scattering operator (i.e., the reflection operator) for the material. This is an integral operator represented by

$$\begin{aligned} p^r(x,t,\theta) &= \int_0^t \tilde{R}(\tau,\theta) p^i\left(t - \tau + \frac{x \cos \theta}{c_0}\right) d\tau \\ &= \tilde{R}(t,\theta) * p^i(t) * \delta\left(t + \frac{x \cos \theta}{c_0}\right). \end{aligned} \tag{22}$$

In Eq. (22) function \tilde{R} is the reflection kernel for incidence from the left. Note that the lower limit of integration in Eqs. (22) is set to 0, which is equivalent to assuming that the incident wave front impinges on the material at $t=0$.

The expression for the reflection-scattering operator taking into account n -multiple reflections in the material is given by

$$\begin{aligned} \tilde{R}(t,\theta) &= \left(\frac{1-E}{1+E}\right) \sum_{n \geq 0} \left(\frac{1-E}{1+E}\right)^{2n} \left[F\left(t, 2n \frac{L}{c}\right) \right. \\ &\quad \left. - F\left(t, (2n+2) \frac{L}{c}\right) \right], \end{aligned} \tag{23}$$

with

$$E = \frac{\phi\sqrt{1 - \sin^2\theta/\alpha_\infty}}{\sqrt{\alpha_\infty \cos \theta}}, \tag{24}$$

and F is the medium's Green function^{4,40} given by

$$F(t,k) = \begin{cases} 0 & \text{if } 0 \leq t \leq k, \\ \Xi(t) + \Delta \int_0^{t-k} h(t,\xi) d\xi & \text{if } t \geq k, \end{cases} \tag{25}$$

with

$$\Xi(t) = \frac{b'}{4\sqrt{\pi}} \frac{k}{(t-k)^{3/2}} \exp\left(-\frac{b'^2 k^2}{16(t-k)}\right), \tag{26}$$

where $h(\tau,\xi)$ has the following form:

$$\begin{aligned} h(\xi,\tau) &= -\frac{1}{4\pi^{3/2}} \frac{1}{\sqrt{(\tau-\xi)^2 - k^2}} \frac{1}{\xi^{3/2}} \\ &\quad \times \int_{-1}^1 \exp\left(-\frac{\chi(\mu,\tau,\xi)}{2}\right) \\ &\quad \times [\chi(\mu,\tau,\xi) - 1] \frac{\mu d\mu}{\sqrt{1-\mu^2}}, \end{aligned} \tag{27}$$

and $\chi(\mu,\tau,\xi) = [\Delta\mu\sqrt{(\tau-\xi)^2 - k^2} + b'(\tau-\xi)]^2/8\xi$, $b' = Bc_0^2\sqrt{\pi}$, $c' = C.c_0^2$ and $\Delta = b'^2 - 4c'$. In most cases of air-saturated porous materials, multiple reflection effects may be negligible because of the high attenuation of sound waves in these media. This depends on the thickness L and physical properties of the porous material. For common air-saturated porous materials, only the wave reflected by the first interface is seen experimentally³² and all contributions from the bulk of the porous material are neglected. So, by taking into account only the first reflections at interfaces $x=0$ and $x=L$, the expression of the reflection operator will be given by

$$\tilde{R}(t,\theta) = r(t,\theta) + \mathfrak{R}(t,\theta), \tag{28}$$

with

$$r(t,\theta) = \left(\frac{1-E}{1+E}\right) \delta(t)$$

and

$$\mathfrak{R}(t,\theta) = -\frac{4E(1-E)}{(1+E)^3} F\left(t, \frac{2L}{c}\right), \tag{29}$$

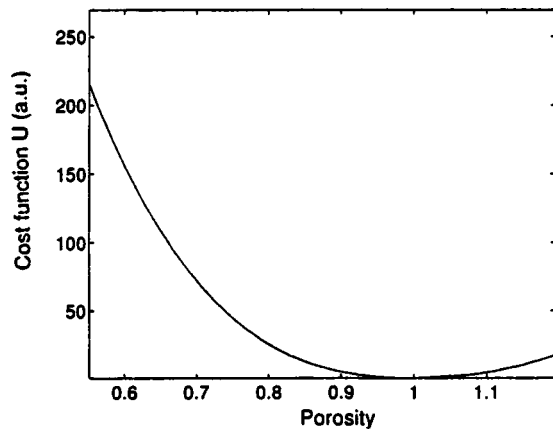
with $r(t,\theta)$ the instantaneous response of the porous material corresponding to the reflection contribution at the first interface ($x=0$). $\mathfrak{R}(t,\theta)$ is equivalent to reflection at interface $x=L$, which is the bulk contribution to reflection. The part of the wave corresponding to $r(t,\theta)$ is not subjected to dispersion but simply multiplied by the factor $(1-E)/(1+E)$.

The reflection coefficient at the first interface vanishes for a critical angle θ_c ,

$$r(t,\theta) = 0 \Rightarrow \sin \theta_c = \sqrt{\frac{\alpha_\infty(\alpha_\infty - \phi^2)}{\alpha_\infty^2 - \phi^2}}.$$

Figure 2 shows the variation of the reflection coefficient at first interface r with incident angle θ for fixed porosity value of $\phi=0.9$, and for different values of tortuosity, $\alpha_\infty=1.99$ (solid line), 1.75 (star), 1.5 (dashed line), 1.24 (dash-dotted line), and 1 (circle).

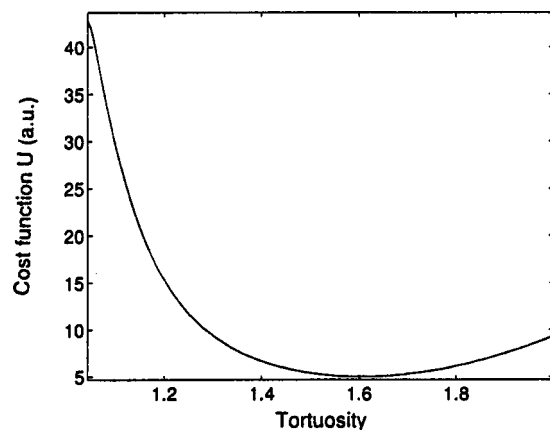
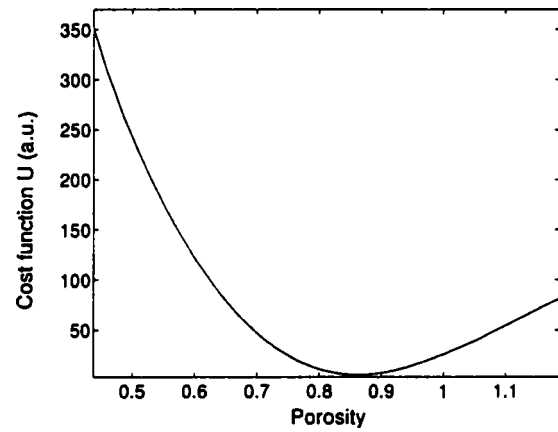
Figure 3 shows the variation of r with the incident angle, for a fixed tortuosity of $\alpha_\infty=1.1$, and for different values of porosity $\phi=0.99$ (solid line), 0.75 (star), 0.50 (dashed line), 0.25 (dash-dotted line), and 0.01 (circle).

FIG. 8. Variation of cost function U with the porosity for plastic foam M2.

When the incident angle is $\theta < \theta_c$, the reflection coefficient decreases slowly with the incident angle, and when it is $\theta > \theta_c$, the reflection coefficient increases quickly with the angle. It can also be seen from Figs. 2 and 3 that the sensitivity of porosity variation is more important than the sensitivity of tortuosity on the reflection coefficient at the first interface.

V. INVERSE PROBLEM

The propagation of acoustic waves in a slab of porous material in the high frequency asymptotic domain is characterized by four parameters: porosity ϕ , tortuosity α_∞ , viscous characteristic length Λ , and thermal characteristic length Λ' , the values of which are crucial to the behavior of sound waves in such materials. It is of some importance to work out new experimental methods and efficient tools for estimation of them. The basic inverse problem associated with the slab may be stated as follows: from measurement of the signals transmitted and/or reflected outside the slab, find the values of the medium's parameters. The inverse problem has been solved in transmission mode in Refs. 4–6 and an estimate of α_∞ , Λ , and Λ' made therein that gives a good correlation between experience and theory. Porosity was not estimated in this mode because of its weak sensitivity. Porosity was estimated in reflected mode in our previous

FIG. 9. Variation of cost function U with the tortuosity for plastic foam M3.FIG. 10. Variation of cost function U with the porosity for plastic foam M3.

studies^{7,8,31,32} by solving the inverse problem at normal⁸ and oblique incidence³¹ for a fixed value of tortuosity. Porosity and tortuosity were measured^{7,32} simultaneously for plastic foams and air-saturated random packings of beads by measuring reflected waves for each pair of incident angles.

In this article, we determine the porosity and tortuosity by solving the inverse problem for waves reflected by the first interface, and by taking into account experimental data for all measured incident angles.

The inverse problem is to find values of parameters ϕ and α_∞ , which minimize the function

$$U(\phi, \alpha_\infty) = \sum_{\theta_i} \sum_{t_i} [p^r(x, \theta_i, t_i) - r(\theta_i, t_i) * p^i(x, \theta_i, t_i)]^2, \quad (30)$$

where $p^r(x, \theta_i, t_i)$ represents the discrete set of values of the experimental reflected signal for different incident angles θ_i , $r(\theta_i, t_i)$ is the reflection coefficient at the first interface, and $p^i(x, \theta_i, t_i)$ is the experimental incident signal. The term $r(\theta_i, t_i) * p^i(x, \theta_i, t_i)$ represents the predicted reflected signal. The inverse problem is solved numerically by the least-square method.

Experiments were performed in air with two broadband Ultrat NCT202 transducers with 190 kHz center frequency in air and 6 dB bandwidth that extends from 150 to 230 kHz. An optical goniometer was used to position the transducers. Pulses of 400 V were provided by a 5052PR Panametrics pulser/receiver. The signals received were amplified to 90 dB and filtered above 1 MHz to avoid high frequency noise. Electronic interference was removed by 1000 acquisition averages. The experimental setup is shown in Fig. 4. The duration of the signal plays an important role: its spectrum must verify the condition of the high frequency approximation referred to in Sec. IV.

TABLE II. Reconstructed values of porosity and tortuosity by solving the inverse problem.

Material	M1	M2	M3
Tortuosity	1.12	1.1	1.6
Porosity	0.96	0.99	0.85

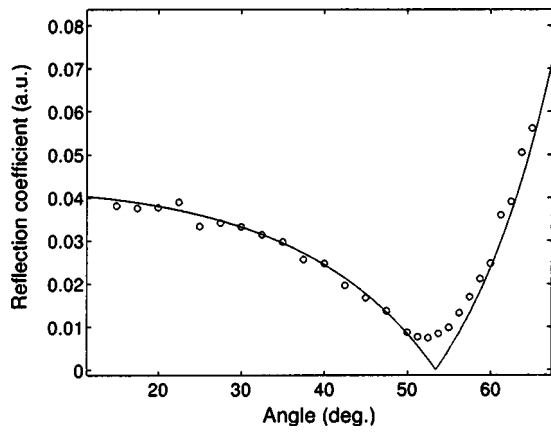


FIG. 11. Comparison between the simulated reflection coefficient at the first interface using reconstructed values of the porosity and tortuosity (solid line) and experimental data for the reflection coefficient at the first interface (circle) for plastic foam M1.

Let us consider three samples of plastic foam, M1, M2, and M3. Their tortuosity and porosity were measured using classical methods^{4-6,9-11} (Table I). We solved the inverse problem for these samples via waves reflected at the first interface and for different incident angles. Figures 5 and 6, 7 and 8, and 9 and 10 show variation of the cost function, U , with the tortuosity and porosity for samples M1, M2, and M3, respectively. The reconstructed values of porosity and tortuosity corresponding to the positions of the minima of these cost functions are given in Table II. Figures 11-13 show comparisons between simulated reflection coefficients at the first interface using reconstructed values of the porosity and tortuosity (solid line) and experimental data for the reflection coefficient at the first interface (circle) for plastic foams M1, M2, and M3, respectively. The correspondence between experiment and theory is good, which leads us to conclude that this method based on solution of the inverse problem is appropriate for estimating the porosity and tortuosity of porous materials with a rigid frame.

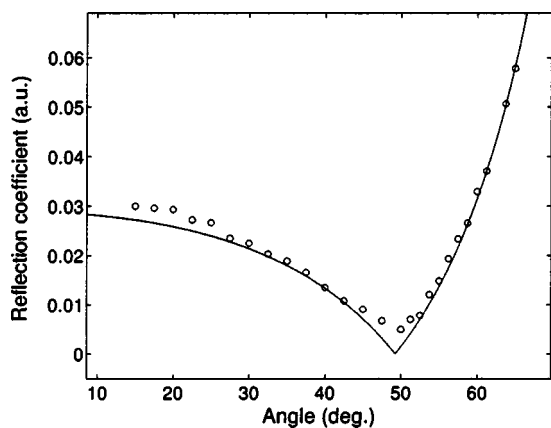


FIG. 12. Comparison between the simulated reflection coefficient at the first interface using reconstructed values of the porosity and tortuosity (solid line) and experimental data for the reflection coefficient at the first interface (circle) for plastic foam M2.

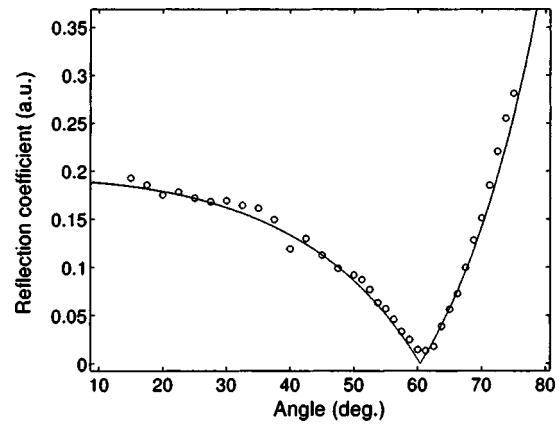


FIG. 13. Comparison between the simulated reflection coefficient at the first interface using reconstructed values of the porosity and tortuosity (solid line) and experimental data for the reflection coefficient at the first interface (circle) for plastic foam M3.

VI. CONCLUSION

In this article, an inverse scattering estimate of the porosity and tortuosity was given by solving the inverse problem for waves reflected by the first interface, and by taking into account experimental data for all measured incident angles. The inverse problem is solved numerically by the least-square method. The reconstructed values of the porosity and tortuosity are close to those using classical methods. Generally, it is easy to evaluate the tortuosity from transmitted waves, but that is not the case for porosity because of its weak sensitivity in transmitted mode. This method is an alternative to the usual method that involves the use of a porosimeter, introduced by Beranek, and improved by Champoux *et al.* or the other ultrasonic methods based on transmitted mode.

The direct problem is based upon the propagation equation in the time domain in a slab of porous material with a rigid frame in the high frequency asymptotic domain. Interaction of the sound pulse with the fluid-saturated porous material was described by a time-domain equivalent fluid model.

The sensitivity of the porosity and tortuosity was studied and it showed their effect on the reflection coefficient at the first interface. This study has shown that porosity is much more sensitive than tortuosity to reflection, especially when the incident angle is less than its critical value, at which the reflection coefficient vanishes.

We hope, in the future, to extend this method to porous media with an elastic frame, such as cancellous bone saturated with viscous fluid, in order to estimate other parameters which play an important role in acoustic propagation. The advantage of the data concept using waves reflected by the first interface is its simple analysis, and similar to the transmitted wave concept which, however, is more complicated. The wave reflected by the first interface is not subject to dispersion but is simply attenuated, its frequency and temporal bandwidths are the same as the incident signal, and experimental detection of it is easy for resistive media compared to transmitted data.

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