Direct and inverse scattering of transient acoustic waves by a slab of rigid porous material

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(Received 23 May 2002; revised 9 October 2002; accepted 21 October 2002)

This paper provides a temporal model of the direct and inverse scattering problem for the propagation of transient ultrasonic waves in a homogeneous isotropic slab of porous material having a rigid frame. This new time domain model of wave propagation takes into account the viscous and thermal losses of the medium as described by the model of Johnson *et al.* [D. L. Johnson, J. Koplik, and R. Dashen, J. Fluid. Mech. **176**, 379 (1987)] and Allard [J. F. Allard (Chapman and Hall, London, 1993)] modified by a fractional calculus based method applied in the time domain. This paper is devoted to the analytical calculus of acoustic field in a slab of porous material. The main result is the derivation of the expression of the scattering operators (reflection and transmission) which are the responses of the medium to an incident acoustic pulse. In this model the reflection operator is the sum of two contributions: the first interface and the bulk of the medium. Experimental and numerical results are given as a validation of our model. © *2003 Acoustical Society of America.* [DOI: 10.1121/1.1528592]

PACS numbers: 43.20.Bi, 43.20.Hq [ANN]

I. INTRODUCTION

The ultrasonic characterization of porous materials saturated by air is of a great interest for a large class of industrial applications. These materials are frequently used in the automotive and aeronautics industries or in the building trade. The determination of the properties of a medium from waves that have been reflected by or transmitted through the medium is a classical inverse scattering problem. Such problems are often approached by taking a physical model of the scattering process, generating a synthetic response for some assumed values of the parameters, adjusting these parameters until reasonable agreement is obtained between the synthetic response and the observed data. Some of the recent progresses in this area are reviewed in Allard[1] and Lafarge.[2] Many applications like medical imaging or inverse scattering problems require the study of the behavior of pulses traveling into porous media,[3] it is only recently that the response of these media to such excitations has been fully addressed in Fellah *et al.*[4–7] for some elementary configurations in porous media. To efficiently cope with the specific problems appearing in the transient acoustic field propagation, new approaches are required.[8] At present most analysis of signal propagation are carried out in the frequency domain using the Fourier transform to translate the results in the time domain and vice versa. This, however, has several limitations. The first is that the transformation is difficult to compute numerically with sufficient accuracy for non analytical functions. For example, using Fourier transform to obtain time domain results for a lossy material is a more complicated approach than using a true time domain analysis, and the numerical results are less accurate. The second disadvantage is that by working in the frequency domain some numerical information is lost or hard to recover. For example, in case of noisy data it may be difficult to reconstruct the chronological events of a signal by phase unwrapping. Consequently, it is difficult to obtain a deep understanding to transient signal propagation using frequency domain method.

The time domain response of the material is described by an instantaneous response and a ''susceptibility'' kernel responsible of the memory effects. A time domain approach differs from the frequency analysis in that the susceptibility functions of the problem are convolution operators acting on the velocity and pressure fields, and therefore a different algebraic formalism has to be applied to solve the wave equation. In the past, many authors have used the fractional calculus as an empirical method to describe the properties of viscoelastic materials, e.g., in Caputo[9] and Bagley.[10] The observation that the asymptotic expressions of stiffness and damping in porous materials are proportional to fractional powers of frequency suggests the fact that time derivatives of

fractional order might describe the behavior of sound waves in this kind of materials, including relaxation and frequency dependence.

The core of this paper is the analytical calculus in time domain of the acoustic field inside the porous material and the derivation of the expression of the scattering operators (reflection and transmission) which are the responses of the medium to an incident acoustic pulse.

The outline of this paper is as follows. In Sec. II, a time domain model is given, the connection between the fractional derivatives and wave propagation in rigid porous media in high frequency range is established, the basic equations are written in the time domain. Section III is devoted to the direct problem and to the general solution of the propagation wave in this domain. Section IV contains the expressions of the reflection and transmission kernels in the time domain. In Sec. V, a validation via ultrasonic measurements for the direct problem is treated and finally in Sec. VI an approach to inverse problem given the physical parameters describing the propagation of ultrasonic waves in porous materials is given.

II. MODEL

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory.[11,12] In air-saturated porous media the structure is generally motionless and the waves propagate only in the fluid. This case is described by the model of equivalent fluid which is a particular case of the Biot model, in which the interactions between the fluid and the structure are taken into account in two frequency response factors: the dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al.*[13] and the dynamic compressibility of the air included in the porous material $\beta(\omega)$ given by Allard.[1] In the frequency domain, these factors multiply the density of the fluid and its compressibility respectively and represent the deviation from the behavior of the fluid in free space as the frequency increases. In the time domain, they act as operators and in the high frequency approximation their expressions are given by Fellah and Depollier[4,5] and Fellah *et al.*[6,7] as

$$\tilde{\alpha}(t)=\alpha_\infty\left(\delta(t)+\frac{2}{\Lambda}\left(\frac{\eta}{\pi\rho_f}\right)^{1/2}t^{-1/2}\right), \tag{1}$$

$$\tilde{\beta}(t)=\left(\delta(t)+\frac{2(\gamma-1)}{\Lambda'}\left(\frac{\eta}{\pi\,\mathrm{Pr}\,\rho_f}\right)^{1/2}t^{-1/2}\right), \tag{2}$$

in these equations, $\delta(t)$ is the Dirac function, Pr is the Prandtl number, η and ρ_f are, respectively, the fluid viscosity and the fluid density and γ is the adiabatic constant. The relevant physical parameters of the model are the tortuosity of the medium α_∞ initially introduced by Zwikker and Kosten,[14] the viscous and thermal characteristic lengths Λ and Λ' introduced by Johnson *et al.*[13] and Allard.[1] In this model the time convolution of $t^{-1/2}$ with a function is interpreted as a semi derivative operator following the definition of the fractional derivative of order ν given in Samko and colleagues[15]

$$D^\nu[x(t)]=\frac{1}{\Gamma(-\nu)}\int_0^t(t-u)^{-\nu-1}x(u)\,du, \tag{3}$$

where $\Gamma(x)$ is the gamma function.

In this framework, the basic equations of our model can be written as

$$\rho_f\tilde{\alpha}(t)*\frac{\partial v_i}{\partial t}=-\nabla_{ip}\ \text{ and }\ \frac{\tilde{\beta}(t)}{K_a}*\frac{\partial p}{\partial t}=-\nabla\cdot v, \tag{4}$$

where * denotes the time convolution operation, p is the acoustic pressure, v is the particle velocity, and K_a is the bulk modulus of the air. The first equation is the Euler equation, the second one is a constitutive equation obtained from the equation of mass conservation associated with the behavior (or adiabatic) equation.

For a wave propagating along the x-axis, these equations become:

$$\rho_f\alpha_\infty\frac{\partial v}{\partial t}+2\frac{\rho_f\alpha_\infty}{\Lambda}\left(\frac{\eta}{\pi\rho_f}\right)^{1/2}\int_0^t\frac{\partial v/\partial t'}{\sqrt{t-t'}}\,dt'=-\frac{\partial p}{\partial x}, \tag{5}$$

$$\frac{1}{K_a}\frac{\partial p}{\partial t}+2\frac{\gamma-1}{K_a\Lambda'}\left(\frac{\eta}{\pi\,\mathrm{Pr}\,\rho_f}\right)^{1/2}\int_0^t\frac{\partial p/\partial t'}{\sqrt{t-t'}}\,dt'=-\frac{\partial v}{\partial x}, \tag{6}$$

in these equations the convolutions express the dispersive nature of the porous material. They take into account the memory effects due the fact that the response of the medium to the wave excitation is not instantaneous but needs more time to become effective. The retarding force is no longer proportional to the time derivative of the acoustic velocity but is found to be proportional to the fractional derivative of order 1/2 of this quantity. This occurs because the volume of fluid participating to the motion is not the same during the whole length of the signal as it is in the case of a fully developed steady flow. The phenomena may be understood by considering such a volume of fluid in a pore to be in harmonic motion. At high frequencies, only a thin layer of fluid is excited: the average shear stress is high. At a lower frequency, the same amplitude of fluid motion allows a thicker layer of fluid to participate in the motion and consequently the shear stress is less. The penetration distance of the viscous forces and therefore the excitation of the fluid depends on frequency. In the time domain, such a dependence is associated with a fractional derivative.

III. DIRECT PROBLEM

The direct scattering problem is that of determining the scattered field as well as the internal field, that arises when a known incident field impinges on the porous material with known physical properties. To compute the solution of the direct problem one need to know the Green's function[6] of the modified wave equation in the porous medium. In that case, the internal field is given by the time convolution of the

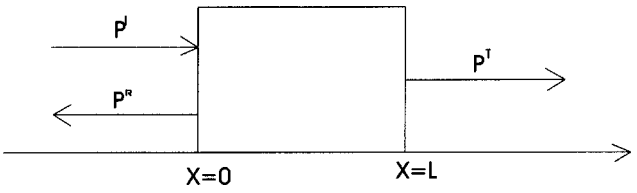

FIG. 1. Geometry of the problem.

Green's function with the incident wave and the reflected and transmitted fields are deduced from the internal field and the boundary conditions.

The generalized lossy wave equation in the time domain is derived from the basic equations (4) by elementary calculation in the following form:

$$\frac{\partial^2 p}{\partial x^2} - A\frac{\partial^2 p}{\partial t^2} - B\int_0^t \frac{\partial^2 p/\partial t'^2}{\sqrt{t-t'}}\,dt' - C\frac{\partial p}{\partial t} = 0, \tag{7}$$

where the coefficients A, B and C are constants, respectively, given by;

$$A = \frac{\rho_f \alpha_\infty}{K_a}, \quad B = \frac{2\alpha_\infty}{K_a}\sqrt{\frac{\rho_f \eta}{\pi}}\left(\frac{1}{\Lambda} + \frac{\gamma - 1}{\sqrt{\mathrm{Pr}}\Lambda'}\right),$$
$$C = \frac{4\alpha_\infty(\gamma-1)\eta}{K_a \Lambda \Lambda' \sqrt{\mathrm{Pr}}}, \tag{8}$$

the first one is related to the velocity $c = 1/\sqrt{\rho_f \alpha_\infty / K_a}$ of the wave in the air included in the porous material. α_∞ appears as the refractive index of the medium which changes the wave velocity from $c_0 = \sqrt{K_a/\rho_f}$ in free space to $c = c_0/\sqrt{\alpha_\infty}$ in the porous medium. The other coefficients are essentially dependent of the characteristic lengths Λ and Λ' and express the viscous and thermal interactions between the fluid and the structure. The constant B governs the spreading of the signal while C is responsible of the attenuation of the wave. Obviously, a knowledge of these three coefficients allows the determination of the parameters α_∞, Λ and Λ'. One way to solve Eq. (7) with suitable initial and boundary conditions is by using the Laplace transform. The approach is quite simple although the inverse Laplace transform require tedious calculus.[16] A suitable setting for the introduction of the time domain solution of the modified wave propagation equation (7) is provided by the following model.

A. General solution of the propagation equation

In this section some notation is introduced. The geometry of the problem is shown in Fig. 1. An homogeneous porous material occupies the region $0 \leqslant x \leqslant L$. This medium is assumed to be isotropic and to have a rigid frame. A short sound pulse impinges normally on the medium from the left. It gives rise to an acoustic pressure field $p(x,t)$ and an acoustic velocity field $v(x,t)$ within the material, which satisfying the propagation equation (7) written also as:

$$\frac{\partial^2 p(x,t)}{\partial x^2} - \left(\frac{1}{c^2}\,\delta(t) + CH(t) + \frac{\beta}{\sqrt{t}}\right) * \frac{\partial^2 p(x,t)}{\partial t^2} = 0, \tag{9}$$

where $H(t)$ is the Heaviside function:[17] $H(t) = 0$ for $t < 0$, $H(0) = 1/2$ and $H(t) = 1$, for $t > 0$.

It is assumed that the pressure field is continuous at the boundary of the material

$$p(0^+,t) = p(0^-,t), \quad p(L^-,t) = p(L^+,t), \tag{10}$$

(where $\pm$ superscript denotes the limit from the left and the right, respectively) and to the initial conditions

$$p(x,t)|_{t=0} = 0 \quad \left.\frac{\partial p}{\partial t}\right|_{t=0} = 0, \tag{11}$$

which means that the medium is idle for $t = 0$.

If the incident sound wave is launched in the region $x \leqslant 0$, then the general solution of Eq. (9) in the region to the left of the material is the sum of the incident and reflected fields

$$p_1(x,t) = p^i\left(t - \frac{x}{c_0}\right) + p_r\left(t + \frac{x}{c_0}\right), \quad x < 0, \tag{12}$$

here, $p_1(x,t)$ is the field in the region $x < 0$, p^i and p^r denote the incident and the reflected field, respectively. In addition, a transmitted field is produced in the region at the right of the material. This has the form

$$p_3(x,t) = p^t\left(t - \frac{L}{c} - \frac{(x-L)}{c_0}\right), \quad x > L. \tag{13}$$

[$p_3(x,t)$ is the field in the region $x > L$ and p^t is the transmitted field.]

The incident and scattered fields are related by the scattering operators (i.e., reflection and transmission operators) for the material. These are integral operators represented by

$$p^r(x,t) = \int_0^t \tilde{R}(\tau) p^i\left(t - \tau + \frac{x}{c_0}\right) d\tau$$
$$= \tilde{R}(t) * p^i(t) * \delta\left(t + \frac{x}{c_0}\right), \tag{14}$$

$$p^t(x,t) = \int_0^t \tilde{T}(\tau) p^i\left(t - \tau - \frac{L}{c} - \frac{(x-L)}{c_0}\right) d\tau$$
$$= \tilde{T}(t) * p^i(t) * \delta\left(t - \frac{L}{c} - \frac{(x-L)}{c_0}\right). \tag{15}$$

In Eqs. (14) and (15) the functions $\tilde{R}$ and $\tilde{T}$ are the reflection and the transmission kernels, respectively, for incidence from the left. Note that the lower limit of integration in Eqs. (14), (15) is chosen to be 0, which is equivalent to assuming that the incident wave front first impinges on the material at $t = 0$.

The scattering operators given in Eqs. (14) and (15) are independent of the incident field used in scattering experiment and depend only on the properties of the materials. In the region $x \leqslant 0$, the field $p_1(x,t)$ is given by:

$$p_1(x,t) = \left[\delta\left(t - \frac{x}{c_0}\right) + \tilde{R}(t) * \delta\left(t + \frac{x}{c_0}\right)\right] * p^i(t). \tag{16}$$

Equation (9) are solved by the Laplace transform method by taking into account to the conditions (10) and (11). We note $P(x,z)$ the Laplace transform of $p(x,t)$ defined by

$$P(x,z)=\mathcal{L}[p(x,t)]=\int_0^\infty \exp(-zt)p(x,t)dt. \tag{17}$$

Using the following relations

$$\mathcal{L}[\delta(t)]=1, \quad \mathcal{L}[H(t)]=\frac{1}{z}, \text{ and } \mathcal{L}\left[\frac{1}{\sqrt{t}}\right]=\sqrt{\frac{\pi}{z}}, \tag{18}$$

the Laplace transform of the wave equation (9) satisfying the initials conditions (11) becomes:

$$\frac{\partial^2 P_2(x,z)}{\partial x^2}-\frac{f(z)}{c^2}P_2(x,z)=0, \tag{19}$$

where $P_2(x,z)$ is the Laplace transform of the acoustic pressure $p_2(x,t)$ inside the porous material for $0\leqslant x\leqslant L$ and $f(z)=z^2+b'z\sqrt{z}+c'z$, $b'=B\cdot c^2\sqrt{\pi}$, $c'=C\cdot c^2$.

The Laplace transform of the field outside the materials is given by

$$P_1(x,z)=\left[\exp\left(-z\frac{x}{c_0}\right)+R(z)\exp\left(z\frac{x}{c_0}\right)\right]\varphi(z), \quad x\leqslant 0, \tag{20}$$

$$P_3(x,z)=T(z)\exp\left[-\left(\frac{L}{c}+\frac{(x-L)}{c_0}\right)z\right]\varphi(z), \quad x\geqslant L. \tag{21}$$

Here $P_1(x,z)$ and $P_3(x,z)$ are, respectively, the Laplace transform of the field at the left and the right of the material, $\varphi(z)$ denotes the Laplace transform of the incident field $p^i(t)$ and finally $R(z)$ and $T(z)$ are the Laplace transform of the reflection and transmission kernels respectively. The Laplace transform of the continuous conditions (10) are written as

$$P_2(0^+,z)=P_1(0^-,z) \text{ and } P_2(L^-,z)=P_3(L^+,z), \tag{22}$$

where $P_1(0^-,z)$ and $P_3(L^+,z)$ are the Laplace transform of $p_1(x,t)$ and $p_3(x,t)$, respectively, given by

$$P_1(0^-,z)=(1+R(z))\varphi(z), \tag{23}$$

$$P_3(L^-,z)=T(z)\exp\left(-\frac{L}{c}\right)z\varphi(z) \tag{24}$$

from Eqs. (19) and (22), we deduce the expression of the field inside the material $P_2(x,z)$

$$P_2(x,z)=P_1(0^-,z)\frac{\sinh\left(\frac{L-x}{c}\sqrt{f(z)}\right)}{\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)}+P_3(L^+,z)\frac{\sinh\left(\frac{x}{c}\sqrt{f(z)}\right)}{\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)}, \tag{25}$$

where sinh is the hyperbolic sine function.

The inverse Laplace transform of $\exp(-k\sqrt{f(z)})$, where k is a positive constant, gives the Green function of the medium:[6]

$$F(t,k)=\begin{cases}0 & \text{if } 0\leqslant t\leqslant k\\ \Xi(t)+\Delta\int_0^{t-k}h(t,\xi)d\xi & \text{if } t\geqslant k'\end{cases} \tag{26}$$

with

$$\Xi(t)=\frac{b'}{4\sqrt{\pi}}\frac{k}{(t-k)^{3/2}}\exp\left(-\frac{b'^2k^2}{16(t-k)}\right), \tag{27}$$

where $h(\tau,\xi)$ has the following form:

$$h(\xi,\tau)=-\frac{1}{4\pi^{3/2}}\frac{1}{\sqrt{(\tau-\xi)^2-k^2}}\frac{1}{\xi^{3/2}}\times\int_{-1}^{1}\exp\left(-\frac{\chi(\mu,\tau,\xi)}{2}\right)\times(\chi(\mu,\tau,\xi)-1)\frac{\mu\,d\mu}{\sqrt{1-\mu^2}}, \tag{28}$$

and where $\chi(\mu,\tau,\xi)=(\Delta\mu\sqrt{(\tau-\xi)^2-k^2}+b'(\tau-\xi))^2/8\xi$, $b'=Bc_0^2\sqrt{\pi}$, $c'=Cc_0^2$, and $\Delta=b'^2-4c'$. The inverse Laplace transform of $P_2(x,z)$ gives the complete solution of the wave equation in time domain in the porous material taking into account the multiple reflections at the interfaces $x=0$ and $x=L$. (Appendix A).

$$p_2(x,t)=\sum_{n\geqslant 0}\left[F\left(t,2n\frac{L}{c}+\frac{x}{c}\right)-F\left(t,(2n+2)\frac{L}{c}-\frac{x}{c}\right)\right]*p_1(0,t)+\sum_{n\geqslant 0}\left[F\left(t,(2n+1)\frac{L}{c}-\frac{x}{c}\right)-F\left(t,(2n+1)\frac{L}{c}+\frac{x}{c}\right)\right]*p_3(L,t), \tag{29}$$

which can be written as:

$$p_2(x,t)=\sum_{n\geqslant 0}\int_{2nL/c+x/c}^{t}F\left(\tau,2n\frac{L}{c}+\frac{x}{c}\right)p_1(0,t-\tau)d\tau-\sum_{n\geqslant 0}\int_{(2n+2)L/c-x/c}^{t}F\left(\tau,(2n+2)\frac{L}{c}-\frac{x}{c}\right)p_1(0,t-\tau)d\tau$$
$$+\sum_{n\geqslant 0}\int_{(2n+1)L/c-x/c}^{t}F\left(\tau,(2n+1)\frac{L}{c}-\frac{x}{c}\right)p_3(L,t-\tau)d\tau$$
$$-\sum_{n\geqslant 0}\int_{(2n+1)L/c+x/c}^{t}F\left(\tau,(2n+1)\frac{L}{c}+\frac{x}{c}\right)p_3(L,t-\tau)d\tau. \tag{30}$$

IV. REFLECTION AND TRANSMISSION SCATTERING OPERATORS

To derive the reflection and transmission coefficients the boundary conditions flow velocity at the interfaces $x=0$ and $x=\dot{L}$ are needed. The Euler equation is written in the regions (1) $(x\leqslant 0)$ and (2) $(0\leqslant x\leqslant L)$ as:

$$\rho_f\left.\frac{\partial v_1(x,t)}{\partial t}\right|_{x=0}=-\left.\frac{\partial p_1(x,t)}{\partial x}\right|_{x=0},\quad x\leqslant 0, \tag{31}$$

$$\rho_f\tilde{\alpha}(t)*\left.\frac{\partial v_2(x,t)}{\partial t}\right|_{x=0}=-\left.\frac{\partial p_2(x,t)}{\partial x}\right|_{x=0},\quad 0\leqslant x\leqslant L, \tag{32}$$

where $v_1(x,t)$ and $v_2(x,t)$ are the acoustic velocity field in the regions (1) and (2), respectively. In the free space [region (1)], the tortuosity operator is equal to 1.

The equation of the flow continuity at $x=0$ is written as:

$$v_1(x,t)=\phi v_2(x,t), \tag{33}$$

where ϕ is the porosity of the medium. From Eqs. (31), (32), and (33) it is easy to write:

$$\tilde{\alpha}(t)*\left.\frac{\partial p_1(x,t)}{\partial x}\right|_{x=0}=\phi\left.\frac{\partial p_2(x,t)}{\partial x}\right|_{x=0}, \tag{34}$$

with

$$\left.\frac{\partial p_1(x,t)}{\partial x}\right|_{x=0}=\frac{1}{c_0}(-\delta(t)+\tilde{R}(t))*\frac{\partial p^i(t)}{\partial t}. \tag{35}$$

The Laplace transform of Eq. (34) gives a relation between the reflection and transmission coefficient

$$(R(z)-1)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)=\frac{\phi c_0}{c}\frac{\sqrt{f(z)}}{z\alpha(z)}\left[T(z)\exp\left(\frac{-Lz}{c}\right)-(1+R(z))\cosh\left(\frac{L}{c}\sqrt{f(z)}\right)\right], \tag{36}$$

where $\alpha(z)$ is the Laplace transform of $\tilde{\alpha}(t)$.

At the interface $x=L$, the Euler equation is written in the two regions (2) and (3) $(x\geqslant L)$ as:

$$\rho_f\tilde{\alpha}(t)*\left.\frac{\partial v_2(x,t)}{\partial t}\right|_{x=L^-}=-\left.\frac{\partial p_2(x,t)}{\partial x}\right|_{x=L^-},$$
$$\rho_f\left.\frac{\partial v_3(x,t)}{\partial t}\right|_{x=L^+}=-\left.\frac{\partial p_3(x,t)}{\partial x}\right|_{x=L^+}. \tag{37}$$

At $x=L$, the continuity of the flow velocity leads to the relation

$$v_3(L^+,t)=\phi v_2(L^-,t). \tag{38}$$

From Eqs. (37)–(38), we have:

$$\tilde{\alpha}(t)*\left.\frac{\partial p_3(x,t)}{\partial x}\right|_{x=L^+}=\phi\left.\frac{\partial p_2(x,t)}{\partial x}\right|_{x=L^-}, \tag{39}$$

with

$$\left.\frac{\partial p_3(x,t)}{\partial x}\right|_{x=L^+}=-\frac{1}{c_0}\tilde{T}(t)*\left.\frac{\partial p^i}{\partial t}\right|_{t=L/c}, \tag{40}$$

the Laplace transform of Eq. (39) gives:

$$T(z)\exp\left(-\frac{L}{c}z\right)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)=\frac{\phi c_0}{c}\frac{\sqrt{f(z)}}{z\alpha(z)}\left[-T(z)\exp\left(-\frac{L}{c}z\right)\times\cosh\left(\frac{L}{c}\sqrt{f(z)}\right)+1+R(z)\right]. \tag{41}$$

The functions $R(z)$ and $T(z)$ following from Eqs. (36) and (41) are the reflection coefficient $[R(z)]$ and the transmission coefficient $(T(z))$ given by:

$$R(z)=\frac{\left(-\frac{\phi^2}{\alpha_\infty}+1\right)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)}{2\frac{\phi}{\sqrt{\alpha_\infty}}\cosh\left(\frac{L}{c}\sqrt{f(z)}\right)+\left(\frac{\phi^2}{\alpha_\infty}+1\right)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)}, \tag{42}$$

$$T(z)=\frac{2\frac{\phi}{\sqrt{\alpha_\infty}}\exp\left(\frac{L}{c}\right)z}{2\frac{\phi}{\sqrt{\alpha_\infty}}\cosh\left(\frac{L}{c}\sqrt{f(z)}\right)+\left(\frac{\phi^2}{\alpha_\infty}+1\right)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)}. \tag{43}$$

The development of these expressions in exponential series (Appendix B) and the inverse Laplace transform lead to the reflection and transmission scattering kernels

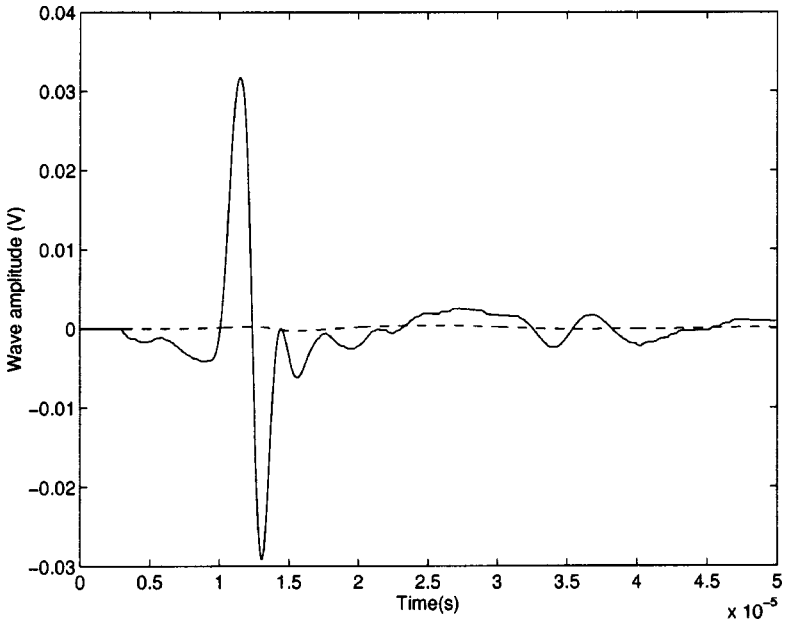

FIG. 2. Simulated contributions of the interface $x=0$ (solid line) and of the bulk material (dashed line) to the reflected wave.

$$\tilde{R}(t)=\left(\frac{-\phi+\sqrt{\alpha_\infty}}{\phi+\sqrt{\alpha_\infty}}\right)\sum_{n\geqslant 0}\left(\frac{\phi-\sqrt{\alpha_\infty}}{\phi+\sqrt{\alpha_\infty}}\right)^{2n}\times\left[F\left(t,2n\frac{L}{c}\right)-F\left(t,(2n+2)\frac{L}{c}\right)\right], \tag{44}$$

$$\tilde{T}(t)=\frac{4\phi\sqrt{\alpha_\infty}}{(\sqrt{\alpha_\infty}+\phi)^2}\sum_{n\geqslant 0}\left(\frac{\phi-\sqrt{\alpha_\infty}}{\phi+\sqrt{\alpha_\infty}}\right)^{2n}\times F\left(t+\frac{L}{c_0},(2n+1)\frac{L}{c}\right). \tag{45}$$

These expressions takes into account the n-multiple reflections in the material. In most cases, in porous materials saturated by air, the multiply reflection effects are negligible be cause of the high attenuation of sound waves in these media. So, by taking into account only the first reflections at the interfaces $x=0$ and $x=L$, the pressure $p_2(x,t)$ in the material becomes

$$p_2(x,t)=\left[F\left(t,\frac{x}{c}\right)-F\left(t,\frac{2L}{c}-\frac{x}{c}\right)\right]*p_1(0,t)+\left[F\left(t,\frac{L}{c}-\frac{x}{c}\right)-F\left(t,\frac{L}{c}+\frac{x}{c}\right)\right]*p_3(L,t), \tag{46}$$

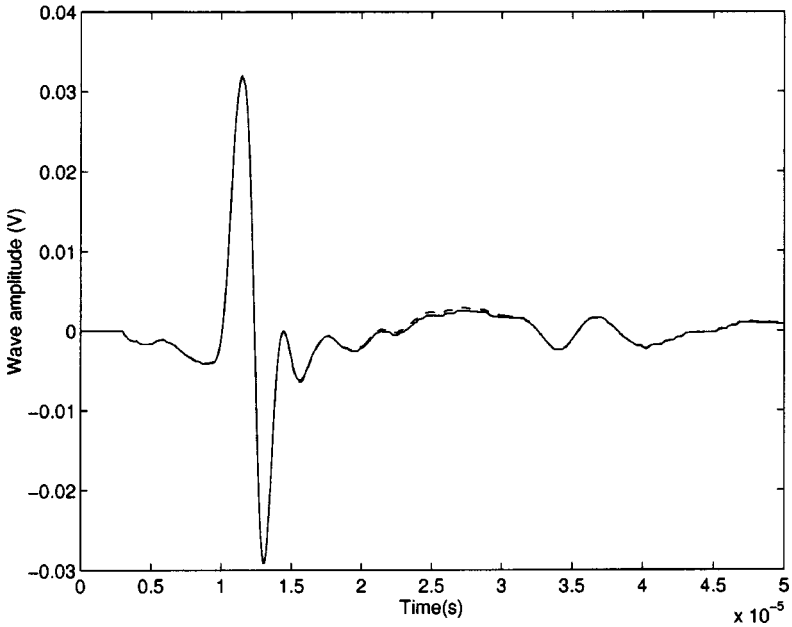

FIG. 3. Reflected wave at the interface $x=0$ (solid line) and the total reflected wave (dashed line).

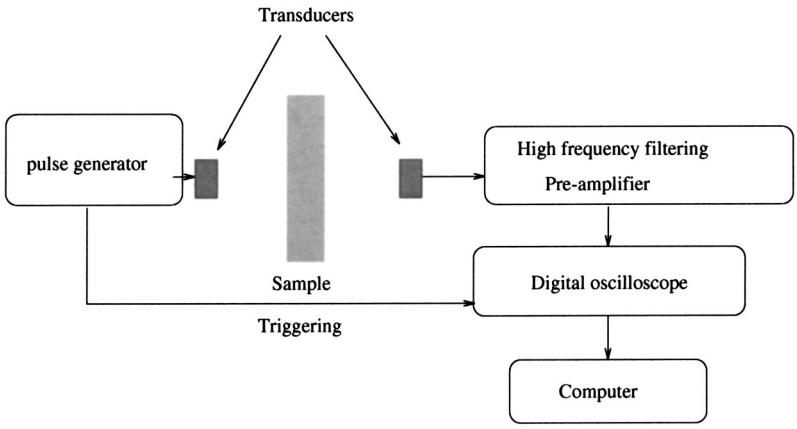

FIG. 4. Experimental setup of the ultrasonic measurements in transmitted mode.

and can also be written as

$$p_2(x,t)=\int_{x/c}^{t} F\left(\tau,\frac{x}{c}\right)p_1(0,t-\tau)\,d\tau - \int_{2L/c-x/c}^{t} F\left(\tau,\frac{2L}{c}-\frac{x}{c}\right) \times p_1(0,t-\tau)\,d\tau+\int_{L/c-x/c}^{t} F\left(\tau,\frac{L}{c}-\frac{x}{c}\right) \times p_3(L,t-\tau)\,d\tau-\int_{L/c+x/c}^{t} F\left(\tau,\frac{L}{c}+\frac{x}{c}\right) \times p_3(L,t-\tau)\,d\tau. \tag{47}$$

So, the kernels of reflection and transmission operators are given by

$$\tilde{R}(t)=\frac{\sqrt{\alpha_\infty}-\phi}{\sqrt{\alpha_\infty}+\phi}\,\delta(t)-\frac{4\phi\sqrt{\alpha_\infty}(\sqrt{\alpha_\infty}-\phi)}{(\sqrt{\alpha_\infty}+\phi)^3}F\left(t,\frac{2L}{c}\right), \tag{48}$$

$$\tilde{T}(t)=\frac{4\phi\sqrt{\alpha_\infty}}{(\phi+\sqrt{\alpha_\infty})^2}F\left(t+\frac{L}{c},\frac{L}{c}\right). \tag{49}$$

In Eq. (48) the first term is equivalent to the reflection at the interface $x=0$. The part of the wave corresponding to this term is not subjected to the dispersion but it is just multiplied by the factor $(\sqrt{\alpha_\infty}-\phi)/(\sqrt{\alpha_\infty}+\phi)$. This shows that although the tortuosity is a bulk parameter, it may be evaluated from the wave reflected at the first interface when the porosity is known. The second term: $-[4\phi\sqrt{\alpha_\infty}(\sqrt{\alpha_\infty}-\phi)/(\sqrt{\alpha_\infty}+\phi)^3]F(t,2L/c)$ in Eq. (48) is the bulk contribution to the reflection. Figure 2 shows each contribution to the reflected wave simulated from the expression (48) for a plastic foam M1. The solid line curve corresponds to the reflection at the first interface $x=0$ and the dashed line curve corresponds to the reflected wave (bulk contribution) at the second interface $x=L$. The parameters used in the simulation namely thickness: 5 cm, $\phi=0.98$, $\alpha_\infty=1.04$, $\Lambda=200\ \mu$m, and $\Lambda'=600\ \mu$m have been determined by classical methods.[18–20] As we can see, the bulk contribution to the reflected wave is negligible when it is compared to the first interface contribution. In Fig. 3 we show by numerical simulation the difference between the reflected wave at the first interface [due to the term: $(\sqrt{\alpha_\infty}-\phi)/(\sqrt{\alpha_\infty}+\phi)$ in Eq. (48)] and the total reflected wave [all terms in Eq. (48)]. The difference between the two curves is weak and the reflected wave by the porous material may be approximated by the reflected wave by the first interface with a good accuracy.

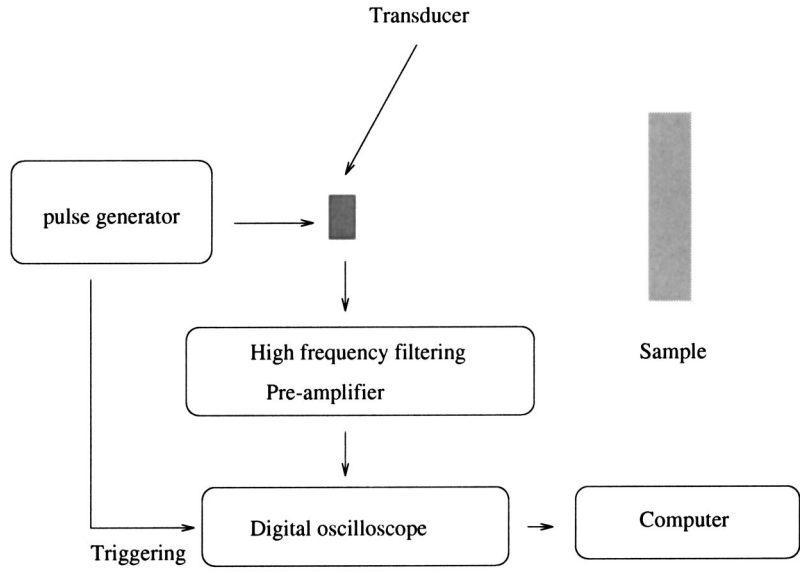

FIG. 5. Experimental setup of the ultrasonic measurements in reflected mode.

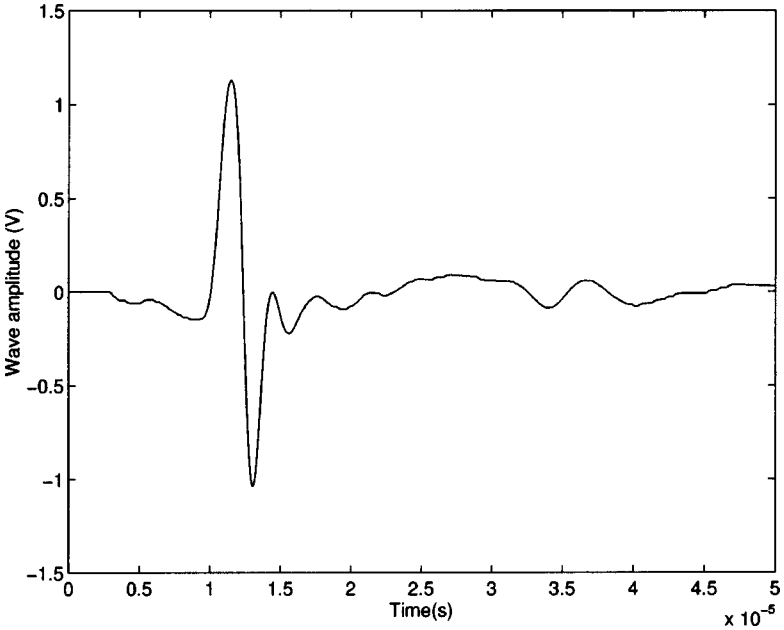

FIG. 6. Incident signal given out by the transducer.

V. ULTRASONIC MEASUREMENTS

As an application of this model, some numerical simulations are compared to experimental results. Experiments are done in air with two broadband Panametrics V389 piezoelectric transducers having a 250 kHz central frequency in air and a bandwith at 6 dB extending from 60 kHz to 420 kHz. Pulses of 900 V are provided by a 5058 PR Panametrics pulser/receiver. The received signals are amplified up to 90 dB and filtered above 1 MHz to avoid high frequency noise (energy is totally filtered by the sample in this upper frequency domain). Electronics perturbations are removed by 1000 acquisition averages. The experimental setup is shown in Fig. 4.

Reflected waves are processed by an other experimental set up given in Fig. 5. One transducer is used alternatively as a transmitter and receiver in order to detect the reflected wave.

Measurements have been performed on plastic foam M1. Figure 6 shows the incident signal generated by the transducer. Numerical simulation and experimental results (transmitted signal) are presented in Fig. 7. The numerical results are obtained from the convolution of the transmission operator with the signal generated by the transducer shown in Fig. 6. A good agreement between experimental data and theory is observed, which allows the validation of our model and the expression of the kernel of the transmission operator.

VI. INVERSE PROBLEM

A slab of porous material is characterized by four parameters, namely, the porosity ϕ, the tortuosity α_∞, the viscous characteristic length Λ, and the thermal characteristic length Λ', the values of which are crucial for the behavior of the sound waves in such materials. So, it is of some importance to work out new experimental methods and efficient

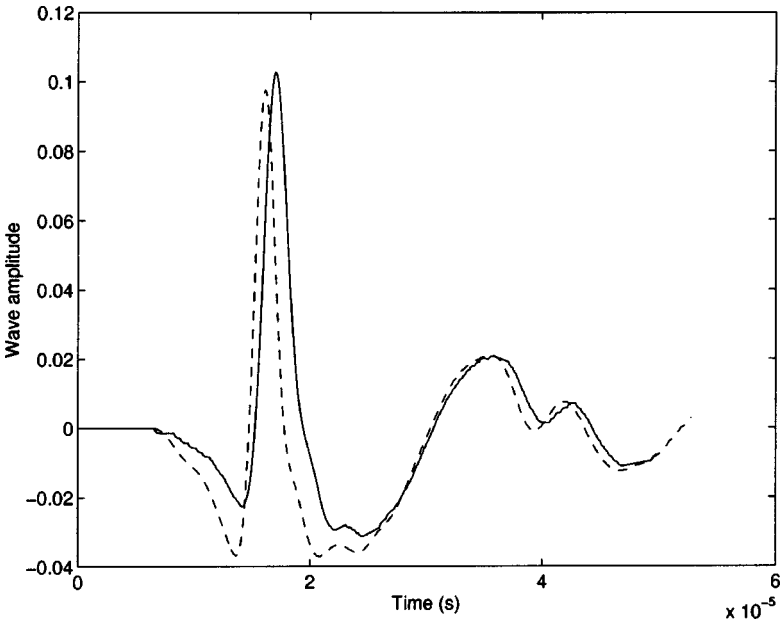

FIG. 7. Direct problem: experimental (solid line) and simulated transmitted signals (dashed line).

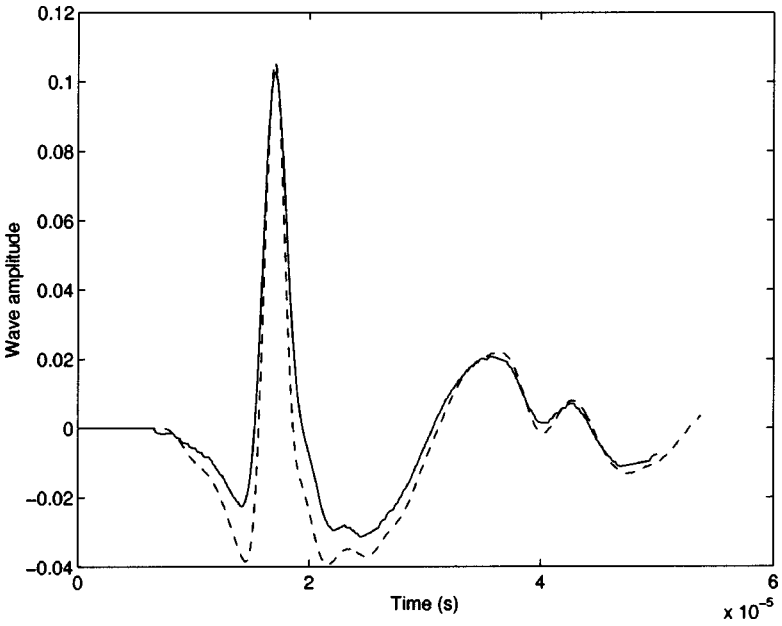

FIG. 8. Inverse problem: experimental (solid line) and simulated transmitted signals (dashed line).

tools for their estimation. Therefore, a basic inverse problem associated with the slab may be stated as follows: from the measurements of the transmitted and/or reflected signals outside the slab, find the values of the parameters of the medium. As shown in Sec. III, the solution of the direct problem is the system of two operators expressed as functions on ϕ, α_∞, Λ and Λ'. The inversion algorithm for finding the values of the parameters of the slab is based on a fitting procedure: find the values of the parameters ϕ, α_∞, Λ and Λ' such that the transmitted and reflected signal describes the scattering problem in the best possible way (e.g., in the least-squares sense). The inverse problem is to find values of parameters ϕ, α_∞, Λ and Λ' which minimize the functions

$$U_1(\phi,\alpha_\infty,\Lambda,\Lambda')=\int_0^t (r(t)-p^r(x,t))^2\,dt,$$

$$U_2(\phi,\alpha_\infty,\Lambda,\Lambda')=\int_0^t (s(t)-p^t(x,t))^2\,dt,$$

where $r(t)$ is the experimentally determined reflected signal, $p^r(x,t)$ is reflected wave predicted from Eq. (14), $s(t)$ is the experimentally determined transmitted signal and $p^t(x,t)$ is the transmitted wave predicted from Eq. (15). However, because of the nonlinearity of the equations, the analytical solution of the inverse problem by the conventional least-squares methods is tedious. In our case, one can seek the numerical solution of the least-square method which minimize the $U_{1,2}(\phi,\alpha_\infty,\Lambda,\Lambda')$ defined by

$$U_1(\phi,\alpha_\infty,\Lambda,\Lambda')=\sum_{i=1}^{i=N} (r_i-p^r(x,t_i))^2,$$

$$U_2(\phi,\alpha_\infty,\Lambda,\Lambda')=\sum_{i=1}^{i=N} (s_i-p^r(x,t_i))^2,$$

where $r_i=r(t_i)_{i=1,2,\ldots,N}$ [resp. $s_i=s(t_i)_{i=1,2,\ldots,N}$] represents the discrete set of values of the reflected (resp. transmitted) experimental signal and $p^r(x,t_i)_{i=1,2,\ldots,N}$ (resp. $p^t(x,t_i)_{i=1,2,\ldots,N}$) is the discrete set of values of the simulated reflected (resp. transmitted) signal. Figure 8 shows a comparison between experimental transmitted signal and simulated signal obtained by optimization from the inverse problem. The optimized parameters are $\alpha_\infty=1.05$, $\Lambda=208$ μm, and $\Lambda'=624$ μm. The comparison between Figs. 7 and 8 shows that the values of the acoustic parameters obtained by solving the inverse problem in the time-domain method leads to better results than those given by classical methods.[18–22]

Now, we will try to estimate the porosity via the measurement of reflected waves knowing the value of the tortu-

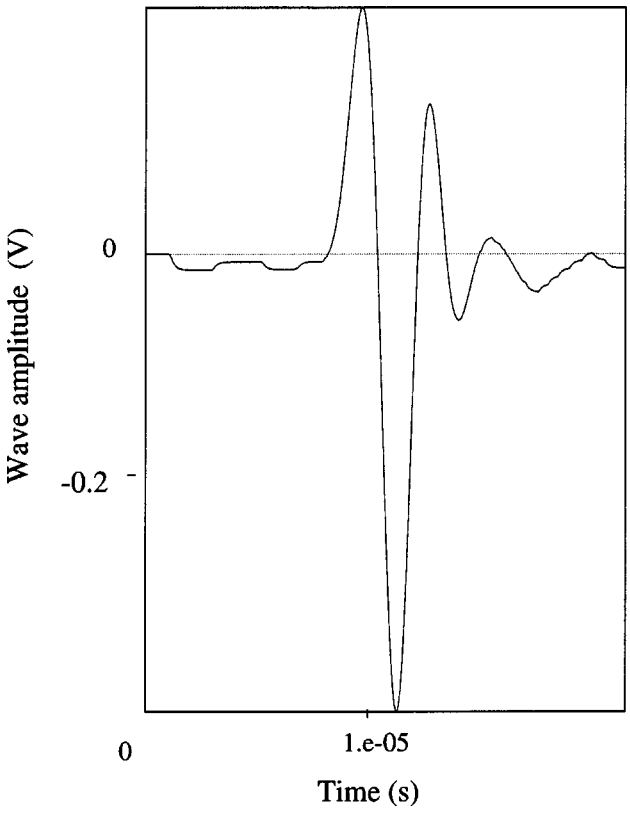

FIG. 9. Incident signal in reflected mode.

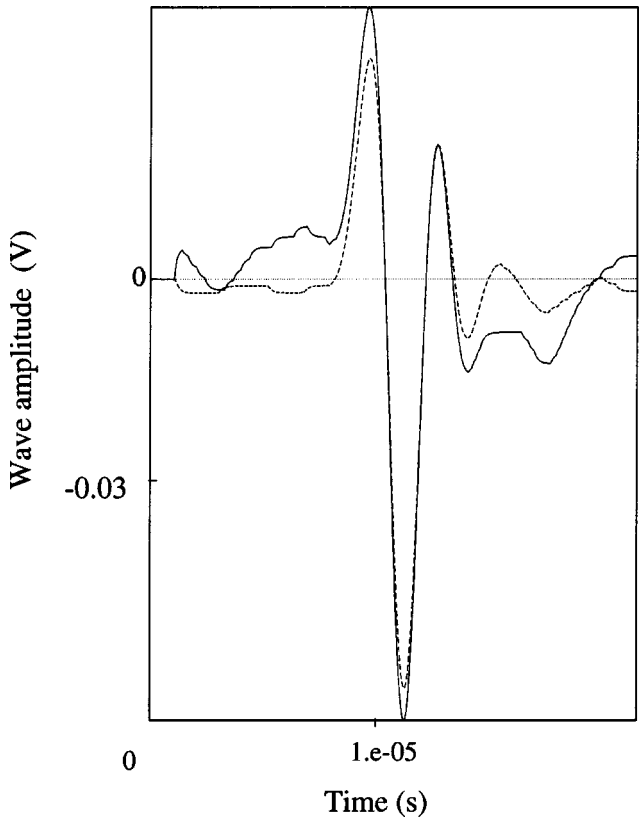

FIG. 10. Experimental reflected signal (solid line) and simulated reflected signal (dashed line).

osity (optimized by solving the inverse problem for the transmitted wave) and using the fact that the measured reflected wave is essentially due to the reflected wave at the first interface. The experimental setup using is shown in Fig. 5.

Figure 9 shows the incident signal generated by the transducer and Fig. 10 shows the comparison between experimental reflected signal by the foam M1 and simulated signal obtained by optimization of the inverse problem, the optimized value of the porosity is $\phi=0.97$.

In the future, one hopes to use this new method to measure the tortuosity and the porosity of plastic foams which are at the moment measured by a classical static methods.[20–22]

VII. CONCLUSION

In this paper the propagation equation in time domain in a slab of porous material having a rigid frame is considered in the high frequency range. A time domain model of wave propagation in such material is worked out from the concept of fractional calculus. The analytical general solution of the wave propagation in the time domain is established for a slab of porous medium. The kernels of the reflection and transmission scattering operators are derived giving simple relations between these operators and the parameters of the medium. Is is shown that the reflection scattering operator is equal to the sum of two contributions: the first interface and the bulk of the porous medium.

An experimental validation of the theoretical expressions of the scattering operators illustrates the good agreement between numerical and experimental results and shows that this time domain model is well suited for the characterization of porous media via acoustic wave propagation. To progress in this area, coming works will must improve experimental methods and inversion algorithms.

APPENDIX A: EXPRESSION OF THE ACOUSTIC FIELD INSIDE THE POROUS MATERIAL

The expression of the acoustic field inside the porous material taking into account the multiple reflection at the interfaces $x=0$ and $x=L$ is given from the Eq. (25)

$$P_2(x,z)=P_1(0^-,z)\frac{\sinh(\alpha\sqrt{f(z)})}{\sinh(\beta\sqrt{f(z)})},$$

$$+P_3(L^+,z)\frac{\sinh(\theta\sqrt{f(z)})}{\sinh(\beta\sqrt{f(z)})},$$

with $\alpha=L-x/c$, $\beta=L/c$ and $\theta=x/c$ by the following development in series:

$$\frac{\sinh(\alpha\sqrt{f(z)})}{\sinh(\beta\sqrt{f(z)})}=\frac{\exp(\alpha\sqrt{f(z)})-\exp(-\alpha\sqrt{f(z)})}{\exp(\beta\sqrt{f(z)})-\exp(-\beta\sqrt{f(z)})}$$

$$=\frac{\exp(\alpha\sqrt{f(z)})-\exp(-\alpha\sqrt{f(z)})}{\exp(\beta\sqrt{f(z)})[1-\exp(-2\beta\sqrt{f(z)})]}$$

$$=\frac{\exp(\alpha\sqrt{f(z)})-\exp(-\alpha\sqrt{f(z)})}{\exp(\beta\sqrt{f(z)})}$$

$$\times\sum_{n\geqslant 0}\exp(-2n\beta\sqrt{f(z)})$$

$$=[\exp(\alpha\sqrt{f(z)})-\exp(-\alpha\sqrt{f(z)})]$$

$$\times\sum_{n\geqslant}\exp(-(2n+1)\beta\sqrt{f(z)})$$

$$=\sum_{n\geqslant 0}[\exp(-[(2n+1)\beta-\alpha]\sqrt{f(z)})$$

$$-\exp(-[(2n+1)\beta+\alpha]\sqrt{f(z)})],$$

and the inverse Laplace transform of $P_2(x,z)$:

$$p_2(x,t)=\mathcal{L}^{-1}P_2(x,z)$$

$$=p_1(0,t)*\mathcal{L}^{-1}\left[\frac{\sinh(\alpha\sqrt{f(z)})}{\sinh(\beta\sqrt{f(z)})}\right]$$

$$+p_3(L,t)*\mathcal{L}^{-1}\left[\frac{\sinh(\theta\sqrt{f(z)})}{\sinh(\beta\sqrt{f(z)})}\right],$$

where

$$\mathcal{L}^{-1}\left[\frac{\sinh(\alpha\sqrt{f(z)})}{\sinh(\beta\sqrt{f(z)})}\right]=\sum_{n\geqslant 0}[F(t,(2n+1)\beta-\alpha)$$

$$-F(t,(2n+1)\beta+\alpha)]$$

and

$$F(t,k)=\mathcal{L}^{-1}[\exp(-k\sqrt{f(z)})]$$

is given by Eq. (26). The field $p_2(x,t)$ inside the porous medium is then given by:

$$p_2(x,t)=p_1(0,t)*\sum_{n\geqslant 0}[F(t,(2n+1)\beta-\alpha)-F(t,(2n+1)\beta+\alpha)]+p_3(L,t)*\sum_{n\geqslant 0}[F(t,(2n+1)\beta-\theta)-F(t,(2n+1)\beta+\theta)].$$

By substituting α, β and θ by their values, we find the expression (29).

APPENDIX B: EXPRESSION OF THE REFLECTION AND TRANSMISSION OPERATORS

To write the time domain expressions of the reflection and transmission kernels we consider their Laplace transforms. In the domain of validity of this model,[4] $(c/\phi c_0)\times[z\alpha(z)/\sqrt{f(z)}]\approx(\sqrt{\alpha_\infty}/\phi)$. Putting $a=(\sqrt{\alpha_\infty}/\phi)$, from Eq. (36) and Eq. (41), we get the system:

$$a(-1+R(z))\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)=T(z)\exp\left(-\frac{L}{c}z\right)-(1+R(z))\cosh\left(\frac{L}{c}\sqrt{f(z)}\right),$$

$$aT(z)\exp\left(-\frac{L}{c}z\right)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)=-T(z)\exp\left(-\frac{L}{c}z\right)+R(z)+1,$$

the solution of which is given by:

$$R(z)=\frac{(a^2-1)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)}{2a\cosh\left(\frac{L}{c}\sqrt{f(z)}\right)+(a^2+1)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)},$$

$$T(z)=\frac{2a\exp\left(\frac{L}{c}z\right)}{2a\cosh\left(\frac{L}{c}\sqrt{f(z)}\right)+(2+1)\sinh\left(\frac{L}{c}\sqrt{f(z)}\right)},$$

which corresponds to the expressions given in Eq. (42) and Eq. (43). The reflection coefficient $R(z)$ can be written as

$$R(z)=\frac{(a^2-1)\left[\exp\left(\frac{L}{c}\sqrt{f(z)}\right)-\exp\left(-\frac{L}{c}\sqrt{f(z)}\right)\right]}{2a\left[\exp\left(\frac{L}{c}\sqrt{f(z)}\right)+\exp\left(-\frac{L}{c}\sqrt{f(z)}\right)\right]+(a^2+1)\left[\exp\left(\frac{L}{c}\sqrt{f(z)}\right)-\exp\left(-\frac{L}{c}\sqrt{f(z)}\right)\right]}$$

and is equivalent to

$$R(z)=\left[\frac{a^2-1}{(a+1)^2}\right]\frac{1-\exp\left(-2\frac{L}{c}\sqrt{f(z)}\right)}{1-\left(\frac{a-1}{a+1}\right)^2\exp\left(-2\frac{L}{c}\sqrt{f(z)}\right)}.$$

From the identity

$$\frac{1}{1-\left(\frac{a-1}{a+1}\right)^2\exp\left(-2\frac{L}{c}\sqrt{f(z)}\right)}=\sum_{n\geqslant 0}\left(\frac{a-1}{a+1}\right)^{2n}\exp\left(-2n\frac{L}{c}\sqrt{f(z)}\right)$$

$R(z)$ has the following form:

$$R(z)=\left(\frac{a-1}{a+1}\right)\sum_{n\geqslant 0}\left(\frac{a-1}{a+1}\right)^{2n}\left[\exp\left(-2n\frac{L}{c}\sqrt{f(z)}\right)-\exp\left(-2(n+1)\frac{L}{c}\sqrt{f(z)}\right)\right]. \quad (50)$$

The kernel of the reflection scattering operator $\tilde{R}(t)$ is given by the inverse Laplace transform of this equation

$$\tilde{R}(t)=\left(\frac{a-1}{a+1}\right)\sum_{n\geqslant 0}\left(\frac{a-1}{a+1}\right)^{2n}\left[F\left(t,2n\frac{L}{c}\right)-F\left(t,2(n+1)\frac{L}{c}\right)\right], \quad (51)$$

where $F(t,k)$ is given by Eq. (26). In the same manner, the transmission coefficient is given by:

$$T(z)=\frac{4a\exp\left(\frac{L}{c}z\right)}{2a\left[\exp\left(\frac{L}{c}\sqrt{f(z)}\right)+\exp\left(-\frac{L}{c}\sqrt{f(z)}\right)\right]+(a^2+1)\left[\exp\left(\frac{L}{c}\sqrt{f(z)}\right)-\exp\left(-\frac{L}{c}\sqrt{f(z)}\right)\right]},$$

which can be written as:

$$T(z)=\frac{4a\exp\left(\frac{L}{c}z\right)\exp\left(-\frac{L}{c}\sqrt{fz}\right)}{(a+1)^2-(a-1)^2\exp-\left(2\frac{L}{c}\sqrt{f(z)}\right)}.$$

Expanding this relation in series, one gets

$$T(z)=\frac{4a}{(a+1)^2}\exp\left(\frac{L}{c}z\right)\sum_{n\geqslant 0}\left(\frac{a-1}{a+1}\right)^{2n}\times\exp\left(-(2n+1)\frac{L}{c}\sqrt{f(z)}\right),$$

which by inverse Laplace transform leads to the kernel of the transmission scattering operator:

$$\tilde{T}(t)=\frac{4a}{(a+1)^2}\sum_{n\geqslant 0}\left(\frac{a-1}{a+1}\right)^{2n}F\left(t+\frac{L}{c},(2n+1)\frac{L}{c}\right). \tag{52}$$

The reflection operator is obtained with $n=0$ and $x=L$ in Eq. (51):

$$\tilde{R}(t)=\left(\frac{a-1}{a+1}\right)\left[\delta(t)-\frac{4a}{(a+1)^2}F\left(t,2\frac{L}{c}\right)\right].$$

The transmission operator is obtained with $n=0$ and $x=L$ in Eq. (52):

$$\tilde{T}(t)=\frac{4a}{(a+1)^2}F\left(t+\frac{L}{c},\frac{L}{c}\right).$$

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