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Higher-moments estimation of industry-level cost of equity capital with errors-in-variables

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Abstract

Despite several anomalies documented in the literature, the CAPM is still widely used in practice to estimate discount rates for the valuation of firms and projects. In this paper, we conduct an empirical exercise to address the errors-in-variables (EIV) problem in the estimation of the industry-level cost of equity capital. We compare the results obtained using naive OLS against those from an instrumental variables approach developed by Dagenais and Dagenais (1997). Although the presence of EIV is statistically not detectable in some cases, we argue that the differences in the output from the two estimation techniques can lead to considerable gaps when it comes to use the results in any discounted cash-flow analysis.

Keywords: Cost of equity capital, CAPM, Fama-French 3-factor model, instrumental variables, higher-moments estimator, errors-in-variables

JEL classification: G10, G12, G31

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1 Introduction

The Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM) stands at the center of the finance theory (Sharpe, 1964; Lintner, 1965; Mossin, 1966). The CAPM's basic yet essential premise is that expected excess returns on securities are a linear function of a single systematic risk factor. More than a half century after its inception, the debate surrounding the theoretical and empirical success of the CAPM remains inconclusive. However, this does not obscure the popularity of the model among not only academics but also, and particularly, practitioners who still widely employ the model for various purposes. In fact, to paraphrase the prominent British statistician G.E.P. Box, although the CAPM is, or may, not be the *true* model of the expected returns, it is widely acknowledged not only by academics but also practitioners as a *useful* model.

In this paper, we analyze the effect of measurement error on the estimation of discount rates using factor models as an econometric issue that is relatively less discussed in the empirical literature, taking the CAPM as a (very) popular tool to estimate the cost of equity capital. While considering the CAPM as the baseline model of the determination of discount factors may seem a narrow choice, it is worthwhile to study its empirical properties in light of its persistent popularity. It is not surprising to note that the CAPM is indeed still an integral component of leading investment decision support tools such as Bloomberg, Thomson Reuters or FactSet. Additionally, we emphasize that our goal in this work is not to give a discussion of the existing methods developed so far for estimating the cost of equity capital. For a selection of recent discussions of the topic see, among others, Lutzenberger (2017), Breuer et al. (2014), Da et al. (2012) or Artmann et al. (2012).

As underlined by Graham and Harvey (2001), Graham and Harvey (2002), Anand (2002), Rosenbaum and Pearl (2009), or Da et al. (2012), among many others, the traditional, and widely employed, practice consists in calculating the cost of equity capital as the sum of a certain rate of return plus a compensation for bearing systematic risk proportional to the *project's* or *corporation's* beta β . While the risk-free rate of return is generally set as the yield on a basket of public debt instruments, institutions and practitioners concentrate their efforts mostly on the determination of the systematic risk premia and their respective risk loadings, i.e. β 's. As far as the CAPM-based formula is concerned, the appropriate systematic risk premium is the return on the *market portfolio* of risky assets in excess of the risk-free rate. The factor loading on the market risk premium is then called the *market beta*, or simply the beta, and is defined as the ratio of the covariance between the returns on the test asset and the market risk factor to the variance of the latter. As discussed by Rosenbaum and Pearl (2009, p. 128), there is no a consensus, just like the risk-free rate, in the industry regarding the determination of these two inputs but practitioners have so far come up with several guidelines as to the proper implementation of the CAPM formula.

These rules-of-thumb collectively guarantee that the CAPM still remains the industry standard to estimate the discount rates necessary in valuing companies using discounted cash-flow analysis. In a frequently cited paper, Graham and Harvey (2001) report that the CAPM is *by far the most popular method of estimating the cost of equity capital: 73.5% of respondents always or almost always use the CAPM* (Graham and Harvey, 2001, p. 201).¹ In line with Graham and Harvey (2001), Bruner et al. (1998) also note survey-based evidence data that the CAPM *is the dominant model for estimating the cost of equity* and no firms cited specifications of the CAPM to adjust for any empirical shortcomings of the model (Bruner et al., 1998, p. 15-16).²

In this paper, we conduct a simple empirical exercise to highlight the potential effects of measurement error in predictor variables on the economic significance of model estimates. Specifically, we focus on estimating the cost of equity capital for an array of industry-level portfolios using the CAPM approach and provide a short and comparative discussion of two sets of output. One set is obtained from the widely used OLS estimation method, while the other set considers the potential measurement errors in the observed risk factor premium. To do this, we employ an instrumental variables estimation method developed by Dagenais and Dagenais (1997) that uses the higher-moments of the data as valid instruments to mitigate the EIV bias. As noted by Carmichael and Coen (2008, p. 780), financial models offer a natural laboratory for Dagenais-Dagenais higher-moment estimator (DDHME), particularly if asset returns exhibit systematic non-Gaussianity. As a matter of fact, the DDHME estimator has been used in several studies like Coen and Racicot (2007), Coen and Hübner (2009), Coen et al. (2010), and was later extended to a GMM setting by Racicot and Theoret (2012).

We run time-series regressions of monthly excess returns on 44 Fama-French industry portfolios to a set of Fama-French research factors representing estimates of systematic risk premiums from March 2003 to February 2022. We consider two data generating processes, namely the one factor model, i.e. the CAPM and the Fama-French three factor model, a straightforward choice given the large scale popularity of the three-factor model. The former holds that expected returns are a linear function of a market risk premium while the latter includes two additional factors as source of systematic risk determining expected returns. We present the results for two models and two competing estimation methods. Our results suggest that, as pointed out by Cragg (1994, p. 794), overlooking the potential errors-in-variables bias in the explanatory variables tends to *distort seriously the interpretation that should be given to standard econometric*

¹The authors also published a compressed version of their original paper, see Graham and Harvey (2002).

²In passing, we would like to mark a side-note on the *joint* popularity of CAPM-based calculation of the cost of equity and the implementation of the model using OLS among practitioners. The reason would arguably be the availability of OLS as the unique regression estimator openly available within the world's most popular spreadsheet tool, that is MS Excel. In fact, the default installation of the Microsoft Office comes with several tools and functions that collectively make the OLS estimation the default, and easily accessible, choice for calculating regression parameters.

estimates. Out of the 44 industries, the OLS estimates of the market risk premium coefficient are lower than the DDHME estimates for 30 cases with the one-factor model and 28 cases with the 3-factor model. Although the test results for the presence of EIV do not suggest a systematic rejection of the null of no measurement error in the data, estimates of the industry-level (annualized) cost of equity of capital are significantly lower with OLS relative to DDHME. Broadly speaking, ignoring the downward bias in the risk factor loadings due to measurement error would have substantial impact on any corporate decision-making process using discounted cash-flows analysis.

The paper is organized as follows: In the next section, there is a brief discussion of the relationship between factor models of asset returns and the measurement error. Section 3 introduces the econometric methodology and provides an overview of the higher-moment estimation approach developed by Dagenais and Dagenais (1997). Section 4 presents the data. We discuss our results in section 5. Finally, section 6 concludes.

2 Factor models with measurement error

Our goal is to estimate the equity cost of capital k_j of an industry portfolio j . The baseline model for k_j consists in breaking it down as,

$$k_j = E(R_j) - R_f = \sum_{j=1}^K \beta_{jk} E(\tilde{F}_k) \quad (1)$$

for $j = 1, \dots, n$ and where R_f is a risk-free rate of interest, $E(\cdot)$ the expectation operator, \tilde{F}_k a systematic risk factor and β_{jk} the sensitivity coefficient associated with k th factor, $k = 1, \dots, K$.³ The empirical version of the model can be expressed as,

$$R_{jt} - R_{ft} = \alpha_j + \sum_{k=1}^K \beta_{jk} F_{tk} + \epsilon_{jt} \quad (2)$$

Setting $K = 1$ and defining F as the *market risk premium*, we obtain the well-known CAPM formula that can be written as the following straight-line equation,

$$r_{jt} = \alpha_j + \beta_j f_t + \epsilon_{jt} \quad (3)$$

where α_j , β_j and $\sigma^2(\epsilon_j)$ are unknown parameters that must be estimated using data. In this formulation, smallcase letters r and f denote the dependent and independent variables in excess of the risk-free rate as $r_j = R_j - R_f$ and $f_t = R_{M^*} - R_f$ respectively. In CAPM's vocabulary, the appropriate systematic risk factor M^* corresponds to the market portfolio of risky assets, which includes all risky assets in proportion to their

³In financial regressions, this is commonly called as *factor loading*.

relative total market capitalization.

When it comes to put the model into action, however, the very definition of M^* is a little tricky. In fact, because the true M^* is, by definition, unobservable, which has long been acknowledged in the literature since Roll's famous paper (Roll, 1977), one would rightfully suspect from possible endogeneity in estimations due to measurement error in M^* . As long as M^* cannot be identified but observed with error as $M = M^* + \delta$, this drives one to a strand of empirical literature in which attempts have so far been made to mitigate the errors-in-variables problem (EIV) in the estimation of CAPM parameters. Carmichael and Coen (2008, p. 779) argue that Roll's critique is *a reminder that EIV should be endemic in linear regression models of asset returns* and that applies not only to CAPM but also to more general models such as Fama and French (1992), Fama and French (1993) or Carhart (1997).

Meanwhile, as noted by Coen and Racicot (2007, p. 444), the errors-in-variables problem has so far been at the center of relatively fewer theoretical and applied efforts in the literature. Carmichael and Coen (2008) enumerate a number of reasons for this observation. First, the typical textbook treatment of EIV models are mostly limited to the single variable case where it is possible to derive exact relationships to illustrate the impact of measurement error in the right-hand-side variable on parameter estimates. Once the setup includes two or more predictors, this is no longer possible even if there is only one variable measured with error while the observations on other variables correspond to their true values.⁴ Second, that the EIV bias in the single predictor regression, which Hausman (2001) called the *attenuation bias*, does not change the sign of the estimate can also be one reason of the disregard against it. In fact, at first sight, the presence of badly measured variables yields *at worst* to conservative estimates and, consequently, EIV would be seen as a *much ado about nothing*.

We do not agree with this view. From the outset, the objective of bringing into play any theoretical model of asset returns is to come up with reasonable estimates of a parameter of interest and

In this word, we consider here the cost of capital as an important by-product of CAPM as the most popular factor model of asset returns, which, despite ample criticisms, is still widely used in practice and can provide reasonable cost of capital estimates (Da et al., 2012, p. 217-218). Consequently, any systematic bias in the key parameter of the model should not be overlooked for it automatically generates poor estimates of the cost of equity. Such an effect should even be taken more seriously as long as the bias is downwards so that the implies an underestimation of the risks associated with the future cash-flows on the project's or firm's equity.

⁴Cragg (1994) labeled this phenomenon as the *contamination effect*.

3 Higher-moments estimation of EIV models

Following the short discussion on how endemic EIV is in factor models of asset returns in general and in CAPM specifically, we outline in this section the higher-moments estimation of the EIV models. Our discussion follow closely Dagenais and Dagenais (1997), Davidson and MacKinnon (2004), and Racicot and Theoret (2012).

Suppose the model is,

$$\mathbf{Y} = \boldsymbol{\alpha}\mathbf{1}_T + \mathbf{X}^*\boldsymbol{\beta} + \boldsymbol{\epsilon} \quad (4)$$

where \mathbf{X}^* is a $T \times K$ matrix of exogenous variables, $\mathbf{1}_T$ a $T \times 1$ vector of ones, and \mathbf{Y} a $T \times 1$ vector of dependent variables. The objective is to estimate the $K \times 1$ vectors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\epsilon}$. Assuming the matrix \mathbf{X}^* is not directly observable but only available as

$$\mathbf{X} = \mathbf{X}^* + \boldsymbol{\Delta}$$

where $\boldsymbol{\Delta}$ is a $T \times K$ matrix of measurement errors assumed to be uncorrelated with the regression errors $\boldsymbol{\epsilon}$, the model can be rewritten as,

$$\mathbf{Y} = \boldsymbol{\alpha}\mathbf{1}_T + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega} \quad (5)$$

where $\boldsymbol{\omega} = \boldsymbol{\epsilon} - \boldsymbol{\Delta}\boldsymbol{\beta}$. The direct consequence of the error in the right-hand-side variables is that the least-squares estimators $\hat{\boldsymbol{\beta}}^{LS}$ will be biased and inconsistent. To overcome this problem, Dagenais and Dagenais (1997) suggested an instrumental variables estimation approach based on sample moments of order higher than two. Specifically, the HM estimator of the model parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta}^\top)^\top$ is derived using,

$$E_{T \rightarrow \infty} \left[\frac{\mathbf{Z}^\top \boldsymbol{\omega}}{\sqrt{T}} \right] = 0 \quad (6)$$

where the matrix $\mathbf{Z} = [\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4, \mathbf{z}_5, \mathbf{z}_6, \mathbf{z}_7]$ has the columns,

- $\mathbf{z}_0 = \mathbf{1}_T$
- $\mathbf{z}_1 = \mathbf{x} \otimes \mathbf{x}$
- $\mathbf{z}_2 = \mathbf{x} \otimes \mathbf{y}$
- $\mathbf{z}_3 = \mathbf{y} \otimes \mathbf{y}$
- $\mathbf{z}_4 = \mathbf{x} \otimes \mathbf{x} \otimes \mathbf{x} - 3\mathbf{x} (E[(\mathbf{x}^\top \mathbf{x})/T] \otimes \mathbf{1}_K)$
- $\mathbf{z}_5 = \mathbf{x} \otimes \mathbf{x} \otimes \mathbf{y} - 2\mathbf{x} (E[(\mathbf{x}^\top \mathbf{y})/T] \otimes \mathbf{1}_K) - \mathbf{y} (\mathbf{1}_K^\top E[(\mathbf{x}^\top \mathbf{x})/T] \otimes \mathbf{1}_K)$
- $\mathbf{z}_6 = \mathbf{x} \otimes \mathbf{y} \otimes \mathbf{y} - \mathbf{x} (E[(\mathbf{y}^\top \mathbf{y})/T] \otimes \mathbf{1}_K) - 2\mathbf{y} (E[(\mathbf{y}^\top \mathbf{x})/T] \otimes \mathbf{1}_K)$
- $\mathbf{z}_7 = \mathbf{y} \otimes \mathbf{y} \otimes \mathbf{y} - 3\mathbf{y} (E[(\mathbf{y}^\top \mathbf{y})/T] \otimes \mathbf{1}_K)$

where small-case letters \mathbf{x} and \mathbf{y} denote the matrices \mathbf{X} and \mathbf{Y} in deviations from the mean form and the symbol \otimes is the element-by-element (i.e. Hadamard) matrix product. Following Davidson and MacKinnon (2004, p. 60-62), the HM estimator of $(\boldsymbol{\alpha}, \boldsymbol{\beta}^\top)^\top$ can be obtained by applying OLS to the following *augmented* model,

$$\mathbf{Y} = \boldsymbol{\alpha}\mathbf{1}_T + \mathbf{X}\boldsymbol{\beta} + \widehat{\boldsymbol{\omega}}\boldsymbol{\psi} + \boldsymbol{\epsilon} \quad (7)$$

where $\widehat{\boldsymbol{\omega}} = \mathbf{X} - \mathbf{P}_Z\mathbf{X}$ and $\mathbf{P}_Z = \mathbf{Z}(\mathbf{Z}^\top\mathbf{Z})^{-1}\mathbf{Z}^\top$ the orthogonal projection matrix onto the subspace spanned by the column of instrument set \mathbf{Z} .

Following the discussion in Carmichael and Coen (2008), we consider below a subset of \mathbf{Z} that includes \mathbf{z}_0 , \mathbf{z}_1 and \mathbf{z}_4 . As noted by the authors, the selection of instruments involves two conflicting objectives. While asymptotic efficiency advocates the use of all of the seven instruments listed above on one hand, increasing the number of instruments would induce finite-sample bias in the IV estimates on the other hand. In addition, Dagenais and Dagenais (1997) reported that the root mean square errors of parameter estimates are smaller with this subset of \mathbf{Z} . We therefore follow the advice of these studies and select \mathbf{z}_0 , \mathbf{z}_1 and \mathbf{z}_4 as the instruments for the explanatory variables of our regressions. Doing so, we first start by constructing estimates of the possibly mis-measured right-hand-side variables in equation (2) using artificial regressions where the risk factor premiums F_k are modeled as a linear response variable to our instrument set $\mathbf{Z} = (\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_4)$. The residuals from these regressions $\widehat{\boldsymbol{\omega}}$ are in the second step plugged back into the original model and the estimating model becomes,

$$R_{jt} - R_{ft} = \alpha_j + \sum_{k=1}^K \beta_{jk}F_{tk} + \sum_{k=1}^K \psi_{jk}\widehat{\omega}_{tk} + \epsilon_{it} \quad (8)$$

To paraphrase Carmichael and Coen (2008, p. 781), it is worth to emphasize that equation (8) is not an alternative to (2) because the variables $\widehat{\omega}_k$ are not additional risk factors, nor ψ_{jk} new risk factor loadings but estimates of the measurement errors on predictor variables.

4 Data and estimations

We consider a dataset of monthly excess returns on industry-level portfolios from March 2002 to February 2022 obtained from Kenneth French’s data library.⁵ Our selection includes 240 observations. The original list includes 48 industry portfolios but following the tradition in empirical studies of asset pricing and as advocated by, among others, Fama and French (1992, p. 429) or Foerster and Sapp (2005), we drop four financials, namely “Banks”, “Insurance”, “Real Estate” and “Financials” from our set of test

⁵https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

assets. Foerster and Sapp (2005) suggest that the exclusion of financial services does not influence the inferences from various asset pricing models. Fama and French (1992, p. 429) justify their choices by stating that financial firms exhibit high leverage, which is normal for these firms but probably does not have the same meaning as for non-financial firms, where high leverage more likely indicates distress. A complete list of the industry definitions with the SIC codes is given in the appendix.

Because higher-moments estimation of the EIV model requires non-normality in the data, a first step naturally consists in controlling for the statistical significance of the higher-moments of the data. This is given in Table 1. Aside from the usual descriptive values like the sample average or the standard deviation, we also provide the sample skewness and the sample (excess) kurtosis for each variable. The t -statistics of the estimates are given in the columns next to the estimates. Finally, to assess the extent of non-normality, we implement a Jarque-Bera test (Jarque and Bera, 1987) and give the p -value of the test statistic in the last column.⁶ Looking at the last column of the table, we notice that the null of normality is, as expected, rejected for nearly all of the series under consideration except three industries with p -values larger 10%. In addition, we also observe negative skewness to a large extent, albeit not always statistically significant. Therefore, Dagenais & Dagenais higher-moment estimation (DDHME) appears to be suitable for our purpose.

We fit time-series regressions of industry portfolios excess returns considering two common data generating process for expected returns, namely the CAPM and the Fama-French 3-factor model. Therefore, we consider expected excess returns as proportional to the market risk premium for CAPM regressions,

$$r_{jt} = \alpha_j + \beta_j r_{Mt} + \epsilon_{jt} \quad (9)$$

augmented by two additional systematic factors for the Fama-French 3-factor regressions as,

$$r_{jt} = \alpha_j + \beta_j r_{Mt} + s_j SMB_t + h_j HML_t + \epsilon_{jt} \quad (10)$$

where SMB and HML are Fama-French factors capturing the size premium and the value premium, respectively. Then, each model is fitted to the data first by OLS and then using the DDHME methods. The latter yields the following set of estimates:

$$r_{jt} = \alpha_j + \beta_j r_{Mt} + \psi^M \widehat{w}_{Mt} + \epsilon_{jt} \quad (11)$$

$$r_{jt} = \alpha_j + \beta_j r_{Mt} + s_j SMB_t + h_j HML_t + \psi_1^M \widehat{w}_{Mt} + \psi_2^H \widehat{w}_{Ht} + \psi_3^S \widehat{w}_{St} + \epsilon_{jt} \quad (12)$$

⁶Given a random sample of size T , the Jarque-Bera test statistic is calculated as $JB = \frac{\widehat{S}k^2}{6/T} + \frac{(\widehat{K}-3)^2}{24/T}$ and is distributed as a χ^2 random variable at 2 degrees of freedom.

5 Results

Tables 2 to 4 present the output. We report the coefficient estimates and the t -statistics next to each estimate for our test assets. The t -statistics are obtained using White’s heteroskedasticity and autocorrelation consistent covariance matrix estimates (White, 1980). The column $F(\psi = 0)$ in table 2 shows the value of the squared t -statistic of the fitted ψ coefficient in equation (11). The column DWH in tables 3 and 4 shows the Durbin-Wu-Hausman test to check the null that there is no EIV in the data. For the three-variable case, DWH statistic follows a χ^2 distribution at 3 degrees of freedom.⁷

We turn to the discussion of the results. Overall, these are quite different whether we use OLS or the higher-moment estimation. According to table 2, for 16 industries the market β is estimated more than 20% lower with OLS relative to HM. For the one-factor model, the fitted risk factor loading using OLS are lower than those obtained from DDHME for 30 industries out of 44 as suggested in table 2. The average $\hat{\beta}^{OLS}$ is 1.05 and the average $\hat{\beta}^{HM}$ is 1.18. For some portfolios, the difference is substantial. Although the downward bias in market betas appears to attenuate with the 3-factor model, we still observe that the $\hat{\beta}$ ’s are lower than their DDHME counterparts in 28 cases. This in turn spills over to the estimates of cost of equity. To highlight the extent to which a *conservative* estimate in the market β can distort the output, we consider the difference between the (annualized) CAPM-based industry cost of equity estimates between the OLS vs. DDHME methods. Table 5 shows the calculation results where the numbers are given in annual terms. In addition, we also visualize the same output in figures 1 and 2.

For ease of exposition, we divide the forty-four industries into two groups. In this figure each bar shows the difference between CAPM-based estimate of the industry cost of equity obtained using OLS vs. DDHME estimations. A bar above the zero line indicates an underestimation of the cost equity, which we calculate as the sum of average risk-free rate of return over the entire period plus the fitted market beta times the average systematic risk premium, $k_j = \bar{R}_f + \hat{\beta}_j (\bar{R}_M - \bar{R}_f)$. A quick look to the figure suggest that OLS estimates of the equity cost of capital that does not take into account potential measurement error in the systematic risk factor can be seriously flawed. The difference between OLS vs. HM estimations can diverge by up to nearly 6% on an annual basis. From this perspective, it is clear that overlooking EIV as a minor issue in formulating expected excess returns on industry portfolios would have important consequences for valuation of projects based on discounted cash-flow analysis.

Finally, we should also put our results into perspective in light of the general lack of statistical evidence suggested by the EIV tests. Specifically, in tables 2 to 4, the Durbin-Wu-Hausman specification test does not systematically point out to the pres-

⁷For simplicity, we do not report other regression statistics like the R^2 . The full set of results as spreadsheet or R.data files are available upon request.

ence of measurement error. In table 2, the F -test of no EIV is rejected for only one fourth of the test assets. The evidence against the null of no EIV is not heavily emphasized neither in the three factor model where nearly half of the cases suggest that the data exhibit EIV while the remaining half the opposite given the DWH statistics.⁸ Therefore, although the final output of interest derived from our factor models differ depending on whether one takes into account the EIV or not, the estimation results remain inconclusive regarding the presence of EIV in the model variables. This casts doubt on our choice of instruments $\mathbf{Z} = (\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_4)$ as it is a well-known fact that poor instruments tend to do more harm than good on the statistical properties of the estimates.

6 Conclusion

Despite the flourishing literature on alternative data generating processes of expected returns, the CAPM still remains at the center of most practical work in corporate financial decision-making. The model sits on sound theoretical grounds, but it is long known that empirical implementation of CAPM comes with several conceptual and empirical difficulties. Errors-in-variables (EIV) is one such issue endemic in the implementation of CAPM. In this paper, using a sample of monthly returns on Fama-French industry portfolios over a twenty-years long period, we have presented empirical evidence that EIV can lead to conservative estimates of the cost of equity capital. We compare two sets of estimations, one that overlooks the EIV, i.e. the OLS, and another one based on Dagenais and Dagenais (1997) higher-moments method (DDHME). We find that OLS time-series regressions of CAPM-based formula to estimate the cost of equity capital is in most cases likely to give misleading discount rates for project evaluation. Running the estimations using DDHME pulls the results upward numerically. Switching from CAPM to Fama-French 3-factor model seems to attenuate the difference in the outputs from OLS vs. DDHME, but the differences in the annualized cost of equity capital estimates remain non-neutral from the perspective of corporate financial decision-making.

Our empirical exercise is simple and can certainly be extended in several grounds. One key limitation to our work is that it is not possible to claim that one should definitely drop OLS in favor of DDHME due to the presence of EIV in the data. It is simply impossible to know with certainty the CAPM, or any other factor model presentation, is the best description of the population model for the data under consideration. A second path that needs further investigation is about the selection of higher-moments of the data used as instruments. In line with the recommendations of previous studies like Carmichael and Coen (2008) or Racicot and Theoret (2012), we have selected a subset from the full list proposed by Dagenais and Dagenais (1997) as our instruments

⁸With the 3-factor model, we can compare the DWH statistic to the 90th quantile of a χ^2 random variable at 3 degrees of freedom. This is equal to 6.25.

to obtain a filtered version of the error-prone variables. As long as the selection of instruments is a crucial step in instrumental variables estimations, we believe additional efforts must be done to study other possible sets of higher-moments or cross-moments of the data as possible instruments to correct the EIV bias. In fact, if it turns out that instruments involving higher cross-moments of the data, i.e. \mathbf{z}_5 or \mathbf{z}_6 , are not weakly correlated with the instrumented risk premiums, it may also be interesting to check the results by combining with these additional instruments. We consider this question as a departure point for a future simulation-based and/or empirical study in which it would be interesting to study the behavior of DDHME method with other approaches like Satman and Diyarbakirlioglu (2015), Andersson and Moen (2016) or Racicot et al. (2019) recently developed in the literature addressing the EIV.

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Table 1: Descriptive Statistics

The table shows the descriptive statistics for the industry portfolio monthly excess returns from 2002-03 to 2022-02 (240 observations). t -statistics for sample skewness and sample (excess) kurtosis are calculate using the asymptotic variance of the sample estimates. JB stands for the Jarque-Bera χ^2 statistic (Jarque and Bera, 1987) to test the null that the data is normal. The p-value of the JB test is given in the last column.

	avg	st dev	min	max	skew	t-stat	kurt	t-stat	J-B	p-value
<i>Industry portfolios</i>										
Agric	0.0094	0.0640	-0.1820	0.2097	0.4184	2.65	0.8533	2.70	14.93	0.0006
Food	0.0065	0.0363	-0.1111	0.0966	-0.4223	-2.67	0.8024	2.54	14.19	0.0008
Soda	0.0098	0.0580	-0.2246	0.2848	-0.2817	-1.78	3.5991	11.38	136.76	0.0000
Beer	0.0076	0.0388	-0.1476	0.1100	-0.3482	-2.20	1.0898	3.45	17.55	0.0002
Smoke	0.0111	0.0622	-0.2218	0.3238	-0.2188	-1.38	3.4457	10.90	124.42	0.0000
Toys	0.0065	0.0709	-0.2340	0.2277	-0.1283	-0.81	0.7928	2.51	7.47	0.0239
Fun	0.0105	0.0804	-0.3160	0.3930	-0.1796	-1.14	2.9206	9.24	89.53	0.0000
Books	0.0034	0.0628	-0.2527	0.3073	0.2681	1.70	3.8611	12.21	156.47	0.0000
Hshld	0.0068	0.0378	-0.1375	0.1141	-0.3767	-2.38	1.0600	3.35	17.72	0.0001
Clths	0.0109	0.0602	-0.2169	0.2128	-0.1389	-0.88	1.2003	3.80	16.05	0.0003
Hlth	0.0068	0.0584	-0.1942	0.1655	-0.3240	-2.05	0.9903	3.13	14.73	0.0006
MedEq	0.0095	0.0480	-0.1924	0.1392	-0.8252	-5.22	2.1771	6.88	76.89	0.0000
Drugs	0.0066	0.0414	-0.1110	0.1314	-0.2251	-1.42	0.0800	0.25	2.16	0.3388
Chems	0.0091	0.0609	-0.2106	0.1905	-0.2895	-1.83	1.6555	5.24	32.11	0.0000
Rubbr	0.0099	0.0604	-0.2084	0.3194	0.0298	0.19	3.5380	11.19	129.12	0.0000
Txtls	0.0079	0.0941	-0.3608	0.5892	0.5294	3.35	7.8022	24.67	634.32	0.0000
BldMt	0.0094	0.0688	-0.3189	0.3440	-0.3368	-2.13	4.7766	15.10	239.03	0.0000
Cnstr	0.0088	0.0734	-0.3214	0.2186	-0.5022	-3.18	1.7699	5.60	42.97	0.0000
Steel	0.0085	0.0913	-0.3299	0.2624	-0.2805	-1.77	0.9064	2.87	12.01	0.0025
FabPr	0.0088	0.0854	-0.3262	0.3037	-0.2262	-1.43	1.1452	3.62	16.00	0.0003
Mach	0.0109	0.0679	-0.2983	0.2302	-0.4685	-2.96	1.9904	6.29	50.19	0.0000
ElcEq	0.0089	0.0653	-0.2478	0.2287	-0.3470	-2.19	1.3935	4.41	25.34	0.0000
Autos	0.0116	0.0949	-0.3650	0.4956	1.1401	7.21	6.1016	19.30	434.32	0.0000
Aero	0.0100	0.0652	-0.3599	0.3250	-0.5626	-3.56	5.7191	18.09	348.32	0.0000
Ships	0.0125	0.0780	-0.2261	0.2915	-0.1523	-0.96	1.8938	5.99	38.38	0.0000
Gums	0.0111	0.0561	-0.2177	0.1396	-0.6546	-4.14	1.4748	4.66	40.23	0.0000
Gold	0.0086	0.1035	-0.3361	0.3390	0.3485	2.20	0.8709	2.75	13.08	0.0014
Mines	0.0135	0.0899	-0.3483	0.2695	-0.4217	-2.67	1.0823	3.42	19.67	0.0001
Coal	0.0070	0.1331	-0.4084	0.3630	-0.1987	-1.26	0.4963	1.57	4.36	0.1129
Oil	0.0080	0.0720	-0.3480	0.3292	0.0416	0.26	4.2573	13.46	186.54	0.0000
Util	0.0072	0.0399	-0.1313	0.1035	-0.8387	-5.30	1.3856	4.38	48.73	0.0000
Telcm	0.0058	0.0493	-0.1630	0.2120	-0.2342	-1.48	1.8342	5.80	37.37	0.0000
PerSv	0.0047	0.0587	-0.2621	0.1859	-0.4465	-2.82	2.1852	6.91	57.75	0.0000
BusSv	0.0083	0.0519	-0.2403	0.1816	-0.6285	-3.98	2.6841	8.49	90.63	0.0000
Hardw	0.0098	0.0656	-0.1956	0.2075	-0.1618	-1.02	0.4936	1.56	3.80	0.1499
Softw	0.0101	0.0551	-0.1776	0.2383	-0.1815	-1.15	1.4373	4.55	23.08	0.0000
Chips	0.0114	0.0697	-0.2455	0.2274	-0.3605	-2.28	0.8172	2.58	12.48	0.0020
LabEq	0.0110	0.0575	-0.2313	0.1618	-0.4600	-2.91	0.9761	3.09	18.76	0.0001
Paper	0.0056	0.0505	-0.1831	0.2310	-0.0916	-0.58	2.0217	6.39	42.93	0.0000
Boxes	0.0099	0.0544	-0.1964	0.1277	-0.5366	-3.39	1.0354	3.27	23.10	0.0000
Trans	0.0088	0.0545	-0.1662	0.1709	-0.2629	-1.66	0.7519	2.38	8.94	0.0115
Whsl	0.0079	0.0496	-0.2109	0.1593	-0.5980	-3.78	2.1496	6.80	62.57	0.0000
Rtail	0.0087	0.0465	-0.1464	0.1864	-0.0613	-0.39	1.0474	3.31	11.85	0.0027
Meals	0.0105	0.0473	-0.2246	0.1864	-0.5513	-3.49	2.7418	8.67	90.15	0.0000
<i>Risk factors</i>										
MktRF	0.0077	0.0437	-0.1723	0.1365	-0.5563	-3.52	1.4273	4.51	33.98	0.0000
SMB	0.0017	0.0263	-0.0832	0.0706	0.1444	0.91	0.0904	0.29	0.98	0.6127
HML	-0.0002	0.0293	-0.1402	0.1279	-0.0537	-0.34	3.8323	12.12	151.40	0.0000

Table 2: CAPM time-series regressions

The table shows OLS vs. DDHME estimation results from CAPM time-series regressions of industry portfolios excess returns on market risk premium $R_{jt} - R_{ft} = \alpha_j + \beta_j (R_{Mt} - R_{ft}) + \epsilon_{jt}$. $R_M - R_f$ is the market risk premium. Monthly data cover the period from 2002-03 to 2022-02. t -statistics based on White (1980) HCCME standard errors are shown next to coefficient estimates.

	OLS				DDHME					
	$\hat{\alpha}$	t -stat	$\hat{\beta}$	t -stat	$\hat{\alpha}$	t -stat	$\hat{\beta}$	t -stat	$\hat{\psi}$	$F(\psi = 0)$
Agric	0.0032	0.93	0.7998	9.13	0.0041	1.14	0.6779	5.83	0.1397	0.83
Food	0.0022	1.21	0.5509	12.65	0.0017	0.90	0.6125	6.77	-0.0706	0.61
Soda	0.0039	1.24	0.7716	8.62	0.0008	0.26	1.1690	9.93	-0.4555	9.30
Beer	0.0034	1.69	0.5365	9.70	0.0014	0.70	0.7965	11.67	-0.2981	12.09
Smoke	0.0062	1.68	0.6340	6.75	0.0064	1.54	0.6160	2.89	0.0206	0.01
Toys	-0.0026	-0.82	1.1781	17.25	-0.0040	-1.31	1.3593	15.04	-0.2077	2.72
Fun	-0.0009	-0.29	1.4750	15.53	-0.0018	-0.48	1.5839	8.16	-0.1248	0.34
Books	-0.0055	-2.35	1.1524	15.15	-0.0068	-2.49	1.3259	10.04	-0.1989	1.64
Hshld	0.0021	1.19	0.6113	14.06	0.0011	0.59	0.7422	9.78	-0.1500	3.32
Clths	0.0026	1.07	1.0777	17.12	0.0019	0.74	1.1640	9.90	-0.0989	0.55
Hlth	0.0000	-0.01	0.8853	10.66	-0.0023	-0.72	1.1809	8.27	-0.3389	3.83
MedEq	0.0028	1.37	0.8638	15.54	0.0014	0.63	1.0500	11.55	-0.2135	5.69
Drugs	0.0013	0.70	0.6855	14.02	0.0026	1.32	0.5176	5.37	0.1924	3.16
Chem	-0.0003	-0.17	1.2149	23.13	-0.0003	-0.17	1.2171	16.16	-0.0026	0.00
Rubbr	0.0011	0.51	1.1356	15.75	0.0012	0.45	1.1296	9.09	0.0069	0.00
Txtls	-0.0041	-1.00	1.5527	10.54	-0.0065	-1.21	1.8709	6.24	-0.3648	1.16
BldMt	-0.0011	-0.46	1.3555	16.73	-0.0039	-1.54	1.7207	15.71	-0.4185	8.88
Cnstr	-0.0014	-0.47	1.3292	16.51	-0.0033	-0.95	1.5769	7.26	-0.2839	1.69
Steel	-0.0041	-1.11	1.6373	20.14	-0.0060	-1.68	1.8907	15.79	-0.2904	3.90
FabPr	-0.0011	-0.25	1.2712	11.69	-0.0037	-0.78	1.6172	5.92	-0.3965	1.82
Mach	0.0004	0.20	1.3583	22.21	-0.0009	-0.36	1.5277	11.51	-0.1942	1.87
ElcEq	-0.0012	-0.61	1.3132	25.95	-0.0019	-0.90	1.3958	17.71	-0.0947	1.00
Autos	-0.0015	-0.43	1.6906	13.19	-0.0011	-0.23	1.6453	5.60	0.0519	0.02
Aero	0.0012	0.42	1.1496	11.40	-0.0007	-0.14	1.3932	3.11	-0.2792	0.40
Ships	0.0020	0.61	1.3562	13.24	0.0017	0.44	1.3972	7.11	-0.0470	0.05
Guns	0.0060	1.81	0.6599	6.87	0.0017	0.50	1.2095	7.90	-0.6299	15.10
Gold	0.0056	0.82	0.3894	2.10	0.0008	0.10	1.0187	1.84	-0.7212	1.57
Mines	0.0028	0.62	1.3941	12.67	0.0005	0.12	1.6799	6.88	-0.3276	1.60
Coal	-0.0039	-0.49	1.4187	7.49	-0.0063	-0.75	1.7207	4.24	-0.3461	0.68
Oil	-0.0002	-0.05	1.0576	8.89	-0.0026	-0.51	1.3690	3.42	-0.3569	0.80
Util	0.0031	1.41	0.5288	9.08	0.0004	0.13	0.8871	4.91	-0.4107	5.06
Telcm	-0.0015	-0.84	0.9447	19.72	-0.0013	-0.69	0.9209	16.36	0.0273	0.12
PerSv	-0.0025	-0.85	0.9299	10.40	-0.0012	-0.30	0.7696	2.42	0.1837	0.33
BusSv	-0.0003	-0.23	1.1105	30.41	-0.0016	-1.20	1.2824	17.20	-0.1971	6.41
Hardw	0.0003	0.13	1.2197	17.21	0.0025	0.89	0.9434	6.68	0.3168	4.21
Softw	0.0017	0.91	1.0915	21.60	0.0033	1.49	0.8881	6.48	0.2332	2.67
Chips	0.0012	0.46	1.3105	18.79	0.0039	1.38	0.9695	7.21	0.3908	7.82
LabEq	0.0023	1.19	1.1316	21.72	0.0024	0.98	1.1139	6.18	0.0203	0.01
Paper	-0.0019	-1.06	0.9627	19.64	-0.0018	-0.91	0.9565	10.15	0.0071	0.00
Boxes	0.0022	1.01	0.9877	17.11	0.0015	0.64	1.0790	9.46	-0.1047	0.82
Trans	0.0008	0.38	1.0356	19.11	0.0021	0.93	0.8667	7.00	0.1936	2.32
Whlsl	0.0002	0.10	1.0096	24.58	-0.0012	-0.74	1.1794	16.94	-0.1946	6.42
Rtail	0.0020	1.12	0.8791	18.01	0.0032	1.51	0.7129	4.91	0.1905	1.73
Meals	0.0039	2.02	0.8576	15.55	0.0026	0.89	1.0354	4.31	-0.2038	0.73

Table 3: Fama-French 3-factor time-series regressions - I

The table shows OLS vs. DDHME estimation results from Fama-French 3-factor model time-series regressions of industry portfolios excess returns on market risk premium $R_{jt} - R_{ft} = \alpha_j + \beta_j (R_{Mt} - R_{ft}) + s_j SMB_t + h_j HML_t + \epsilon_{jt}$. $R_M - R_f$ is the market risk premium. SMB is the size premium. HML is the value premium. Monthly data cover the period from 2002-03 to 2022-02. t -statistics based on White (1980) HCCME standard errors are shown next to coefficient estimates.

Industry	OLS				DDHME							DWH
	$\hat{\alpha}$	$\hat{\beta}$	\hat{s}	\hat{h}	$\hat{\alpha}$	\hat{b}	\hat{s}	\hat{h}	$\hat{\psi}_1$	$\hat{\psi}_2$	$\hat{\psi}_3$	
Agric	0.0032 (0.94)	0.7579 (7.95)	0.1959 (1.35)	-0.0015 (-0.01)	0.0035 (1.04)	0.8389 (5.38)	-0.3875 (-0.6)	0.0688 (0.31)	-0.0449	0.6204	-0.0625	1.48
Food	0.0024 (1.34)	0.5790 (13.11)	-0.2111 (-2.83)	0.1557 (2.09)	0.0022 (1.25)	0.6983 (6.99)	-0.6733 (-1.79)	0.1069 (0.86)	-0.1193	0.4828	0.0997	3.22
Soda	0.0042 (1.33)	0.7960 (9.38)	-0.2366 (-1.6)	0.2388 (2.22)	0.0033 (1.06)	0.7908 (3.31)	0.3148 (0.36)	0.2665 (2.34)	-0.0892	-0.5882	-0.0909	5.74
Beer	0.0035 (1.75)	0.6085 (9)	-0.3569 (-3.5)	0.0415 (0.54)	0.0024 (1.25)	0.7726 (6.32)	-0.4697 (-0.93)	0.1003 (1.03)	-0.2671	0.1099	-0.0987	21.41
Smoke	0.0066 (1.89)	0.7184 (7.51)	-0.5603 (-3.26)	0.3244 (2.9)	0.0074 (2.08)	0.5613 (3.1)	-0.3105 (-0.45)	0.3289 (1.61)	0.2251	-0.2544	-0.0092	2.31
Toys	-0.0028 (-0.91)	1.0746 (16.03)	0.5657 (4.39)	-0.1627 (-1.35)	-0.0041 (-1.37)	1.0344 (9.43)	1.5026 (3.17)	-0.3600 (-3.08)	-0.0732	-1.0071	0.2152	13.39
Fun	-0.0011 (-0.35)	1.3925 (15.14)	0.4491 (2.8)	-0.1259 (-1.02)	-0.0019 (-0.62)	1.4021 (6.21)	0.8647 (0.88)	-0.4191 (-2.6)	-0.0589	-0.4574	0.3996	4.84
Books	-0.0052 (-2.41)	1.0273 (16.84)	0.4283 (3.85)	0.2996 (4.04)	-0.0058 (-2.68)	0.9418 (5.86)	1.2110 (1.87)	0.1998 (1.81)	0.0209	-0.8344	0.0892	4.81
Hshld	0.0022 (1.26)	0.6499 (13.43)	-0.2259 (-2.87)	0.0900 (1.21)	0.0015 (0.89)	0.7216 (7.82)	-0.1750 (-0.49)	-0.0357 (-0.39)	-0.1189	-0.0654	0.1742	5.72
Clths	0.0026 (1.08)	1.0072 (15.89)	0.3432 (3.39)	-0.0289 (-0.32)	0.0017 (0.73)	1.1094 (8.47)	0.3653 (0.73)	-0.1046 (-0.93)	-0.1711	-0.0350	0.0982	4.56
Hlth	0.0000 (-0.02)	0.7562 (8.65)	0.5916 (4.28)	0.0182 (0.19)	-0.0013 (-0.51)	0.7568 (6.36)	1.2823 (3.03)	-0.3160 (-2.69)	-0.0868	-0.7527	0.4371	18.99
MedEq	0.0025 (1.3)	0.8439 (14.99)	0.2120 (2.52)	-0.2322 (-2.84)	0.0018 (0.97)	1.0484 (13.29)	-0.3170 (-1.07)	-0.2742 (-2.72)	-0.2542	0.5476	0.0847	10.34
Drugs	0.0010 (0.59)	0.7468 (16.93)	-0.1490 (-1.8)	-0.2642 (-4.57)	0.0015 (0.86)	0.7635 (7.87)	-0.4778 (-1.2)	-0.2272 (-3.72)	0.0244	0.3519	-0.0276	3.93
Chems	-0.0002 (-0.09)	1.1748 (20.91)	0.1092 (1.37)	0.1500 (1.94)	-0.0006 (-0.31)	1.3026 (14.25)	-0.2343 (-0.67)	0.0702 (0.99)	-0.1513	0.3538	0.1343	5.56
Rubbr	0.0013 (0.59)	1.0571 (16.53)	0.2970 (2.6)	0.1324 (1.32)	0.0010 (0.51)	1.0573 (7.93)	0.3851 (0.53)	-0.1412 (-1.07)	0.0119	-0.1056	0.4013	7.12
Txtls	-0.0035 (-0.98)	1.3142 (12.15)	0.8391 (4.31)	0.5253 (3.44)	-0.0047 (-1.27)	1.2325 (4.43)	1.8745 (1.46)	0.4128 (1.87)	-0.0278	-1.1055	0.0847	6.31
BldMt	-0.0008 (-0.38)	1.2216 (18.73)	0.4941 (5.2)	0.2505 (3.09)	-0.0020 (-1.03)	1.2905 (8.24)	0.8886 (1.35)	0.1526 (1.15)	-0.1761	-0.4316	0.1019	6.76
Cnstr	-0.0013 (-0.47)	1.1754 (16.67)	0.6437 (5.23)	0.1404 (1.21)	-0.0022 (-0.81)	1.3461 (11.48)	0.4152 (0.73)	0.1522 (0.61)	-0.2537	0.2311	-0.0187	5.17
Steel	-0.0037 (-1.02)	1.4737 (15.52)	0.5589 (3.75)	0.3938 (2.58)	-0.0036 (-0.98)	1.4408 (6.46)	0.6616 (0.92)	0.2875 (1.29)	0.0492	-0.1116	0.1539	0.51
FabPr	-0.0010 (-0.27)	1.0111 (10.04)	1.1663 (6.75)	0.0864 (0.62)	-0.0020 (-0.55)	1.0414 (6.35)	1.6130 (2.44)	-0.1494 (-0.6)	-0.1054	-0.4899	0.3083	3.85
Mach	0.0005 (0.24)	1.2559 (20.14)	0.4448 (5.95)	0.0616 (0.79)	0.0003 (0.12)	1.2880 (7.58)	0.4420 (1.23)	0.0898 (0.64)	-0.0571	0.0016	-0.0460	0.26
ElcEq	-0.0013 (-0.71)	1.2121 (26.52)	0.5101 (6.87)	-0.0766 (-1.18)	-0.0018 (-0.98)	1.1611 (14.92)	0.9965 (3.23)	-0.1534 (-1.92)	0.0109	-0.5194	0.0773	4.73

Table 4: Fama-French 3-factor time-series regressions - II

The table shows OLS vs. DDHME estimation results from Fama-French 3-factor model time-series regressions of industry portfolios excess returns on market risk premium $R_{jt} - R_{ft} = \alpha_j + \beta_j (R_{Mt} - R_{ft}) + s_j SMB_t + h_j HML_t + \epsilon_{jt}$. $R_M - R_f$ is the market risk premium. SMB is the size premium. HML is the value premium. Monthly data cover the period from 2002-03 to 2022-02. t -statistics based on White (1980) HCCME standard errors are shown next to coefficient estimates.

Industry	OLS				DDHME							DWH
	$\hat{\alpha}$	$\hat{\beta}$	\hat{s}	\hat{h}	$\hat{\alpha}$	\hat{b}	\hat{s}	\hat{h}	$\hat{\psi}_1$	$\hat{\psi}_2$	$\hat{\psi}_3$	
Auto	-0.0016 (-0.45)	1.5919 (11.77)	0.4832 (2.42)	-0.0460 (-0.33)	-0.0034 (-0.94)	1.4581 (7.3)	2.1167 (2.01)	-0.3845 (-1.11)	-0.0194	-1.7505	0.3751	13.48
Aero	0.0016 (0.61)	1.0901 (12.85)	0.0797 (0.62)	0.3819 (2.95)	0.0017 (0.62)	0.9699 (4.61)	0.6161 (0.76)	0.7035 (2.94)	0.0808	-0.5508	-0.5145	6.13
Ships	0.0024 (0.8)	1.1866 (12.05)	0.5967 (4.2)	0.3742 (3.27)	0.0028 (0.92)	1.1050 (6.53)	0.7656 (1.3)	0.2522 (1.53)	0.1226	-0.1792	0.1776	2.02
Guns	0.0063 (1.93)	0.6428 (5.98)	-0.0622 (-0.38)	0.2752 (2.39)	0.0051 (1.56)	0.8224 (4.46)	-0.1602 (-0.25)	0.3373 (1.46)	-0.2966	0.0929	-0.1071	4.39
Gold	0.0057 (0.83)	0.3925 (2.01)	-0.0325 (-0.12)	0.0349 (0.16)	0.0012 (0.18)	1.4624 (4.92)	-2.2973 (-2.21)	-0.1797 (-0.67)	-1.4175	2.3276	0.3899	19.10
Mines	0.0030 (0.67)	1.2906 (10.58)	0.3806 (2.21)	0.1967 (1.3)	0.0020 (0.44)	1.5393 (6.51)	-0.1710 (-0.25)	0.1596 (0.94)	-0.3266	0.5685	0.0738	1.49
Coal	-0.0035 (-0.45)	1.1830 (5.72)	0.8686 (2.62)	0.4442 (1.46)	-0.0040 (-0.53)	1.3934 (5.38)	0.2613 (0.23)	0.5886 (1.49)	-0.2696	0.6386	-0.1878	0.92
Oil	0.0005 (0.14)	0.9298 (8.65)	0.2691 (1.72)	0.6314 (3.17)	-0.0004 (-0.1)	1.1462 (3.64)	-0.1519 (-0.17)	1.0677 (2.96)	-0.3412	0.4510	-0.6422	5.87
Util	0.0032 (1.46)	0.5550 (9.29)	-0.1617 (-1.64)	0.0766 (0.84)	0.0026 (1.17)	0.6340 (6.04)	-0.1330 (-0.32)	0.1088 (0.85)	-0.1432	-0.0357	-0.0609	3.97
Telcm	-0.0014 (-0.82)	0.9844 (16.89)	-0.2129 (-2.22)	0.0540 (0.92)	-0.0011 (-0.67)	0.8991 (9.85)	0.0270 (0.08)	0.1322 (1.92)	0.0967	-0.2465	-0.1278	3.84
PerSv	-0.0024 (-0.9)	0.8236 (10.58)	0.4611 (3.93)	0.0659 (0.59)	-0.0016 (-0.55)	0.8515 (4.38)	-0.1478 (-0.21)	0.1848 (0.55)	0.0455	0.6539	-0.1261	2.26
BusSv	-0.0005 (-0.42)	1.0819 (28.46)	0.2300 (4.2)	-0.1876 (-3.99)	-0.0012 (-1.08)	1.2557 (24.11)	-0.1429 (-0.66)	-0.0196 (-0.3)	-0.2499	0.3919	-0.2396	45.90
Hardw	0.0001 (0.04)	1.2268 (17.21)	0.0753 (0.79)	-0.2102 (-2.21)	0.0013 (0.52)	1.0325 (8.05)	0.2711 (0.68)	-0.2741 (-2.45)	0.3051	-0.1965	0.1027	6.69
Softw	0.0012 (0.76)	1.1663 (22.51)	-0.1211 (-1.41)	-0.4400 (-7.22)	0.0018 (1.19)	1.1678 (13.64)	-0.4741 (-1.43)	-0.2328 (-4.51)	0.0376	0.3859	-0.2781	12.62
Chips	0.0007 (0.29)	1.3452 (19.48)	0.0721 (0.64)	-0.4531 (-4.75)	0.0025 (1.04)	1.1741 (11.78)	-0.1559 (-0.48)	-0.1932 (-1.64)	0.3060	0.2684	-0.3462	17.89
LabEq	0.0018 (1.1)	1.0977 (23.65)	0.3737 (4.73)	-0.4188 (-7.05)	0.0020 (1.15)	1.2691 (10.85)	-0.4803 (-1.29)	-0.3631 (-5.75)	-0.1513	0.9020	-0.0277	8.51
Paper	-0.0017 (-0.98)	0.9290 (19.35)	0.0822 (0.9)	0.1450 (2.12)	-0.0016 (-0.88)	0.9181 (7.56)	0.0483 (0.09)	0.1068 (1)	0.0283	0.0355	0.0616	0.45
Boxes	0.0023 (1.04)	0.9520 (15.02)	0.1348 (1.34)	0.0609 (0.6)	0.0027 (1.17)	1.0864 (8.21)	-0.6649 (-1.48)	0.2225 (1.11)	-0.1081	0.8512	-0.1863	4.16
Trans	0.0010 (0.5)	0.9681 (17.7)	0.2178 (2.45)	0.1871 (2.59)	0.0017 (0.87)	0.8157 (7.3)	0.5122 (1.27)	0.2045 (1.8)	0.2081	-0.3019	-0.0332	3.86
Whlsl	0.0002 (0.18)	0.9271 (26.88)	0.3389 (5.57)	0.0881 (1.3)	-0.0001 (-0.07)	1.0516 (13.54)	-0.0242 (-0.08)	0.1542 (1.84)	-0.1569	0.3811	-0.0817	5.17
Rtail	0.0018 (1.04)	0.8930 (18.8)	0.0293 (0.41)	-0.1826 (-2.7)	0.0017 (1.01)	0.9689 (11.66)	-0.2834 (-1.05)	-0.2740 (-3.5)	-0.0666	0.3246	0.1553	4.93
Meals	0.0040 (2.08)	0.8262 (15.07)	0.1256 (1.39)	0.0405 (0.46)	0.0034 (1.69)	0.7774 (5.02)	0.6842 (1.53)	0.1187 (0.61)	-0.0210	-0.5903	-0.1616	6.27

Table 5: Cost of equity capital estimates

The table shows the industry equity cost of capital estimates from CAPM- and Fama-French 3-factor models using OLS vs. DDHME methods. The (annualized) estimations are calculated as $k_j = \bar{R}_f + \sum_{k=1}^K \widehat{\beta}_{jk} \bar{F}_k$ where a bar over the parameter indicates the time-series average.

Industry	CAPM		F-F 3-factor	
	OLS	DDHME	OLS	DDHME
Agric	0.086	0.074	0.086	0.081
Food	0.063	0.068	0.060	0.062
Soda	0.083	0.120	0.080	0.090
Beer	0.061	0.085	0.061	0.073
Smoke	0.070	0.069	0.066	0.056
Toys	0.121	0.137	0.123	0.139
Fun	0.148	0.158	0.150	0.160
Books	0.118	0.134	0.115	0.123
Hshld	0.068	0.080	0.067	0.075
Clths	0.111	0.119	0.112	0.122
Hlth	0.094	0.121	0.094	0.109
MedEq	0.092	0.109	0.095	0.103
Drugs	0.075	0.059	0.078	0.073
Chems	0.124	0.124	0.122	0.127
Rubbr	0.117	0.116	0.115	0.118
Txtls	0.155	0.185	0.149	0.163
BldMt	0.137	0.171	0.134	0.149
Cnstr	0.135	0.158	0.133	0.144
Steel	0.163	0.187	0.158	0.158
FabPr	0.129	0.161	0.129	0.141
Mach	0.137	0.153	0.137	0.140
ElcEq	0.133	0.141	0.134	0.140
Autos	0.168	0.164	0.169	0.191
Aero	0.118	0.141	0.113	0.112
Ships	0.137	0.141	0.132	0.129
Guns	0.073	0.124	0.069	0.083
Gold	0.048	0.106	0.047	0.101
Mines	0.141	0.167	0.138	0.150
Coal	0.143	0.171	0.137	0.144
Oil	0.109	0.138	0.101	0.112
Util	0.061	0.094	0.059	0.067
Telcm	0.099	0.097	0.098	0.095
PerSv	0.098	0.083	0.097	0.087
BusSv	0.114	0.130	0.117	0.125
Hardw	0.125	0.099	0.127	0.113
Softw	0.113	0.094	0.118	0.111
Chips	0.133	0.101	0.139	0.118
LabEq	0.116	0.115	0.122	0.120
Paper	0.101	0.100	0.099	0.097
Boxes	0.103	0.111	0.102	0.098
Trans	0.107	0.092	0.105	0.097
Whsl	0.105	0.121	0.104	0.108
Rtail	0.093	0.078	0.095	0.096
Meals	0.091	0.107	0.090	0.097

Figure 1: Differences in CAPM-based cost of equity estimates

The numbers shows the difference between CAPM-based estimate of the industry cost of equity obtained using OLS vs. DDHME methods as $k_j = \bar{R}_f + \hat{\beta}_j (\bar{R}_M - \bar{R}_f)$. A bar above the zero line indicates an underestimation of the cost of equity capital.

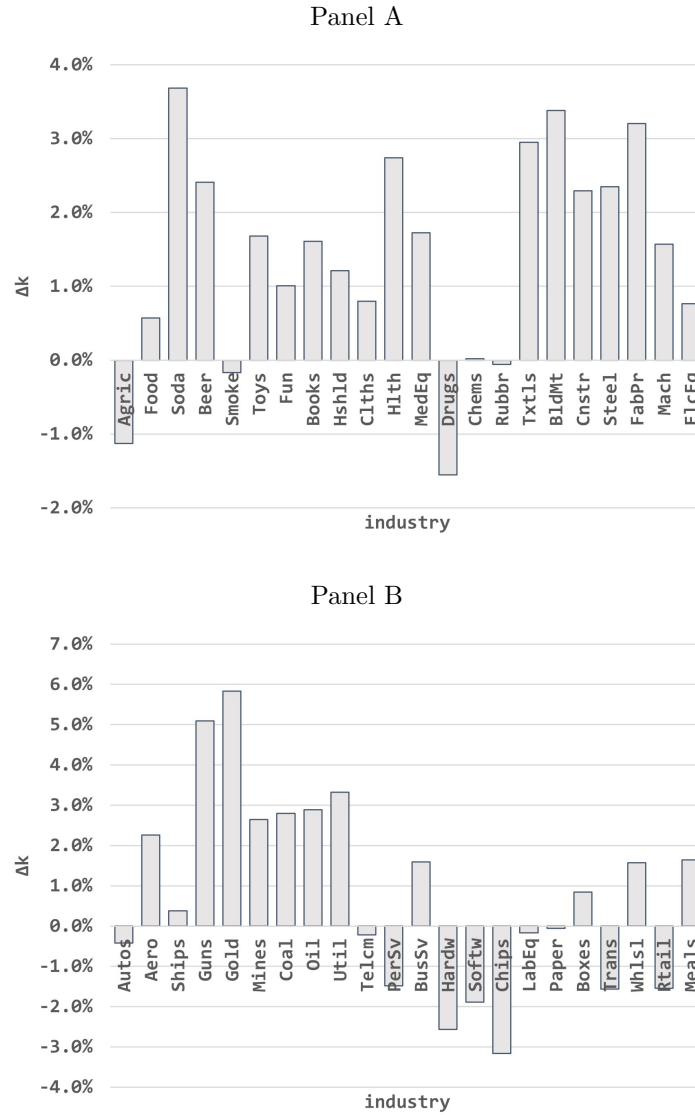
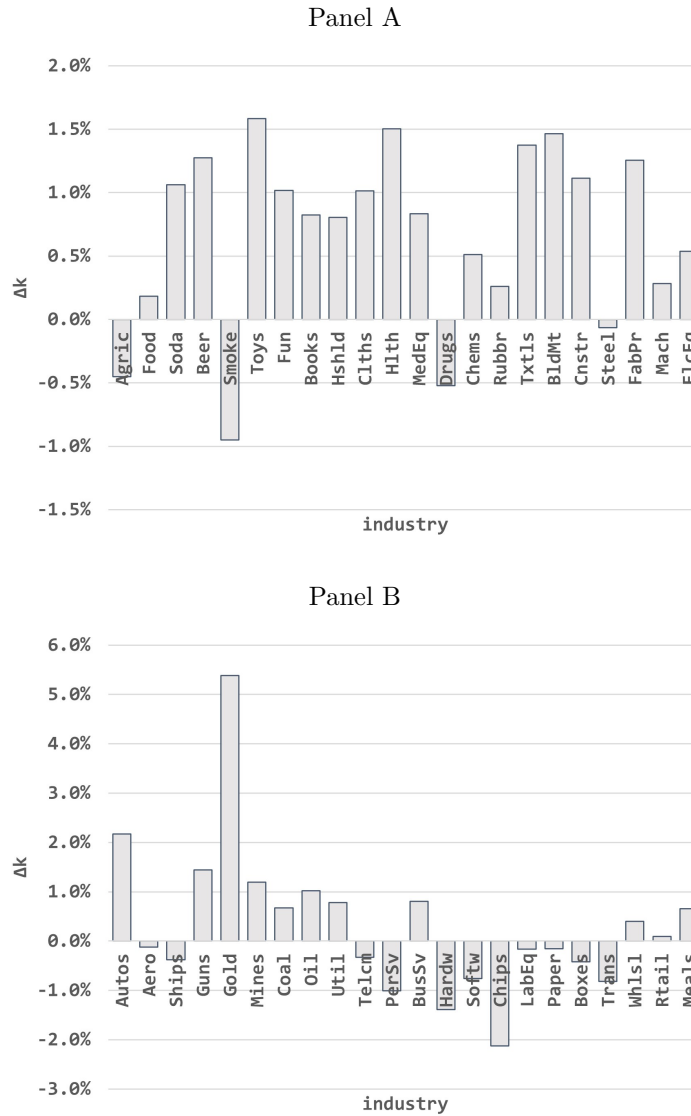


Figure 2: Differences in F-F 3-factor model-based cost of equity estimates

The numbers show the difference between Fama-French 3-factor model-based estimate of the industry cost of equity obtained using OLS vs. DDHME methods as $k_j = \bar{R}_f + \hat{\beta}_j (\bar{R}_M - \bar{R}_f) + \hat{s}_j \overline{SMB} + \hat{h}_j \overline{HML}$. A bar above the zero line indicates an underestimation of the cost of equity capital.



A Fama-French industry portfolios and sub-sectors

This appendix gives a complete list of Fama-French industry portfolio definitions with the SIC codes of the corresponding sub-sectors.

1. **Agric** Agriculture

0100-0199 Agricultural production – crops; 0200-0299 Agricultural production – livestock; 0700-0799 Agricultural services; 0910-0919 Commercial fishing; 2048-2048 Prepared feeds for animals

2. **Food** Food products

2000-2009 Food and kindred products; 2010-2019 Meat products; 2020-2029 Dairy products; 2030-2039 Canned & preserved fruits & vegetables; 2040-2046 Flour and other grain mill products; 2050-2059 Bakery products; 2060-2063 Sugar and confectionery products; 2070-2079 Fats and oils; 2090-2092 Misc food preparations and kindred products; 2095-2095 Roasted coffee; 2098-2099 Misc food preparations

3. **Soda** Candy & Soda

2064-2068 Candy and other confectionery; 2086-2086 Bottled-canned soft drinks; 2087-2087 Flavoring syrup; 2096-2096 Potato chips; 2097-2097 Manufactured ice

4. **Beer** Beer & Liquor

2080-2080 Beverages; 2082-2082 Malt beverages; 2083-2083 Malt; 2084-2084 Wine; 2085-2085 Distilled and blended liquors

5. **Smoke** Tobacco products

2100-2199 Tobacco products

6. **Toys** Recreation

0920-0999 Fishing, hunting & trapping; 3650-3651 Household audio visual equipment; 3652-3652 Phonograph records; 3732-3732 Boat building and repairing; 3930-3931 Musical instruments; 3940-3949 Toys

7. **Fun** Entertainment

7800-7829 Services - motion picture production and distribution; 7830-7833 Services - motion picture theaters; 7840-7841 Services - video rental; 7900-7900 Services - amusement and recreation; 7910-7911 Services - dance studios; 7920-7929 Services - bands, entertainers; 7930-7933 Services - bowling centers; 7940-7949 Services - professional sports; 7980-7980 Amusement and recreation services; 7990-7999 Services - Misc entertainment

8. **Books** Printing and publishing

2700-2709 Printing publishing and allied; 2710-2719 Newspapers: publishing-printing; 2720-2729 Periodicals: publishing-printing; 2730-2739 Books: publishing-

printing; 2740-2749 Misc publishing; 2770-2771 Greeting card; 2780-2789 Book-binding; 2790-2799 Service industries for the print trade

9. **Hshld** Consumer goods

2047-2047 Dog and cat food; 2391-2392 Curtains, home furnishings; 2510-2519 Household furniture; 2590-2599 Misc furniture and fixtures; 2840-2843 Soap & other detergents; 2844-2844 Perfumes, cosmetics and other toilet preparations; 3160-3161 Luggage; 3170-3171 Handbags and purses; 3172-3172 Personal leather goods, except handbags and purses; 3190-3199 Leather goods; 3229-3229 Pressed and blown glass; 3260-3260 Pottery and related products; 3262-3263 China and earthenware table articles; 3269-3269 Pottery products; 3230-3231 Glass products; 3630-3639 Household appliances; 3750-3751 Motorcycles, bicycles and parts (Harley & Huffy); 3800-3800 Misc instruments, photo goods & watches; 3860-3861 Photographic equipment (Kodak etc, but also Xerox); 3870-3873 Watches, clocks and parts; 3910-3911 Jewelry, precious metals; 3914-3914 Silverware; 3915-3915 Jewelers' findings and materials; 3960-3962 Costume jewelry and novelties; 3991-3991 Brooms and brushes; 3995-3995 Burial caskets

10. **Clths** Apparel

2300-2390 Apparel and other finished products; 3020-3021 Rubber and plastics footwear; 3100-3111 Leather tanning and finishing; 3130-3131 Boot & shoe cut stock & findings; 3140-3149 Footwear, except rubber; 3150-3151 Leather gloves and mittens; 3963-3965 Fasteners, buttons, needles, pins

11. **Hlth** Healthcare

8000-8099 Services - health

12. **MedEq** Medical equipment

3693-3693 X-ray, electromedical app; 3840-3849 Surgical, medical, and dental instruments and supplies; 3850-3851 Ophthalmic goods

13. **Drugs** Pharmaceutical products

2830-2830 Drugs; 2831-2831 Biological products; 2833-2833 Medicinal chemicals; 2834-2834 Pharmaceutical preparations; 2835-2835 In vitro, in vivo diagnostic substances; 2836-2836 Biological products, except diagnostic substances

14. **Chems** Chemicals

2800-2809 Chemicals and allied products; 2810-2819 Industrial inorganic chemicals; 2820-2829 Plastic material & synthetic resin/rubber; 2850-2859 Paints; 2860-2869 Industrial organic chemicals; 2870-2879 Agriculture chemicals; 2890-2899 Misc chemical products

15. **Rubbr** Rubber and plastic products
3031-3031 Reclaimed rubber; 3041-3041 Rubber & plastic hose & belting; 3050-3053 Gaskets, hoses, etc.; 3060-3069 Fabricated rubber products; 3070-3079 Misc rubber products; 3080-3089 Misc plastic products; 3090-3099 Misc rubber and plastic products
16. **Txtls** Textiles
2200-2269 Textile mill products; 2270-2279 Floor covering mills; 2280-2284 Yarn and thread mills; 2290-2295 Misc textile goods; 2297-2297 Non-woven fabrics; 2298-2298 Cordage and twine; 2299-2299 Misc textile products; 2393-2395 Textile bags, canvas products; 2397-2399 Misc textile products
17. **BldMt** Construction materials
0800-0899 Forestry; 2400-2439 Lumber and wood products; 2450-2459 Wood buildings & mobile homes; 2490-2499 Misc wood products; 2660-2661 Building paper and board mills; 2950-2952 Paving & roofing materials; 3200-3200 Stone, clay, glass, concrete, etc.; 3210-3211 Flat glass; 3240-3241 Cement, hydraulic; 3250-3259 Structural clay products; 3261-3261 Vitreous China plumbing fixtures; 3264-3264 Porcelain electrical supplies; 3270-3275 Concrete, gypsum & plaster products; 3280-3281 Cut stone and stone products; 3290-3293 Abrasive and asbestos products; 3295-3299 Misc nonmetallic mineral products; 3420-3429 Cutlery, hand tools and general hardware; 3430-3433 Heating equipment & plumbing fixtures; 3440-3441 Fabricated structural metal products; 3442-3442 Metal doors, frames; 3446-3446 Architectural or ornamental metal work; 3448-3448 Prefabricated metal buildings and components; 3449-3449 Misc structural metal work; 3450-3451 Screw machine products; 3452-3452 Bolts, nuts, screws, rivets and washers; 3490-3499 Misc fabricated metal products; 3996-3996 Hard surface floor coverings
18. **Cnstr** Construction
1500-1511 Build construction - general contractors; 1520-1529 General building contractors – residential; 1530-1539 Operative builders; 1540-1549 General building contractors - non-residential; 1600-1699 Heavy construction - not building contractors; 1700-1799 Construction - special contractors
19. **Steel** Steel works etc.
3300-3300 Primary metal industries; 3310-3317 Blast furnaces & steel works; 3320-3325 Iron & steel foundries; 3330-3339 Primary smelting & refining of nonferrous metals; 3340-3341 Secondary smelting & refining of nonferrous metals; 3350-3357 Rolling, drawing & extruding of nonferrous metals; 3360-3369 Nonferrous foundries and casting; 3370-3379 Steel works etc.; 3390-3399 Misc primary metal products

20. **FabPr** Fabricated products
3400-3400 Fabricated metal, except machinery and trans eq.; 3443-3443 Fabricated plate work; 3444-3444 Sheet metal work; 3460-3469 Metal forgings and stampings; 3470-3479 Coating, engraving and allied services
21. **Mach** Machinery
3510-3519 Engines & turbines; 3520-3529 Farm and garden machinery and equipment; 3530-3530 Construction, mining & material handling machinery & equipment; 3531-3531 Construction machinery & equipment; 3532-3532 Mining machinery & equipment, except oil field; 3533-3533 Oil & gas field machinery & equipment; 3534-3534 Elevators & moving stairways; 3535-3535 Conveyors & conveying equipment; 3536-3536 Cranes, hoists and monorail systems; 3538-3538 Machinery; 3540-3549 Metalworking machinery & equipment; 3550-3559 Special industry machinery; 3560-3569 General industrial machinery & equipment; 3580-3580 Refrigeration & service industry machinery; 3581-3581 Automatic vending machines; 3582-3582 Commercial laundry and dry cleaning machines; 3585-3585 Air conditioning, warm air heating and refrigeration equipment; 3586-3586 Measuring and dispensing pumps; 3589-3589 Service industry machinery; 3590-3599 Misc industrial and commercial equipment and machinery
22. **ElcEq** Electrical equipment
3600-3600 Electronic & other electrical equipment; 3610-3613 Electric transmission and distribution equipment; 3620-3621 Electrical industrial apparatus; 3623-3629 Electrical industrial apparatus; 3640-3644 Electric lighting & wiring equipment; 3645-3645 Residential electric lighting fixtures; 3646-3646 Commercial, industrial and institutional electric lighting fixtures; 3648-3649 Misc lighting equipment; 3660-3660 Communications equipment; 3690-3690 Misc electrical machinery and equipment; 3691-3692 Storage batteries; 3699-3699 Misc electrical machinery, equipment and supplies
23. **Autos** Automobiles and trucks
2296-2296 Tire cord and fabric; 2396-2396 Automotive trimmings, apparel findings & related products; 3010-3011 Tires and inner tubes; 3537-3537 Industrial trucks, tractors, trailers & stackers; 3647-3647 Vehicular lighting equipment; 3694-3694 Electrical equipment for internal combustion engines; 3700-3700 Transportation equipment; 3710-3710 Motor vehicles and motor vehicle equipment; 3711-3711 Motor vehicles & passenger car bodies; 3713-3713 Truck & bus bodies; 3714-3714 Motor vehicle parts & accessories; 3715-3715 Truck trailers; 3716-3716 Motor homes; 3792-3792 Travel trailers and campers; 3790-3791 Misc transportation equipment; 3799-3799 Misc transportation equipment
24. **Aero** Aircraft

- 3720-3720 Aircraft & parts; 3721-3721 Aircraft; 3723-3724 Aircraft engines & engine parts; 3725-3725 Aircraft parts; 3728-3729 Misc aircraft parts & auxiliary equipment
25. **Ships** Shipbuilding, railroad equipment
3730-3731 Ship building and repairing; 3740-3743 Railroad Equipment
26. **Guns** Defense
3760-3769 Guided missiles and space vehicles and parts; 3795-3795 Tanks and tank components; 3480-3489 Ordnance & accessories
27. **Gold** Precious metals
1040-1049 Gold & silver ores
28. **Mines** Non-metallic and industrial metal mining
1000-1009 Metal mining; 1010-1019 Iron ores; 1020-1029 Copper ores; 1030-1039 Lead and zinc ores; 1050-1059 Bauxite and other aluminum ores; 1060-1069 Ferroalloy ores; 1070-1079 Mining; 1080-1089 Metal mining services; 1090-1099 Misc metal ores; 1100-1119 Anthracite mining; 1400-1499 Mining and quarrying non-metallic minerals
29. **Coal** Coal
1200-1299 Bituminous coal and lignite mining
30. **Oil** Petroleum and natural gas
1300-1300 Oil and gas extraction; 1310-1319 Crude petroleum & natural gas; 1320-1329 Natural gas liquids; 1330-1339 Petroleum and natural gas; 1370-1379 Petroleum and natural gas; 1380-1380 Oil and gas field services; 1381-1381 Drilling oil & gas wells; 1382-1382 Oil & gas field exploration services; 1389-1389 Misc oil & gas field services; 2900-2912 Petroleum refining; 2990-2999 Misc products of petroleum & coal
31. **Util** Utilities
4900-4900 Electric, gas & sanitary services; 4910-4911 Electric services; 4920-4922 Natural gas transmission; 4923-4923 Natural gas transmission & distribution; 4924-4925 Natural gas distribution; 4930-4931 Electric and other services combined; 4932-4932 Gas and other services combined; 4939-4939 Misc combination utilities; 4940-4942 Water supply
32. **Telcm** Communication
4800-4800 Communications; 4810-4813 Telephone communications; 4820-4822 Telegraph and other message communication; 4830-4839 Radio & TV broadcasters;

4840-4841 Cable and other pay TV services; 4880-4889 Communications; 4890-4890 Communication services (Comsat); 4891-4891 Cable TV operators; 4892-4892 Telephone interconnect; 4899-4899 Misc communication services

33. **PerSv** Personal services

7020-7021 Rooming and boarding houses; 7030-7033 Camps and recreational vehicle parks; 7200-7200 Services – personal; 7210-7212 Services - laundry, cleaning & garment services; 7214-7214 Services - diaper service; 7215-7216 Services - coin-operated cleaners, dry cleaners; 7217-7217 Services - carpet & upholstery cleaning; 7219-7219 Services - Misc laundry & garment services; 7220-7221 Services - photographic studios, portrait; 7230-7231 Services - beauty shops; 7240-7241 Services - barber shops; 7250-7251 Services - shoe repair shops & shoeshine parlors; 7260-7269 Services - funeral service & crematories; 7270-7290 Services – Misc; 7291-7291 Services - tax return; 7292-7299 Services – Misc; 7395-7395 Services - photofinishing labs (School pictures); 7500-7500 Services - auto repair, services & parking; 7520-7529 Services - automobile parking; 7530-7539 Services - automotive repair shops; 7540-7549 Services - automotive services, except repair (car washes); 7600-7600 Services - Misc repair services; 7620-7620 Services - Electrical repair shops; 7622-7622 Services - Radio and TV repair shops; 7623-7623 Services - Refrigeration and air conditioning service & repair shops; 7629-7629 Services - Electrical & electronic repair shops; 7630-7631 Services - Watch, clock and jewelry repair; 7640-7641 Services - Reupholster & furniture repair; 7690-7699 Services - Misc repair shops & related services; 8100-8199 Services – legal; 8200-8299 Services – educational; 8300-8399 Services - social services; 8400-8499 Services - museums, art galleries, botanical and zoological gardens; 8600-8699 Services - membership organizations; 8800-8899 Services - private households; 7510-7515 Services - truck & auto rental and leasing

34. **BusSv** Business services

2750-2759 Commercial printing; 3993-3993 Signs & advertising specialties; 7218-7218 Services - industrial launderers; 7300-7300 Services - business services; 7310-7319 Services – advertising; 7320-7329 Services - consumer credit reporting agencies, collection services; 7330-7339 Services - mailing, reproduction, commercial art & photography; 7340-7342 Services - services to dwellings & other buildings; 7349-7349 Services - building cleaning & maintenance; 7350-7351 Services - Misc equipment rental and leasing; 7352-7352 Services - medical equipment rental and leasing; 7353-7353 Services - heavy construction equipment rental and leasing; 7359-7359 Services - equipment rental and leasing; 7360-7369 Services - personnel supply services; 7374-7374 Services - computer processing, data preparation and processing; 7376-7376 Services - computer facilities management service; 7377-7377 Services - computer rental and leasing; 7378-7378 Services - computer main-

tenance and repair; 7379-7379 Services - computer related services; 7380-7380 Services - Misc business services; 7381-7382 Services - security; 7383-7383 Services - news syndicates; 7384-7384 Services - photofinishing labs; 7385-7385 Services - telephone interconnect systems; 7389-7390 Services - Misc business services; 7391-7391 Services - R&D labs; 7392-7392 Services - management consulting & P.R.; 7393-7393 Services - detective and protective (ADT); 7394-7394 Services - equipment rental & leasing; 7396-7396 Services - trading stamp services; 7397-7397 Services - commercial testing labs; 7399-7399 Services - business services; 7519-7519 Services - utility trailer & recreational vehicle rental; 8700-8700 Services - engineering, accounting, research, management; 8710-8713 Services - engineering, accounting, surveying; 8720-8721 Services - accounting, auditing, bookkeeping; 8730-8734 Services - research, development, testing labs; 8740-8748 Services - management, public relations, consulting; 8900-8910 Services - Misc; 8911-8911 Services - Misc engineering & architect; 8920-8999 Services - Misc; 4220-4229 Public warehousing and storage

35. **Hardw** Computers

3570-3579 Computer & office equipment; 3680-3680 Computers; 3681-3681 Computers - mini; 3682-3682 Computers - mainframe; 3683-3683 Computers - terminals; 3684-3684 Computers - disk & tape drives; 3685-3685 Computers - optical scanners; 3686-3686 Computers - graphics; 3687-3687 Computers - office automation systems; 3688-3688 Computers - peripherals; 3689-3689 Computers - equipment; 3695-3695 Magnetic and optical recording media

36. **Softw** Computer software

7370-7372 Services - computer programming and data processing; 7375-7375 Services - information retrieval services; 7373-7373 Computer integrated systems design

37. **Chips** Electronic equipment

3622-3622 Industrial controls; 3661-3661 Telephone and telegraph apparatus; 3662-3662 Communications equipment; 3663-3663 Radio & TV broadcasting & communications equipment; 3664-3664 Search, navigation, guidance systems; 3665-3665 Training equipment & simulators; 3666-3666 Alarm & signaling products; 3669-3669 Communication equipment; 3670-3679 Electronic components & accessories; 3810-3810 Search, detection, navigation, guidance, aeronautical & nautical systems, instruments & equipment; 3812-3812 Search, detection, navigation, guidance, aeronautical & nautical systems & instruments

38. **LabEq** Measuring and control equipment

3811-3811 Engr laboratory and research equipment; 3820-3820 Measuring and controlling equipment; 3821-3821 Laboratory apparatus and furniture; 3822-3822

Automatic controls for regulating residential & commercial environments & appliances; 3823-3823 Industrial measurement instruments & related products; 3824-3824 Totalizing fluid meters & counting devices; 3825-3825 Instruments for measuring & testing of electricity & electrical instruments; 3826-3826 Lab analytical instruments; 3827-3827 Optical instruments and lenses; 3829-3829 Misc measuring and controlling devices; 3830-3839 Optical instruments and lenses

39. **Paper** Business supplies

2520-2549 Office furniture and fixtures; 2600-2639 Paper and allied products; 2670-2699 Paper and allied products; 2760-2761 Manifold business forms; 3950-3955 Pens, pencils & other artists' supplies

40. **Boxes** Shipping containers

2440-2449 Wood containers; 2640-2659 Paperboard containers, boxes, drums, tubs; 3220-3221 Glass containers; 3410-3412 Metal cans and shipping containers

41. **Trans** Transportation

4000-4013 Railroads, line-haul operating; 4040-4049 Railway express service; 4100-4100 Local & suburban transit & interurban highway passenger transportation; 4110-4119 Local & suburban passenger transportation; 4120-4121 Taxicabs; 4130-4131 Intercity & rural bus transportation (Greyhound); 4140-4142 Bus charter service; 4150-4151 School buses; 4170-4173 Motor vehicle terminals & service facilities; 4190-4199 Misc transit and passenger transportation; 4200-4200 Trucking & warehousing; 4210-4219 Trucking & courier services, except air; 4230-4231 Terminal & joint terminal maintenance; 4240-4249 Transportation; 4400-4499 Water transport; 4500-4599 Air transportation; 4600-4699 Pipelines, except natural gas; 4700-4700 Transportation services; 4710-4712 Freight forwarding; 4720-4729 Arrangement of passenger transportation; 4730-4739 Arrangement of transportation of freight and cargo; 4740-4749 Rental of railroad cars; 4780-4780 Misc services incidental to transportation; 4782-4782 Inspection and weighing services; 4783-4783 Packing and crating; 4784-4784 Misc fixed facilities for vehicles; 4785-4785 Motor vehicle inspection; 4789-4789 Misc transportation services

42. **Whlsl** Wholesale

5000-5000 Wholesale - durable goods; 5010-5015 Wholesale - automotive vehicles & automotive parts & supplies; 5020-5023 Wholesale - furniture and home furnishings; 5030-5039 Wholesale - lumber and construction materials; 5040-5042 Wholesale - professional and commercial equipment and supplies; 5043-5043 Wholesale - photographic equipment & supplies; 5044-5044 Wholesale - office equipment; 5045-5045 Wholesale - computers & peripheral equipment & software; 5046-5046 Wholesale - commercial equipment; 5047-5047 Wholesale - medical, dental & hospital equipment; 5048-5048 Wholesale - ophthalmic goods; 5049-5049 Wholesale -

professional equipment and supplies; 5050-5059 Wholesale - metals and minerals, except petroleum; 5060-5060 Wholesale - electrical goods; 5063-5063 Wholesale - electrical apparatus and equipment; 5064-5064 Wholesale - electrical appliance, TV and radio sets; 5065-5065 Wholesale - electronic parts & equipment; 5070-5078 Wholesale - hardware, plumbing & heating equipment; 5080-5080 Wholesale - machinery, equipment & supplies; 5081-5081 Wholesale - machinery & equipment; 5082-5082 Wholesale - construction and mining machinery & equipment; 5083-5083 Wholesale - farm and garden machinery & equipment; 5084-5084 Wholesale - industrial machinery & equipment; 5085-5085 Wholesale - industrial supplies; 5086-5087 Wholesale - service establishment machinery & equipment; 5088-5088 Wholesale - transportation equipment, except motor vehicles; 5090-5090 Wholesale - Misc durable goods; 5091-5092 Wholesale - sporting goods & toys; 5093-5093 Wholesale - scrap and waste materials; 5094-5094 Wholesale - jewelry, watches, precious stones & metals; 5099-5099 Wholesale - durable goods; 5100-5100 Wholesale - nondurable goods; 5110-5113 Wholesale - paper and paper products; 5120-5122 Wholesale - drugs & drug proprietaries; 5130-5139 Wholesale - apparel, piece goods & notions; 5140-5149 Wholesale - groceries & related products; 5150-5159 Wholesale - farm product raw materials; 5160-5169 Wholesale - chemicals & allied products; 5170-5172 Wholesale - petroleum and petroleum products; 5180-5182 Wholesale - beer, wine & distilled alcoholic beverages; 5190-5199 Wholesale - Misc nondurable goods

43. **Rtail** Retail

5200-5200 Retail - retail-building materials, hardware, garden supply; 5210-5219 Retail - lumber & other building materials; 5220-5229 Retail; 5230-5231 Retail - paint, glass & wallpaper stores; 5250-5251 Retail - hardware stores; 5260-5261 Retail - nurseries, lawn & garden supply stores; 5270-5271 Retail - mobile home dealers; 5300-5300 Retail - general merchandise stores; 5310-5311 Retail - department stores; 5320-5320 Retail - general merchandise stores; 5330-5331 Retail - variety stores; 5334-5334 Retail - catalog showroom; 5340-5349 Retail; 5390-5399 Retail - Misc general merchandise stores; 5400-5400 Retail - food stores; 5410-5411 Retail - grocery stores; 5412-5412 Retail - convenience stores; 5420-5429 Retail - meat & fish markets; 5430-5439 Retail - fruit and vegetable markets; 5440-5449 Retail - candy, nut & confectionary stores; 5450-5459 Retail - dairy products stores; 5460-5469 Retail - bakeries; 5490-5499 Retail - Misc food stores; 5500-5500 Retail - automotive dealers and gas stations; 5510-5529 Retail - automotive dealers; 5530-5539 Retail - automotive and home supply stores; 5540-5549 Retail - gasoline service stations; 5550-5559 Retail - boat dealers; 5560-5569 Retail - recreation vehicle dealers; 5570-5579 Retail - motorcycle dealers; 5590-5599 Retail - automotive dealers; 5600-5699 Retail - apparel & accessory stores; 5700-5700 Re-

tail - home furniture and equipment stores; 5710-5719 Retail - home furnishings stores; 5720-5722 Retail - household appliance stores; 5730-5733 Retail - radio, TV and consumer electronic stores; 5734-5734 Retail - computer and computer software stores; 5735-5735 Retail - record and tape stores; 5736-5736 Retail - musical instrument stores; 5750-5799 Retail; 5900-5900 Retail - Misc; 5910-5912 Retail - drug & proprietary stores; 5920-5929 Retail - liquor stores; 5930-5932 Retail - used merchandise stores; 5940-5940 Retail - Misc; 5941-5941 Retail - sporting goods stores & bike shops; 5942-5942 Retail - book stores; 5943-5943 Retail - stationery stores; 5944-5944 Retail - jewelry stores; 5945-5945 Retail - hobby, toy and game shops; 5946-5946 Retail - camera and photographic supply stores; 5947-5947 Retail - gift, novelty & souvenir shops; 5948-5948 Retail - luggage & leather goods stores; 5949-5949 Retail - sewing & needlework stores; 5950-5959 Retail; 5960-5969 Retail - non-store retailers (catalogs, etc); 5970-5979 Retail; 5980-5989 Retail - fuel dealers & ice stores (Penn Central Co); 5990-5990 Retail - Misc retail stores; 5992-5992 Retail - florists; 5993-5993 Retail - tobacco stores and stands; 5994-5994 Retail - newsdealers and news stands; 5995-5995 Retail - optical goods stores; 5999-5999 Misc retail stores

44. **Meals** Restaurants, hotels, motels

5800-5819 Retail - eating places; 5820-5829 Restaurants, hotels, motels; 5890-5899 Eating and drinking places; 7000-7000 Hotels & other lodging places; 7010-7019 Hotels & motels; 7040-7049 Membership hotels and lodging houses; 7213-7213 Services - linen supply

45. **Banks** Banking

6000-6000 Depository institutions; 6010-6019 Federal reserve banks; 6020-6020 Commercial banks; 6021-6021 National commercial banks; 6022-6022 State commercial banks - Fed Res System; 6023-6024 State commercial banks - not Fed Res System; 6025-6025 National commercial banks - Fed Res System; 6026-6026 National commercial banks - not Fed Res System; 6027-6027 National commercial banks, not FDIC; 6028-6029 Misc commercial banks; 6030-6036 Savings institutions; 6040-6059 Banks; 6060-6062 Credit unions; 6080-6082 Foreign banks; 6090-6099 Functions related to depository banking; 6100-6100 Non-depository credit institutions; 6110-6111 Federal credit agencies; 6112-6113 FNMA; 6120-6129 S&Ls; 6130-6139 Agricultural credit institutions; 6140-6149 Personal credit institutions (Beneficial); 6150-6159 Business credit institutions; 6160-6169 Mortgage bankers and brokers; 6170-6179 Finance lessors; 6190-6199 Financial services

46. **Insur** Insurance

6300-6300 Insurance; 6310-6319 Life insurance; 6320-6329 Accident and health insurance; 6330-6331 Fire, marine & casualty insurance; 6350-6351 Surety insurance;

6360-6361 Title insurance; 6370-6379 Pension, health & welfare funds; 6390-6399 Misc insurance carriers; 6400-6411 Insurance agents, brokers & service

47. **REst** Real estate

6500-6500 Real estate; 6510-6510 Real estate operators and lessors; 6512-6512 Operators - non-resident buildings; 6513-6513 Operators - apartment buildings; 6514-6514 Operators - other than apartment; 6515-6515 Operators - residential mobile home; 6517-6519 Lessors of railroad & real property; 6520-6529 Real estate; 6530-6531 Real estate agents and managers; 6532-6532 Real estate dealers; 6540-6541 Title abstract offices; 6550-6553 Land subdividers & developers; 6590-6599 Real estate; 6610-6611 Combined real estate, insurance, etc.

48. **Fin** Trading

6200-6299 Security and commodity brokers, dealers, exchanges & services; 6700-6700 Holding & other investment offices; 6710-6719 Holding offices; 6720-6722 Management investment offices, open-end; 6723-6723 Management investment offices, closed-end; 6724-6724 Unit investment trusts; 6725-6725 Face-amount certificate offices; 6726-6726 Unit investment trusts, closed-end; 6730-6733 Trusts; 6740-6779 Investment offices; 6790-6791 Misc investing; 6792-6792 Oil royalty traders; 6793-6793 Commodity traders; 6794-6794 Patent owners & lessors; 6795-6795 Mineral royalty traders; 6798-6798 REIT; 6799-6799 Investors, NEC