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# Measuring the porosity of porous materials having a rigid frame via reflected waves: A time domain analysis with fractional derivatives

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An ultrasonic reflectivity method is proposed for measuring porosity of porous materials having a rigid frame. Porosity is the relative fraction by volume of the air contained within a material. It is important as one of the several parameters required by acoustical theory to characterize porous materials like plastic foams and fibrous or granular materials. The proposed method is based on a temporal model of the direct and inverse scattering problem for the propagation of transient ultrasonic waves in a homogeneous isotropic slab of porous material having a rigid frame. This time domain model of wave propagation was initially introduced by the authors [Z. E. A. Fellah and C. Depollier, *J. Acoust. Soc. Am.* **107**, 683 (2000)]. The viscous and thermal losses of the medium are described by the model devised by Johnson *et al.* [D. L. Johnson, J. Koplik, and R. Dashen, *J. Fluid. Mech.* **176**, 379 (1987)] and Allard [J. F. Allard, Chapman and Hall, London, (1993)] modified by a fractional calculus-based method applied in the time domain. Reflection and transmission scattering operators for a slab of porous material are derived from the responses of the medium to an incident acoustic pulse. The porosity is determined from the expressions of these operators. Experimental and numerical validation results of this method are presented. This method has the advantage of being simple, rapid, and efficient. © 2003 American Institute of Physics. [DOI: 10.1063/1.1524025]

## I. INTRODUCTION

The ultrasonic characterization of porous materials saturated by air such as plastic foams, fibrous or granular materials is of great interest for a wide range of industrial applications. These materials are frequently used in the automotive and aeronautics industries and in the building trade. The determination of the properties of a medium using waves that have been reflected by or transmitted through the medium is a classical inverse scattering problem. Such problems are often approached by taking a physical model of the scattering process, generating a synthetic response for a number of assumed values for the parameters, and adjusting these parameters until a reasonable level of correspondence is attained between the synthetic response and the data observed. One important parameter which appears in theories of sound propagation in porous materials<sup>1-13</sup> is porosity. Porosity is the relative fraction, by volume, of the air contained within the material. Unlike other parameters included in the description of different various phenomena occurring in the acoustical propagation of porous media at high frequency range such as tortuosity,<sup>11</sup> viscous characteristic length,<sup>5</sup> and

thermal characteristic length,<sup>12</sup> or at low frequency range such as flow resistivity<sup>11</sup> and thermal permeability,<sup>13</sup> porosity is a key parameter that plays an important role in propagation at all frequencies. As such, in studies of acoustical properties of porous materials, it is extremely useful to be able to measure this parameter.

Beranek<sup>10</sup> described an apparatus (porosimeter) used to measure the porosity of porous materials. This device was based on the equation of state for ideal gases at constant temperature (i.e., Boyle's law). Porosity can be determined by measuring the change in air pressure occurring with a known change in volume of the chamber containing the sample. In the Beranek apparatus, both pressure change and volume change are monitored using a U-shaped fluid-filled manometer. An alternate technique for measuring porosity is a dynamic method proposed by Leonard.<sup>14</sup> Techniques that use water as the pore-filling fluid, rather than air, are common in geophysical studies.<sup>15,16</sup> Mercury has been used as the pore-filling fluid in other applications.<sup>17</sup> However, for many materials, the introduction of liquids into the material is not appropriate. Recently a similar device to that of Beranek, involving the use of an electronic pressure transducer, was introduced by Champoux *et al.*<sup>18</sup> This device can be used to measure very slight changes in pressure accurately, and the output can be recorded by a computer.

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In this work, we present a simple method of measuring porosity by measuring a wave reflected by a slab of porous material. This method is based on a temporal model of the direct and inverse scattering problem for the propagation of transient ultrasonic waves in a homogeneous isotropic slab of porous material having a rigid frame. This work was initially introduced by the authors of Refs. 1–4. The viscous and thermal losses of the medium are described by the Johnson *et al.*<sup>5</sup> and Allard<sup>6</sup> model modified by a fractional calculus based method in order to be used in the time domain. Reflection and transmission scattering operators of a slab of porous material are derived and thus the responses of the medium to an incident acoustic pulse are obtained.

The outline of this work is as follows. In Sec. II, a time domain model is given, the connection between the fractional derivatives and wave propagation in rigid porous media in high frequency range is established and the basic equations are written in the time domain. Section III is devoted to the direct problem and to the expressions of the reflection and transmission kernels in the time domain and numerical results are given. Section IV deals with the inverse problem and an expression of porosity based on the incident and reflected waves from a slab of porous materials is established. Finally in Sec. V, the experimental results are given to validate this method.

## II. MODEL

In the acoustics of porous materials, a distinction can be made between two situations depending on whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid frame and the fluid are clearly described by the Biot theory.<sup>7,8</sup> In air-saturated porous media the structure is generally motionless and the waves propagate only in the fluid. This case is described by the model of equivalent fluid which is a particular case in the Biot model, in which the interactions between the fluid and the structure are taken into account in two frequency response factors: the dynamic tortuosity of the medium  $\alpha(\omega)$  given by Johnson *et al.*<sup>5</sup> and the dynamic compressibility of the air included in the porous material  $\beta(\omega)$  given by Allard.<sup>6</sup> In the frequency domain, these factors multiply the density of the fluid and its compressibility respectively and represent the deviation from the behavior of the fluid in free space as the frequency increases. In the time domain, they act as operators and in the high frequency approximation their expressions are given by Fellah and Depollier<sup>1,2</sup> and Fellah *et al.*<sup>3,4</sup> as

$$\tilde{\alpha}(t) = \alpha_\infty \left[ \delta(t) + \frac{2}{\Lambda} \left( \frac{\eta}{\pi \rho_f} \right)^{1/2} t^{-1/2} \right], \tag{1}$$

$$\tilde{\beta}(t) = \left[ \delta(t) + \frac{2(\gamma-1)}{\Lambda'} \left( \frac{\eta}{\pi \text{Pr} \rho_f} \right)^{1/2} t^{-1/2} \right]. \tag{2}$$

In these equations,  $\delta(t)$  is the Dirac function, Pr is the Prandtl number,  $\eta$  and  $\rho_f$  the fluid viscosity and the fluid density, respectively, and  $\gamma$  is the adiabatic constant. The relevant physical parameters of the model are the tortuosity of the medium  $\alpha_\infty$  initially introduced by Zwikker and

Kosten<sup>11</sup> and the viscous and the thermal characteristic lengths  $\Lambda$  and  $\Lambda'$  introduced by Johnson *et al.*<sup>5</sup> and Allard.<sup>6</sup> In this model, the time convolution of  $t^{-1/2}$  with a function is interpreted as a semi-derivative operator following the definition of the fractional derivative of order  $\nu$  given in Samko *et al.*<sup>19</sup>

$$D^\nu[x(t)] = \frac{1}{\Gamma(-\nu)} \int_0^t (t-u)^{-\nu-1} x(u) du, \tag{3}$$

where  $\Gamma(x)$  is the gamma function.

In this framework, the basic equations of our model can be expressed as follows:

$$\rho_f \tilde{\alpha}(t) * \frac{\partial v_i}{\partial t} = -\nabla_i p$$

and

$$\frac{\tilde{\beta}(t)}{K_a} * \frac{\partial p}{\partial t} = -\nabla v, \tag{4}$$

where  $*$  denotes the time convolution operation,  $p$  is the acoustic pressure,  $v$  is the particle velocity, and  $K_a$  is the bulk modulus of the air. The first equation is the Euler equation; the second is a constitutive equation obtained from the equation of mass conservation associated with the behavior (or adiabatic) equation.

For a wave propagating along the  $x$  axis, these equations become

$$\rho_f \alpha_\infty \frac{\partial v}{\partial t} + 2 \frac{\rho_f \alpha_\infty}{\Lambda} \left( \frac{\eta}{\pi \rho_f} \right)^{1/2} \int_0^t \frac{\partial v / \partial t'}{\sqrt{t-t'}} dt' = -\frac{\partial p}{\partial x}, \tag{5}$$

$$\frac{1}{K_a} \frac{\partial p}{\partial t} + 2 \frac{\gamma-1}{K_a \Lambda'} \left( \frac{\eta}{\pi \text{Pr} \rho_f} \right)^{1/2} \int_0^t \frac{\partial p / \partial t'}{\sqrt{t-t'}} dt' = -\frac{\partial v}{\partial x}. \tag{6}$$

In these equations, the convolutions express the dispersive nature of the porous material. They take into account the memory effects due to the fact that the response of the medium to the wave excitation is not instantaneous but needs more time to take effect. The retarding force is no longer proportional to the time derivative of the acoustic velocity but is found to be proportional to the fractional derivative of order 1/2 of this quantity. This occurs because the volume of fluid involved in the motion is not the throughout the whole length of the signal as it is in the case of a fully developed steady flow. The phenomenon may be understood by considering such a volume of fluid in a pore to be in harmonic motion. At high frequencies, only a thin layer of fluid is excited and so the average shear stress is high. At a lower frequency, the same amplitude of fluid motion allows a thicker layer of fluid to participate in the motion and consequently the shear stress is lower. The penetration distance of the viscous forces and therefore the excitation of the fluid depends on frequency. In the time domain, such dependence is associated with a fractional derivative.

## III. DIRECT PROBLEM

The direct scattering problem is that of determining the scattered field, as well as the internal field, that arises when a

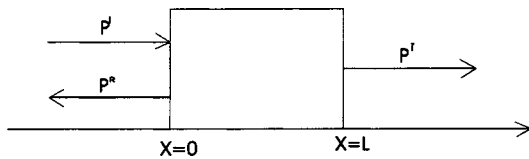


FIG. 1. Geometry of the problem.

known incident field impinges on the porous material with known physical properties. To compute the solution of the direct problem, it is necessary to know the reflection and/or the transmission scattering operators which depends on the Green<sup>3</sup> function of the porous medium. In that case, the reflected field is given by the convolution of the reflection operator with the incident field and the transmitted field is given by the convolution of the transmission operator by the incident field.

Consider a homogeneous slab of porous material which occupies the region  $0 \leq x \leq L$ . The geometry of the problem is shown in Fig. 1. This medium is assumed to be isotropic and to have a rigid frame. A short sound pulse impinges normally on the medium from the left. It gives rise to an acoustic pressure field  $p(x, t)$  and an acoustic velocity field  $v(x, t)$  within the material, which satisfies the propagation equation<sup>1</sup>

$$\frac{\partial^2 p}{\partial x^2} - A \frac{\partial^2 p}{\partial t^2} - B \int_0^t \frac{\partial^2 p / \partial t'^2}{\sqrt{t-t'}} dt' - C \frac{\partial p}{\partial t} = 0, \tag{7}$$

where the coefficients  $A$ ,  $B$ , and  $C$  are constants expressed, respectively, by

$$\begin{aligned} A &= \frac{1}{c^2} = \frac{\rho_f \alpha_\infty}{K_a}, \\ B &= \frac{2\alpha_\infty}{K_a} \sqrt{\frac{\rho_f \eta}{\pi}} \left( \frac{1}{\Lambda} + \frac{\gamma-1}{\sqrt{\text{Pr}\Lambda'}} \right), \\ C &= \frac{4\alpha_\infty(\gamma-1)\eta}{K_a \Lambda \Lambda' \sqrt{\text{Pr}}}. \end{aligned} \tag{8}$$

The first one is related to the velocity  $c = 1/\sqrt{\rho_f \alpha_\infty / K_a}$  of the wave in the air included in the porous material.  $\alpha_\infty$  is the refractive index of the medium which changes the wave velocity from  $c_0 = \sqrt{K_a / \rho_f}$  in free space to  $c = c_0 / \sqrt{\alpha_\infty}$  in the porous medium. The other coefficients are essentially dependent on the characteristic lengths  $\Lambda$  and  $\Lambda'$  and express the viscous and thermal interactions between the fluid and the structure. The constant  $B$  governs the spreading of the signal while  $C$  is responsible for the attenuation of the wave.

To derive the reflection and transmission scattering operators, it is assumed that the pressure field and the flow velocity are continuous at the boundary of the material

$$\begin{aligned} p(0^+, t) &= p(0^-, t), & p(L^-, t) &= p(L^+, t), \\ v(0^-, t) &= \phi v(0^+, t), & v(L^+, t) &= \phi v(L^-, t), \end{aligned} \tag{9}$$

where  $\phi$  is the porosity of the medium and  $\pm$  superscript denotes the limit from the left and the right, respectively. The initials conditions are given by

$$\begin{aligned} p(x, t)|_{t=0} &= 0, \\ \frac{\partial p}{\partial t} \Big|_{t=0} &= 0, \end{aligned} \tag{10}$$

which means that the medium is idle for  $t=0$ .

If the incident sound wave is launched in the region  $x \leq 0$ , then the general solution of Eq. (7) in the region to the left of the material is the sum of the incident and reflected field

$$p_1(x, t) = p^i \left( t - \frac{x}{c_0} \right) + p^r \left( t + \frac{x}{c_0} \right), \quad x < 0. \tag{11}$$

Here,  $p_1(x, t)$  is the field in the region  $x < 0$ ,  $p^i$  and  $p^r$  denote the incident and the reflected field, respectively. In addition, a transmitted field is produced in the region to the right of the material. This has the form

$$p_3(x, t) = p^t \left( t - \frac{L}{c} - \frac{(x-L)}{c_0} \right), \quad x > L, \tag{12}$$

where  $p_3(x, t)$  is the field in the region  $x > L$  and  $p^t$  is the transmitted field.

The incident and scattered fields are related by the scattering operators (i.e., reflection and transmission operators) for the material. These are integral operators represented by

$$\begin{aligned} p^r(x, t) &= \int_0^t \tilde{R}(\tau) p^i \left( t - \tau + \frac{x}{c_0} \right) d\tau \\ &= \tilde{R}(t) * p^i(t) * \delta \left( t + \frac{x}{c_0} \right), \\ p^t(x, t) &= \int_0^t \tilde{T}(\tau) p^i \left( t - \tau - \frac{L}{c} - \frac{(x-L)}{c_0} \right) d\tau \\ &= \tilde{T}(t) * p^i(t) * \delta \left( t - \frac{L}{c} - \frac{(x-L)}{c_0} \right). \end{aligned} \tag{13}$$

In Eqs. (13) the functions  $\tilde{R}$  and  $\tilde{T}$  are the reflection and the transmission kernels, respectively, for incidence of waves from the left. Note that the lower limit of integration in Eq. (13) is set 0, which is equivalent to assuming that the incident wave front first impinges on the material at  $t=0$ . The scattering operators given in Eqs. (13) are independent of the incident field used in scattering experiment and depend only on the properties of the materials. Using Eqs. (4), (7), (9), (13), and Laplace transform calculus, we can derive the reflection and transmission scattering operators given by

$$\begin{aligned} \tilde{R}(t) &= \left( \frac{-\phi + \sqrt{\alpha_\infty}}{\phi + \sqrt{\alpha_\infty}} \right) \sum_{n \geq 0} \left( \frac{\phi - \sqrt{\alpha_\infty}}{\phi + \sqrt{\alpha_\infty}} \right)^{2n} \\ &\quad \times \left[ F \left( t, 2n \frac{L}{c} \right) - F \left( t, (2n+2) \frac{L}{c} \right) \right], \end{aligned} \tag{14}$$

$$\begin{aligned} \tilde{T}(t) &= \frac{4\phi\sqrt{\alpha_\infty}}{(\sqrt{\alpha_\infty} + \phi)^2} \sum_{n \geq 0} \left( \frac{\phi - \sqrt{\alpha_\infty}}{\phi + \sqrt{\alpha_\infty}} \right)^{2n} \\ &\quad \times F \left( t + \frac{L}{c_0}, (2n+1) \frac{L}{c} \right). \end{aligned} \tag{15}$$

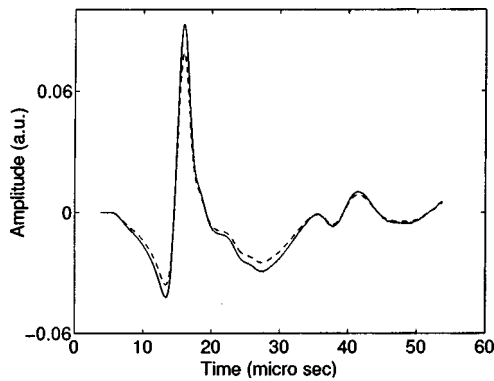


FIG. 2. Transmitted simulated signals for  $\phi=0.98$  (solid line) and for  $\phi=0.49$  (dashed line).

These expressions takes into account the  $n$ -multiple reflections in the material. For a positive constant  $k$ ,  $F(t,k)$  is the Green function of the porous material given in Ref. 3 (see the Appendix A).

In most cases, for porous materials saturated by air, the multiple reflection effects are negligible because of the high attenuation of sound waves in this kind of media. So, by taking into account only the reflections at the interfaces  $x=0$  and  $x=L$ , the kernel of transmission is given by

$$\tilde{T}(t) = \frac{4\phi\sqrt{\alpha_\infty}}{(\phi + \sqrt{\alpha_\infty})^2} F\left(t + \frac{L}{c}, \frac{L}{c}\right), \tag{16}$$

and the kernel of reflection by

$$\tilde{R}(t) = r(t) + \mathcal{R}(t), \tag{17}$$

with

$$r(t) = \frac{\sqrt{\alpha_\infty} - \phi}{\sqrt{\alpha_\infty} + \phi} \delta(t)$$

and

$$\mathcal{R}(t) = -\frac{4\phi\sqrt{\alpha_\infty}(\sqrt{\alpha_\infty} - \phi)}{(\sqrt{\alpha_\infty} + \phi)^3} F\left(t, \frac{2L}{c}\right), \tag{18}$$

in the case of a semi-infinite medium when  $L \rightarrow \infty$ ;  $F(t, 2L/c) \rightarrow 0$  and  $\tilde{R}(t) \rightarrow r(t)$  (see Appendix). This means that  $r(t)$  is equivalent to the reflection at the interface  $x=0$  and that  $\mathcal{R}(t)$  is equivalent to reflection at the interface  $x=L$ , which is the bulk contribution to the reflection. The part of the wave corresponding to  $r(t)$  is not subjected to the dispersion but it is simply multiplied by the factor  $(\sqrt{\alpha_\infty} - \phi)/(\sqrt{\alpha_\infty} + \phi)$ . This shows that although tortuosity is a bulk parameter, it may be evaluated from the wave reflected at the first interface when the porosity is known and vice versa. Although generally speaking it is easy to evaluate the tortuosity from transmitted waves<sup>3,4,20,21</sup> this is not the case for porosity because of its weak sensitivity in transmitted mode. Figure 2 shows two simulated transmitted signals for a plastic foam M1, the first one (solid line) corresponds to the value of porosity  $\phi_1=0.98$  and the second one (dashed line) corresponds to  $\phi_2=0.49$ . The parameters used in the simulation namely thickness: 1.1 cm,  $\phi=0.92$ ,  $\alpha_\infty=1.25$ ,

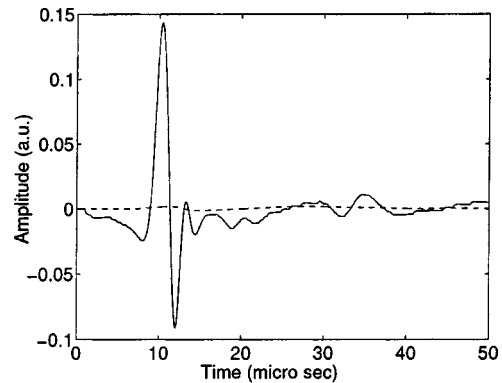


FIG. 3. Numerical simulations of the reflections contributions  $r(t)$  (solid line) and  $\mathcal{R}(t)$  (dashed line).

$\Lambda = 50 \mu\text{m}$ , and  $\Lambda' = 150 \mu\text{m}$  have been determined using conventional methods.<sup>3,4,19,20</sup> Readers can see the slight difference between the two curves for a 50% difference in porosity values, which is due to the dispersion phenomenon that is governed by viscous, thermal, and inertial effects contributed by  $\alpha_\infty$ ,  $\Lambda$ , and  $\Lambda'$ , and plays a more important role in the Green function  $F(t,k)$  than  $\phi$ .

Figure 3 shows the two contributions to the reflection:  $r(t)$  and  $\mathcal{R}(t)$  for a plastic foam M1. The contribution of  $\mathcal{R}(t)$  to the reflection is negligible when it is compared to the contribution of  $r(t)$ . In Fig. 4 we show by numerical simulation, the difference between the reflected wave at the first interface and the total reflected wave. The difference between the two curves is slight. This means that the wave reflected by the slab may be approximated by the reflected wave by the first interface  $r(t)$  with a good level of accuracy.

#### IV. INVERSE PROBLEM

A porous material having a rigid frame is characterized by four parameters in the high frequency range, namely, porosity  $\phi$ , tortuosity  $\alpha_\infty$ , the viscous characteristic length  $\Lambda$ , and the thermal characteristic length  $\Lambda'$ . These values are crucial for the behavior of the sound waves in such materials. So, it is therefore fairly important to work out experimental methods and efficient tools to assess them. Therefore, a basic inverse problem associated with the slab may be stated as follows: to find the values of the parameters of the medium from the measurements of the transmitted and/or reflected signals outside the slab. As shown in Sec. II, the solution to the direct problem is the system of two operators expressed as functions on  $\phi$ ,  $\alpha_\infty$ ,  $\Lambda$ , and  $\Lambda'$ . The inversion algorithm for finding the values of the parameters of the slab is based on a fitting procedure. To find the values of the parameters  $\phi$ ,  $\alpha_\infty$ ,  $\Lambda$ , and  $\Lambda'$  such that the transmitted and reflected signal describes the scattering problem in the best possible way (e.g., in the least-squares sense). Solving the inverse problem using experimental data from transmitted waves is treated<sup>3,4,22</sup> and an estimation of  $\alpha_\infty$ ,  $\Lambda$ , and  $\Lambda'$  has been given with a good level of correspondence from ultrasonic measurements. Here we will try to solve the inverse problem

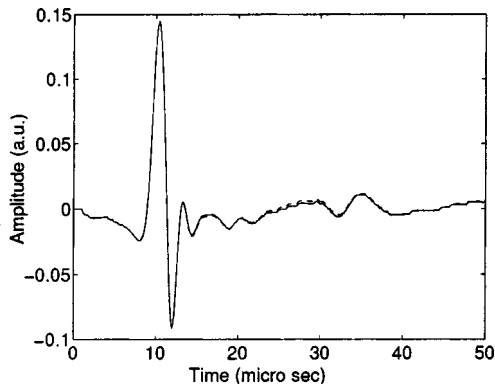


FIG. 4. Reflected wave at the interface  $x=0$  (solid line) and the total reflected wave (dashed line).

using experimental reflected wave data, with the aim of returning to the porosity which, because of its weak sensitivity, cannot be characterized in transmitted mode.

The inverse problem is to find the porosity value  $\phi$  which minimizes the function

$$U = \sum_{i=1}^{i=N} (p^r(t_i) - p_{\text{exp}}^r(t_i))^2, \quad (19)$$

where  $p^r(t_i)_{i=1,2,\dots,N}$  represents the discrete set of values of the simulated reflected signal given by Eq. (13) and  $p_{\text{exp}}^r(t_i)_{i=1,2,\dots,N}$  is the discrete set of values of the experimental signal. In the approximation that the reflected wave is essentially due to the reflection at the first interface  $x=0$

$$\tilde{R}(t) = r(t) = \frac{a-1}{a+1} \delta(t). \quad (20)$$

where  $a = \sqrt{\alpha_\infty} / \phi$  and

$$p^r(t_i) = \frac{a-1}{a+1} p^i(t_i). \quad (21)$$

$p^i(t_i)$  is the discrete set of values of the experimental incident signal given out by the transducer. To minimize  $U(a)$  we must have

$$\frac{\partial U}{\partial a} = 0,$$

and

$$\frac{\partial^2 U}{\partial a^2} > 0, \quad (22)$$

$$\frac{\partial U}{\partial a} = \frac{\partial}{\partial a} \sum_{i=1}^{i=N} \left( \frac{a-1}{a+1} p^i(t_i) - p_{\text{exp}}^r(t_i) \right)^2 = 0, \quad (23)$$

then

$$2 \sum_{i=1}^{i=N} \frac{(a+1) - (a-1)}{(a+1)^2} p^i(t_i) \left( \frac{a-1}{a+1} p^i(t_i) - p_{\text{exp}}^r(t_i) \right) = 0, \quad (24)$$

and

$$\sum_{i=1}^{i=N} p^i(t_i) \left( \frac{a-1}{a+1} p^i(t_i) - p_{\text{exp}}^r(t_i) \right) = 0, \quad (25)$$

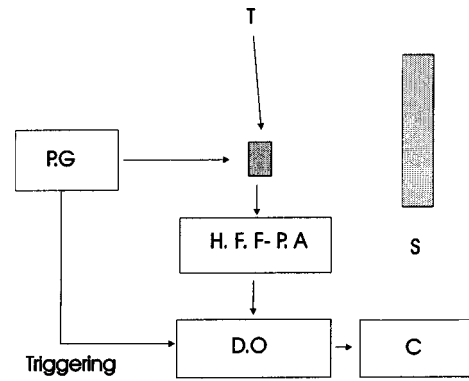


FIG. 5. Experimental setup of the ultrasonic measurements in reflected mode: (P.G) pulse generator, (H. F. F. P. A) high frequency filtering-preamplifier, (D. O) digital oscilloscope, (C) computer, (T) transducer, and (S) sample.

which is equivalent to

$$a = \frac{\sum_{i=1}^{i=N} p^i(t_i)(p^i(t_i) + p_{\text{exp}}^r(t_i))}{\sum_{i=1}^{i=N} p^i(t_i)(p^i(t_i) - p_{\text{exp}}^r(t_i))} = \frac{\sqrt{\alpha_\infty}}{\phi}, \quad (26)$$

the expression of the porosity is

$$\phi = \frac{\sum_{i=1}^{i=N} p^i(t_i)(p^i(t_i) - p_{\text{exp}}^r(t_i))}{\sum_{i=1}^{i=N} p^i(t_i)(p^i(t_i) + p_{\text{exp}}^r(t_i))} \sqrt{\alpha_\infty}, \quad (27)$$

the second derivative term

$$\begin{aligned} \frac{\partial^2 U}{\partial a^2} = & -\frac{8}{(a+1)^3} \sum_{i=1}^{i=N} \left( \frac{a-1}{a+1} (p^i(t_i))^2 - p_{\text{exp}}^r(t_i) \cdot p^i(t_i) \right) \\ & + \frac{8}{(a+1)^4} \sum_{i=1}^{i=N} (p^i(t_i))^2, \end{aligned} \quad (28)$$

from Eq. (24) we have

$$\frac{\partial^2 U}{\partial a^2} = -\frac{2}{(a+1)} \frac{\partial U}{\partial a} + \frac{8}{(a+1)^4} \sum_{i=1}^{i=N} (p^i(t_i))^2, \quad (29)$$

when  $\partial U / \partial a = 0$ , the condition  $\partial^2 U / \partial a^2 > 0$  is always verified because  $a > 1$ ,  $(\alpha_\infty > 1$  and  $\phi < 1)$ .

Relation (27) is the expression of the porosity as a function of the tortuosity, a discrete set of values of the incident and reflected signal, we will use this relation in Sec. V to estimate the value of the porosity from experimental incident and reflected data.

### V. ULTRASONIC MEASUREMENTS

In application of this model, some numerical simulations are compared to experimental results. Experiments are carried out in air with a broadband Ultrasonics NCT202 transducer having a 190 kHz central frequency in air and a bandwidth at 6 dB extending from 150 to 230 kHz. This transducer is used simultaneously as a transmitter and receiver in order to detect the reflected wave. Pulses of 400 V are provided by a 5052PR Panametrics pulser/receiver. The received signals are amplified up to 90 dB and filtered above 1 MHz to avoid high frequency noise (energy is totally filtered by the sample in this upper frequency domain in transmitted mode but not in reflected mode). Electronic perturbations are removed by

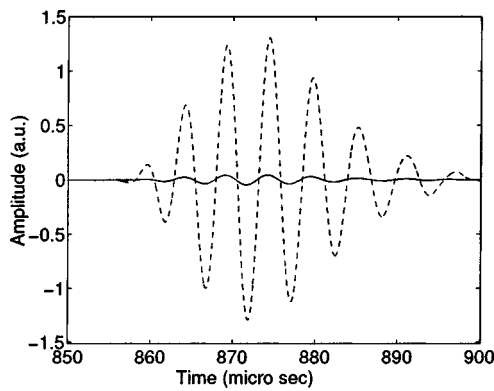


FIG. 6. Experimental incident signal (dashed line) and experimental reflected signal (solid line).

1000 acquisition averages. The experimental setup is showed in Fig. 5. The duration of the signal plays an important role in solving the inverse problem, its spectrum must verify the condition of high frequency approximation referred to in the previous section.

Figure 6 shows the incident signal given out by the transducer and the reflected signal by the plastic foam *M2* (thickness 4.1 cm,  $\alpha_\infty = 1.07 \pm 0.01$ ,  $\Lambda = 230 \mu\text{m}$ , and  $\Lambda' = 690 \mu\text{m}$ ). Figure 7 shows their spectrums. From the spectrums of the two signals, the reader can see that they have the same bandwidths which means that there is no dispersion. This concurs the theory below which predicts that the reflected wave from the first interface  $x=0$  is measured and simply attenuated by the factor  $(\sqrt{\alpha_\infty} - \phi) / (\sqrt{\alpha_\infty} + \phi)$ .

Using the incident and reflected experimental data for the foam *M2* and relation (27), the value of the porosity optimized by solving the inverse problem is  $\phi = 0.968 \pm 0.005$ . This value of the porosity is obtained on the basis of an average of a set of values, and its accuracy is calculated knowing the accuracy of the tortuosity using the relation  $\delta\phi/\phi = (1/2)(\delta\alpha_\infty/\alpha_\infty)$ , where  $\delta\phi$  is the accuracy of the porosity and  $\delta\alpha_\infty$  is the accuracy of the tortuosity.

Figure 8 shows the comparison between experimental reflected signals and simulated reflected signals for the optimized value of the porosity, the difference between the two curves is slight which leads us to conclude that the optimized value of the porosity is good. The value of the porosity of the

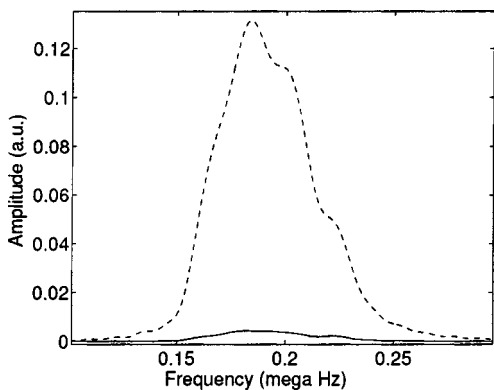


FIG. 7. Spectrum of experimental incident signal (dashed line) and spectrum of experimental reflected signal (solid line).

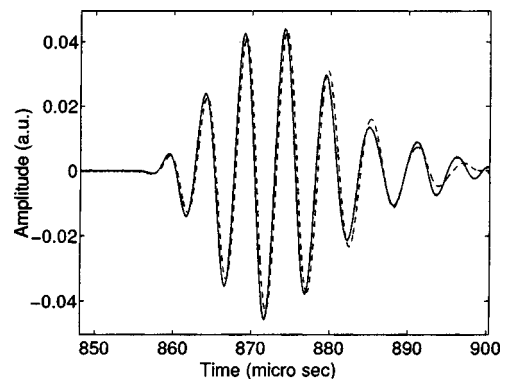


FIG. 8. Comparison between experimental reflected signal (solid line) and simulated reflected signal for  $\phi = 0.968$  (dashed line).

foam *M2* given by the porosimeter<sup>10,18</sup> is  $\phi = 0.97 \pm 0.02$ . The flow resistivity of the plastic foam *M2* is  $\sigma = 3000 \text{ N m}^{-4} \text{ s}$ . This value is low compared with other plastics foams, and is the reason why the value of the porosity is very close to 1.

Another plastic foam *M3* is studied (thickness 0.6 cm). This has a high level of flow resistivity,  $\sigma = 125\,000 \text{ N m}^{-4} \text{ s}$  and the ultrasonic parameters are  $\alpha_\infty = 1.70 \pm 0.01$ ,  $\Lambda = 23 \mu\text{m}$ ,  $\Lambda' = 69 \mu\text{m}$ . Figure 9 shows the incident and reflected signal and Fig. 10 shows their spectra respectively. Here again, we can conclude from the two spectra that there is no dispersion. By solving the inverse problem from the incident and reflected data and relation (27) the optimized value of the porosity is  $\phi = 0.821 \pm 0.005$ . Figure 11 shows the comparison between simulated reflected signal obtained by the optimized value of the porosity and the experimental signal. The readers can see the slight difference between the two curves that proves the optimized value of the porosity.

The main advantage of this method is its simplicity compared with the porosimeter,<sup>18</sup> which involves more complicated measurement steps and a more expensive experimental setup.

## VI. CONCLUSION

A simple method for the measurement of the porosity has been proposed. This method is based on a concept of

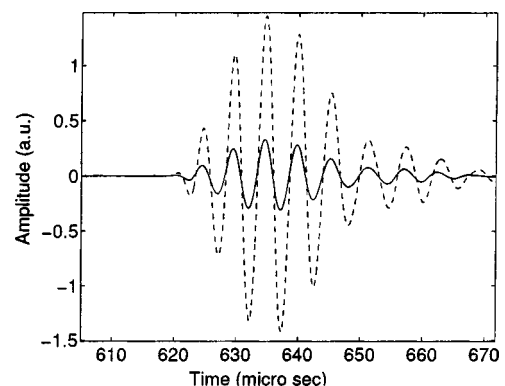


FIG. 9. Experimental incident signal (dashed line) and experimental reflected signal (solid line).

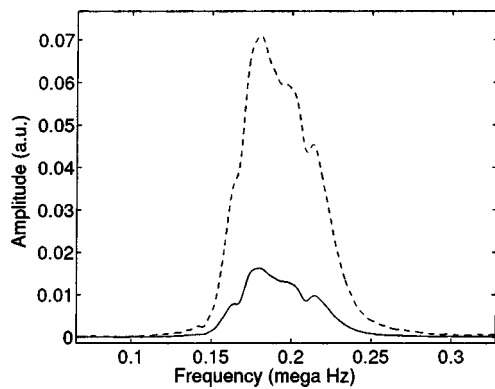


FIG. 10. Spectrum of experimental incident signal (dashed line) and spectrum of experimental reflected signal (solid line).

measuring waves reflected by the porous media. It is an alternative to the usual method involving the use of a porosimeter introduced by Beranek and improved by Champoux *et al.*

The method is based upon the propagation equation in the time domain in a slab of porous material having a rigid frame at high frequency range. A time domain model of wave propagation in such material is worked out from the concept of fractional calculus. The kernels of the reflection and transmission scattering operators are derived giving simple relations between these operators and the parameters of the medium. Because of the high attenuation in such media, the wave reflected by the porous material can be approximated by the wave reflected at the first interface. As a consequence, it gives a simple relation between the porosity, tortuosity and reflected wave, thereby proving that all physical parameters of acoustic material can be evaluated using acoustic measurements.

An experimental validation of the theoretical expression of the reflection scattering operators illustrates the high level of correspondence between numerical and experimental results and shows that this time domain model is well suited for the characterization of porous media via acoustic wave propagation. As part of our future plan, we hope to deal with the problem of oblique incidence that can give us more data for the reflected mode of the inverse problem.

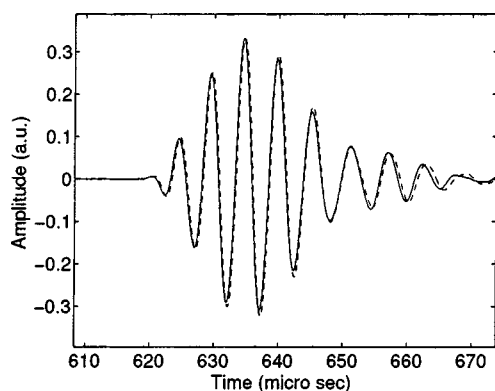


FIG. 11. Comparison between experimental reflected signal (solid line) and simulated reflected signal for  $\phi=0.821$  (dashed line).

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## APPENDIX: GREEN FUNCTION OF THE MEDIUM

The Green function of the propagation equation (7) is given in Ref. 3 by

$$F(t, k) = \begin{cases} 0 & \text{if } 0 \leq t \leq k \\ \Xi(t) + \Delta \int_0^{t-k} h(t, \xi) d\xi, & \text{if } t \geq k \end{cases},$$

with

$$\Xi(t) = \frac{b'}{4\sqrt{\pi}} \frac{k}{(t-k)^{3/2}} \exp\left(-\frac{b'^2 k^2}{16(t-k)}\right),$$

where  $h(\tau, \xi)$  has the following form:

$$h(\xi, \tau) = -\frac{1}{4\pi^{3/2}} \frac{1}{\sqrt{(\tau-\xi)^2 - k^2}} \frac{1}{\xi^{3/2}} \int_{-1}^1 \times \exp\left(-\frac{\chi(\mu, \tau, \xi)}{2}\right) [\chi(\mu, \tau, \xi) - 1] \frac{\mu d\mu}{\sqrt{1-\mu^2}},$$

and where

$$\chi(\mu, \tau, \xi) = [\Delta \mu \sqrt{(\tau-\xi)^2 - k^2} + b'(\tau-\xi)]^2 / 8\xi,$$

$$b' = Bc_0^2 \sqrt{\pi}, c' = C \cdot c_0^2$$

and  $\Delta = b'^2 - 4c'$  when  $k \rightarrow \infty$ ,  $\Xi$ , and  $h(\xi, \tau)$  tends to zero, then the Green function  $F(t, k)$  tends also to zero.

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